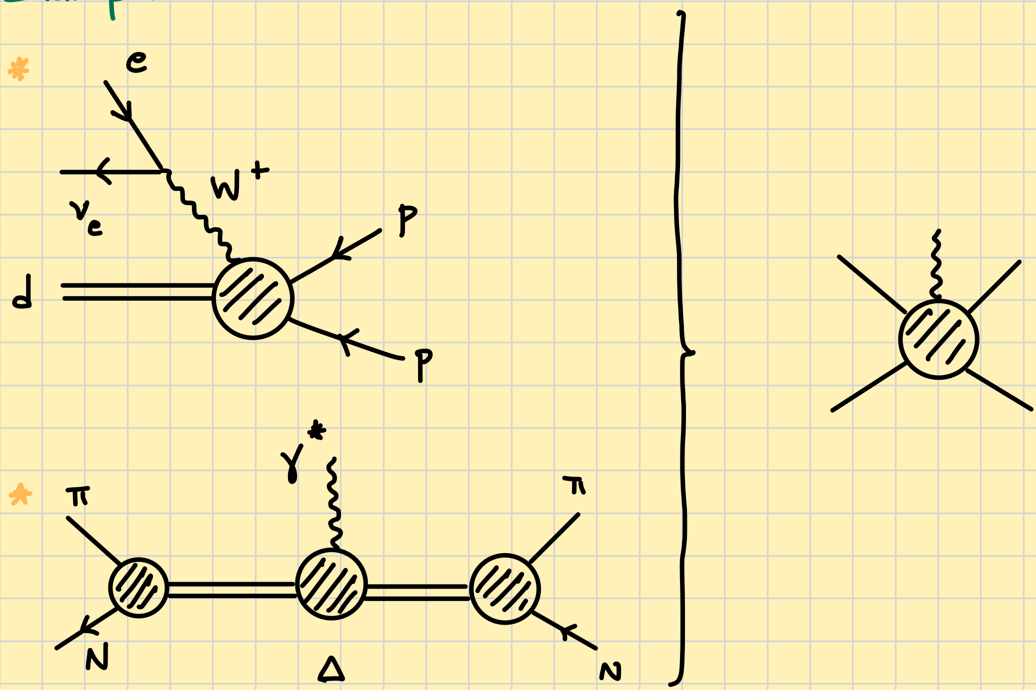
Efficient Truncations of $SU(N_c)$ LGT For QC (arXiv:2503.11888)

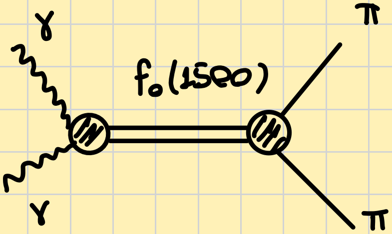
1. Why Quantum Computing for Nuclear Physics?

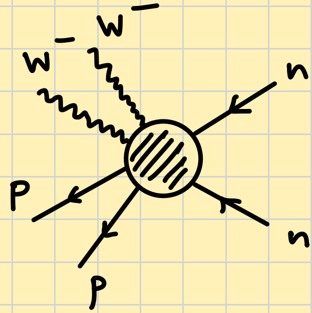
q.c.

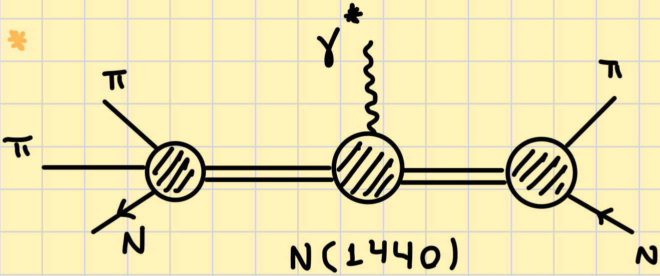
It is important to have theoretical predictions for Hadronic scattering

Examples:



* 

* 

* 

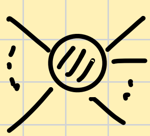
... but, QCD is non-perturbative!

