CS/MATH 111 Quiz 1 Responses

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Problem 1:

a)

1)

i) $5n^4 - 3n^3 + 10^5 + 7n$ $\quad \theta(n^4)$

ii) $2n\log(n^7) + n^2\log^3(n) + \frac{10n}{5^n}$ $\quad \Theta(n^2\log(n))$

iii) $\frac{10n^4}{n\sqrt{n}} + 5n^2\log n + 12n^2$ $\quad \theta(n^2\log(n))$

iv) $15n^3 \cdot 5^n + \left(\frac{10}{3}\right)^n$ $\quad \theta(n^3 \cdot 5^n)$

b) not enough time

Problem 2:

a)

2a)

i) $\sum_{i=0}^{20} \sum_{j=1}^{i} 1$

- First loop runs 20 times
- second runs i times for each j.

ii) $\sum_{i=0}^{n} \sum_{j=1}^{\log n} 1 + \sum_{k=1}^{2n} 1$

$i=0$: n times $+1$; $j=1$: $\log n$ times; separate for loop

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iii) $\sum_{i=1}^{24n^2} (\log_3(n))$

while loop runs $\log_3(n)$ times

iv) $\sum_{i=1}^{2n^3} 1$

First while loop will cause j to be $> n^2$.

b)

2b)

i) $\Theta(1)$ does not depend on n

ii) $O(n \log n)$

- First loop runs n
  - $\log n$ for every n

iii) $\Theta(n^2 \log n)$

- $\log n$ is summed $24n^2$ times
  - we drop constants and bases

iv) $\Theta(n^3)$

- goes from i to $2n^3$

Problem 3:

3)

$$P(n): \sum_{i=1}^{n} i \cdot 2^{i+1} \geq 2^n \cdot (n-1)$$

for $n \geq 1$.

base case

$P(1): 1 \cdot 2^2 \geq 2^1 (0)$ ✓

Inductive step

hypothesis $P(k): \sum_{i=1}^{k} i \cdot 2^{i+1} \geq 2^k (k-1)$

Proving $P(K) \rightarrow P(K+1)$

$$\sum_{i=1}^{K+1} i \cdot 2^{i+1} \geq 2^{K+1}(K)$$

$$\sum_{i=1}^{K} i \cdot 2^{i+1} + \sum_{i=K+1}^{K+1} i \cdot 2^{i+1} \geq 2^{K+1}(K)$$

$$\sum_{i=1}^{K} i \cdot 2^{i+1} + \sum_{i=K+1}^{K+1} i \cdot 2^{i+1} \geq 2^{K+1}(K)$$

$$\sum_{i=1}^{K} i \cdot 2^{i+1} + (K+1) \cdot 2^{K+2} \geq 2^{K+1}(K)$$

- Not enough time for steps after