

Seminar announcement

August 28, 2020

1 Goal

The moduli space of Higgs bundles over a Riemann surface was introduced by Hitchin in [H2, H1] and has turned out to be connected to many subjects of interest in geometry, topology and quantum field theory. The first goal of the seminar is to understand the basic topological and geometric properties of this space, after developing some necessary background. The second goal will be to explore some of its applications and connections to other subjects, according to the interests of the participants.

2 Timing

Starting the week of September 14. Approximately two hours a week. Meeting time is to be selected by collecting preferences from the audience.

3 Background

We expect that students who want to participate in the seminar familiarize themselves with the following topics.

- 1) Kähler manifolds, [CS, Lecture 16].
- 2) Definition of a holomorphic vector bundle. [Huy, 2.2]
- 3) Basics of Hodge theory, [CS, Lecture 17].
- 4) Basics on Hamiltonian systems, [CS, Lecture 18].
- 5) Basics on Lie groups, Lie algebras and their actions on manifolds, [CS, 21.1-21.3].

4 Seminar topics

Here is a tentative list of topics for the seminar.

1. Introductory lecture: many reasons to love moduli spaces of Higgs bundles.
By Gurbir Dhillon.

2. Symplectic and Kahler quotients, a connection to Geometric Invariant theory. By Do Kien Hoang.
3. Hyperkähler manifolds and hyperkäler quotients. By Yaochen Wu.
4. Moduli space of G -bundles over a Riemann surface. Narasimhan-Seshadri theorem. Construction as an infinite dimensional Kähler quotient. References: [S] for an overview, original references are [AB, Das, NS, Don].
5. Moduli space of Higgs bundles over a Riemann surface, its structure as a complex integrable system. Reference: [H2].
6. Interpretation of moduli of Higgs bundles as a hyperkahler quotient by infinite-dimensional gauge group, nonabelian Hodge correspondence.
7. Further topics may include: computation of Betti numbers of character variety, relations to Teichmuller theory, relation to opers, relation to mirror symmetry, ...

References

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