5 2. Exercises

Exercise 5 1

We already showed in the previous homework that

$$\left(\frac{\partial}{\partial\theta^\alpha}\right)_{(x,\theta,\bar\theta)} = i\sigma^m{}_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\left(\frac{\partial}{\partial y^m}\right)_{(y,\theta,\bar\theta)} + \left(\frac{\partial}{\partial\theta^\alpha}\right)_{(y,\theta,\theta)},$$

$$\left(\frac{\partial}{\partial\bar\theta_{\dot\alpha}}\right)_{(x,\theta,\bar\theta)} = -i\bar\sigma^{m\dot\alpha\alpha}\theta_\alpha\left(\frac{\partial}{\partial y^m}\right)_{(y,\theta,\bar\theta)} + \left(\frac{\partial}{\partial\bar\theta_{\dot\alpha}}\right)_{(y,\theta,\bar\theta)},$$

$$\left(\frac{\partial}{\partial x^m}\right)_{(x,\theta,\bar\theta)} = \left(\frac{\partial}{\partial y^m}\right)_{(y,\theta,\bar\theta)}$$

Thus in the coordnates $(y,\theta,\bar\theta)$ we have

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + 2i\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\frac{\partial}{\partial y^m},$$

$$\bar D^{\dot\alpha} = \frac{\partial}{\partial\bar\theta_{\dot\alpha}}$$

Finally, notice that

$$\varepsilon_{\dot\alpha\dot\beta}\frac{\partial\bar\theta_{\dot\gamma}}{\partial\bar\theta_{\dot\beta}} = \varepsilon_{\dot\alpha\dot\beta}\delta^{\dot\beta}_{\dot\gamma} = \varepsilon_{\dot\alpha\dot\gamma} = -\varepsilon_{\dot\gamma\dot\alpha} = -\varepsilon_{\dot\gamma\dot\beta}\delta^{\dot\beta}_{\dot\alpha} = -\varepsilon_{\dot\gamma\dot\beta}\frac{\partial\bar\theta^{\dot\beta}}{\partial\bar\theta^{\dot\alpha}} = -\frac{\partial\bar\theta_{\dot\gamma}}{\partial\bar\theta^{\dot\alpha}}$$

Thus

$$\bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}}$$

Exercise 52

Due to the previous exercise, we know that $\bar{D}_{\dot\alpha}\Phi = 0$ implies $\Phi = \Phi(y,\theta) = \varphi(y) + \sqrt{2}\,\theta\psi(y) + \theta\theta F(y)$ Now consider an arbitrary superfield

$$U(y,\theta,\bar\theta) = A(y) + \theta B(y) + \bar\theta C(y) + \theta\theta D(y) + \bar\theta\bar\theta E(y) + \theta\sigma^m\bar\theta F_m(y)$$

$$+ \theta\theta\bar\theta\bar H(y) + \bar\theta\bar\theta\theta I(y) + \theta\theta\bar\theta\bar\theta J(y)$$

Since

$$(\bar D\bar D)(\bar\theta\bar\theta) = \bar D_{\dot\alpha}\left(\frac{\partial\bar\theta_{\dot\beta}}{\partial\bar\theta_{\dot\alpha}}\bar\theta^{\dot\beta} - \bar\theta_{\dot\beta}\frac{\partial\bar\theta^{\dot\beta}}{\partial\bar\theta_{\dot\alpha}}\right) = \bar D_{\dot\alpha}\left(\bar\theta^{\dot\alpha} + \bar\theta^{\dot\beta}\delta^{\dot\alpha}_{\dot\beta}\right)$$

$$= 2\bar D_{\dot\alpha}\bar\theta^{\dot\alpha} = -2\delta^{\dot\alpha}_{\dot\alpha} = -4,$$

we have

$$(\bar D\bar D)U(y,\theta,\bar\theta) = -4\left(E(y) + \theta I(y) + \theta\theta J(y)\right)$$

Thus $\Phi = (\bar D\bar D)U$ if

$$U = -\frac{1}{4}\bar\theta\bar\theta\Phi$$

Exercise 53

Following the calculations above $(g \leftrightarrow U)$

$$\int d^4x\, d^2\theta\, f = \int d^4x\, d^2\theta\, \bar D\bar D g = -4\int d^4x\, J(y) = -4\int d^4x\, d^4\theta\, g$$

