

# SYMPLECTIC REFLECTION ALGEBRAS

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Symplectic reflection algebras are certain associative algebras introduced by P.E. and Ginzburg in the beginning of 2000's. They are connected to other objects in Representation theory (quivers, Hecke algebras) but also to Algebraic Geometry (resolutions and deformations of symplectic quotient singularities, plane curves), Combinatorics (Macdonald polynomials, Catalan numbers) and Integrable systems (Calogero-Moser type systems). We plan to explain some foundations of SRA's and then proceed to two topics of current research that connect SRA's to affine Lie algebras in very different ways:

- (1) P.E.'s conjecture on counting representations with given support of the cyclotomic Rational Cherednik algebras (proved by Shan and Vasserot) and of a P.E. type conjecture on counting finite dimensional irreducible representations for quantized quiver varieties (proved by R.B. and I.L.).
- (2) Varagnolo-Vasserot category equivalence conjecture (proved by I.L. and also by Rouquier, Shan, Varagnolo, Vasserot).

We concentrate on (1) and discuss (2) time permitting. Besides, we will discuss some other popular and important topics in Representation theory, such as Schur-Weyl dualities (in the modular and quantum setting), categorical Kac-Moody actions, and fusion products of representations of affine Kac-Moody algebras. Some topics of interest for algebraic geometers (derived equivalences between different crepant resolutions) will also be discussed. Also there will be lectures by R.B. on a connection of P.E.'s conjectures to geometry.

Students are especially encouraged to attend!

*Preliminary (optimistic!) program:*

- 1) Symplectic reflection algebras as deformations of skew-group algebras (P.E.).
- 2) Nakajima quiver varieties (I.L.).
- 3) Rational Cherednik algebras and their categories  $\mathcal{O}$  (P.E.).
- 4) Quantizations of quiver varieties. Relation to Symplectic reflection algebras (I.L.)
- 5) The main results (Varagnolo-Vasserot and Etingof conjectures). (P.E., I.L.)
- 6) Induction and restriction functors (P.E.).
- 7) Procesi bundles and their deformations (I.L.).
- 8) KZ functor and its properties (P.E.).
- 9) Quantum Schur-Weyl duality and Rouquier's equivalences of categories (I.L., P.E.).
- 10) Categorical Kac-Moody actions (I.L.).
- 11) Webster's categorical action for quantized quiver varieties (I.L.).
- 12) Outline of a proof of Etingof's conjecture in the cyclotomic case. (I.L.)
- 13) Wall-crossing functors. Outline of a proof of the Etingof type conjecture for quantized quiver varieties. (I.L.)
- 14\*) Fusion products for affine Lie algebras. (P.E.)
- 15\*) Outline of a proof of the Varagnolo-Vasserot conjecture. (I.L.)
- 14) and 15) will be covered time permitting.