Emergent Time from Quantum Variability

(Tiempo emergente de la variabilidad cuántica)

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Abstract

Operator algebras, as general frameworks for the description of the observables of physical systems, have been an important tool towards rigorous investigations in information theory, statistical physics, and quantum field theory[1, 2, 3, 4]. In this project we will familiarize with these techniques and explore Connes' proposal for the emergence of time as a quantum phenomenon[5]. By building from [6], we will begin through the study of a generalization of the Radon-Nikodým theorem to von Neumann algebras[7]. This central result will yield a canonical dynamical mapping $\mathbb{R} \to \mathrm{Out}(\mathcal{M})$ due to the noncommutativity of the von Neumann algebra $\mathcal{M}$. For type III von Neumann algebras, which are intimately related to the physics of systems with an infinite number of degrees of freedom[8], this prescription provides a class of dynamical evolutions which differ only locally from each other. It is in this sense that $\mathcal{M}$ should be regarded by itself as a dynamical object. After understanding the theoretical and mathematical details of this construction, we will aim to present in great detail physically meaningful examples. We hope that this exercise will shed light into the physical meaning of Connes' mathematical proposal and guide research towards more concrete formulations.

Resumen

Las álgebras de operadores, como marcos generales para la descripción de observables en sistemas físicos, han sido una herramienta importante para investigaciones rigurosas en teoría de la información, física

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estadística y teoría cuántica de campos[1, 2, 3, 4]. En este proyecto vamos a familiarizarnos con estas técnicas y exploraremos la propuesta de Connes sobre la emergencia del tiempo como un fenómeno cuántico[5]. Partiendo de [6], empezaremos estudiando una generalización del teorema de Radon-Nỳkodym para álgebras de von Neumann[7]. Este resultado central presentará un mapa dinámico canónico $\mathbb{R} \to \mathrm{Out}(\mathcal{M})$ debido a la no-conmutatividad del álgebra de von Neumann $\mathcal{M}$. Para álgebras de von Neumann tipo III, las cuales están intimamente relacionadas con la física de sistemas con un número infinito de grados de libertad[8], esta prescripción provee una clase de evoluciones dinámicas que solo difieren localmente entre si. Es en este sentido que $\mathcal{M}$ debe de ser entendido como un objeto dinámico por si mismo. Después de entender los detalles teóricos y matemáticos de esta construcción, pretenderemos mostrar en detalle ejemplos de importancia física. Esperamos que este ejercicio esclarezca el significado físico de la propuesta matemática de Connes y guíe la investigación hacía formulaciones más concretas.

1. State of the Art

2. Objectives

3. Theoretical Framework

In quantum mechanics observables are identified with operators on a separable Hilbert space $\mathcal{H}$. The spectrum of such an operator is then interpreted as the possible set of values the observation can yield. Therefore, the operators representing observables must be selfadjoint. Given that real observations are represented by concrete measurement devices whose range of outcomes is always bounded, we conclude that the observables must be in the bounded operators $\mathcal{B}(\mathcal{H})$[1]. The following mathematical structures implement these requirements and are thus candidate structures for the algebra of observables of physical systems.

Definition 3.1 *A C^*-algebra $\mathcal{A}$ is a Banach *-algebra which satisfies the C^*-condition, that is, for every $a \in \mathcal{A}$ we have*

$$\|a^*a\| = \|a\|^2. \tag{1}$$

A von Neumann algebra (W^*-algebra for short) $\mathcal{M}$ is a C^*-algebra for which there exists a predual, that is, a Banach space $\mathcal{M}_*$ such that $\mathcal{M} = (\mathcal{M}_*)^*$ as Banach spaces.

[1] One can alternatively claim that, in light of the spectral theorem[9], every unbounded operator can be understood in terms of its spectral projections. These are in particular bounded operators. This point of view corresponds to adopting the position that every measurement can be ultimately broken down into a (possibly uncountably infinite) set of propositions.

On the other hand, physical states, which constitute the physical information required to evaluate the expected value of observables, can aslo be defined in this setting.

Definition 3.2 *Let $\mathcal{A}$ be a C^*-algebra. A state on $\mathcal{A}$ is a normalized positive linear functional $\omega : \mathcal{A} \to \mathbb{C}$. If $\mathcal{M}$ is a von Neumann algebra, a state ω on $\mathcal{M}$ is said to be normal if $\omega \in \mathcal{M}_*$ under the canonical embedding $\mathcal{M}_* \subseteq \mathcal{M}^*$.*

Note that in the context of von Neumann algebras and through the restriction to normal states, the structure of observables and states is complementary. This is evidence of the fact that each of them is meaningless without the other and, in fact, determines it. Finally, we can also consider dynamical evolutions in this framework. For definiteness we will work in the Heisenberg picture.

Definition 3.3 *A dynamical evolution on a C^*-algebra $\mathcal{A}$ is a strongly continuous representation $\alpha : \mathbb{R} \to \operatorname{Aut}\mathcal{A}$. A dynamical evolution on a W^*-algebra $\mathcal{M}$ is a weakly continuous representation $\alpha : \mathbb{R} \to \operatorname{Aut}\mathcal{M}$.*

The relationship between this framework and the one usually used in classical and quantum formulations is given by the Gelfand-Naimark theorem and its commutative version[**?**], Sakai's theorem on the relation between abstract and concrete W^*-algebras[**?**], its commutative analogue, Riez theorem[**?**, **?**, 10], Gleason's theorem[**?**].

4. Methodology

5. Timetable

6. Expected Results

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