Exercise 54

Let us define

$$\tilde{B}(\Phi) = -\frac{1}{4} DDA(\Phi, \bar{\Phi}) + B(\Phi)$$

Then, due to the previous exercise,

$$\int d^2\theta \, \tilde{B}(\Phi) = \int d^4\theta \, A(\Phi, \bar{\Phi}) + \int d^2\theta \, B(\Phi)$$

Thus, in terms of $\tilde{B}$, the action can be rewritten as

$$S = \int d^4x \left(\int d^2\theta \, \tilde{B}(\Phi) + \int d^2\bar{\theta} \, \bar{B}(\bar{\Phi}) \right)$$

Starting from this action, the EOM (5 29) is

$$0 = \frac{\partial \tilde{B}}{\partial \Phi}$$

However, noticing that

$$\frac{\partial}{\partial \Phi}(\bar{D}A(\Phi,\bar{\Phi})) = \frac{\partial}{\partial \Phi}\left(\frac{\partial A(\Phi,\bar{\Phi})}{\partial \Phi} \cancelto{0}{\bar{D}\Phi} + \frac{\partial A(\Phi,\bar{\Phi})}{\partial \bar{\Phi}} \bar{D}\bar{\Phi} \right)$$

$$= \frac{\partial^2 A(\Phi,\bar{\Phi})}{\partial \Phi \partial \bar{\Phi}} \bar{D}\bar{\Phi} + \frac{\partial A(\Phi,\bar{\Phi})}{\partial \bar{\Phi}} \cancelto{0}{\frac{\partial}{\partial \Phi}(\bar{D}\bar{\Phi})} = \bar{D} \frac{\partial A(\Phi,\bar{\Phi})}{\partial \Phi},$$

we see that this new EOM is equivalent to the previous

$$\frac{\partial \tilde{B}(\Phi)}{\partial \Phi} = -\frac{1}{4} \bar{D}\bar{D} \frac{\partial A(\Phi,\bar{\Phi})}{\partial \Phi} + \frac{\partial B(\Phi)}{\partial \Phi}$$

Similarly, since $\bar{D}B(\Phi) = 0$ (since Φ doesn't depend on $\bar{\theta}$), we obtain through exercise 52 a function U st $B(\Phi) = DDU_{\Phi}$ Then, by defining

$$\tilde{A}(\Phi,\bar{\Phi}) = A(\Phi,\bar{\Phi}) - 4U_{\Phi},$$

we obtain

$$\int d^4\theta\, \tilde{A}(\Phi,\bar{\Phi}) = \int d^4\theta\, A(\Phi,\bar{\Phi}) + \int d^2\theta\, B(\Phi)$$

The action can then be written as

$$S = \int d^4x \left(\int d^4\theta \tilde{A}(\Phi,\bar{\Phi}) + \int d^2\bar{\theta}\, \bar{B}(\bar{\Phi}) \right)$$

The corresponding EOM is

$$0 = -\frac{1}{4}\bar{D}\bar{D}\frac{\partial\hat{A}(\Phi,\bar{\Phi})}{\partial\Phi} = -\frac{1}{4}\bar{D}\bar{D}\frac{\partial A(\Phi,\bar{\Phi})}{\partial\Phi} + \bar{D}\bar{D}\frac{\partial U_{\Phi}}{\partial\Phi}$$

$$= -\frac{1}{4}\bar{D}\bar{D}\frac{\partial\hat{A}(\Phi,\bar{\Phi})}{\partial\Phi} + \frac{\partial B(\Phi)}{\partial\Phi},$$

consistently with (5 29)

Exercise 5 5

Notice that in coordinates $(y,\theta,\bar{\theta})$

$$\bar{D}^{\dot\alpha} = \varepsilon^{\dot\alpha\dot\beta}\bar{D}_{\dot\beta} = -\varepsilon^{\dot\alpha\dot\beta}\frac{\partial}{\partial\bar{\theta}^{\dot\beta}} - 2i\varepsilon^{\dot\alpha\dot\beta}\theta^{\beta}\sigma^{m}{}_{\beta\dot\beta}\frac{\partial}{\partial\bar{y}^{m}}$$

$$= +\frac{\partial}{\partial\bar{\theta}_{\dot\alpha}} - 2i\varepsilon^{\dot\alpha\dot\beta}\varepsilon^{\beta\alpha}\theta_{\alpha}\sigma^{m}{}_{\beta\dot\beta}\frac{\partial}{\partial\bar{y}^{m}}$$

$$= +\frac{\partial}{\partial\bar{\theta}_{\dot\alpha}} + 2i\bar{\sigma}^{m\dot\alpha\alpha}\frac{\partial}{\partial\bar{y}^{m}}$$

