

Lectures by Joel Kamnitzer.

Title: Categorical \mathfrak{sl}_2 actions in geometric settings and the resulting equivalences

Abstract:

One can define an action of \mathfrak{sl}_2 on a category to be a sequence of categories with functors between them satisfying certain relations. Such actions were studied by Chuang-Rouquier in the context of representations of the symmetric group in positive characteristic. They used this action to build an equivalence of categories which settled a case of Broue's conjecture.

Later, Cautis, Licata, and the speaker studied an \mathfrak{sl}_2 action where the categories involved were derived categories of coherent sheaves on cotangent bundles to Grassmannians. Following the ideas of Chuang-Rouquier, we used this \mathfrak{sl}_2 action to construct an equivalence of derived categories between different cotangent bundles of Grassmannians. This settled an open problem posed by Namikawa.

Another related setting for \mathfrak{sl}_2 actions is D-modules on Grassmannians, which has been studied extensively by Rouquier, Webster and others. A bridge between D-modules on Grassmannians and coherent sheaves on their cotangent bundles is provided by the theory of mixed Hodge modules. Recently, Dodd, Cautis, and the speaker used this bridge to relate the two situations. A nice consequence is an explicit computation for the associated graded of the weight filtration on the mixed Hodge module coming from the open locus in the product of complementary Grassmannians.

Suggested prerequisites:

Fourier-Mukai transforms in algebraic geometry (Huybrechts) sections 3, 5, 8.1-8.2, 11.4

D-modules, perverse sheaves and representation theory (HTT) section 1-3 (for the 3rd lecture).