EXAMEN PARCIAL 2 DE FÍSICA DE PARTÍCULAS – FISI_3152

NOMBRE: Solución detallada (en exceso) código: 201720 NOTA: ______ /20

1. Demuestre, para transformaciones de Lorentz en general incluyendo rotaciones propias e impropias, que la cantidad $\overline{\psi}\gamma^5\gamma^\mu\psi$ es un pseudo-4-vector o, simplemente, un vector axial.
2. A partir de la ecuación de Dirac obtenga la ecuación adjunta respectiva. ¿Es posible construir una corriente de probabilidad, donde la densidad de probabilidad es positiva definida, la probabilidad total es constante y normalizable? ¿Hay posibilidad de una interpretación alterna?
3. Evalúe $Tr(\gamma^5\gamma^\mu\gamma^\nu)$.
4. Evalúe $Tr[\gamma^\mu(\not{p}_1 + mc)\gamma^\nu(\not{p}_3 + mc)]$.

Problema similar al tratado en clase 14/11/2017: $\overline{\psi}\gamma^\mu\psi$

SOLUCIÓN:

1. $\overline{\psi}\gamma^5\gamma^\mu\psi \xrightarrow{S(\Lambda)} \overline{\psi}'\gamma^5\gamma^\mu\psi' = \psi'^\dagger\gamma^0\gamma^5\gamma^\mu\psi' = (S\psi)^\dagger\gamma^0\gamma^5\gamma^\mu S\psi = \psi^\dagger S^\dagger\gamma^0\gamma^5\gamma^\mu S\psi$

$= \psi^\dagger\gamma^0\gamma^0 S^\dagger\gamma^0\gamma^5\gamma^\mu S\psi = \overline{\psi}S^{-1}\gamma^5\gamma^\mu S\psi$

$\{\gamma^5,\gamma^\mu\} = 0 \Rightarrow S^{-1}\gamma^5 = \gamma^5 S^{-1} \Rightarrow \overline{\psi}'\gamma^5\gamma^\mu\psi' = \overline{\psi}\gamma^5 S^{-1}\gamma^\mu S\psi = \Lambda^\mu{}_\nu\overline{\psi}\gamma^5\gamma^\nu\psi$

$\rightarrow$ 4-vector bajo transformaciones propias de Lorentz (subgrupo propio ortocrono)

Respecto a las transformaciones de Paridad:

$\overline{\psi}\gamma^5\gamma^\mu\psi \xrightarrow{P} \overline{\psi}'\gamma^5\gamma^\mu\psi' = \psi'^\dagger\gamma^0\gamma^5\gamma^\mu\psi' = (P\psi)^\dagger\gamma^0\gamma^5\gamma^\mu P\psi = (\gamma^0\psi)^\dagger\gamma^0\gamma^5\gamma^\mu\gamma^0\psi$

$\overset{\gamma^{0\dagger}=\gamma^0}{=} \psi^\dagger\gamma^0\gamma^0\gamma^5\gamma^\mu\gamma^0\psi = -\overline{\psi}\gamma^5\gamma^0\gamma^\mu\gamma^0\psi = -\overline{\psi}\gamma^5\gamma^{\mu\dagger}\psi$

$= \begin{cases} -\overline{\psi}\gamma^5\gamma^\mu\psi & \mu = 0 \\ +\overline{\psi}\gamma^5\gamma^\mu\psi & \mu = i \end{cases} \rightarrow$ Vector axial

2. $(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0 \Rightarrow \overline{\psi}(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0$ (1)

$\overline{(i\hbar\gamma^\mu\partial_\mu - mc)\psi} = \overline{0} \Rightarrow (i\hbar\gamma^\mu\partial_\mu\psi - mc\psi)^\dagger\gamma^0 = 0$