↳ We need lattice QCD (i.e. computers)

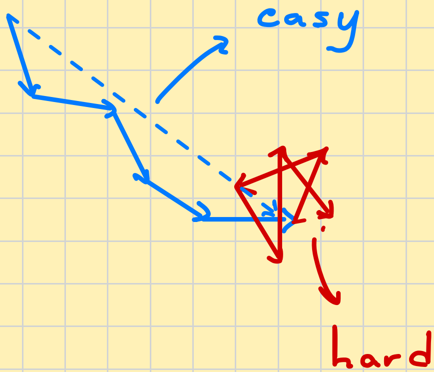
Sign Problem

Computing scattering processes inolves

$$\underbrace{\langle f | e^{-iHt} | i \rangle}_{\text{Quantum Computers!}} = f$$

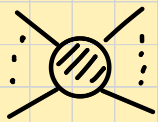


$$i = \underbrace{\cancel{\int D\phi \, e^{iS}}}_{\text{Traditional Computers}}$$



Caveat: Traditional computers are good at

$$\int D\phi\, e^{-S_E} \xRightarrow[\text{Lüscher-like formalism}]{}$$

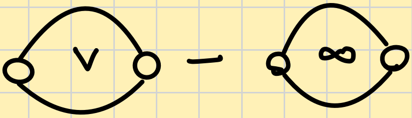


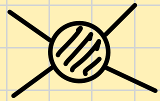
$$\downarrow$$

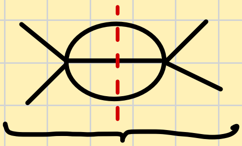
Gets harder as the processes get more involved

Example: 1+1D $2 \to 2$ scattering

$$\left.(F + M^{-1})\right|_{\text{Finite volume energies}} = 0$$





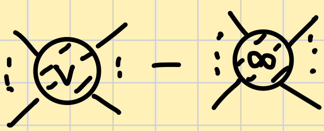
Key:  + ...

Below three particle threshold this is analytic

Publicity: As long as one has $i\varepsilon$ turned on

Fourier transform of real-time (i.e. q.c. correlations)

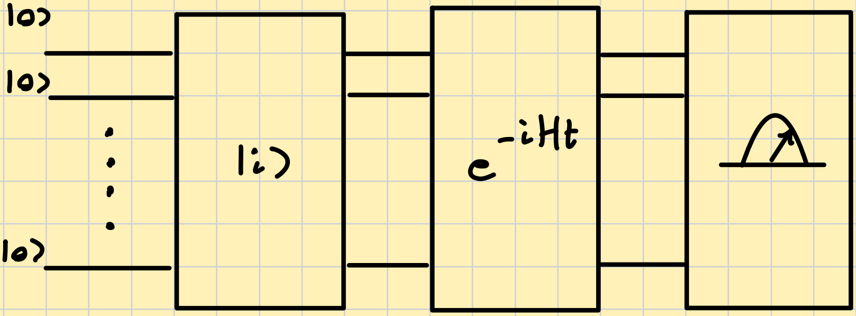
Finite volume effects:



$$= \begin{cases} e^{-mL} & \text{below threshold} \\ e^{-\varepsilon L} & \text{above thresholds} \end{cases}$$

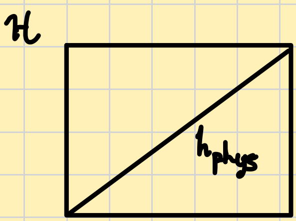
Coming in $\mathcal{O}(1)$ months w/ Raul, Marco, Anthony & Rana!

2. Key difficulties in QCD through q.c.'s



How to describe the Hilbert space of QCD as $|001\dots10100\rangle$?

- $\mathcal{H} \longrightarrow \mathcal{H}'$, $\dim \mathcal{H}' < \infty$
- $\mathcal{H} \supseteq h_{phys}$ satisfies Gauss' law

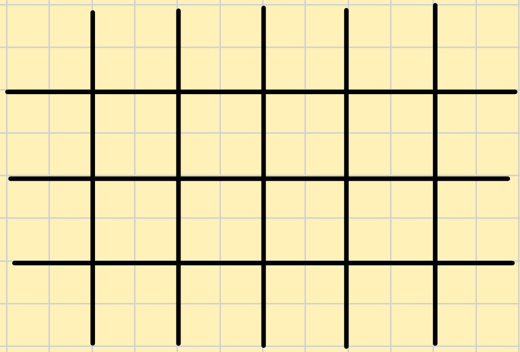


Tension!!!

How to set up time evolution efficiently ?

