HECKE ALGEBRA AND CATEGORY

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1. Plan

We proceed to the final part of the course – the Hecke algebra/category:
(1) Representations of $\mathrm{GL}_n(\mathbb{F}_q)$ (Lecture 18).
(2) The generic Hecke algebra for S_n.
(3) Generalizations: Kac-Moody Lie algebras and Coxeter groups.
(4) Kazhdan-Lusztig basis and Kazhdan-Lusztig conjecture.
(5) Category O.
(6) Soergel bimodules and modules.
(7) The geometry behind Soergel modules.
(8) Discussions: How to guess the Kazhdan-Lusztig conjectures? How people proved them? What happens in positive characteristic? Representations of semisimple Lie algebras in positive characteristic.

2. Brief overview of the literature

This is to be completed. For now, here are two references quoted in Lecture 18, both are monographs that deal with the finite groups of Lie type.

[St] mostly deals with the structure theory (and a bit with the representation theory).

[C] deals with the representation theory including its geometric aspects such as the Deligne-Lusztig theory.

References

[C] R.W. Carter, *Finite groups of Lie type. Conjugacy classes and complex characters.* Pure and Applied Mathematics (New York). A Wiley-Interscience Publication. John Wiley & Sons, Inc., New York, 1985.

[St] R. Steinberg, *Lectures on Chevalley groups.* Notes prepared by John Faulkner and Robert Wilson. Revised and corrected edition of the 1968 original. With a foreword by Robert R. Snapp. University Lecture Series, 66. American Mathematical Society, Providence, RI, 2016.