

Category Theory

Ivan Murashko

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Introduction

There is an introduction to Category Theory.
Several examples use GAP [\[1\]](#).

Chapter 1

Base definitions

1.1 Definitions

1.1.1 Object

Definition 1.1 (Class). A class is a collection of sets (or sometimes other mathematical objects) that can be unambiguously defined by a property that all its members share.

Definition 1.2 (Object). In category theory object is considered as something that does not have internal structure (aka point) but has a property that makes different objects belong to the same [Class](#)

Remark 1.3 (Class of Objects). The [Class](#) of [Objects](#) will be marked as $\text{ob}(C)$

1.1.2 Morphism

Morphism is a kind of relation between 2 [Objects](#).

Definition 1.4 (Morphism). A relation between two [Objects](#) a and b

$$f_{ab} : a \rightarrow b$$

is called *morphism*. Morphism assumes a direction i.e. one [Object](#) (a) is called *source* and another one (b) *target*.

[Morphisms](#) have several properties. ¹

¹The properties don't have any proof and postulated as axioms

Property 1.5 (Composition). If we have 3 *Objects* a, b and c and 2 *Morphisms*

$$f_{ab} : a \rightarrow b$$

and

$$f_{bc} : b \rightarrow c$$

then there exists *Morphism*

$$f_{ac} : a \rightarrow c$$

such that

$$f_{ac} = f_{bc} \circ f_{ab}$$

Remark 1.6 (Composition). The equation

$$f_{ac} = f_{bc} \circ f_{ab}$$

means that we apply f_{ab} first and then we apply f_{bc} to the result of the application i.e. if our objects are sets and $x \in a$ then

$$f_{ac}(x) = f_{bc}(f_{ab}(x)),$$

where $f_{ab}(x) \in b$.

Property 1.7 (Associativity). The *Morphisms Composition* (*Property 1.5*) should follow associativity property:

$$f_{ce} \circ (f_{bc} \circ f_{ab}) = (f_{ce} \circ f_{bc}) \circ f_{ab} = f_{ce} \circ f_{bc} \circ f_{ab}.$$

Definition 1.8 (Identity morphism). For every *Object* a we define a special *Morphism* $1a : a \rightarrow a$ with the following properties: $\forall f_{ab} : a \rightarrow b$

$$1a \circ f_{ab} = f_{ab}$$

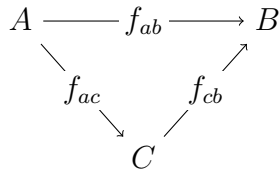
and $\forall f_{ba} : b \rightarrow a$

$$f_{ba} \circ 1a = f_{ba}.$$

This morphism is called *identity morphism*.

Definition 1.9 (Commutative diagram). A commutative diagram is a diagram of *Objects* (also known as vertices) and *Morphisms* (also known as arrows or edges) such that all directed paths in the diagram with the same start and endpoints lead to the same result by composition

The following diagram commutes if $f_{ab} = f_{cb} \circ f_{ac}$.



Remark 1.10 (Class of Morphisms). The [Class of Morphisms](#) will be marked as $\text{hom}(C)$

Definition 1.11 (Monomorphism). If $\forall g_1, g_2$ the equation

$$f \circ g_1 = f \circ g_2$$

leads to

$$g_1 = g_2$$

then f is called *monomorphism*.

Definition 1.12 (Epimorphism). If $\forall g_1, g_2$ the equation

$$g_1 \circ f = g_2 \circ f$$

leads to

$$g_1 = g_2$$

then f is called *epimorphism*.

1.1.3 Category

TBD

1.2 Examples

1.2.1 Set category

TBD

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Bibliography

- [1] Gap - groups, algorithms, programming - a system for computational discrete algebra.—<https://www.gap-system.org/>.