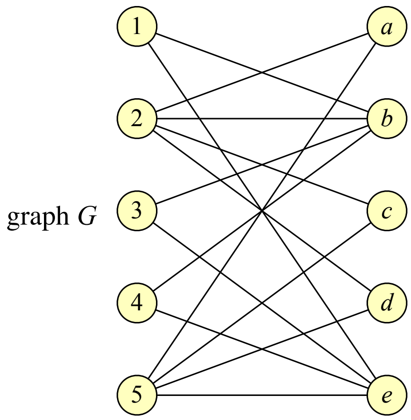
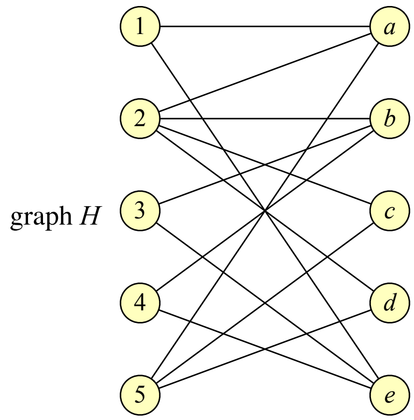
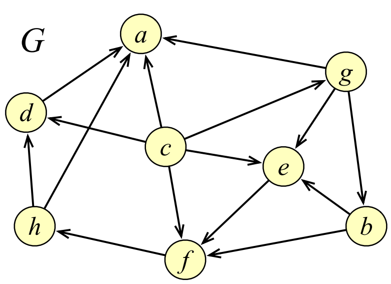
CS/MATH 111 Spring 2021
Practice Final

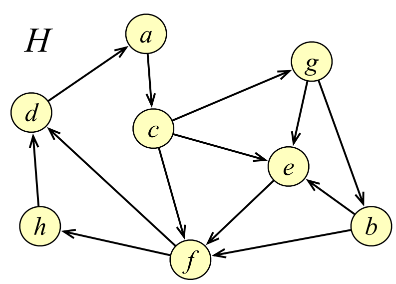
Problem 1: Determine whether the two bipartite graphs below have a perfect matching. Justify your answer, either by showing a perfect matching or using Hall's theorem to prove that it does not exist.



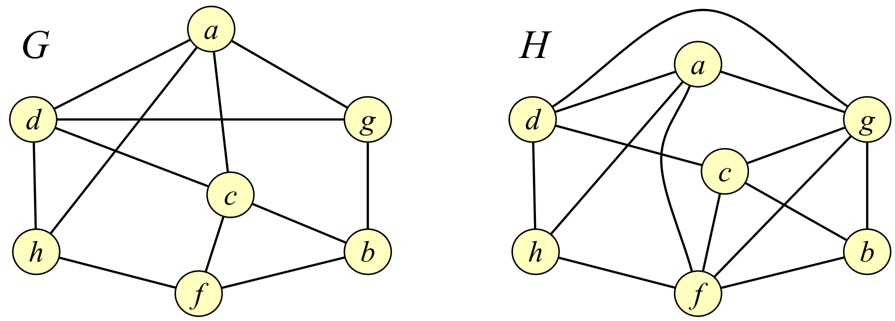


Problem 2: For each of the two directed graphs below, determine whether they are acyclic. If a digraph is not acyclic, give at least one cycle in this digraph. If a digraph is acyclic, provide its topological ordering.





Problem 3: For each of the two graphs below, determine whether they are planar. Justify your answer: if the graph is planar, show its planar embedding. If a graph is not planar, use Kuratowski's theorem to justify it.



Problem 4: For each degree sequence below, determine whether there is a graph with 5 vertices that have these degrees. If a graph exists, draw it. If it doesn't, justify that it doesn't exist.

(a) $4, 4, 3, 3, 2$.
(b) $5, 3, 2, 2, 2$.
(c) $4, 4, 3, 2, 2$.
(d) $4, 4, 3, 2, 1$.

Problem 5: Recall that K_n is the complete graph with n vertices[1]. Let u, v be two different vertices in K_n. Give the formula for the number of hamiltonian paths[2] from u to v. Justify your answer.

Problem 6: Let G be a graph with k connected components $C_1, C_2, ..., C_k$, where $k \geq 2$. Each connected component has at least three vertices and has an Euler tour. Is it possible to always add edges between these connected components so that G has an Euler tour? If so, what is the minimum number of edges needed to accomplish this[3]. Justify your answer.

Problem 7: Let G be a graph in which the sum of vertex degrees is equal 30.

(a) What is the smallest possible number of vertices in G?
(b) What is the largest possible number of vertices in G if G is connected?

You need to give a complete justification for your answers.

Problem 8: Let G be a connected planar graph with n vertices and f faces. Prove that $f \leq 2n - 4$.

Problem 9: Let T be a tree with n vertices. Prove that T has at least $n/2$ vertices whose degree is less than 3.

[1] K_n is also called an n-clique. It has an edge between any pair of vertices.
[2] Recall that a hamiltonian path is a path that visits each vertex exactly once.
[3] To make sure, as defined in class, multiple edges are not allowed in graphs.

Problem 10: Consider a graph G with 7 vertices whose degrees are $6, 6, 5, 4, 3, 3, 3$. (a) Show that G can be colored with 4 colors. (b) Show that G cannot be colored with 3 colors. (For partial credit, you can show that G can be colored with 5 or 6 colors, or that it cannot be colored with 2 colors.)