CS215 ASSIGNMENT 2

Due Wednesday, February 7, 11:59PM

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Problem 1: Is the following decision problem decidable? Give a complete proof. (It is sufficient to give an appropriate reduction and justify its correctness.)

TwoInARow:

Instance A TM M, a word w;

Query: Is M ever in the same state twice in a row in the computation on w?

To clarify, the decision problems asks whether M, in the computation on w, ever executes a transition of the form $\&(q, a) = (q, b, \mu)$, staying in the same state.

Solutions:

The preceding decision problem is undecidable.

Assume this problem is decidable. Meaning, there is a turing machine L that takes as input $< M, w >$ and:

1. L *accepts* when M is ever in the same state twice in a row
2. L *rejects* when M is never in the same state twice in a row

L can easily be fitted to do item 1 above, by simply adding counter states in L to simulate M reaching the same state immediately after a state.

For item 2, consider M being stuck on an infinite loop between different states, never having been in the same state twice in a row. The problem of deciding item 2 reduces to deciding $HALT_{TM}$. By Theorem 5.1, the halting problem is undecidable.

Deciding this decision problem implies deciding $HALT_{TM}$, which is a contradiction. By contradiction, I conclude that this decision problem is undecidable.

Problem 2: Let $u \in \Sigma^*$ be any fixed string. Prove that the decision problem below is undecidable. Give a complete proof. (It is sufficient to give an appropriate reduction and justify its correctness.)

AllWithPrefix$_u$:

Instance: A context-free grammar G with alphabet Σ;

Query: Is $L(G) = \{uv : v \in \Sigma^*\}$

In other words, the decision problem asks whether G generates all strings starting with u (and no other strings).

Solutions:

Since $u \in \Sigma^*$, assume $u = \epsilon$. In this case, $L(G) = \{uv : v \in \Sigma^*\} = \{v : v \in \Sigma^*\}$.

I shall assume that there is a turing machine F that decides the problem of whether $L(G) = \{v : v \in \Sigma^*\}$. Since v is any word in Σ^*, F rejects on an infinitely looping input, and rejects on a rejecting input. Similarly, it accepts in an accepting input. This is equal to A_{TM}, which in Theorem 4.11 we prove to be undecidable.

Assuming the decision problem to be decidable implies A_{TM} is decidable, which it is not.

By contradiction, this decision problem is undecidable, due to being reducible to A_{TM}.

Academic integrity / collaboration statement

I did not use any external tools or resources for this homework.

The assigned Introduction to the Theory of Computation textbook was used to aid my proofs by using some known theorems (Theorems 5.1 and 4.11). I also used their method of reducibility to prove that the problems were undecidable.