

Discussion 1: Review

- Logic
- Counting
- Basic algebra

Logic

For each sentence below, which of the following sentences is equivalent to it

1. If X is green or pink, then X is a vegetable.

- A. X is green and X is a vegetable.
- B. X is not green and X is not a vegetable.
- C. X is not green or X is a vegetable.
- D. X is not green and X is a vegetable.
- E. None of the above

2. X is a pig, and Y or Z is a bird.

- A. Either X is not a pig, or Y and Z are not birds.
- B. Either X is a pig and Y is a bird, or X is a pig and Z is a bird.
- C. X is not a pig, and neither Y nor Z is a bird.
- D. X is a pig, and both Y and Z are birds.
- E. None of the above.

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Logic

For each sentence below, which of the following sentences is its negation

3. $\forall x \exists y : y < x^2 + 17$
- A. $\forall x \exists y : y \geq x^2 + 17$
- B. $\forall x \exists y : x^2 + 17 < y$
- C. $\exists x \exists y : y > x^2 + 17$
- D. $\exists x \forall y : y \geq x^2 + 17$
- E. None of the above

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- ☒ D. $\exists x \forall y : y \geq x^2 + 17$
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Logic

For each sentence below, which of the following sentences its negation

4. Some of us can write but cannot spell.

- A. All of us cannot write and can spell.
- B. Some of us cannot write but can spell.
- C. Some of us can spell but cannot write.
- D. All of us either can spell or cannot write.
- E. None of the above

5. For any X, if X moos then X is a cow.

- A. There exists an X that moos and is not a cow.
- B. There is no X that does not moo and is not a cow.
- C. For any X, X does not moo and X is not a cow.
- D. For any X, if X does not moo then X is not a cow.
- E. None of the above

Logic

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4. Some of us can write but cannot spell.

- A. All of us cannot write and can spell.
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- D. For any X, if X does not moo then X is not a cow.
- E. None of the above

Counting

Let X be a set of 10 distinct items. Give formulas for the following quantities:

1. What is the total number of subsets of X ?
2. In how many ways we can choose 6 items from X if the items in the choices are ordered and repetition is **not** allowed?
3. In how many ways we can choose 6 items from X if the items in the choices are ordered and repetition is allowed?
4. In how many ways we can choose 6 items from X if the items in the choices are **not** ordered and repetition is **not** allowed?
5. In how many ways we can order X ?

Counting

Let X be a set of 10 distinct items. Give formulas for the following quantities:

1. What is the total number of subsets of X ? 2^{10}
2. In how many ways we can choose 6 items from X if the items in the choices are ordered and repetition is **not** allowed? $P(10,6) = \frac{10!}{(10-6)!} = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 151200$
3. In how many ways we can choose 6 items from X if the items in the choices are ordered and repetition is allowed? 10^6
4. In how many ways we can choose 6 items from X if the items in the choices are **not** ordered and repetition is **not** allowed? $C(10,6) = \frac{10!}{(10-6)!6!} = 210$
5. In how many ways we can order X ? $P(10,10) = 10!$

Basic Algebra: Quadratic Equations

Solve

1. $x^2 - 4 = 0$

2. $x^2 + 5x = 0$

3. $x^2 - 6x = 16$

4. $x^2 + 5x + 6 = 0$

Basic Algebra: Quadratic Equations

Solve

$$1. x^2 - 4 = 0 \qquad x = 2, \text{ or } x = -2$$

$$2. x^2 + 5x = 0 \qquad x = 0, \text{ or } x = -5$$

$$3. x^2 - 6x = 16 \qquad x = -2, \text{ or } x = 8$$

$$4. x^2 + 5x + 6 = 0 \qquad x = -2, \text{ or } x = -3$$

Basic Algebra: Cubic Equations

Solve $x^3 - 3x - 2 = 0$

Candidate of integer roots: $-1, 1, -2, 2$

Plug in equation: $-1, 2$ are roots

Factor:

$$x^3 - 3x - 2 = 0 : 0$$

$$\Rightarrow x^2(x - 2) + 2x(x - 2) + (x - 2) = 0$$

$$\Rightarrow (x - 2)(x^2 + 2x + 1) = 0$$

$$\Rightarrow (x - 2)(x + 1)^2 = 0$$

Finish: $x = -1, x = 2$

Basic Algebra: Cubic Equations

Solve $x^3 + 4x^2 + x - 6 = 0$

Basic Algebra: Cubic Equations

Solve $x^3 + 4x^2 + x - 6 = 0$ $x = 1, x = -2, x = -3$

Basic Algebra: System of Equations

$$\text{Solve: } \begin{cases} 2x + y + 3z = 1 \\ 2x + 6y + 8z = 3 \\ 6x + 8y + 18z = 5 \end{cases}$$

$$\Rightarrow \begin{cases} 2x + y + 3z = 1 \\ 2x + 6y + 8z - (2x + y + 3z) = 3 - 1 \\ 6x + 8y + 18z - 3(2x + y + 3z) = 5 - 3 \cdot 1 \end{cases} \Rightarrow \begin{cases} 2x + y + 3z = 1 \\ 5y + 5z = 2 \\ 5y + 9z = 2 \end{cases}$$

$$\Rightarrow \begin{cases} 2x + y + 3z = 1 \\ 5y + 5z = 2 \\ 4z = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{3}{10} \\ y = \frac{2}{5} \\ z = 0 \end{cases}$$