Emerging Gauge Symmetries and Quantum Operations

I.M. Burbano[1]*
Joint work with: A.P. Balachandran[2] A.F. Reyes-Lega[1] S. Tabban[1]

[1]Departamento de Física, Universidad de los Andes, A.A. 4976-12340, Bogotá, Colombia

[2]Department of Physics, Syracuse University, Syracuse, New York 13244-1130, USA

February 15, 2019

*Contact: ivanmbur@gmail.com
Currently at Instituto de Física Teórica, UNESP.

Motivation

- The original motivation was to study entanglement entropy in systems which are not a tensor product of its subsystems.

Preliminary work: Balachandran, et al. 2013

Motivation

- The original motivation was to study entanglement entropy in systems which are not a tensor product of its subsystems.
- The algebraic approach leads to a satisfactory notion of restriction of states to such subsystems. However, how do we define the entropy of algebraic states?

Preliminary work: Balachandran, et al. 2013

Building Blocks

Physical systems are described by:

1. observables a,
2. states ω, and
3. a pairing $\omega(a)$ which describes the expectation value of a in the state ω.

Building Blocks

Physical systems are described by:

1. observables a,
2. states ω, and
3. a pairing $\omega(a)$ which describes the expectation value of a in the state ω.

In the algebraic formulation (see Strocchi 2008):

1. observables are the selfadjoint elements of a von Neumann algebra $\mathcal{A}$ (we will consider $n \times n$ complex matrices),
2. states are the positive normalized linear functionals $\omega : \mathcal{A} \to \mathbb{C}$ (we will consider faithful states), and
3. the expectation value of an observable $a = a^\dagger \in \mathcal{A}$ in the state ω is given by $\omega(a)$

Purification **C** and the Emergence of an Environment

1. Hilbert space $\mathcal{H} = \mathcal{A}$ with $\langle a|b\rangle = \omega(a^\dagger b)$.

Purification **C** and the Emergence of an Environment

1. Hilbert space $\mathcal{H} = \mathcal{A}$ with $\langle a|b\rangle = \omega(a^{\dagger} b)$.
2. Algebra $\mathcal{F} = \mathfrak{B}(\mathcal{H})$.

Purification **C** and the Emergence of an Environment

1. Hilbert space $\mathcal{H} = \mathcal{A}$ with $\langle a|b\rangle = \omega(a^\dagger b)$.
2. Algebra $\mathcal{F} = \mathfrak{B}(\mathcal{H})$.
3. Representation $\pi(a)\,|b\rangle = |ab\rangle$.

Purification C and the Emergence of an Environment

1. Hilbert space $\mathcal{H} = \mathcal{A}$ with $\langle a|b\rangle = \omega(a^\dagger b)$.
2. Algebra $\mathcal{F} = \mathfrak{B}(\mathcal{H})$.
3. Representation $\pi(a)\,|b\rangle = |ab\rangle$.
4. Notice that if $|\Omega\rangle = |\mathbb{1}_\mathcal{A}\rangle$ then

$$\langle\Omega|\pi(a)|\Omega\rangle = \langle\mathbb{1}_\mathcal{A}|\pi(a)|\mathbb{1}_\mathcal{A}\rangle = \langle\mathbb{1}_\mathcal{A}|a\rangle = \omega(a). \tag{1}$$

Purification C and the Emergence of an Environment

1. Hilbert space $\mathcal{H} = \mathcal{A}$ with $\langle a|b\rangle = \omega(a^\dagger b)$.
2. Algebra $\mathcal{F} = \mathfrak{B}(\mathcal{H})$.
3. Representation $\pi(a)\,|b\rangle = |ab\rangle$.
4. Notice that if $|\Omega\rangle = |\mathbb{1}_{\mathcal{A}}\rangle$ then

$$\langle\Omega|\pi(a)|\Omega\rangle = \langle\mathbb{1}_{\mathcal{A}}|\pi(a)|\mathbb{1}_{\mathcal{A}}\rangle = \langle\mathbb{1}_{\mathcal{A}}|a\rangle = \omega(a). \tag{1}$$

5. A new system **B** emerges whose observable algebra is $\pi(\mathcal{A})'$ the set of all operators in $\mathcal{F}$ which commute with $\pi(\mathcal{A})$.

