Iván Mauricio Burbano Aldana

Prof. Nathan Berkovits

Instituto de Física Teórica, UNESP

Supersymmetry

Homework 10

We want to diagonalize the quadratic form

$$\frac{M}{2}\psi_3\psi_3 + \sqrt{\mu}\,\psi_2\psi_3 + \sqrt{\mu}\,\psi_1\psi_3 .$$

In order to do this, we note that

$$\sqrt{\mu}\,\psi_2\psi_3 + \sqrt{\mu}\,\psi_1\psi_3 = \sqrt{\mu}\,(\psi_1+\psi_2)\psi_3 .$$

We are now in a similar position to equation (9.19). Thus we note that

$$\frac{\sqrt{\mu}}{2}\left(\frac{(\psi_1+\psi_2+\psi_3)}{\sqrt{2}}\frac{(\psi_1+\psi_2+\psi_3)}{\sqrt{2}} + \frac{i(\psi_1+\psi_2-\psi_3)}{\sqrt{2}}\frac{i(\psi_1+\psi_2-\psi_3)}{\sqrt{2}}\right)$$

$$= \frac{\sqrt{\mu}}{2}\Big(\frac{1}{2}\big(\cancel{\psi_1\psi_1} + \cancel{\psi_1\psi_2} + \psi_1\psi_3 + \cancel{\psi_2\psi_1} + \cancel{\psi_2\psi_2} + \psi_2\psi_3 + \psi_3\psi_1 + \psi_3\psi_2 + \cancel{\psi_3\psi_3}$$

$$- \cancel{\psi_1\psi_1} - \cancel{\psi_1\psi_2} + \psi_1\psi_3 - \cancel{\psi_2\psi_1} - \cancel{\psi_2\psi_2} + \psi_2\psi_3 + \psi_3\psi_1 + \psi_3\psi_2 - \cancel{\psi_3\psi_3}\big)\Big)$$

$$= \frac{\sqrt{\mu}}{4}\left(2\psi_1\psi_3 + 2\psi_2\psi_3 + 2\psi_3\psi_1 + 2\psi_3\psi_2\right) = \sqrt{\mu}\,(\psi_1\psi_3 + \psi_2\psi_3).$$

In here we used the fact that $\chi\xi = \chi^\alpha\xi_\alpha = -\xi_\alpha\chi^\alpha = \xi^\alpha\chi_\alpha = \xi\chi$ for all spinors χ and ξ. Thus, our initial quadratic form is equal to

$$\frac{M}{2}\psi_3\psi_3 + \frac{\sqrt{\mu}}{2}\frac{(\psi_1+\psi_2+\psi_3)}{\sqrt{2}}\frac{(\psi_1+\psi_2+\psi_3)}{\sqrt{2}} + \frac{\sqrt{\mu}}{2}\frac{i(\psi_1+\psi_2-\psi_3)}{\sqrt{2}}\frac{i(\psi_1+\psi_2-\psi_3)}{\sqrt{2}} .$$