- Set up the interactions locally
- Good approximation to continuum physics

3. Kogut-Susskind and derived formulations

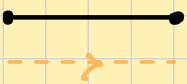


SPATIAL lattice

Hilbert Space

$$|\psi\rangle : u \mapsto \underbrace{\langle u|\psi\rangle}$$

Amplitude for link having a Wilson line u

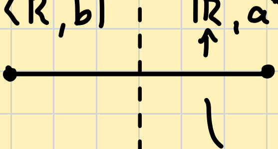


$u \in SU(3)$
$= P\exp \int_{\text{link}} A$

Continuous $\leftarrow$ $\mathcal{H} = L^2(SU(3))$

Electric Representation

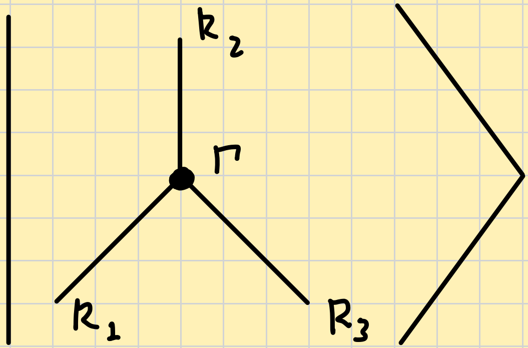
$\langle R, b|$ $|R, a\rangle$



$$\langle u|\psi\rangle := \langle R, b|R(u)|R, a\rangle$$

Irreducible Representation of $SU(3) \longrightarrow$ Discrete

Gauge invariant states



$$= \sum_{a_1, a_2, a_3} |R_1, a_1; R_2, a_2; R_3, a_3\rangle$$

$$\langle R_1, a_1; R_2, a_2; R_3, a_3 | 1, \Gamma \rangle$$

: linear combination that yields the Γ-th trivial irrep in $R_1 \otimes R_2 \otimes R_3$

Dynamics

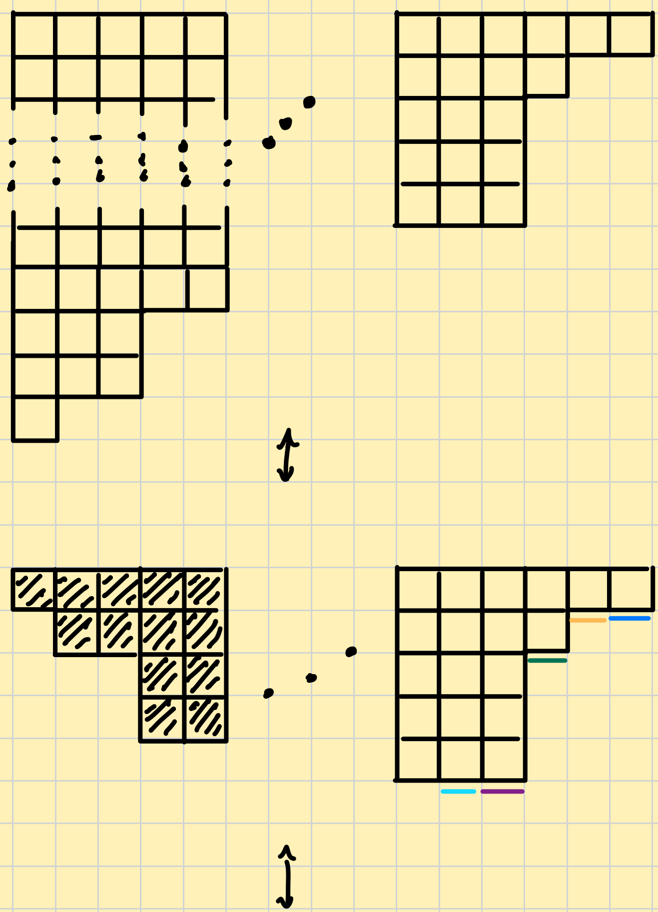
$$H = \frac{g^2}{2} \underbrace{\sum_{\ell} E_\ell^2}_{\text{links}} + \frac{1}{2g^2} \underbrace{\sum_{p} \mathrm{tr}(U)}_{\text{plaquettes}}$$