$\Rightarrow (-i\hbar\partial_\mu\psi^\dagger\gamma^{\mu\dagger} - mc\psi^\dagger)\gamma^0 = -i\hbar\partial_\mu\psi^\dagger\gamma^0\gamma^0\gamma^{\mu\dagger}\gamma^0 - mc\overline{\psi} = 0$

obtenido en una clase

$\Rightarrow \boxed{i\hbar\partial_\mu\overline{\psi}\gamma^\mu + mc\overline{\psi} = 0}$ ✓ $i\hbar(\partial_\mu\overline{\psi})\gamma^\mu\psi + mc\overline{\psi}\psi = 0$ (2)

(1) + (2) $\Rightarrow i\hbar[\overline{\psi}\gamma^\mu\partial_\mu\psi + (\partial_\mu\overline{\psi})\gamma^\mu\psi] = 0 \Rightarrow \boxed{\partial_\mu(\overline{\psi}\gamma^\mu\psi) = 0}$

$J^\mu = J^\mu(x) \equiv \overline{\psi}\gamma^\mu\psi \Rightarrow \boxed{\partial_\mu J^\mu = 0}$ J^μ: corriente conservada.

$\Rightarrow \frac{\partial}{\partial x^0}J^0 + \partial_i J^i = \frac{1}{c}\frac{\partial J^0}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0 \rightarrow \int_V \frac{1}{c}\frac{\partial J^0}{\partial t}dV = -\int_V \vec{\nabla}\cdot\vec{J}dV = \oint_{S\to\infty}\vec{J}\cdot d\vec{S} = 0$

$\Rightarrow \frac{d}{dt}\int_V J^0 dV = 0 \Rightarrow \int_V J^0(x)dV = \text{const.}$ (3) ✓ $\boxed{J^0(x) = \overline{\psi}\gamma^0\psi = \psi^\dagger\psi}$

$J^0 \rightarrow$ densidad de probabilidad definida-positiva ✓, probabilidad total (3) constante y por lo tanto normalizable ✓ $\longleftrightarrow J^\mu(x)$: corriente de probabilidad. ✓

Interpretación alterna: $q\overline{\psi}\gamma^\mu\psi \rightarrow$ corriente cargada. ✓

3. Evaluar $Tr(\gamma^5\gamma^\mu\gamma^\nu)$ [Parte de la tarea 10]

Solución:

caso $\mu=\nu \Rightarrow \gamma^\mu\gamma^\nu = \gamma^{\mu 2} = \pm \mathbb{1} \Rightarrow Tr(\gamma^5\gamma^\mu\gamma^\nu) = \pm Tr(\gamma^5) = 0$ en la rep. de Pauli-Dirac de γ^5 (ec. 7.64). En general siempre es válido. En efecto: $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\gamma^3\gamma^0\gamma^1\gamma^2$ (anticonmutación)

$\Rightarrow Tr(\gamma^5) = Tr(i\gamma^0\gamma^1\gamma^2\gamma^3) = i\,Tr(\gamma^0\gamma^1\gamma^2\gamma^3) = -i\,Tr(\gamma^3\gamma^0\gamma^1\gamma^2)$

$= -i\,Tr(\gamma^0\gamma^1\gamma^2\gamma^3)$ por la propiedad de ciclicidad de la traza.

$= -Tr(\gamma^5) \Rightarrow \boxed{Tr(\gamma^5) = 0 \text{ y } Tr(\gamma^5\gamma^\mu\gamma^\nu) = 0 \text{ para } \mu=\nu}$

caso $\mu\neq\nu$: $\mu\neq\nu = 0,1,2$ ó $3 \Rightarrow \gamma^5\gamma^\mu\gamma^\nu = \pm\gamma^\alpha\gamma^\beta$ $\begin{cases}\alpha\neq\beta\\ \alpha \text{ y } \beta \neq \mu \text{ y } \nu\end{cases}$

$\Rightarrow Tr(\gamma^5\gamma^\mu\gamma^\nu) = \pm Tr(\gamma^\alpha\gamma^\beta) = \mp Tr(\gamma^\beta\gamma^\alpha)$ anticonmutación

$= \mp Tr(\gamma^\alpha\gamma^\beta)$ propiedad cíclica de la traza

$\Rightarrow Tr(\gamma^\alpha\gamma^\beta) = -Tr(\gamma^\alpha\gamma^\beta) = 0 \Leftrightarrow Tr(\gamma^5\gamma^\mu\gamma^\nu) = 0$ para $\mu\neq\nu$.

