Recap on Lusztig's form.

1) Quantum Frobenius:

$l \in \mathbb{Z}_{>0}$, odd (for G_2 not divisible by 3), $\varepsilon :=$ lth primitive root of 1. $\rightsquigarrow$ Lusztig's form $\dot{U}_{\mathbb{C},\varepsilon}$ ($\mathbb{C}$-algebra) generated by:

$K_\alpha^{\pm 1}, E_\alpha, F_\alpha, E_\alpha^{(l)}, F_\alpha^{(l)}$

(if $\mathbb{C} \rightsquigarrow$ general ring R also need $E_\alpha^{(n)}, F_\alpha^{(n)}$ $\forall n > 0$).

Quantum Frobenius $Fr: \dot{U}_{\mathbb{C},\varepsilon} \longrightarrow \dot{U}_{\mathbb{C},1}$

$Fr: K_\alpha \to K_\alpha,\ E_\alpha, F_\alpha \mapsto 0,\ E_\alpha^{(l)} \longrightarrow E_\alpha,\ F_\alpha^{(l)} \longrightarrow F_\alpha$

& $\binom{K_\alpha;0}{l} \longmapsto \binom{K_\alpha;0}{1}$.

Notice: $K_\alpha^l \in \dot{U}_{\mathbb{C},\varepsilon}$ is central & $(K_\alpha^l)^2 = 1$ $\rightsquigarrow$

$$\dot{U}^1_{\mathbb{C},\varepsilon} = \dot{U}_{\mathbb{C},\varepsilon} / (K_\alpha^l = 1\ \forall \alpha \in \Pi)$$

$$Fr: \dot{U}^1_{\mathbb{C},\varepsilon} \twoheadrightarrow \dot{U}^1_{\mathbb{C},1} = U(\mathfrak{g}).$$

2) Small quantum group:

$u_\varepsilon :=$ subalg. of $\dot{U}_\varepsilon$ gen'd by $K_\alpha^{\pm 1}, E_\alpha, F_\alpha$ $(\alpha \in \Pi)$ – Hopf subalgebra

$$u_\varepsilon = \operatorname{im}(\underbrace{U_\varepsilon}_{DK} \longrightarrow \dot{U}_\varepsilon)$$

It's an analog of $Dist(G_1)$

• $\ker Fr: \dot{\mathcal{U}}_{\mathbb{C},\varepsilon} \longrightarrow \mathcal{U}(\mathfrak{g})$
is generated by augmentation ideal of u_ε
(quantum analog of exact sequence $1 \to G_1 \to G \to G^{(1)} \to 1$)

PBW basis in u_ε: $\prod_{\alpha\in R_+} E_{-\alpha}^{k_\alpha} \prod_{\beta\in\Pi} K_\beta^{m_\beta} \prod_{\alpha\in R_+} E_\alpha^{n_\alpha}$ w. $k_\alpha, n_\alpha \in \{0,1,\dots \ell-1\}$
$m_\beta \in \{0,\dots 2\ell-1\}$.

3) $\dot{\mathcal{U}}_\varepsilon^1$ vs $Dist(G_{\mathbb{F}_p})$.

Let $\ell = p$ be a prime, $\mathcal{A} = \mathbb{Z}[x]/(1+x+\dots+x^{p-1})$
$\rightsquigarrow \dot{\mathcal{U}}^1_{\mathcal{A},\varepsilon}$

• $Frac(\mathcal{A}) = \mathbb{Q}(\varepsilon) \Rightarrow \mathbb{Q}(\varepsilon)\otimes_{\mathcal{A}} \dot{\mathcal{U}}^1_{\mathcal{A},\varepsilon}$ (as good as $\dot{\mathcal{U}}^1_{\mathbb{C},\varepsilon}$)

• $\mathfrak{m} = (p, x-1)$, $\mathcal{A}/\mathfrak{m} = \mathbb{F}_p$
$\mathbb{F}_p \otimes_{\mathcal{A}} \dot{\mathcal{U}}^1_{\mathcal{A},\varepsilon} = Dist(G_{\mathbb{F}_p})$