Thus

$$\bar{D}\bar{D} = \left(-\frac{\partial}{\partial\bar{\theta}^{\dot\alpha}} - 2i\theta^{\alpha}\sigma^{m}{}_{\alpha\dot\alpha}\frac{\partial}{\partial\bar{y}^{m}}\right)\left(\frac{\partial}{\partial\bar{\theta}_{\dot\alpha}} + 2i\bar{\sigma}^{n\dot\alpha\beta}\theta_{\beta}\frac{\partial}{\partial\bar{y}^{n}}\right)$$

$$= -\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\alpha}} - 2i\theta^\alpha\sigma^m{}_{\alpha\dot\alpha}\frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar y^m} + \underbrace{2i\bar\sigma^{m\dot\alpha\beta}\theta_\beta\frac{\partial}{\partial\bar\theta^{\dot\alpha}}}\frac{\partial}{\partial\bar y^m}$$

$$\underbrace{+4\theta^\alpha\sigma^m{}_{\alpha\dot\alpha}\bar\sigma^{n\dot\alpha\beta}\theta_\beta\frac{\partial^2}{\partial\bar y^m\partial\bar y^n}} \qquad \varepsilon^{\dot\alpha\dot\beta}\varepsilon^{\beta\alpha}\sigma^m{}_{\alpha\dot\beta}\theta_\beta\frac{\partial}{\partial\bar\theta^{\dot\alpha}} = -\theta^\alpha\sigma^m{}_{\alpha\dot\beta}\frac{\partial}{\partial\bar\theta_{\dot\beta}}$$

$$2(\sigma^{(m}\bar\sigma^{n)})_\alpha{}^\beta\theta^\alpha\theta_\beta\frac{\partial^2}{\partial\bar y^m\partial\bar y^n} = -4(\theta\theta)\Box_{\bar y}$$

$$= -\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\alpha}} - 4i\theta^\alpha\sigma^m{}_{\alpha\dot\alpha}\frac{\partial}{\partial\bar\theta_{\dot\alpha}}\frac{\partial}{\partial\bar y^m} - 4(\theta\theta)\Box_{\bar y}$$

Since

$$\frac{\partial}{\partial\bar\theta_{\dot\alpha}}(\bar\theta\bar\theta) = \frac{\partial}{\partial\bar\theta_{\dot\alpha}}(\bar\theta_{\dot\beta}\bar\theta^{\dot\beta}) = \delta^{\dot\alpha}_{\dot\beta}\bar\theta^{\dot\beta} - \bar\theta_{\dot\beta}\varepsilon^{\dot\beta\dot\alpha} = 2\bar\theta^{\dot\alpha},$$

and

$$\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta_{\dot\alpha}}(\bar\theta\bar\theta) = 2\delta^{\dot\alpha}_{\dot\alpha} = 4,$$

we can use the expansion into components

$$\bar\Phi(\bar y,\bar\theta) = \bar\varphi(\bar y) + \sqrt{2}\,\bar\theta\bar\psi(\bar y) + \bar\theta\bar\theta\bar F(\bar y),$$

to calculate

$$(\bar D\bar D)\bar\Phi(\bar y,\bar\theta) = -4\bar F(\bar y) - 4\sqrt{2}\,i(\theta\sigma^m\partial_m\bar\psi(\bar y)) - 8i(\theta\sigma^m\bar\theta)\partial_m\bar F(\bar y) - 4(\theta\theta)\Box\bar\varphi(\bar y)$$

$$-4\sqrt{2}(\theta\theta)(\bar\theta\Box\bar\psi(\bar y)) - 4(\theta\theta)(\bar\theta\bar\theta)\Box\bar F(\bar y)$$

A function in space gets extended into

$$f(x \pm i\theta\sigma\bar\theta) = f(x) \pm i(\theta\sigma^m\bar\theta)\partial_m f(x) - \frac{1}{2}(\theta\sigma^m\bar\theta)(\theta\sigma^n\bar\theta)\partial_m\partial_n f(x)$$

$$= f(x) \pm i(\theta\sigma^m\bar\theta)\partial_m f(x) + \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\Box f(x)$$