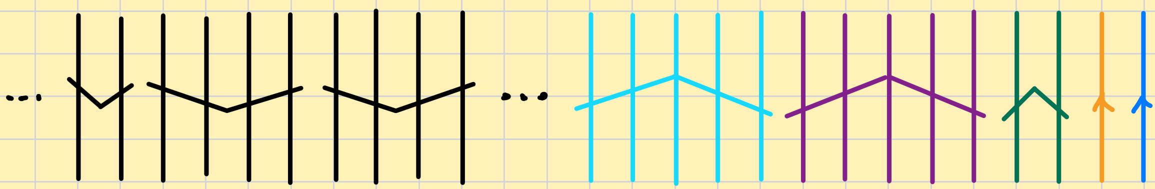
Diagonal in electric basis | Important near the continuum

4. Simplifications @ $SU(N \to \infty)$

Representation theory

Irreps $\longleftrightarrow$

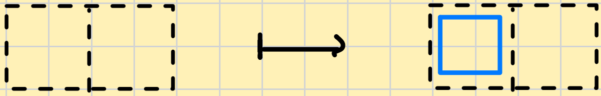




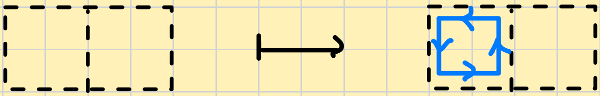
Allowed Transitions

4:

4.1.

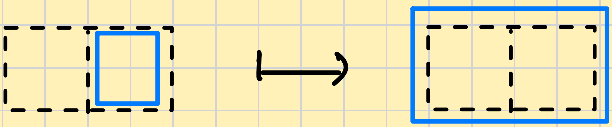


4.1.1.

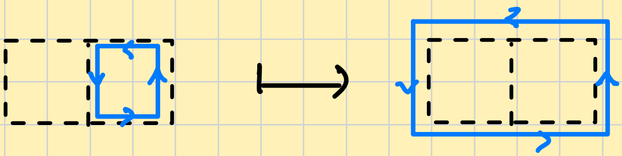


6.

6.1.

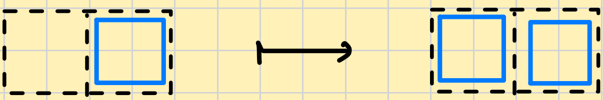


6.1.1.

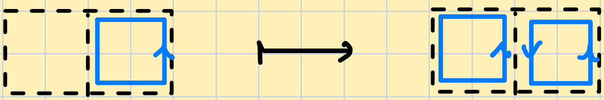


8

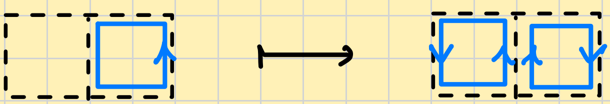
8.1.



8.1.1.



8.1.2.



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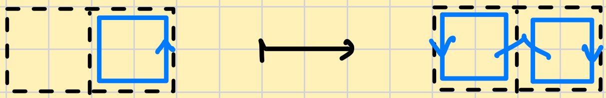
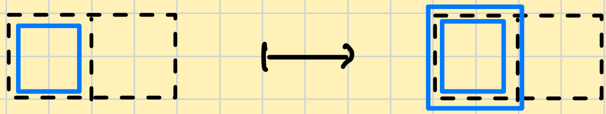


8.2.



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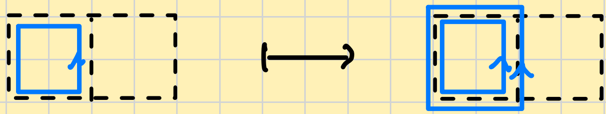

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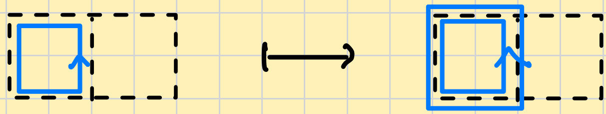

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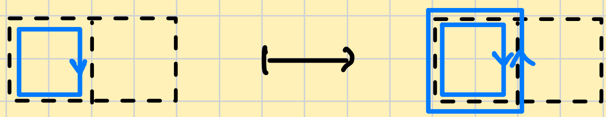

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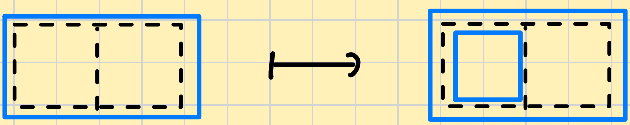
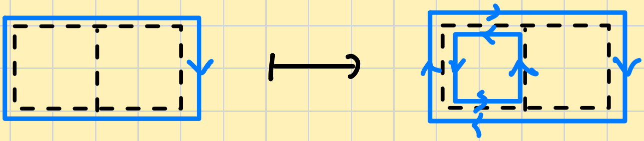
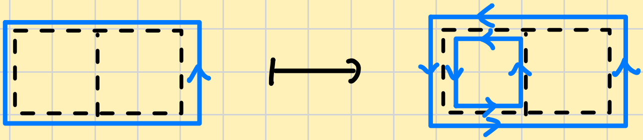
10.1.



