Probability paradoxes

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Introduction

The goal for the article is to demonstrate several paradoxes that are related to probability theory and how can they can be solved.

1 Base definitions of probability theory

I am going to provide several definitions. I will give the both formal and informal definitions and show how they are related each other.

We will start with the simplest example.

Example 1. *In the example we have (see fig. 1) $N = 5$ balls. There are $N_G = 2$ green balls and N_R red balls. I.e. $N = N_G + N_R$.*

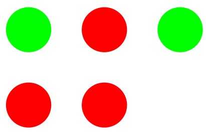

Figure 1: Probability example

We can define the probability to get green ball as

$$P_G = \frac{N_G}{N} = \frac{2}{5}$$

and the probability to get red ball as

$$P_R = \frac{N_R}{N} = \frac{3}{5}.$$

We can get only a red or a green ball and

$$P_G + P_R = 1.$$

TBD [1]

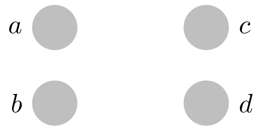

Figure 2: Probability space. It consists of elementary events: a, b, c and d, each of them has equal probability $p_{a,b,c,d} = \frac{1}{4}$

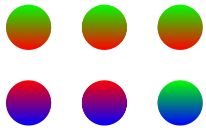

Figure 3: Condition probability. Original probability space. $P(R) = \frac{5}{6}$, $P(B) = \frac{3}{6}$, $P(G) = \frac{4}{6}$

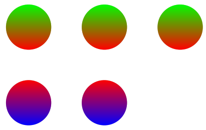

Figure 4: Condition probability. $P(G|R) = \frac{3}{5}$, $P(B|R) = \frac{2}{5}$

Figure 5: Condition probability. $P(R|B) = \frac{2}{3}$, $P(G|B) = \frac{1}{3}$

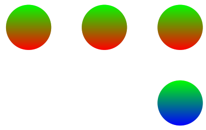

Figure 6: Condition probability. $P(B|G) = \frac{1}{4}$, $P(R|G) = \frac{3}{4}$

2 Monty Hall problem

TBD

3 Waiting time on a bus stop

TBD

References

[1] А. Н. Колмогоров. Основные понятия теории вероятностей / А. Н. Колмогоров. — Москва: Наука, 1974.