

Homework 2 Solutions

Problem 1

Problem 2

Problem 1 $y = \frac{4}{x}$

a) $\frac{dy}{dx} = \frac{y}{x} + e$

$u = \frac{y}{x} = y \left(\frac{1}{x} \right)$

$u' = y' \left(\frac{1}{x} \right) + y \left(-\frac{1}{x^2} \right)$

$u' = y' \left(\frac{1}{x} \right) - \frac{y}{x^2} = \frac{y' - \frac{y}{x}}{x} = \frac{(y/x)' - u}{x}$

$\frac{du}{dx} = \frac{y' - u}{x}$

$\frac{dx}{x} = \frac{1}{y' - u} du = \frac{1}{u + e^{4u} - u} du = \frac{1}{e^{4u}} du$

$\int \frac{dx}{x} = \int e^{-4u} du \rightarrow \ln(-4 \ln x + C) = -4u$

b) $\ln x + C = -\frac{1}{4} e^{-4u}$

c) $-4 \ln x + C = e^{-4u}$

$u = \ln(-4 \ln x + C)$

$y = \frac{\ln(-4 \ln x + C)}{-4} x$

$3 = \frac{\ln C}{-4} \rightarrow -12 = \ln C$

$e^{-12} = C$

$y = \frac{\ln(-4 \ln x + e^{-12})}{-4} x$

*Abbreviated Terms and Conditions
Welcome Checking Cash Bonus Offer
Available in select markets.
So long as you have not owned a CIT checking account in the last 180 days, meet Tax Requirements, and are at least 18 years old, you are eligible to participate in this offer. Tax Requirements: Must furnish or have a valid IRS Form 9-BBEN on file with CIT Bank and are not subject to backup withholding. To fulfill the required activities of this offer, enroll in the Welcome Checking Cash Bonus Offer on the same day you open your new Checking Account to the instructions provided. Open a "Checking Account" in a physical branch OR use a residential address at the time you apply for your account on CIT's website. See CIT's website for details.
On the 20th Day After cardholder's account opening.

Problem 2

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b) $xy \frac{dy}{dx} = x^2 + 2y^2$

$$\frac{dy}{dx} = \frac{x^2 + 2y^2}{xy} = \frac{x(x + \frac{2y^2}{x})}{xy} = \frac{x + \frac{2y^2}{x}}{y} = \frac{x}{y} + \frac{2y}{x}$$

$$u = \frac{y}{x} \rightarrow \frac{du}{dx} = \frac{y' - u}{x} \Rightarrow \int \frac{1}{dx} = \frac{1}{\frac{1}{u} + 2u - u} du = \int \frac{1}{\frac{1}{u} + u} du$$

$$\ln x + C = \int \frac{1}{\frac{1}{u}(1+u^2)} du = \int \frac{u}{1+u^2} du$$

$$v = 1 + u^2$$

$$\frac{dv}{du} = 2u \rightarrow du = \frac{dv}{2u} \rightarrow \int \frac{u}{v} \frac{dv}{2u} = \frac{1}{2} \int \frac{dv}{v} = \frac{1}{2} \ln v$$

$$\ln x + C = \frac{1}{2} \ln(1+u^2)$$

$$e^C e^{\ln x} = e^{\frac{1}{2} \ln(1+u^2)}$$

$$C_x = (1+u^2)^{1/2}$$

$$(C_x)^2 = 1 + u^2$$

$$2 C_x^2 - 1 = u^2$$

$$u = \sqrt{2C_x^2 - 1}$$

$$y = \sqrt{2C_x^2 - 1} x$$

$$y = \sqrt{2C_x^2 - 1} x$$

$$8 = -C - 1 \quad (-1)$$

$$C = -8$$

$$9 = 2C + 1$$

$$9 = 2C + 1$$

$$2C = 8$$

$$C = 8$$

$$y = \sqrt{-8x^2 - 1} (x)$$

c)

$$\frac{dy}{dx} = \frac{y^2 + 2xy}{x^2} = \frac{y^2}{x^2} + \frac{2xy}{x^2} = \left(\frac{y}{x}\right)^2 + 2\left(\frac{y}{x}\right)$$

$$u = \frac{y}{x} \rightarrow \frac{du}{dx} = \frac{y' - u}{x} \rightarrow \int \frac{1}{dx} = \int \frac{1}{u^2 + 2u - u} du$$

$$\ln x + C = \int \frac{1}{u^2 + u} du = \int \frac{1}{u} du - \int \frac{1}{u+1} du$$

$$\ln x + C = \ln u - \ln(u+1)$$

$$Cx = \frac{u}{u+1}$$

$$Cx u + Cx = u$$

$$Cx = u(1 - Cx)$$

$$u = \frac{Cx}{1 - Cx}$$

$$y = \frac{Cx}{1 - Cx} x$$

$$1 = \frac{2C}{1 - 2C} \quad (2)$$

$$1 - 2C = 4C$$

$$1 = 6C$$

$$C = \frac{1}{6}$$

$$y = \frac{\frac{1}{6} x^2}{1 - \frac{1}{6} x} x$$

Problem 2

Problem 2

$$\frac{dy}{dt} = 0.3y \quad a)$$

$$\int \frac{dy}{y} = \int 0.3 dt$$

$$\ln y = 0.3t + C$$

$$y = C e^{0.3t}$$

$$2 = C$$

$$y = 2e^{0.3t}$$

$$b) y(5) = 2e^{1.5} \approx 8.96349$$

$$c) 100 = 2e^{0.3t}$$

$$\frac{\ln(100)}{2} = 0.3t$$

$$t = \frac{\ln(100)}{2 \times 0.3} \text{ days} \approx 76753 \text{ days}$$