En síntesis: $\boxed{Tr(\gamma^5\gamma^\mu\gamma^\nu) = 0}$ un teorema de las trazas de productos de γ's, q.e.d.

4. Evaluar $Tr[\gamma^\mu(\not{p}_1+mc)\gamma^\nu(\not{p}_3+mc)]$.

Ver parte de la solución pp 253-254 Example 7.6 del texto

Solución: $Tr[\quad \text{"} \quad] =$

$= Tr[\gamma^\mu\not{p}_1\gamma^\nu\not{p}_3 + \gamma^\mu\not{p}_1\gamma^\nu mc + \gamma^\mu mc\gamma^\nu\not{p}_3 + \gamma^\mu mc\gamma^\nu mc]$

$= Tr(\gamma^\mu\not{p}_1\gamma^\nu\not{p}_3) + mc\,Tr(\underbrace{\gamma^\mu\not{p}_1\gamma^\nu}_{\to 0} + \underbrace{\gamma^\mu\gamma^\nu\not{p}_3}_{\to 0}) + m^2c^2\,Tr(\gamma^\mu\gamma^\nu)$

$\to 0$ Traza de un producto de un número impar de γ's

$= p_{1\alpha}p_{3\beta}Tr(\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta) + m^2c^2\,Tr(2g^{\mu\nu} - \gamma^\nu\gamma^\mu)$ (1)

$Tr(\gamma^\mu\gamma^\nu) = \frac{1}{2}[Tr(\gamma^\mu\gamma^\nu) + Tr(\gamma^\mu\gamma^\nu)] = \frac{1}{2}[Tr(\gamma^\mu\gamma^\nu) + Tr(\gamma^\nu\gamma^\mu)] = \frac{1}{2}Tr(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu)$

$= \frac{1}{2}Tr(2g^{\mu\nu}) = Tr(g^{\mu\nu}\mathbb{1}) = g^{\mu\nu}Tr(\mathbb{1}) \Rightarrow \boxed{Tr(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}}$ (2)

$Tr(\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta) = Tr[(2g^{\mu\alpha} - \gamma^\alpha\gamma^\mu)\gamma^\nu\gamma^\beta] = 2g^{\mu\alpha}Tr(\gamma^\nu\gamma^\beta) - Tr[\gamma^\alpha(2g^{\mu\nu} - \gamma^\nu\gamma^\mu)\gamma^\beta]$

$= 8g^{\mu\alpha}g^{\nu\beta} - 2g^{\mu\nu}Tr(\gamma^\alpha\gamma^\beta) + Tr[\gamma^\alpha\gamma^\nu(2g^{\mu\beta} - \gamma^\beta\gamma^\mu)]$

$= 8g^{\mu\alpha}g^{\nu\beta} - 8g^{\mu\nu}g^{\alpha\beta} + 8g^{\mu\beta}g^{\alpha\nu} - Tr(\gamma^\alpha\gamma^\nu\gamma^\beta\gamma^\mu)$

$\Rightarrow 2Tr(\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta) = \underbrace{\qquad\qquad}_{\text{"}}$

$\Rightarrow Tr(\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta) = 4(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\nu}g^{\alpha\beta} + g^{\mu\beta}g^{\alpha\nu})$ (3)

(1), (2) y (3) $\Rightarrow Tr[\gamma^\mu(\not{p}_1+mc)\gamma^\nu(\not{p}_3+mc)] = 4p_{1\alpha}p_{3\beta}(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\nu}g^{\alpha\beta} + g^{\mu\beta}g^{\alpha\nu}) + m^2c^2(8g^{\mu\nu} - 4g^{\nu\mu})$

$\Rightarrow \boxed{Tr[\gamma^\mu(\not{p}_1+mc)\gamma^\nu(\not{p}_3+mc)] = 4[p_1^\mu p_3^\nu + p_1^\nu p_3^\mu + g^{\mu\nu}(m^2c^2 - p_1\cdot p_3)]}$