MIT-NORTHEASTERN GRADUATE SEMINAR ON QUANTUM COHOMOLOGY AND REPRESENTATION THEORY, FALL 2013

The seminar is coorganized by Roman Bezrukavnikov, Pavel Etingof, Paul Seidel (MIT), Ivan Losev and Valerio Toledano Laredo (Northeastern). The seminar will run on Mondays, 5-8pm (two 75 minute lectures with a 30 minute break in the middle) alternating between MIT and Northeastern. The first meeting will be at MIT on September 9. The talks will be given by graduate students mentored by the five organizers.

The seminar will bear on a synthesis of Algebraic and Enumerative geometry, Representation theory and Integrable systems. Specifically, its goal will be to explore the background to conjectures (proposed by Bezrukavnikov, Etingof and Okounkov several years ago) relating representations of quantized coordinate rings of a symplectic singularity to geometry of a crepant resolution of the singularity. The talks will mainly focus on the most studied case of the nilpotent cone in a semi-simple Lie algebra $\mathfrak{g}$ and its relation to representations of $\mathfrak{g}$. They will present classical material from representation theory of semi-simple Lie algebras and related algebraic geometry and will be targeted at the audience interested in those subjects (not necessarily interested in the conjectures).

Preliminary list of topics.
(1) The Springer resolution and its properties.
(2) The degenerate affine Hecke algebra and the Dunkl connection.
(3) The affine Hecke algebra. Monodromy of the Dunkl connection.
(4) Quantization of the Springer resolution. The Beilinson-Bernstein localization theorem.
(5) Derived localization theorem in positive characteristic.
(6) Representations of semisimple Lie algebras in characteristic p. Affine braid group action on the derived category as a generalization of Beilinson-Bernstein action.
(7) Reminders on equivariant cohomology.
(8) Generalities on quantum cohomology.
(9) Generalities on quantum equivariant cohomology.
(10) Quantum connections and their properties. Quantum connections on equivariant quantum cohomology.
(11) Computation of the quantum connection for cotangent bundles of flag varieties (after Braverman, Maulik and Okounkov). Comparison of monodromy with braid group action on the derived category.