TOPICS IN REPRESENTATION THEORY: SYMPLECTIC REFLECTION ALGEBRAS

INSTRUCTOR: IVAN LOSEV

Class info:
MATH 7364-01, CRN: 14815.
Time: MR, 4:00-5.30pm; first meeting Sep. 6.
Location: Forsyth building 242.

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office: 519LA, *office hours*: M 10.30-11.20, 2.30-3.50 and by appointment.
Prerequisites, grades, literature: see below.

Description.
Symplectic reflection algebras were introduced by Etingof and Ginzburg about 10 years ago based on the previous work of Cherednik, Dunkl, Crawley-Boevey and Holland, and others. They happen to be connected to various parts of Mathematics: Representation theory (representations of quivers, classical and quantum Nakajima quiver varieties, Hecke algebras and categorical Kac-Moody actions), Algebraic geometry (resolutions of quotient singularities, geometry of plane curves) Combinatorics (Macdonald polynomials), Deformation theory, Integrable systems (systems of Calogero-Moser type), Knot theory (invariants of toric knots). The main goal of this course is to demonstrate some of these connections.

(Approximate) program.
1) Kleinian singularities and their deformations (a warm-up).
1.1) Kleinian singularities and finite subgroups of $\mathrm{SL}_2(\mathbb{C})$.
1.2) Notion of a deformation. Algebras of Crawley-Boevey and Holland.
1.3) Representations of quivers and categorical quotients. McKay correspondence revisited.
1.4) CBH algebras and deformed preprojective algebras.

2) Construction of SRA (deformation theory).
2.1) Formal deformations and Hochschild cohomology.
2.2) Cohomology of smash-products. Symplectic reflection algebras.
2.3) Proof of flatness of SRA. Symplectic reflection groups.

3) Algebraic properties of SRA, incl. centers and spherical subalgebras (algebra).
3.1) Double centralizer property.
3.2) Commutativity of the spherical subalgebra.
3.3) Satake isomorphism.

4) Calogero-Moser systems and Cherednik algebras (integrable systems).
4.1) Classical Calogero-Moser system, Hamiltonian reduction and Calogero-Moser space.
4.2) Quantization as deformation. Quantum CM system.
4.3) Dunkl operators and first integrals for quantum CM system.

5) Quotient singularities and SRA for wreath-products (algebraic geometry).
5.1) Nakajima quiver varieties.
5.2) Quotient singularities vs quiver varieties.

5.3) Deformation theory of symplectic varieties.
5.4) Procesi bundles and SRA.

6) Categories $\mathcal{O}$ for Cherednik algebras (finally, representation theory).
6.1) Categories $\mathcal{O}$ and their properties.
6.2) KZ functor.
6.3) Hecke algebras.
6.4) Induction and restriction functors.
6.5) Categorical Kac-Moody actions.

Literature.

There is basically one textbook on the subject and several review texts, all available online.
The textbook is: P. Etingof. *Lectures on Calogero-Moser systems.*
http://arxiv.org/abs/math/0606233

There are also notes from MIT class, 2009:
P. Etingof, X. Ma. *Lecture notes on Cherednik algebras.*
http://arxiv.org/abs/1001.0432

One of the review texts:
I. Gordon. *Symplectic reflection algebras.*
http://arxiv.org/abs/0712.1568

I plan to post lecture notes. In the notes some additional references (to original papers) will be provided.

Prerequisites.

Algebra: groups, fields and algebras, the structure theory of semisimple finite dimensional algebras, representations of finite groups, etc.

Algebraic geometry: algebraic varieties, correspondence btw. commutative algebras and varieties, algebra homomorphisms and morphisms of varieties. Dominant and finite morphisms. For parts 4 and 5 we will also need smoothness, tangent spaces, cotangent bundles. Finally, in part 5 we will need Cech and De Rham cohomology in the algebraic setting.

Symplectic geometry: for parts 4 and 5 we will need symplectic manifolds, incl. cotangent bundles, symplectic forms, Hamiltonian vector fields, etc. Also for part 4 we will need basics of classical Hamiltonian mechanics. Most of this was covered in the Spring by B. Webster and will be covered in the Fall by J. Weitsman.

Lie groups and Lie algebras: for parts 4 and 5 it will be useful to understand the correspondence between Lie groups and their Lie algebras, incl. the correspondence between their representations etc. Also throughout the course, a familiarity with Weyl groups, roots, etc. will be helpful.

Category theory: basic notions of the category theory (categories, objects, functors, morphisms of functors, abelian categories, exact functors, projective objects etc.) are required for part 6.

Grades are based on the homework. The homework is divided into two parts: exercises and problems. For each class, there will be a problem set to be handed in class and also posted online containing all problems and exercises for the class. You are responsible for all exercises that constitute 50% of the grade. You also will get an individual assignment consisting of 5 problems. Grade cut-offs and due dates are to be determined.