Measurements and Experimental Results:
Graphical Analysis

Introduction:

Physics, like all science, relies on comparison between experimental observation and scientific hypothesis. The "closeness" of this comparison tells us how accurate and precise our hypothesis was in predicting the observed phenomenon. Physicists usually employ mathematical models to describe their hypotheses because math is a universal language and it allows all scientists across the world to test hypotheses in as close similarity as possible.

The job of a physicist (or a physics student) is, fundamentally, threefold: 1) to develop mathematical models that, to the best of our knowledge, will have the ability to predict some observable phenomenon, 2) to develop a procedure that produces consistently repeatable measurements associated with that observed phenomenon, and 3) to determine and demonstrate a means of comparison between the mathematical model and the measured data. The physicist then returns to the first step and attempts to refine the mathematical model to be more accurate and precise based on the results of the previous comparison.

This guide is intended to be a primer to help develop understanding for one of the most basic means of comparison between mathematical models and measured data, which we refer to as "graphical analysis". This requires the unification of algebra (and occasionally calculus) techniques with experimental science.

Review of the Pertinent Mathematics:

This is not a math textbook, but this section will help reinforce various mathematical techniques that are integral to understanding the process, the purpose, and the result of graphical analysis. The primary skills you will need for the introductory physics lab courses are algebraic manipulation and plotting. To review algebraic manipulation, please reference a textbook like *College Algebra* (available at UCR's Orbach Library).

Reference Acknowledgement: David Cook, Lawrence University

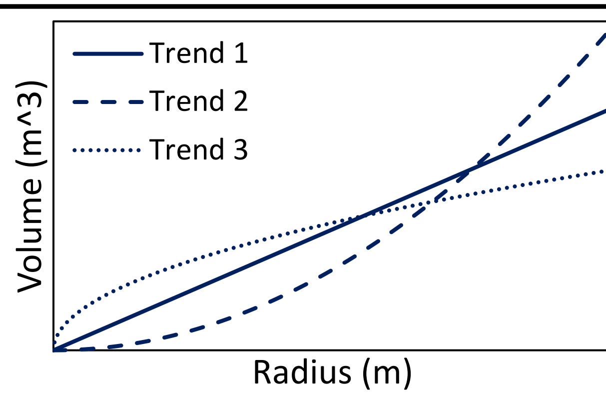
You also need to understand how to relate mathematical constants to physical parameters. For example, the equation for a straight line with non-zero y-intercept looks like: y = A*x + B. In this equation, A and B are called parameters. In this context, these parameters have no *physical* meaning, though they do have *mathematical* meaning (A is "slope" and B is "y-intercept"). In physics, models are developed so that these parameters *do* have physical meaning. For example, you will come across the equation F = m*a, where F is force, m is mass, and a is acceleration. This is a linear equation that looks like y = A*x + B where F = y, m = A, x = a, and B = 0, so it is the same as the mathematical representation, only the terms now have physical meaning, *so they can be measured experimentally!*

In the introductory physics lab courses, you will see several different physical models, but all are characterized by just a few different mathematical models:

Proportional/Linear:	$y = A*x + B$
Quadratic:	$y = A*x^2 + B$
Inverse:	$y = A/x + B = A*x^{-1} + B$
Inverse Square:	$y = A/x^2 + B = A*x^{-2} + B$
Exponential:	$y = A*\exp(B*x) + C$
Sine:	$y = A*\sin(B*x + C)$

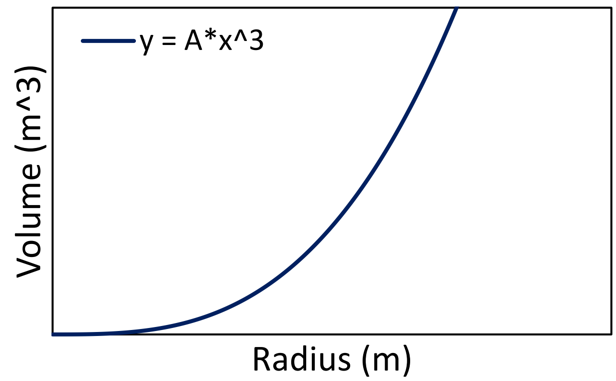
It will be important for you to be able to quickly visualize mathematical functions in graphical form. Make sure you are familiar with each of these functions and their graphical appearance. (*Wolfram Alpha* is a good reference for help.)

Mathematical Models to Describe Physical Hypotheses:



To develop mathematical models that describe physical hypotheses, it is important to be able to visualize the model. For example, if we establish the hypothesis: "*A sphere's volume will increase when the radius is increased*," we infer that the graphical representation of that hypothesis must look something like the figure at right. There are many different ways of graphically representing this hypothesis. But which is correct?

Well, for the hypothesis as it was stated, *all three trends are correct*, because the hypothesis is not detailed enough to tell us what kind of relationship the trend should have! This is where it is crucial to have physics concepts that underlie the mathematical model. We can state a more detailed hypothesis for the example above: *the volume of a sphere is proportional to the cube of the radius*, or mathematically: *Volume* = constant*(*radius*)^3. If we take this mathematical form and plot it, we produce the trend shown at right.

