Category Theory

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Contents

Introduction

There is an introduction to Category Theory.

Several examples use GAP [1].

Chapter 1

Base definitions

1.1 Definitions

1.1.1 Object

Definition 1.1 (Class)**.** A class is a collection of sets (or sometimes other mathematical objects) that can be unambiguously defined by a property that all its members share.

Definition 1.2 (Object)**.** In category theory object is considered as something that does not have internal structure (aka point) but has a property that makes different objects belong to the same Class

Remark 1.3 (Class of Objects)**.** The Class of Objects will be marked as $\mathrm{ob}(C)$

1.1.2 Morphism

Morphism is a kind of relation between 2 Objects.

Definition 1.4 (Morphism)**.** A relation between two Objects a and b

$$f_{ab} : a \to b$$

is called *morphism*. Morphism assumes a direction i.e. one Object (a) is called *source* and another one (b) *target*.

Morphisms have several properties. [1]

[1] The properties don't have any proof and postulated as axioms

Property 1.5 (Composition)**.** *If we have 3 Objects a, b and c and 2 Morphisms*

$$f_{ab} : a \to b$$

and

$$f_{bc} : b \to c$$

then there exists Morphism

$$f_{ac} : a \to c$$

such that

$$f_{ac} = f_{bc} \circ f_{ab}$$

Remark 1.6 (Composition)**.** The equation

$$f_{ac} = f_{bc} \circ f_{ab}$$

means that we apply f_{ab} first and then we apply f_{bc} to the result of the application i.e. if our objects are sets and $x \in a$ then

$$f_{ac}(x) = f_{bc}(f_{ab}(x)),$$

where $f_{ab}(x) \in b$.

Property 1.7 (Associativity)**.** *The Morphisms Composition (Property 1.5) s should follow associativity property:*

$$f_{ce} \circ (f_{bc} \circ f_{ab}) = (f_{ce} \circ f_{bc}) \circ f_{ab} = f_{ce} \circ f_{bc} \circ f_{ab}.$$

Definition 1.8 (Identity morphism)**.** For every Object a we define a special Morphism $\mathbf{1}a : a \to a$ with the following properties: $\forall f_{ab} : a \to b$

$$\mathbf{1}a \circ f_{ab} = f_{ab}$$

and $\forall f_{ba} : b \to a$

$$f_{ba} \circ \mathbf{1}a = f_{ba}.$$

This morphism is called *identity morphism.*

Definition 1.9 (Commutative diagram)**.** A commutative diagram is a diagram of Objects (also known as vertices) and Morphisms (also known as arrows or edges) such that all directed paths in the diagram with the same start and endpoints lead to the same result by composition

The following diagram commutes if $f_{ab} = f_{cb} \circ f_{ac}$.

$$\begin{tikzcd}
A \arrow[rr, "f_{ab}"] \arrow[dr, "f_{ac}"'] & & B \\
& C \arrow[ur, "f_{cb}"'] &
\end{tikzcd}$$

Remark 1.10 (Class of Morphisms)**.** The Class of Morphisms will be marked as $\hom(C)$

Definition 1.11 (Monomorphism)**.** If $\forall g_1, g_2$ the equation

$$f \circ g_1 = f \circ g_2$$

leads to

$$g_1 = g_2$$

then f is called *monomorphism.*

Definition 1.12 (Epimorphism)**.** If $\forall g_1, g_2$ the equation

$$g_1 \circ f = g_2 \circ f$$

leads to

$$g_1 = g_2$$

then f is called *epimorphism.*

1.1.3 Category

TBD

1.2 Examples

1.2.1 Set category

TBD

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Bibliography

[1] Gap - groups, algorithms, programming - a system for computational discrete algebra. — https://www.gap-system.org/.