DAY 2 EXERCISES

1. Quantization of algebras

Exercise 1.1. *Show that the bracket on* $\operatorname{gr}\mathcal{A}$ *is well-defined and is a Poisson bracket.*

Exercise 1.2. *Show that the product in* $\mathcal{A}///_\lambda G$ *is well-defined.*

Remark: You have to show both that the product is well-defined, and that $\mathcal{A}///_\lambda G$ is closed under the product!

2. Quantization of sheaves

Exercise 2.1 (Exercise 2.1 in the notes)**.** *Let* $\mathcal{A}$ *be a (complete and separated) quantization of* A*. Show that if* A *is Noetherian so is* $\mathcal{A}$*.*

Hint: Let $\mathcal{I}$ be a left ideal of $\mathcal{A}$. Pick homogeneous generators $\overline{a}_1, \dots, \overline{a}_n$ of $\operatorname{gr}\mathcal{I}$, and pick lifts $a_1, \dots, a_n \in \mathcal{I}$. Show that $a_1, \dots, a_n$ are generators of $\mathcal{I}$.

Exercise 2.2. *Let* $\mathcal{A}$ *be a filtered algebra, complete and separated. Assume that* $\operatorname{gr}\mathcal{A}$ *is Noetherian. Then any left ideal of* $\mathcal{A}$ *is closed.*

Hint: Use descending induction on degrees.

Exercise 2.3. *Let* G *be a reductive group acting on a vector space* R*. Lift this action to an action on* T^*R*. Consider the action of* $\mathbb{C}^\times$ *on* T^*R *given by* $t.(u, u^*) = (u, t^{-1}u^*)$*, and take a character* $\theta : G \to \mathbb{C}^\times$*. Let* $f \in \mathbb{C}[T^*R]^{G,n\theta}$*. Show that every homogeneous component of* f *(w.r.t. the grading on* $\mathbb{C}[T^*R]$ *induced by the* $\mathbb{C}^\times$*-action) is again in* $\mathbb{C}[T^*R]^{G,n\theta}$*.*

Hint: The actions of $\mathbb{C}^\times$ and G commute.

Exercise 2.4. *Consider a reductive group* G *acting on a vector space* R*. Show that a quantum comoment map for the action of* G *on* $D(R)$ *is* $\xi \mapsto \xi_R$*.*