Quantum Field Theory I: Quiz 10

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Notice that the Dirac operator is in a certain sense the square root of the Klein-Gordon operator

$$
\begin{aligned}
(i\slashed{\partial} - m)(i\slashed{\partial} + m) =& - \slashed{\partial}^2 - m^2 + i\slashed{\partial}m - i\slashed{\partial}m = -\gamma^\mu \gamma^\nu \partial_\mu \partial_\nu - m^2 \\
=& - \gamma^\mu \gamma^\nu \frac{1}{2}(\partial_\mu \partial_\nu + \partial_\nu \partial_\mu) - m^2 \\
=& - \frac{1}{2}(\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu)\partial_\mu \partial_\nu - m^2 = -\frac{1}{2}2\eta^{\mu\nu}\partial_\mu \partial_\nu - m^2 \\
=& - (\partial^2 + m^2).
\end{aligned} \tag{1}
$$

Thus, we have

$$
\begin{aligned}
(i\slashed{\partial}_x - m)S_F(x - y) =& (i\slashed{\partial} - m)(i\slashed{\partial} + m)\Delta_F(x - y) \\
=& - (\partial_x^2 + m^2)\Delta_F(x - y) = i\delta(x - y).
\end{aligned} \tag{2}
$$