

CS/MATH111 ASSIGNMENT 1

Problem 1: Give the exact and asymptotic formula for the number $f(n)$ of letters “A” printed by Algorithm PRINTAS below. Your solution must consist of the following steps:

- (a) First express $f(n)$ using a summation notation \sum .
- (b) Next, give a closed-form formula for $f(n)$.
- (c) Finally, give the asymptotic value of the number of A’s (using the Θ -notation.)

Include justification for each step.

Note: If you need any summation formulas for this problem, you are allowed to look them up, and do not need to prove.

Algorithm PRINTAS (n : integer)

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for  $i \leftarrow 1$  to  $3n + 2$  do
  for  $j \leftarrow 1$  to  $(i + 2)^2$  do print(“A”)
for  $i \leftarrow 1$  to  $(3n + 2)^2$  do
  for  $j \leftarrow 1$  to  $i + 2$  do print(“A”)
```

Problem 2: (a) Use mathematical induction to prove that $3^n \geq n2^n$ for $n \geq 0$. (Note: dealing with the base case may require some thought.)

(b) Consider a sequence defined recursively as $D_0 = 2$, $D_1 = 5$, and $D_n = 3D_{n-1} + D_{n-2}$ for $n \geq 2$. Prove that $D_n = O(3.4^n)$ and $D_n = \Omega(3.2^n)$.

Hint: First, prove by induction that $(3.2)^n \leq D_n \leq 2(3.4)^n$ for all $n \geq 0$.

Problem 3: Give the asymptotic values of the following functions, using the Θ -notation:

- (a) $(n^3 + 5n - 10)(3n^2 - n)$
- (b) $3 + 2\sqrt{n} + \frac{n}{\log^2 n}$
- (c) $7n^2 \log n + 9n \log^5 n + 15n^3$
- (d) $15n^6 + n^2 \cdot 2^n + n^3 \log n$
- (e) $n^3 \cdot \frac{7^n}{6} + n^5 \cdot 4^n$

Justify your answers.

Submission. To submit the homework, you need to upload the pdf file to Gradescope.

Reminders. Remember that only L^AT_EX papers are accepted.