Purification C and the Emergence of an Environment

1. Hilbert space $\mathcal{H} = \mathcal{A}$ with $\langle a|b\rangle = \omega(a^\dagger b)$.
2. Algebra $\mathcal{F} = \mathfrak{B}(\mathcal{H})$.
3. Representation $\pi(a)\,|b\rangle = |ab\rangle$.
4. Notice that if $|\Omega\rangle = |\mathbb{1}_{\mathcal{A}}\rangle$ then

$$\langle\Omega|\pi(a)|\Omega\rangle = \langle\mathbb{1}_{\mathcal{A}}|\pi(a)|\mathbb{1}_{\mathcal{A}}\rangle = \langle\mathbb{1}_{\mathcal{A}}|a\rangle = \omega(a). \tag{1}$$

5. A new system **B** emerges whose observable algebra is $\pi(\mathcal{A})'$ the set of all operators in $\mathcal{F}$ which commute with $\pi(\mathcal{A})$.
6. "$\mathbf{A} + \mathbf{B} = \mathbf{C}$" in the sense that the smallest algebra containing both $\pi(\mathcal{A})$ and $\pi(\mathcal{A})'$ is $\mathcal{F}$.

Tomita-Takesaki and Gauge Group

Tomita-Takesaki Theory allows us to construct an antiunitary operator J on $\mathcal{H}$ such that $J\pi(a)J \in \pi(\mathcal{A})'$ for all $a \in \mathcal{A}$. Consider the group $G = U(\mathcal{A})$ of unitary elements in $\mathcal{A}$. We then have an action of G on $\mathcal{H}$ via $U(g) = J\pi(g)J \in \pi(\mathcal{A})'$. G can be interpreted as a gauge group in the sense of (see Doplicher, Haag, and Roberts 1969)

$$\begin{array}{ccccc} \mathcal{A} & = & \mathcal{F} & \cap & U(G)' \\ \downarrow & \downarrow & \downarrow & & \downarrow \\ \text{Observables} & \text{are} & \text{fields} & & \text{unaffected by } G \end{array}$$

Representation Theory

Consider an orthogonal complete set of projections $P^{(\alpha)} \in \pi(\mathcal{A})'$. Via

$$P_g^{(\alpha)} := U(g)P^{(\alpha)}U(g^\dagger) \tag{2}$$

we get a G-dependent family of such projections.
Obtaining such sets is equivalent to decomposing the GNS representation into subrepresentations

$$\mathcal{H} = \bigoplus_\alpha P_g^{(\alpha)}\mathcal{H}. \tag{3}$$

Quantum Operation

This induces a quantum operation

$$\mathcal{E}_g(\rho) = \sum_{\alpha} P_g^{(\alpha)} \rho P_g^{(\alpha)}, \tag{4}$$

s.t.

$$\mathrm{tr}(\rho\pi(a)) = \mathrm{tr}(\mathcal{E}_g(\rho)\pi(a)) \tag{5}$$

for all observables $a \in \mathcal{A}$ in **A**.

1. $S_{\mathrm{vN}}(\mathcal{E}_g(\rho)) \geq S_{\mathrm{vN}}(\rho)$.
2. If ρ describes ω (i.e. $\rho = |\Omega\rangle\langle\Omega|$) so does $\mathcal{E}_g(\rho)$.

References I

Aiyalam Parameswaran Balachandran, Amilcar Rabelo de Queiroz, and Sachindeo Vaidya. "Entropy of quantum states: Ambiguities". In: *European Physical Journal Plus* 128.10 (2013). ISSN: 21905444. DOI: 10.1140/epjp/i2013-13112-3.

Aiyalam Parameswaran Balachandran, Thupil R. Govindarajan, et al. "Algebraic Approach to Entanglement and Entropy". In: *Physical Review A* 88.2 (2013), p. 022301. ISSN: 1050-2947. URL: http://link.aps.org/doi/10.1103/PhysRevA.88.022301.

Aiyalam Parameswaran Balachandran, Thupil R. Govindarajan, et al. "Entanglement and particle identity: A unifying approach". In: *Physical Review Letters* 110.8 (2013). ISSN: 00319007. DOI: 10.1103/PhysRevLett.110.080503.

References II

Aiyalam Parameswaran Balachandran, Amilcar Rabelo de Queiroz, and Sachindeo Vaidya. "Quantum entropic ambiguities: Ethylene". In: *Physical Review D - Particles, Fields, Gravitation and Cosmology* 88.2 (2013). ISSN: 15507998. DOI: 10.1103/PhysRevD.88.025001.

Franco Strocchi. *An Introduction to the Mathematical Structure of Quantum Mechanics: A Short Course for Mathematicians*. 2nd Editio. Singapore: World Scientific, 2008. ISBN: 981-283-522-9.

Sergio Doplicher, Rudolf Haag, and John E. Roberts. "Fields, Observables and Gauge Transformations I". In: *Communications in Mathematical Physics* 15.3 (1969), pp. 173–200. ISSN: 00103616. DOI: 10.1007/BF01645267.

Thanks!