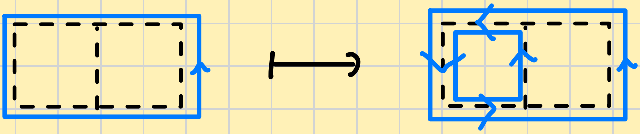
10.1.1



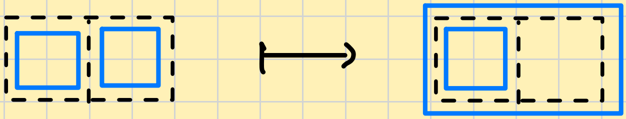
10.1.2



10.1.3



10.2



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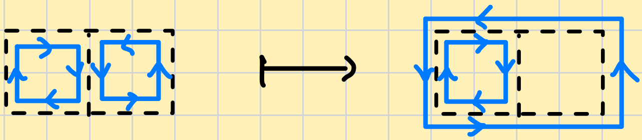

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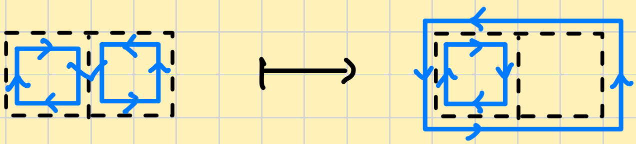

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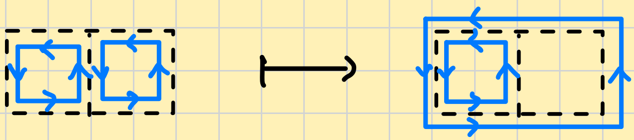

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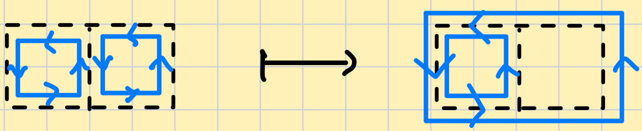

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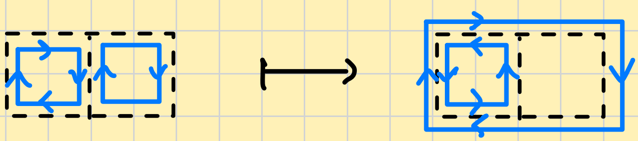

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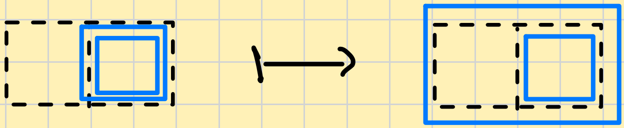

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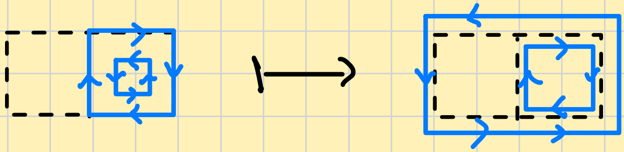
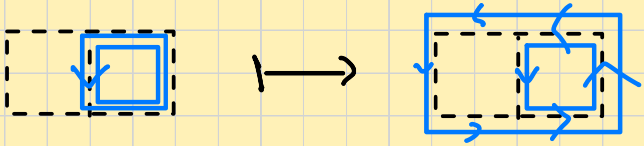


10.3.2



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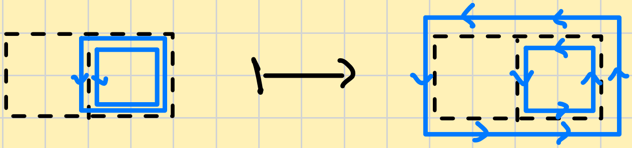

5. Is 3 close enough to ∞?

Mass of glueball in 2+1 D.