Probability paradoxes

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Contents

Introduction

The goal for the article is to demonstrate several paradoxes that are related to probability theory and how can they can be solved.

Chapter 1

Base definitions of probability theory

I am going to provide several definitions. I will give the both formal and informal definitions and show how they are related each other.

We will start with the simplest example.

Example 1.1. In the example we have (see fig. 1.1) $N = 5$ balls. There are $N_G = 2$ green balls and N_R red balls. I.e. $N = N_G + N_R$.

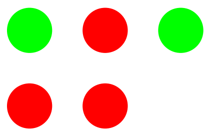

Figure 1.1: Probability example

We can define the probability to get green ball as

$$P_G = \frac{N_G}{N} = \frac{2}{5}$$

and the probability to get red ball as

$$P_R = \frac{N_R}{N} = \frac{3}{5}.$$

We can get only a red or a green ball and

$$P_G + P_R = 1.$$

Now we are ready to give several formal definitions.

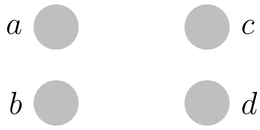

Figure 1.2: Probability space. It consists of elementary events: a, b, c and d, each of them has equal probability $p_{a,b,c,d} = \frac{1}{4}$

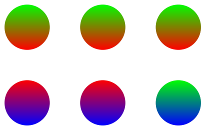

Figure 1.3: Condition probability. Original probability space. $P(R) = \frac{5}{6}$, $P(B) = \frac{3}{6}$, $P(G) = \frac{4}{6}$

Definition 1.2 (σ algebra)**.** Let Ω is a set then a collection $\mathcal{F}$ of sub sets of Ω is called σ algebra if the following conditions are satisfied:

1. $\mathcal{F}$ contains Ω: $\Omega \in \mathcal{F}$
2. TBD
3. TBD

In our example 1.1, σ algebra is a collection of any balls.
TBD [1]

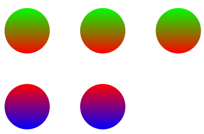

Figure 1.4: Condition probability. $P(G|R) = \frac{3}{5}$, $P(B|R) = \frac{2}{5}$

Figure 1.5: Condition probability. $P(R|B) = \frac{2}{3}$, $P(G|B) = \frac{1}{3}$

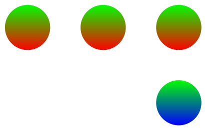

Figure 1.6: Condition probability. $P(B|G) = \frac{1}{4}$, $P(R|G) = \frac{3}{4}$

Chapter 2

Paradoxes

2.1 Monty Hall problem

TBD

2.2 Waiting time on a bus stop

TBD

Bibliography

[1] А. Н. Колмогоров. Основные понятия теории вероятностей / А. Н. Колмогоров. — Москва: Наука, 1974.