Therefore, in coordinates $(x,\theta,\bar\theta)$

$$(\bar D\bar D)\bar\Phi(x - i\theta\sigma\bar\theta, \bar\theta) = -4\bar F(x) + \cancel{4i(\theta\sigma^m\bar\theta)\partial_m\bar F(x)} - (\theta\theta)(\bar\theta\bar\theta)\Box\bar F(x)$$

$$- 4\sqrt{2}\, i(\theta\sigma^m\partial_m\bar\psi(x)) \underbrace{- 4\sqrt{2}(\theta\sigma^m\bar\theta)(\theta\sigma^n\partial_m\partial_n\bar\psi(x))},$$

$$+ 2\sqrt{2}\,\varepsilon^{\alpha\beta}\sigma^m{}_{\beta\dot\beta}\bar\theta^{\dot\beta}\sigma^n{}_{\alpha\dot\alpha}\partial_m\partial_n\bar\psi^{\dot\alpha}(x)(\theta\theta)$$

$$\parallel$$

$$2\sqrt{2}\,\varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\gamma}\sigma^m{}_{\beta\dot\beta}\bar\theta^{\dot\beta}\sigma^n{}_{\alpha\dot\alpha}\partial_m\partial_n\bar\psi_{\dot\gamma}(\theta\theta)$$

$$\parallel$$

$$2\sqrt{2}\,\sigma^m{}_{\beta\dot\beta}\bar\theta^{\dot\beta}\partial_m\partial_n\bar\psi_{\dot\gamma}\bar\sigma^{n\dot\gamma\beta}(\theta\theta)$$

$$- \overset{4}{\cancel{8}}\, i(\theta\sigma^m\bar\theta)\partial_m F(x) - \underbrace{8(\theta\sigma^m\bar\theta)(\theta\sigma^n\bar\theta)\partial_m\partial_n\bar F(x)}$$

$$\cancel{+4(\theta\theta)(\bar\theta\bar\theta)\Box\bar F(x)}$$

$$- 4(\theta\theta)\Box\varphi(x) - 4\sqrt{2}(\theta\theta)(\bar\theta\Box\psi(x)) - \cancel{4(\theta\theta)(\bar\theta\bar\theta)\Box\bar F(x)}$$

On the other hand due to equation (58)

$$\Phi(x + i\theta\sigma\bar\theta, \theta) = \varphi(x) + i(\theta\sigma^m\bar\theta)\partial_m\varphi(x) + \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\Box\varphi(x) + \sqrt{2}\theta\psi(x)$$

$$- \frac{2}{\sqrt{2}}(\theta\theta)(\partial_m\psi(x)\sigma^m\bar\theta) + (\theta\theta)F(x),$$

and

$$\Phi(x + i\theta\sigma\bar\theta, \theta)^2 = \varphi(x)^2 + 2i(\theta\sigma^m\bar\theta)\partial_m\varphi(x)\varphi(x) + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)\Box\varphi(x)\varphi(x) + 2\sqrt{2}(\theta\psi(x))\varphi(x)$$

$$- \sqrt{2}\, i(\theta\theta)(\partial_m\psi(x)\sigma^m\bar\theta)\varphi(x) + 2(\theta\theta)F(x)\varphi(x)$$

$$\underbrace{-(\theta\sigma^m\bar{\theta})(\theta\sigma^n\bar{\theta})\partial_m\varphi(x)\partial_n\varphi(x)} \quad \underbrace{+\,2\sqrt{2}\,i(\theta\psi(x))(\theta\sigma^m\bar{\theta})\partial_m\varphi(x)}$$

$$+\frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})\partial_m\varphi(x)\partial^m\varphi(x) \qquad -\sqrt{2}\,i(\psi(x)\sigma^m\bar{\theta})(\theta\theta)\partial_m\varphi(x)$$

$$\underbrace{+\,2(\theta\psi(x))(\theta\psi(x))}$$

$$-(\theta\theta)(\psi(x)\psi(x))$$

Now consider our action

$$S(\Phi) = \int d^4x \Big(\int d^4\theta\, \Phi(x+i\theta\sigma\bar{\theta},\theta)\bar{\Phi}(x-i\theta\sigma\bar{\theta},\bar{\theta}) + \int d^2\theta \big(\lambda\Phi(x+i\theta\sigma\bar{\theta},\theta)$$

$$+\frac{1}{2}M\Phi(x+i\theta\sigma\bar{\theta},\theta)^2 + \frac{1}{3}g\,\Phi(x+i\theta\sigma\bar{\theta},\theta)^3\big)$$

$$+\int d^2\bar{\theta}\big(\bar{\lambda}\bar{\Phi}(x-i\theta\sigma\bar{\theta},\bar{\theta}) + \frac{1}{2}\bar{M}\bar{\Phi}(x-i\theta\sigma\bar{\theta},\bar{\theta})^2 + \frac{1}{3}\bar{g}\,\bar{\Phi}(x-i\theta\sigma\bar{\theta},\bar{\theta})\big)^3\Big)$$

