A WAY TO BUILD MAPS BETWEEN FUKAYA CATEGORIES

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Talk 1 (survey talk): This talk will be a crash course on pseudoholomorphic curves in symplectic geometry. After introducing the basic notions, I will discuss the basic toolbox for working with moduli spaces of pseudoholomorphic curves: removal of singularity, Gromov compactness, and a sufficient condition for a space of pseudoholomorphic curves to be cut out smoothly. I will then sketch a 1985 result of Gromov's, which gives a restriction on the Lagrangian submanifolds in $\mathbb{C}^n$, and in particular implies the existence of an exotic $\mathbb{C}^n$. Finally, I will introduce the Fukaya category of a compact symplectic manifold. If you know what a symplectic manifold is (or look it up), you will be able to understand this talk.

Talk 2 (research talk): In this talk, I will explain work-in-progress with Katrin Wehrheim in which we aim to bind together the Fukaya categories of many different symplectic manifolds into a single algebraic object. This object is the "symplectic A_∞-2-category", whose objects are symplectic manifolds, and where $\hom(M, N) :=$ $\mathrm{Fuk}(M^- \times N)$. In particular, a Lagrangian correspondence $\Lambda \subset M^- \times N$ will induce an A_∞-functor $F_\Lambda \colon \mathrm{Fuk}(M) \to \mathrm{Fuk}(N)$, improving on a result of Ma'u–Wehrheim–Woodward. At the core of our project are pseudoholomorphic quilts with figure eight singularity, and I will discuss several analytical results related to these objects, including a removal of singularity and a Gromov compactness theorem. If you attend talk 1 or have a basic familiarity with the Fukaya category, you will be able to understand this talk.