

## D-MODULES, HOMEWORK 2

**Problem 1.** Let  $X$  be a smooth affine curve. Prove that every irreducible  $D(X)$ -module is holonomic.

**Problem 2.** Let  $X = \mathbb{A}^n$  and let  $L$  be a lagrangian subspace in  $\text{Span}_{\mathbb{C}}(x^1, \dots, x^n, \partial^1, \dots, \partial^n)$ . Compute the singular support and the characteristic cycle of  $D(\mathbb{A}^n)/D(\mathbb{A}^n)L$ .

**Problem 3.** Let  $a \in \mathbb{C} \setminus \mathbb{Z}$ . Consider the  $D(\mathbb{A}^1)$ -module  $\mathbb{C}[x^{\pm 1}]x^a$  that is identified with  $\mathbb{C}[x^{\pm 1}]$  as a  $\mathbb{C}[x]$ -module and  $\partial$  acts on  $x^n x^a$  via  $\partial(x^n x^a) = (n + a)x^{n-1}x^a$  (here  $x^a$  can be treated as a formal symbol or as an actual multi-valued function). Prove that the  $D(\mathbb{A}^1)$ -module  $\mathbb{C}[x^{\pm 1}]$  is irreducible and compute its singular support and characteristic cycle.