Quantum Field Theory I: Quiz 10

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Considering the Lagrangian $\mathcal{L}=\overline{\psi}(i\partial\!\!\!/-m)\psi=\overline{\psi}_b(i\gamma^{\mu b}{}_a\partial_\mu-m\delta^b{}_a)\psi^a$ we have

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\psi^a}=&-m\overline{\psi}_a,\\ \frac{\partial\mathcal{L}}{\partial\partial_\mu\psi^a}=&i\overline{\psi}_b\gamma^{\mu b}{}_a.\end{aligned}\tag{1}$$

Thus the Euler-Lagrange equations are

$$0=\partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\psi^a}-\frac{\partial\mathcal{L}}{\partial\psi^a}=i\partial_\mu\overline{\psi}_b\gamma^{\mu b}{}_a+m\overline{\psi}_a=\overline{\psi}_b(i\overleftarrow{\partial}_\mu\gamma^{\mu b}{}_a+m\delta^b{}_a).\tag{2}$$

Supressing the Dirac indices we obtain the more familiar form

$$\overline{\psi}(i\overleftarrow{\partial\!\!\!/}+m)\equiv\overline{\psi}(i\overleftarrow{\partial}_\mu\gamma^\mu+m)=0.\tag{3}$$