

Pavel Etingof (MIT)

Periodic pencils of flat connections and their p -curvature

Abstract: A periodic pencil of flat connections on a smooth algebraic variety X is a linear family of flat connections

$$\nabla(s_1, \dots, s_n) = d - \sum_{i=1}^r \sum_{j=1}^n s_j B_{ij} dx_i,$$

where $\{x_i\}$ are local coordinates on X and $B_{ij} : X \rightarrow \text{Mat}_N$ are matrix-valued regular functions. A pencil is periodic if it is generically invariant under the shifts $s_j \mapsto s_j + 1$ up to isomorphism. I will explain that periodic pencils have many remarkable properties, and there are many interesting examples of them, e.g. Knizhnik-Zamolodchikov, Dunkl, Casimir connections and equivariant quantum connections for conical symplectic resolutions with finitely many torus fixed points. I will also explain that in characteristic p , the p -curvature operators $\{C_i, 1 \leq i \leq r\}$ of a periodic pencil ∇ are isospectral to the commuting endomorphisms $C_i^* := \sum_{j=1}^n (s_j - s_j^p) B_{ij}^{(1)}$, where $B_{ij}^{(1)}$ is the Frobenius twist of B_{ij} . This allows us to compute the eigenvalues of the p -curvature for the above examples, and also to show that a periodic pencil of connections always has regular singularities. This is joint work with Alexander Varchenko.