Tarea 5: Mecánica Analítica

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Ejercicio 2

Se define la energía potencial del sistema.

In[3]:=
$$d12 = \sqrt{(x1 - x2)^2 + (a + y2 - y1)^2} - a;$$
$$d13 = \sqrt{(y1 - y3)^2 + (a + x3 - x1)^2} - a;$$
$$d23 = \sqrt{(a + x3 - x2)^2 + (a + y2 - y3)^2} - \sqrt{2}\, a;$$
$$U = \frac{1}{2} k \left(d12^2 + d13^2 + d23^2\right);$$

Se calcula su Hessiana y sus valores propios.

In[7]:=
```
V = Simplify[D[U, {{x1, y1, x2, y2, x3, y3}, 2}] /.
    {x1 → 0, y1 → 0, x2 → 0, y2 → 0, x3 → 0, y3 → 0}, a > 0];
MatrixForm[V]
Eigenvalues[V]
Eigenvectors[V]
```

Out[8]//MatrixForm=
$$\begin{pmatrix} k & 0 & 0 & 0 & -k & 0 \\ 0 & k & 0 & -k & 0 & 0 \\ 0 & 0 & \frac{k}{2} & -\frac{k}{2} & -\frac{k}{2} & \frac{k}{2} \\ 0 & -k & -\frac{k}{2} & \frac{3k}{2} & \frac{k}{2} & -\frac{k}{2} \\ -k & 0 & -\frac{k}{2} & \frac{k}{2} & \frac{3k}{2} & -\frac{k}{2} \\ 0 & 0 & \frac{k}{2} & -\frac{k}{2} & -\frac{k}{2} & \frac{k}{2} \end{pmatrix}$$

Out[9]= {3 k, 2 k, k, 0, 0, 0}

Out[10]= {{1, 1, 1, -2, -2, 1}, {-1, 1, 0, -1, 1, 0}, {-1, -1, 1, 0, 0, 1},
{0, 0, -1, 0, 0, 1}, {1, 0, 1, 0, 1, 0}, {0, 1, 1, 1, 0, 0}}

Ejercicio 5

Verificamos que la transformación es canónica

In[11]:=
$$Q = \text{Log}\left[1 + \sqrt{q}\ \text{Cos}[p]\right];$$
$$P = 2\left(1 + \sqrt{q}\ \text{Cos}[p]\right)\sqrt{q}\ \text{Sin}[p];$$

```
M = Simplify[D[{Q, P}, {{q, p}}]];
J = {{0, 1}, {-1, 0}};
MatrixForm[M]
```

$\text{MatrixForm}[\text{Simplify}[M^T.J]]$

$\text{MatrixForm}[\text{Simplify}[M^T.J.M]]$

Out[15]//MatrixForm=
$$\begin{pmatrix} \frac{\text{Cos}[p]}{2\left(\sqrt{q} + q\,\text{Cos}[p]\right)} & -\frac{\sqrt{q}\ \text{Sin}[p]}{1+\sqrt{q}\ \text{Cos}[p]} \\ \left(\frac{1}{\sqrt{q}} + 2\,\text{Cos}[p]\right)\text{Sin}[p] & 2\left(\sqrt{q}\ \text{Cos}[p] + q\,\text{Cos}[2\,p]\right) \end{pmatrix}$$

Out[16]//MatrixForm=
$$\begin{pmatrix} -\left(\frac{1}{\sqrt{q}} + 2\,\text{Cos}[p]\right)\text{Sin}[p] & \frac{\text{Cos}[p]}{2\left(\sqrt{q} + q\,\text{Cos}[p]\right)} \\ -2\left(\sqrt{q}\ \text{Cos}[p] + q\,\text{Cos}[2\,p]\right) & -\frac{\sqrt{q}\ \text{Sin}[p]}{1+\sqrt{q}\ \text{Cos}[p]} \end{pmatrix}$$

Out[17]//MatrixForm=
$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Ejercicio 6

