CS150 Homework 1

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Problem 1: (3 points) Prove using mathematical induction that $2^p + p < 2^{p+1}$ for all $p \geq 1$.

Solution 1:

We are proving by induction that the preposition P(n): $2^p + p < 2^{p+1}$ is true for all $p \geq 1$.

Base case (p = 1):

$$2^1 + 1 < 2^{1+1}$$
$$3 < 4$$

Thus the preposition P is true for $p = 1$. We will use this as the base case in the induction step.

Inductive step:

Inductive hypothesis:
Suppose that the preposition is true for a number $p = k$ such that the preposition $2^k + 1 < 2^{k+1}$ is true.

Induction:
Now the goal is to prove the preposition $P(k) \to P(k + 1)$.

We begin with the antecedent:

$$2^k + 1 < 2^{k+1}$$

We know this expression holds true, thus adding a constant to the right hand side should not change the truthfulness of the statement:

$$2^k + 1 < 2^{k+1} + \frac{1}{2}$$

Now we perform some steps to show that this expression can be placed in $P(k+1) : 2^{k+1}+1 < 2^{(k+1)+1}$ form:

$$\begin{aligned}
2^k + 1 &< 2^{k+1} + \frac{1}{2} \\
2^k + 1 - \frac{1}{2} &< 2^{k+1} + \frac{1}{2} - \frac{1}{2} \\
2^k + \frac{1}{2} &< 2^{k+1} \\
2\left(2^k + \frac{1}{2}\right) &< 2\left(2^{k+1}\right) \\
2 \cdot 2^k + 2 \cdot \frac{1}{2} &< 2 \cdot 2^{k+1} \\
2^{k+1} + 1 &< 2^{k+2} \\
2^{k+1} + 1 &< 2^{(k+1)+1}
\end{aligned}$$

Thus we have proven that $P(1)$ holds true, and that $P(k) -> P(k+1)$.

Therefore, we have proven by induction that $P(p) : 2^p + 1 < 2^{p+1}$ holds true as well.

Problem 2: (2 points) Draw a DFA accepting the following language over the alphabet

$$\{0,1\} : \{w|w \text{ contains at least three 1s}\}$$

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Solution 2:

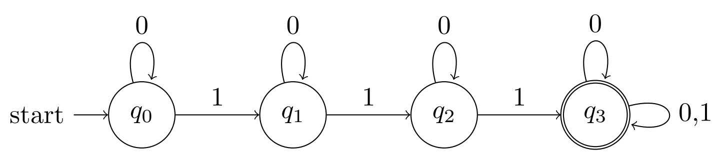

Figure 1: Thank you to the following source for a tutorial on tikz automata (Overleaf): `https://www3.nd.edu/~kogge/courses/cse30151-fa17/Public/other/tikz_tutorial.pdf`

At every state, we wait until all of the 0 inputs are read. Then, if a 1 input is read, the computation will move to the next state. This happens until we have reached our desired amount of 1's (3 1's).z

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Problem 3: (2 points) Draw a DFA accepting the following language over the alphabet

$$\{0, 1\} : \{w | w \text{ does NOT contain the substring 110}\}$$

Solution 3:

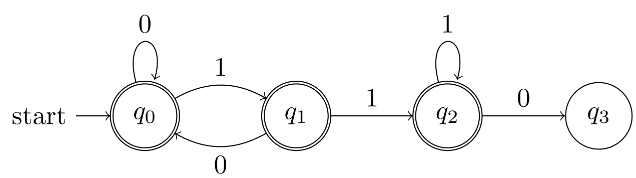

Figure 2: Thank you to the following source for the tikz automata tutorial (Overleaf): `https://www3.nd.edu/~kogge/courses/cse30151-fa17/Public/other/tikz_tutorial.pdf`

State q_0 will catch all prepending 0's. Then, when the machine detects a 1 at q0, it will be on "alert mode" for the substring 110. If after said 1, the machine detects a 0, then it goes back to detecting prepending zeros. However, if the machine detects a 1 instead, it will move to q_2. At q_2, if it detects a 1, then it keeps detecting for a 0 to reject, since there could potentially be one. If the input ends at this point, it will accept because no 110 has been spotted. But if the machine detects a 0 at $q2$ then it rejects because 110 has been spotted.

Problem 4: (3 points total) Draw a DFA for the language over $\{0, 1\}$ in which the third symbol of each string from the right is a 1. For example, "100100" is in the language, but "100000" isn't.

Solution 4:

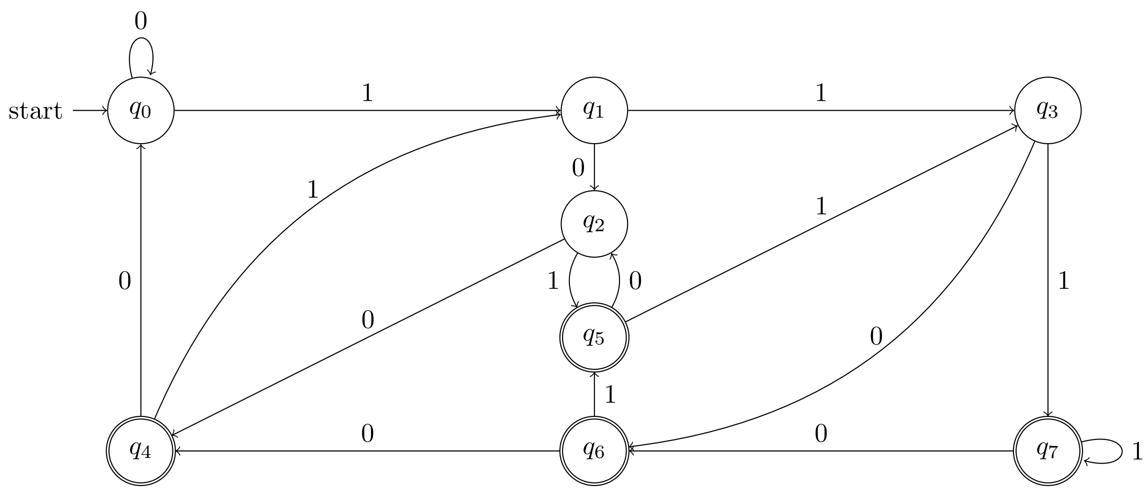

Figure 3: Thanks to the following source for automata tiks tutorial (Overleaf): `https://www3.nd.edu/~kogge/courses/cse30151-fa17/Public/other/tikz_tutorial.pdf`

We have 8 states. The last four states are accepting states, meaning if the sequence ends at a state from q4-q7, the string will be accepted. Otherwise, it'll be rejected. Each number for q represents its binary representation. For example, q0 = 000. This leads to the following states:

$$
\begin{aligned}
q0 &= 000 \\
q1 &= 001 \\
q2 &= 010 \\
q3 &= 011 \\
q4 &= 100 \\
q5 &= 101 \\
q6 &= 110 \\
q7 &= 111
\end{aligned}
$$

At any one point, we only really care about the last three digits. Thus we can produce all of the possible transitions with the following table. For instance, q0 (000) will transition to 001 on 1 and 000 on 0.

Transition Table		
State	Input=0	Input=1
q0(000)	q0(000)	q1(001)
q1(001)	q2(010)	q3(011)
q2(010)	q4(100)	q5(101)
q3(011)	q6(110)	q7(111)
q4(100)	q0(000)	q1(001)
q5(101)	q2(010)	q3(011)
q6(110)	q4(100)	q5(101)
q7(111)	q6(110)	q7(111)