

Emerging Gauge Symmetries and Quantum Operations

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Motivation

- The original motivation was to study entanglement entropy in systems which are not a tensor product of its subsystems.

Preliminary work: Balachandran, et al. 2013

Motivation

- The original motivation was to study entanglement entropy in systems which are not a tensor product of its subsystems.
- The algebraic approach leads to a satisfactory notion of restriction of states to such subsystems. However, how do we define the entropy of algebraic states?

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Building Blocks

Physical systems are described by:

- 1 observables a ,
- 2 states ω , and
- 3 a pairing $\omega(a)$ which describes the expectation value of a in the state ω .

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In the algebraic formulation (see Strocchi 2008):

- 1 observables are the selfadjoint elements of a von Neumann algebra \mathcal{A} (we will consider $n \times n$ complex matrices),
- 2 states are the positive normalized linear functionals $\omega : \mathcal{A} \rightarrow \mathbb{C}$ (we will consider faithful states), and
- 3 the expectation value of an observable $a = a^\dagger \in \mathcal{A}$ in the state ω is given by $\omega(a)$

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- ④ Notice that if $|\Omega\rangle = |\mathbb{1}_{\mathcal{A}}\rangle$ then

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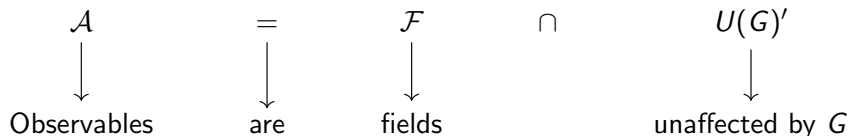
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- ⑥ “**A + B = C**” in the sense that the smallest algebra containing both $\pi(\mathcal{A})$ and $\pi(\mathcal{A})'$ is \mathcal{F} .

Tomita-Takesaki and Gauge Group

Tomita-Takesaki Theory allows us to construct an antiunitary operator J on \mathcal{H} such that $J\pi(a)J \in \pi(\mathcal{A})'$ for all $a \in \mathcal{A}$. Consider the group $G = U(\mathcal{A})$ of unitary elements in \mathcal{A} . We then have an action of G on \mathcal{H} via $U(g) = J\pi(g)J \in \pi(\mathcal{A})'$. G can be interpreted as a gauge group in the sense of (see Doplicher, Haag, and Roberts 1969)



Representation Theory

Consider an orthogonal complete set of projections $P^{(\alpha)} \in \pi(\mathcal{A})'$. Via

$$P_g^{(\alpha)} := U(g)P^{(\alpha)}U(g^\dagger) \quad (2)$$

we get a G -dependent family of such projections.

Obtaining such sets is equivalent to decomposing the GNS representation into subrepresentations

$$\mathcal{H} = \bigoplus_{\alpha} P_g^{(\alpha)} \mathcal{H}. \quad (3)$$

Quantum Operation

This induces a quantum operation

$$\mathcal{E}_g(\rho) = \sum_{\alpha} P_g^{(\alpha)} \rho P_g^{(\alpha)}, \quad (4)$$

s.t.

$$\mathrm{tr}(\rho\pi(a)) = \mathrm{tr}(\mathcal{E}_g(\rho)\pi(a)) \quad (5)$$

for all observables $a \in \mathcal{A}$ in \mathbf{A} .

- ❶ $S_{\mathrm{vN}}(\mathcal{E}_g(\rho)) \geq S_{\mathrm{vN}}(\rho)$.
- ❷ If ρ describes ω (i.e. $\rho = |\Omega\rangle\langle\Omega|$) so does $\mathcal{E}_g(\rho)$.

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Thanks!