NAME: SID:

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Problem 1: (a) Find a particular solution of the recurrence $C_n = 3C_{n-1} + C_{n-2} + 6$. Show your work.

We try $C_n'' = \beta$, for some constant β. We plug it in:

$$\beta = 3\beta + \beta + 6$$

which gives $\beta = -2$. So $C_n'' = -2$.

(b) Find a particular solution of the recurrence $C_n = 3C_{n-1} + C_{n-2} + 3 \cdot 2^n$. Show your work.

We try $C_n'' = \beta \cdot 2^n$, for some constant β. We plug it in:

$$\beta \cdot 2^n = 3\beta \cdot 2^{n-1} + \beta \cdot 2^{n-2} + 3 \cdot 2^n.$$

After dividing by 2^{n-2}, this reduces to

$$\beta \cdot 4 = 3\beta \cdot 2 + \beta + 3 \cdot 4.$$

We solve it for β, which gives $\beta = -4$. So $C_n'' = -4 \cdot 2^n$.

Problem 2: (a) Complete the statement of the inclusion-exclusion principle below:

Let $A_1, A_2, ..., A_k$ be finite sets. Then

$$\left| \bigcup_{i=1}^{k} A_i \right| = \sum_{j=1}^{k} (-1)^{j+1} \sum_{l_1 < l_2 < ... < l_j} \left| A_{l_1} \cap A_{l_2} \cap ... \cap A_{l_j} \right|.$$

(b) We have a group of 46 people, including 24 US citizens, 16 Canadian citizens, and 27 Mexican citizens. The numbers of dual citizens of each type, US and Mexico, or US and Canada, or Mexico and Canada, are all equal. No person has a triple citizenship. How many people have only Mexican citizenship? Show your work.

From the inclusion-exclusion principle, denoting the three sets by U, C and M, we have

$$|U \cup C \cup M| = |U| + |C| + |M| - |U \cap C| - |U \cap M| - |C \cap M| + |U \cap C \cap M|.$$

Let $x = |U \cap C| = |U \cap M| = |C \cap M|$. Then the above equation reduces to

$$46 = 24 + 16 + 27 - x - x - x + 0,$$

so $x = 7$. Therefore the number of people who have only Mexican citizenship is $27 - 2 \cdot 7 = 13$.

Problem 3: For each recurrence equation below, mark (circle) the correct solution.

Recurrence	Solution
(a) $f(n) = 4f(n/5) + 3n$	$\Theta(n^3)$
	$\Theta(\log n)$
	$\Theta(n^{4/5})$
	$\Theta(n^{\log_5 4})$
	$\boxed{\Theta(n)}$
	$\Theta(n^{\log_4 5})$
	$\Theta(n \log n)$
(b) $f(n) = 5f(n/4) + 3n$	$\Theta(n^3)$
	$\Theta(\log n)$
	$\Theta(n^{4/5})$
	$\boxed{\Theta(n^{\log_4 5})}$
	$\Theta(n)$
	$\Theta(n^{\log_5 4})$
	$\Theta(n \log n)$
(c) $f(n) = 4f(n/5) + 3$	$\Theta(n^3)$
	$\Theta(\log n)$
	$\Theta(n^{5/4})$
	$\boxed{\Theta(n^{\log_5 4})}$
	$\Theta(n)$
	$\Theta(n^{\log_4 5})$
	$\Theta(n \log n)$