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Quantum Field Theory II

Homework 2

1.a) We have that the numerator on the r.h.s. of (2) has the perturbative expansion

$$\int \mathcal{D}\phi \exp\left(-\frac{1}{\hbar}S_0(\phi)\right)\exp\left(-\frac{g}{3!\hbar}\int d^dx\, \phi(x)^3\right)\phi(x_1)\cdots\phi(x_N)$$

$$=\sum_{n=0}^{\infty}\frac{1}{n!}\left(-\frac{g}{3!\hbar}\right)^n\int\prod_{i=1}^n d^dz_i\int\mathcal{D}\phi\exp\left(-\frac{1}{\hbar}S_0(\phi)\right)\prod_{i=1}^n\phi(z_i)^3\prod_{i=1}^N\phi(x_i)$$

$$=\sum_{n=0}^{\infty}\left(-\frac{g}{\hbar}\right)^n\frac{1}{3!^n n!}\int\prod_{i=1}^n d^dz_i\left\langle\prod_{i=1}^n\phi(z_i)^3\prod_{i=1}^N\phi(x_i)\right\rangle_0 Z_0,$$

where $S_0(\phi)=\int d^dx\left(\frac{1}{2}(\nabla\phi(x))^2+\frac{1}{2}m^2\phi(x)^2\right),$

$$Z_0 = \int D\phi \exp\left(-\frac{1}{\hbar} S_0(\phi)\right), \quad \text{and}$$

$$\langle A \rangle_0 = \frac{1}{Z_0} \int D\phi \exp\left(-\frac{1}{\hbar} S_0(\phi)\right) A(\phi).$$

Then, we see there is a diagrammatic expansion

$$\langle \phi(x_1) \cdots \phi(x_N) \rangle = \sum_{\substack{\text{Diagrams } G \text{ with} \\ N \text{ external vertices} \\ \text{without vacuum subdiagrams}}} c(G)$$

where $c(G)$ is calculated as follows

1. 

$= \hbar G_0(x-y)$

2. 

$-\frac{g}{\hbar} \int d^d x$

3. Divide by the symmetry factor.

Remarks: • The vacuum subdiagrams are cancelled by the denominator, as can be seen from our perturbative expansion with $N=1$.

• The symmetry factors come from the factor $\frac{1}{3!^n n!}$, the relabelling of the internal vertices $z_1, \dots, z_N$ and the permutation of the wick contractions within each group $\phi(z_i)^3$.

b) We have $\langle \phi(x)\phi(y) \rangle$ is,

$O(g^0)$:

*  $= \hbar G_0(x-y)$,

$O(g^1)$: None, since $\langle \phi(x)\phi(y)\phi(z_1)^3 \rangle$ has an odd number of fields,

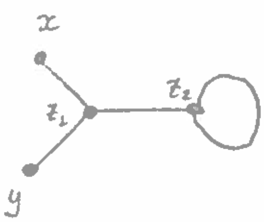
$\mathcal{O}(g^2)$

* 

$$= \frac{1}{2}\left(-\frac{g}{\hbar}\right)^2 \int d^d z_1 d^d z_2 \; \hbar G(x-z_1)\, \hbar G(z_1-z_2) \times$$

$$\hbar G(z_1-z_2)\, \hbar G(z_2-y)$$

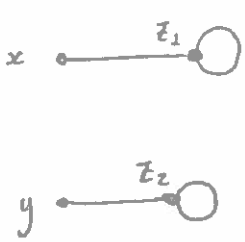
$$= \frac{1}{2} g^2 \hbar^2 \int d^d z_1 d^d z_2 \; G(x-z_1)\, G(z_1-z_2)^2\, G(z_2-y),$$

* 

$$= \frac{1}{2}\left(-\frac{g}{\hbar}\right)^2 \int d^d z_1 d^d z_2 \; \hbar G(x-z_1)\, \hbar G(y-z_1) \times$$

$$\hbar G(z_1-z_2)\; \hbar G(z_2-z_2)$$

$$= \frac{1}{2} g^2 \hbar^2 \int d^d z_1 d^d z_2 \; G(x-z_1) G(y-z_1) G(z_1-z_2) G(0),$$

* 

$$= \frac{1}{4}\left(-\frac{g}{\hbar}\right)^2 \int d^d z_1\, d^d z_2 \; \hbar G(x-z_1)\, \hbar G(z_1-z_1) \times$$

$$\hbar G(y-z_2)\, \hbar G(z_2-z_2)$$

$$= \frac{1}{4} g^2 \hbar^2 \int d^d z_1\, G(x-z_1) \int d^d z_2\, G(y-z_2)\; G(0)^2.$$