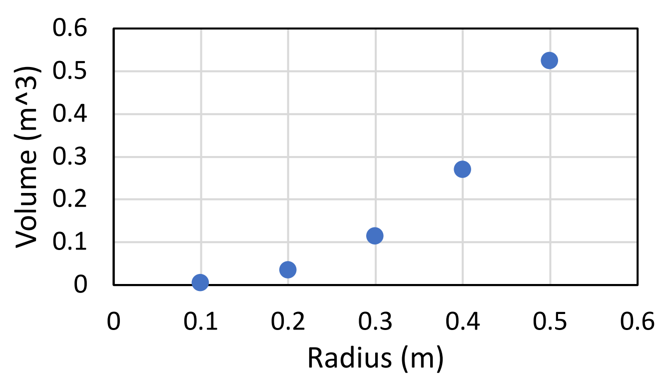


Visualization of Measured Data:

Once you have a sufficiently detailed physical hypothesis and have visualized the mathematical trend that that hypothesis should take, you are ready to collect measurements to test that hypothesis! The visualization of the mathematical trend must come *before* the measurements so that you can then quickly ascertain qualitatively whether or not your measurement procedure has been properly designed or if your hypothesis is clearly invalid.

When you make measurements in any scientific experiment, you are attempting to observe a change in one variable dependent on some controlled change in another variable. Returning to the cube volume example again, to test your physical hypothesis, you might control a change in the side-length of the cube, and you would then measure the resulting change in the cube's volume. You might come up with a data set and accompanying plot that looks like this:

Volume (m^3) *Measured*	Radius (m) *Controlled*
0.004	0.10
0.034	0.20
0.113	0.30
0.268	0.40
0.524	0.50

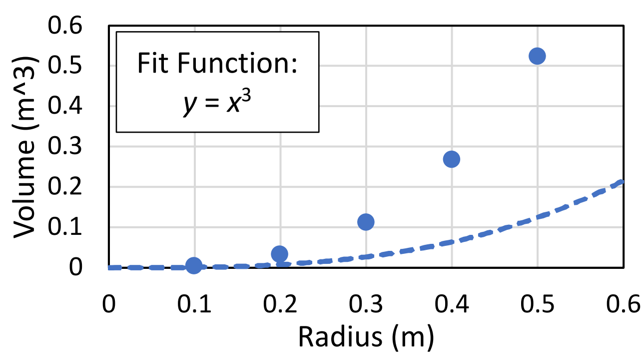


Note the plot convention – always plot your controlled variable on the horizontal axis and your measured variable on the vertical axis. This makes comparisons between mathematical models and measured data more straightforward. The first step, after you have created this visualization of your measured data, is to qualitatively compare your observations with your hypothesis. The hypothesis states that the volume should increase as the side-length is increased, and that it should be a non-linear increase. Both constraints are satisfied, so we can move on to make a more quantitative comparison between measurement and model.

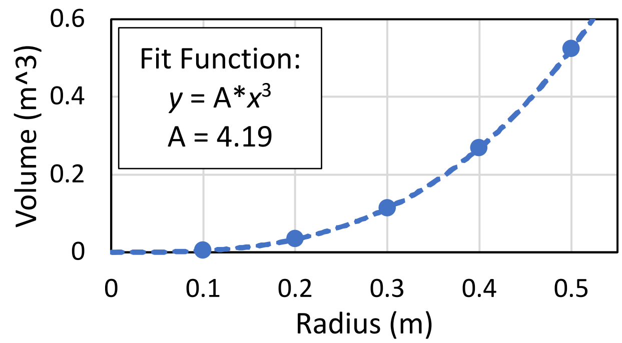
Relating Trends in Measured Data to Physical Hypotheses:

Once you have established a physical hypothesis and acquired some measured data that qualitatively appears to validate that hypothesis, you are ready to move on to the more quantitative comparison between measurement and prediction. To do this, you will usually create a "best-fit line" using some analysis software.

To create the best-fit line, all you have to do is tell your software what kind of mathematical model it should attempt to graph over your data. For the sphere-volume example from above, this means combining the two plots on p. 3 together. Overlaying a mathematical model on top of some measured data is not as trivial as it sounds. This is where the real importance of having well-established parameters is absolutely necessary. If you were just to plot a graph of $y = x^3$ along with your measured data, you would see something that looks like the plot at right. There is clearly no agreement between the model ($y = x^3$) and the measured volumes!



This is why mathematical parameters are so important! Look at the table of different mathematical models on p. 2. You will note that all models include some constants like 'A', 'B', and 'C'. These are the mathematical parameters associated with the model. When you instruct software to generate a best-fit line according to some designated mathematical model, it guesses some initial value of these parameters and then iteratively changes those values until the model represents the measured data as closely as possible. This is why it is called a "best-fit".



The $y = x^3$ model didn't fit our data very accurately because we didn't include any parameters that allow the software to tune the best fit curve. We need to include a parameter that can be adjusted so that the model is a good fit to the data, so the model should actually look like $y = A*x^3$. The result is shown in the figure at right, and the fit is perfect!

The fitting software tells us what value of the parameter yields the closest fit to the data; in this case A = 4.19. This is a *mathematical* value, not a *physical* value! This parameter only has physical meaning if your hypothesis has established it! In this example, we know there must be some value that is multiplied by the cube of the radius, and since it is a sphere, we know that value must include a factor of π. If we divide by π, we get A = 1.33*π, or A = 4/3*π. If we plug this back into our initial model, it looks like *Volume* = 4/3*π*(*radius*)^3, which we know to be valid!

A Final Note of Caution:

When doing experimental physics, you can *never* try to relate a mathematical model to best-fit your measured data without having a physical hypothesis to support the mathematical model! As an example, consider the measured data in the plot below. This is real data that was collected and fit using two different mathematical models. It is impossible to tell which model is "more correct", and both models have very different ramifications on the description of the physics! ALWAYS make sure that you have a physical model underpinning any mathematical trend you seek to fit, otherwise it is scientifically meaningless!