Probability paradoxes

Ivan Murashko

Contents

Introduction

The goal for the article is to demonstrate several paradoxes that are related to probability theory and how can they can be solved.

Chapter 1

Base definitions of probability theory

I am going to provide several definitions. I will give the both formal and informal definitions and show how they are related each other.

1.1 Example and motivation

We will start with the simplest example.

Example 1.1. In the example we have (see fig. 1.1) $N = 5$ balls. There are $N_G = 2$ green balls and N_R red balls. I.e. $N = N_G + N_R$.

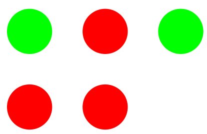

Figure 1.1: Probability example

We can define the probability to get green ball as

$$P_G = \frac{N_G}{N} = \frac{2}{5}$$

and the probability to get red ball as

$$P_R = \frac{N_R}{N} = \frac{3}{5}.$$

We can get only a red or a green ball and

$$P_G + P_R = 1.$$

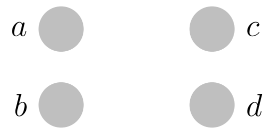

Figure 1.2: Probability space. It consists of elementary events: a, b, c and d, each of them has equal probability $p_{a,b,c,d} = \frac{1}{4}$

1.2 Definitions

Now we are ready to give several formal definitions.

1.2.1 σ-algebra

Definition 1.2 (Power set)**.** Let Ω is a set than the set of all possible subsets of Ω is called *power set* and denoted as $\mathcal{P}(\omega)$.

Definition 1.3 (σ algebra)**.** Let Ω is a set then a subset $\mathcal{F}$ of Power set $\mathcal{P}(\Omega)$ ($\mathcal{F} \subseteq \mathcal{P}(\Omega)$) is called σ algebra if the following conditions are satisfied:

1. $\mathcal{F}$ contains Ω: $\Omega \in \mathcal{F}$
2. TBD
3. TBD

In our example 1.1, σ algebra is a collection of any balls.

1.3 Conditional probability

Definition 1.4 (Conditional probability)**.** The *conditional probability* of event A on event B is defined as follow

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example 1.5 (Conditional probability)**.** Lets consider 6 balls each of them can be either two colors (see fig. 1.3).

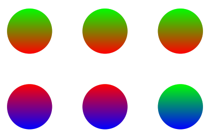

Figure 1.3: Condition probability. Original probability space. $P(R) = \frac{5}{6}$, $P(B) = \frac{3}{6}$, $P(G) = \frac{4}{6}$

Figure 1.5: Condition probability. $P(R|B) = \frac{2}{3}$, $P(G|B) = \frac{1}{3}$

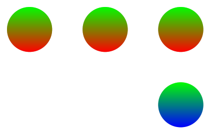

Figure 1.6: Condition probability. $P(B|G) = \frac{1}{4}$, $P(R|G) = \frac{3}{4}$

You can see that the probability to get red ball is $P(R) = \frac{5}{6}$, blue one is $P(B) = \frac{3}{6}$, green one is $P(G) = \frac{4}{6}$.

Now assume that event A is to get a green ball but event B is to get red ball, how we can define $P(A|B)$ in the case.

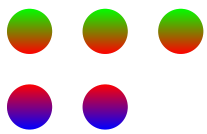

Figure 1.4: Condition probability. $P(G|R) = \frac{3}{5}$, $P(B|R) = \frac{2}{5}$

The situation is displayed on fig. 1.4. We have only 5 possibilities to choose a ball now instead of 6 in the original case. This is because we just got an additional information - "one of the color should be red". Only 3 of the 5 balls are green. Therefore $P(A|B) = P(A = green|B = red) = \frac{3}{5}$.

This result is in correlation with the formal definition of **??**:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} =$$
$$\frac{P(A = green \cap B = red)}{P(B = red)} = \frac{3/6}{5/6} = \frac{3}{5}.$$

TBD

Definition 1.6 (Total probability)**.** The *total probability* is defined as follows

$$P(A) = \sum_i P(A|B_i)$$

Example 1.7. TBD

TBD [1]

Chapter 2

Paradoxes

2.1 Monty Hall problem

TBD

2.2 Waiting time on a bus stop

TBD

Bibliography

[1] А. Н. Колмогоров. Основные понятия теории вероятностей / А. Н. Колмогоров. — Москва: Наука, 1974.