

# Probability paradoxes

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# Introduction

The goal for the article is to demonstrate several paradoxes that are related to probability theory and how can they can be solved.



# Chapter 1

## Base definitions of probability theory

I am going to provide several definitions. I will give the both formal and informal definitions and show how they are related each other.

### 1.1 Example and motivation

We will start with the simplest example.

**Example 1.1.** In the example we have (see fig. 1.1)  $N = 5$  balls. There are  $N_G = 2$  green balls and  $N_R$  red balls. I.e.  $N = N_G + N_R$ .

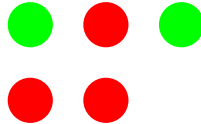


Figure 1.1: Probability example

We can define the probability to get green ball as

$$P_G = \frac{N_G}{N} = \frac{2}{5}$$

and the probability to get red ball as

$$P_R = \frac{N_R}{N} = \frac{3}{5}.$$

We can get only a red or a green ball and

$$P_G + P_R = 1.$$



Figure 1.2: Probability space. It consists of elementary events:  $a$ ,  $b$ ,  $c$  and  $d$ , each of them has equal probability  $p_{a,b,c,d} = \frac{1}{4}$

## 1.2 Definitions

Now we are ready to give several formal definitions.

### 1.2.1 $\sigma$ -algebra

**Definition 1.2** (Power set). Let  $\Omega$  is a set than the set of all possible subsets of  $\Omega$  is called *power set* and denoted as  $\mathcal{P}(\omega)$ .

**Definition 1.3** ( $\sigma$  algebra). Let  $\Omega$  is a set then a subset  $\mathcal{F}$  of [Power set](#)  $\mathcal{P}(\Omega)$  ( $\mathcal{F} \subseteq \mathcal{P}(\Omega)$ ) is called  $\sigma$  algebra if the following conditions are satisfied:

1.  $\mathcal{F}$  contains  $\Omega$ :  $\Omega \in \mathcal{F}$
2. TBD
3. TBD

In our example [1.1](#),  $\sigma$  algebra is a collection of any balls.

## 1.3 Conditional probability

**Definition 1.4** (Conditional probability). The *conditional probability* of event  $A$  on event  $B$  is defined as follow

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**Example 1.5** (Conditional probability). Lets consider 6 balls each of them can be either two colors (see fig. [1.3](#)).

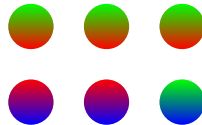


Figure 1.3: Condition probability. Original probability space.  $P(A = \text{red}) = \frac{5}{6}$ ,  $P(A = \text{blue}) = \frac{3}{6}$ ,  $P(A = \text{green}) = \frac{4}{6}$



You can see that the probability  $P(A)$  to get red ball is  $P(A = red) = \frac{5}{6}$ , blue one is  $P(A = blue) = \frac{3}{6}$ , green one is  $P(A = green) = \frac{4}{6}$ .

Now assume that event  $A$  is to get a green ball but event  $B$  is to get red ball, how we can define  $P(A|B)$  in the case.

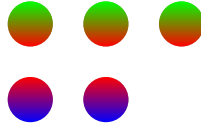


Figure 1.4: Condition probability.  $P(A = green|B = red) = \frac{3}{5}$ ,  $P(A = blue|B = red) = \frac{2}{5}$

The situation is displayed on fig. 1.4. We have only 5 possibilities to choose a ball now instead of 6 in the original case. This is because we just got an additional information - “one of the color should be red”. Only 3 of the 5 balls are green. Therefore  $P(A|B) = P(A = green|B = red) = \frac{3}{5}$ .

This result is in correlation with the formal definition of [Conditional probability](#):

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A = green \cap B = red)}{P(B = red)} = \frac{3/6}{5/6} = \frac{3}{5}.$$

The fig. 1.5 gives as the view if event  $B = blue$  occurs.



Figure 1.5: Condition probability.  $P(A = red|B = blue) = \frac{2}{3}$ ,  $P(A = green|B = blue) = \frac{1}{3}$

In the case we have the following conditional probabilities:  $P(A = red|B = blue) = \frac{2}{3}$ ,  $P(A = green|B = blue) = \frac{1}{3}$ .

Finally, the fig. 1.6 gives as the view if event  $B = green$  occurs.

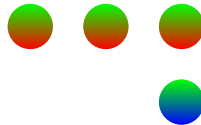


Figure 1.6: Condition probability.  $P(A = blue|B = green) = \frac{1}{4}$ ,  $P(A = red|B = green) = \frac{3}{4}$

**Definition 1.6** (Total probability). The *total probability* is defined as follows

$$P(A) = \sum_i P(A|B_i)$$

**Example 1.7.** TBD

TBD [\[1\]](#)

# Chapter 2

## Paradoxes

### 2.1 Monty Hall problem

TBD

### 2.2 Waiting time on a bus stop

TBD



# Bibliography

- [1] А. Н. Колмогоров. Основные понятия теории вероятностей / А. Н. Колмогоров. — Москва: Наука, 1974.