Thus the EOM (5 29) are

$$0 = -\frac{1}{4}\bar{D}\bar{D}\bar{\Phi}(x-i\theta\sigma\bar{\theta},\bar{\theta}) + \lambda + M\Phi(x+i\theta\sigma\bar{\theta},\theta) + g\,\Phi(x+i\theta\sigma\bar{\theta},\theta)^2$$

By comparing powers in $(\theta,\bar{\theta})$ we have

1 $\quad 0 = \bar{F}(x) + \lambda + M\varphi(x) + g\,\varphi(x)^2 \iff \bar{F}(x) = -\lambda - M\varphi(x) - g\,\varphi(x)^2,$ 1

θ $\quad 0 = \sqrt{2}\,i\,\sigma^m{}_{\alpha\dot\alpha}\partial_m\bar{\psi}^{\dot\alpha}(x) + M\sqrt{2}\,\psi_\alpha(x) + 2\sqrt{2}\,g\,\psi_\alpha(x)\varphi(x)$

$\iff -i\sigma^m{}_{\alpha\dot\alpha}\partial_m\bar{\psi}^{\dot\alpha}(x) = M\psi_\alpha(x) + 2g\,\psi_\alpha(x)\varphi(x),$ 2

$\theta\theta$ $\quad 0 = \Box\bar{\varphi}(x) + MF(x) + 2g\,F(x)\varphi(x) - g(\psi(x)\psi(x))$

$\iff -\Box\bar{\varphi}(x) = MF(x) + 2gF(x)\varphi(x) - g\,\psi(x)\psi(x)$ 3

This are precisely the EOMs (5 22) As we will see, the other eqn's are consequences of the first 3

$$\theta\sigma\bar\theta \quad 0 = i\partial_m \bar F(x) + iM\partial_m \varphi(x) + 2ig\,\varphi(x)\partial_m\varphi(x) = i\partial_m\left(\bar F(x) + M\varphi(x) + g\varphi(x)^2\right) = 0$$

$$(\theta\theta)\bar\theta^{\dot\alpha} \quad 0 = -\frac{1}{\sqrt2}\underbrace{\sigma^m{}_{\alpha\dot\alpha}\partial_m\partial_n\bar\psi_{\dot\beta}\bar\sigma^{n\dot\beta\alpha}} + \frac{iM}{\sqrt2}\partial_m\psi^\alpha\sigma^m{}_{\alpha\dot\alpha} + \sqrt2\, ig\,\partial_m\psi^\alpha\sigma^m{}_{\alpha\dot\alpha}\varphi$$

$$\frac12(\bar\sigma^{(n}\sigma^{m)})^{\dot\beta}{}_{\dot\alpha}\partial_m\partial_n\bar\psi_{\dot\beta} = -\Box\bar\psi_{\dot\alpha}$$

$$+\sqrt2\, ig\,\psi^\alpha\sigma^m{}_{\alpha\dot\alpha}\partial_m\varphi + \underbrace{\sqrt2\,\Box\bar\psi_{\dot\alpha}}$$

$$-\sqrt2\,\sigma^m{}_{\alpha\dot\alpha}\partial_m\partial_n\bar\psi_{\dot\beta}\bar\sigma^{n\dot\beta\alpha}$$

$$= \frac{i}{\sqrt2}\sigma^m{}_{\alpha\dot\alpha}\partial_m\left(-i\partial_n\bar\psi_{\dot\beta}\bar\sigma^{n\dot\beta\alpha} + M\psi^\alpha + 2g\psi^\alpha\varphi\right) = 0$$

Exercise 5 6.

