DAY 3 EXERCISES

1. Modules over quantizations

Exercise 1.1 (Lemma 1.1 in the notes). *Let* $M = \bigcup_{i\in\mathbb{Z}} M_{\leqslant i} = \bigcup_{i\in\mathbb{Z}} M_{\preceq i}$ *be two good filtrations. Then, there exist integers* a, b *such that* $M_{\preceq i+a} \subset M_{\leqslant i} \subset M_{\preceq i+b}$ *for all* $i \in \mathbb{Z}$.

Exercise 1.2. *Let* M, N *be finitely generated modules over a filtered algebra* $\mathcal{A}$. *Assume that good filtrations are given on* M *and* N. *Show that* $\operatorname{Ext}^i_{\mathcal{A}}(M, N)$ *admits a filtration for all* i *and, moreover, that we have a natural inclusion* $\operatorname{gr}\operatorname{Ext}^i_{\mathcal{A}}(M, N) \hookrightarrow \operatorname{Ext}_A(\operatorname{gr} M, \operatorname{gr} N)$.

2. Localization theorems

Exercise 2.1. *Finish the proof of Lemma 2.1 in the notes. That is, show that if* $\operatorname{Loc}^\theta_\lambda$ *is essentially surjective and* Γ *is exact, then abelian localization holds for* (θ, λ).