Verificamos que la transformación es canónica

In[18]:=
$$x = \frac{1}{\alpha}\left(\sqrt{2\,P1}\ \mathrm{Sin}[Q1] + P2\right);$$
$$y = \frac{1}{\alpha}\left(\sqrt{2\,P1}\ \mathrm{Cos}[Q1] + Q2\right);$$
$$px = \frac{\alpha}{2}\left(\sqrt{2\,P1}\ \mathrm{Cos}[Q1] - Q2\right);$$
$$py = -\frac{\alpha}{2}\left(\sqrt{2\,P1}\ \mathrm{Sin}[Q1] - P2\right);$$

```
M = D[{x, y, px, py}, {{Q1, Q2, P1, P2}}];
J = {{0, 0, 1, 0}, {0, 0, 0, 1}, {-1, 0, 0, 0}, {0, -1, 0, 0}};
MatrixForm[M]
MatrixForm[Mᵀ.J]
MatrixForm[Simplify[Mᵀ.J.M]]
```

Out[24]//MatrixForm=
$$\begin{pmatrix} \frac{\sqrt{2}\sqrt{P1}\,\mathrm{Cos}[Q1]}{\alpha} & 0 & \frac{\mathrm{Sin}[Q1]}{\sqrt{2}\sqrt{P1}\,\alpha} & \frac{1}{\alpha} \\ -\frac{\sqrt{2}\sqrt{P1}\,\mathrm{Sin}[Q1]}{\alpha} & \frac{1}{\alpha} & \frac{\mathrm{Cos}[Q1]}{\sqrt{2}\sqrt{P1}\,\alpha} & 0 \\ -\frac{\sqrt{P1}\,\alpha\,\mathrm{Sin}[Q1]}{\sqrt{2}} & -\frac{\alpha}{2} & \frac{\alpha\,\mathrm{Cos}[Q1]}{2\sqrt{2}\sqrt{P1}} & 0 \\ -\frac{\sqrt{P1}\,\alpha\,\mathrm{Cos}[Q1]}{\sqrt{2}} & 0 & -\frac{\alpha\,\mathrm{Sin}[Q1]}{2\sqrt{2}\sqrt{P1}} & \frac{\alpha}{2} \end{pmatrix}$$

Out[25]//MatrixForm=
$$\begin{pmatrix} \frac{\sqrt{P1}\,\alpha\,\mathrm{Sin}[Q1]}{\sqrt{2}} & \frac{\sqrt{P1}\,\alpha\,\mathrm{Cos}[Q1]}{\sqrt{2}} & \frac{\sqrt{2}\sqrt{P1}\,\mathrm{Cos}[Q1]}{\alpha} & -\frac{\sqrt{2}\sqrt{P1}\,\mathrm{Sin}[Q1]}{\alpha} \\ \frac{\alpha}{2} & 0 & 0 & \frac{1}{\alpha} \\ -\frac{\alpha\,\mathrm{Cos}[Q1]}{2\sqrt{2}\sqrt{P1}} & \frac{\alpha\,\mathrm{Sin}[Q1]}{2\sqrt{2}\sqrt{P1}} & \frac{\mathrm{Sin}[Q1]}{\sqrt{2}\sqrt{P1}\,\alpha} & \frac{\mathrm{Cos}[Q1]}{\sqrt{2}\sqrt{P1}\,\alpha} \\ 0 & -\frac{\alpha}{2} & \frac{1}{\alpha} & 0 \end{pmatrix}$$

Out[26]//MatrixForm=
$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

Se aplica la transformación al Hamiltoniano

In[56]:=
```
α = Sqrt[e B]
K = FullSimplify[1/(2 m) Norm[{px, py, 0} - e/2 {0, 0, B} × {x, y, 0}]^2, Q1 ∈ Reals]
```

$$\alpha = \sqrt{e\,B}$$

$$K = \text{FullSimplify}\left[\frac{1}{2\,m}\,\text{Norm}\left[\{px,\,py,\,0\} - \frac{e}{2}\{0,\,0,\,B\}\times\{x,\,y,\,0\}\right]^2,\ Q1 \in \text{Reals}\right]$$

Out[56]= $\sqrt{B\,e}$

Out[57]= $\frac{\text{Abs}[B\,e\,P1]}{m}$

CloudObjectInformation: No CloudObject found at the given address

First: Nonatomic expression expected at position 1 in First[$Failed].

Part: Part specification FileByteCount is not applicable.

Part: Part specification LastModified is not applicable.

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