In order to obtain the EOMs for action (5 28) we need to calculate the variation of the action with respect to Φ. In preparation for this we rewrite the action like

$$S = \int d^4x \int d^4\theta \left(A(\Phi(x,\theta,\bar\theta),\bar\Phi(x,\theta,\bar\theta)) + B(\Phi(x,\theta,\bar\theta))\delta^2(\bar\theta) + \bar B(\bar\Phi(x,\theta,\bar\theta))\delta^2(\theta)\right)$$

Then

$$\frac{\delta S}{\delta\Phi(x,\theta,\bar\theta)} = \int d^4x' \int d^4\theta' \left(\frac{\partial A(\Phi(x',\theta',\bar\theta'),\bar\Phi(x',\theta',\bar\theta'))}{\partial\Phi(x',\theta',\bar\theta')}\frac{\delta\Phi(x',\theta',\bar\theta')}{\delta\Phi(x,\theta,\bar\theta)}\right.$$

$$\left. + \frac{\partial B(\Phi(x',\theta',\bar\theta'))}{\partial\Phi(x',\theta',\bar\theta')}\delta^2(\bar\theta')\frac{\delta\Phi(x',\theta',\bar\theta')}{\delta\Phi(x,\theta,\bar\theta)}\right)$$

We thus see that we obtain the EOM (5 29) if

$$\frac{\delta\Phi(x',\theta',\bar\theta')}{\delta\Phi(x,\theta,\theta)} = -\frac{\bar D\bar D}{4}\left(\delta^4(x-x')\,\delta^2(\theta-\theta')\,\delta^2(\bar\theta-\bar\theta')\right) = \frac{\bar D\bar D}{4}\,\delta^8((x,\theta,\bar\theta)-(x',\theta',\bar\theta')),$$

following Prakash, N Mathematical Perspectives on Theoretical Physics

A Journey from Black Holes to Superstrings, 2003

To be more explicit, for a Grassmann variable θ define

$\delta\theta$ by the requirement that

$$\int d\theta\,\delta(\theta)f(\theta) = f(0)$$

We thus immediately see that $\delta(\theta)=\theta$

Similarly, in superspace

$$\int d^2\theta\, f(\theta)\delta^2(\theta) = f(0), \qquad \int d^2\bar\theta\, f(\bar\theta) = f(0), \qquad \int d^4\theta\, f(\theta)\,\delta^4(\theta) = f(0)$$

Thus

$$\delta^2(\theta) = (\theta\theta), \qquad \delta^2(\bar\theta) = \bar\theta\bar\theta, \qquad \delta^4(\theta) = (\theta\theta)(\bar\theta\bar\theta)$$

Then

$$\bar D\bar D\,\delta^2(\bar\theta) = -\frac{\partial}{\partial\bar\theta^\alpha}\frac{\partial}{\partial\bar\theta_\alpha}(\bar\theta\bar\theta) = -4$$

Then, after integrating by parts and recalling that $\bar D B=0$, we have

$$\frac{\delta S}{\delta \Phi(x,\theta,\bar\theta)} = \int d^4x' d^4\theta' \Big(-\frac{1}{4} \bar D \bar D \frac{\partial A(\Phi(x',\theta',\bar\theta'), \bar\Phi(x',\theta',\bar\theta'))}{\partial \Phi(x',\theta',\bar\theta')}$$

$$- \frac{1}{4} B(\Phi(x',\theta',\bar\theta'))(-4) \Big) \delta^4(x-x') \delta^2(\theta-\theta') \delta^2(\bar\theta - \bar\theta')$$

$$= -\frac{1}{4} \bar D \bar D \frac{\partial A(\Phi(x,\theta,\bar\theta), \bar\Phi(x,\theta,\bar\theta))}{\partial \Phi(x,\theta,\bar\theta)} + B(\Phi(x,\theta,\bar\theta)),$$

which leads to the EOMs (5.29) The reason for our definition is that with this precise choice we have

$$\int d^4x \int d^2\theta\, f(\Phi(x',\theta',\bar\theta')) \frac{\delta \Phi(x',\theta',\bar\theta')}{\delta \Phi(x,\theta,\bar\theta)}$$

$$= \int d^4x\, d^2\theta\, f(\Phi(x',\theta',\bar\theta')) \Big(-\frac{\bar D \bar D}{4} \big(\delta^4(x-x') \delta^2(\theta-\theta') \delta^2(\bar\theta - \bar\theta') \big) \Big)$$

$$= -\int d^4x\, d^2\theta\, \frac{\bar D \bar D}{4} \big(f(\Phi(x',\theta',\bar\theta')) \delta^4(x-x') \delta^2(\theta-\theta') \delta^2(\bar\theta - \bar\theta') \big)$$

$$= \int d^4x\, d^4\theta\, f(\Phi(x',\theta',\bar\theta')) \delta^4(x-x') \delta^2(\theta-\theta') \delta^2(\bar\theta - \bar\theta') = f(\Phi(x,\theta,\bar\theta))$$