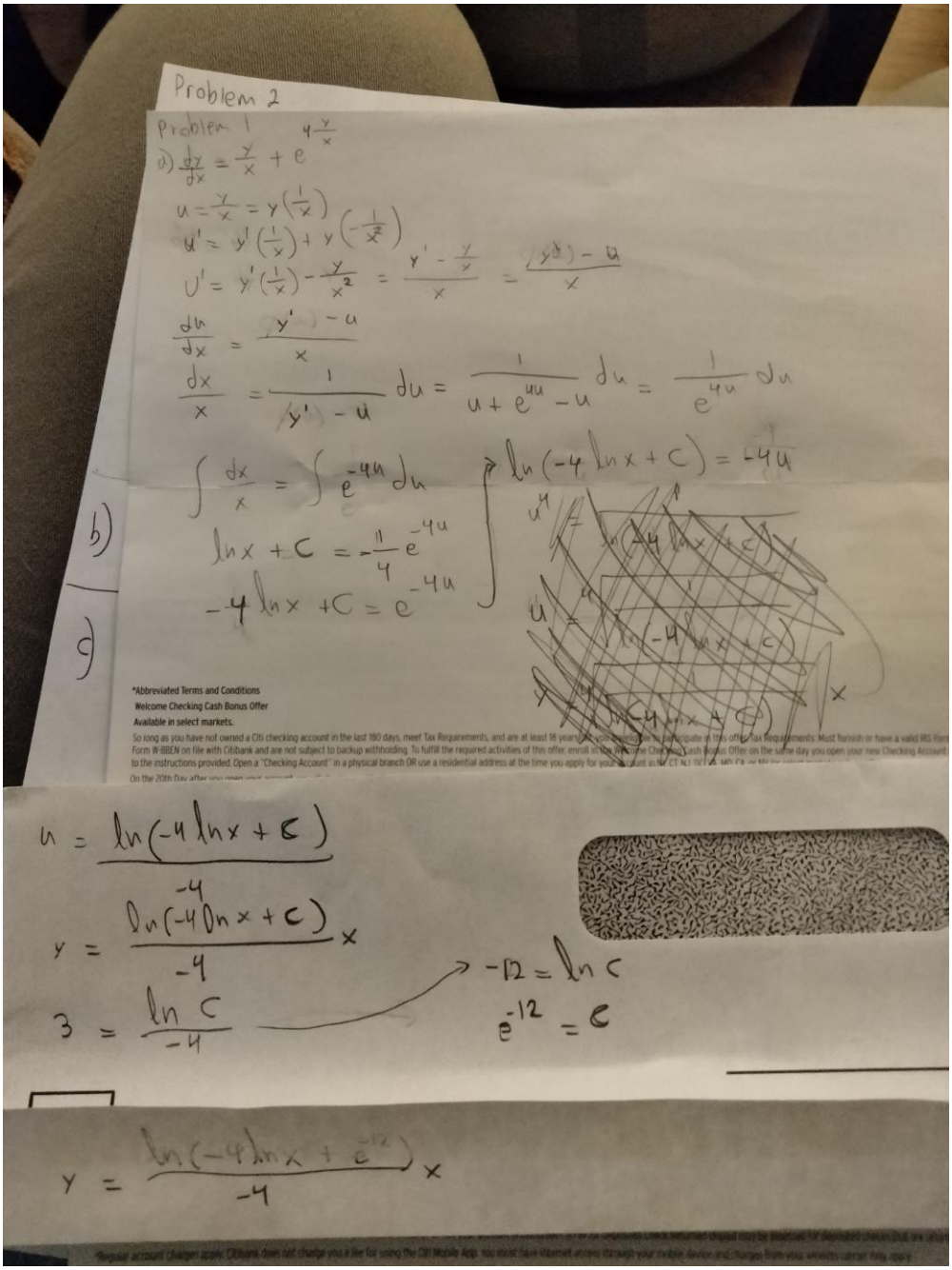
Homework 2 Solutions

Problem 1



Problem 2

b) $xy\frac{dy}{dx} = x^2 + 2y^2$

$$\frac{dy}{dx} = \frac{x^2+2y^2}{xy} = \frac{x\left(x+\frac{2y^2}{x}\right)}{xy} = \frac{x+\frac{2y^2}{x}}{y} = \frac{x}{y} + \frac{2y}{x}$$

$$u = \frac{y}{x} \rightarrow \frac{du}{dx} = \frac{y' - u}{x} \Rightarrow \int \frac{1}{dx} = \frac{1}{\frac{1}{u} + 2u - u} du = \int \frac{1}{\frac{1}{u} + u} du$$

$$\ln x + C = \int \frac{1}{\frac{1}{u}(1+u^2)} du = \int \frac{u}{1+u^2} du$$

$v = 1 + u^2$

$\frac{dv}{du} = 2u \rightarrow du = \frac{dv}{2u}$

$$= \int \frac{u}{v}\frac{dv}{2u} = \frac{1}{2}\int \frac{dv}{v}$$

$$= \frac{1}{2}\ln v$$

$$\ln x + C = \frac{1}{2}\ln(1+u^2)$$

$$e^C e^{\ln x} = e^{\frac{1}{2}\ln(1+u^2)}$$

$$Cx = (1+u^2)^{1/2}$$

$$(Cx)^2 = 1 + u^2$$

$$Cx^2 - 1 = u^2$$

$$u = \sqrt{Cx^2 - 1}$$

$$y = \sqrt{Cx^2 - 1}\ x$$

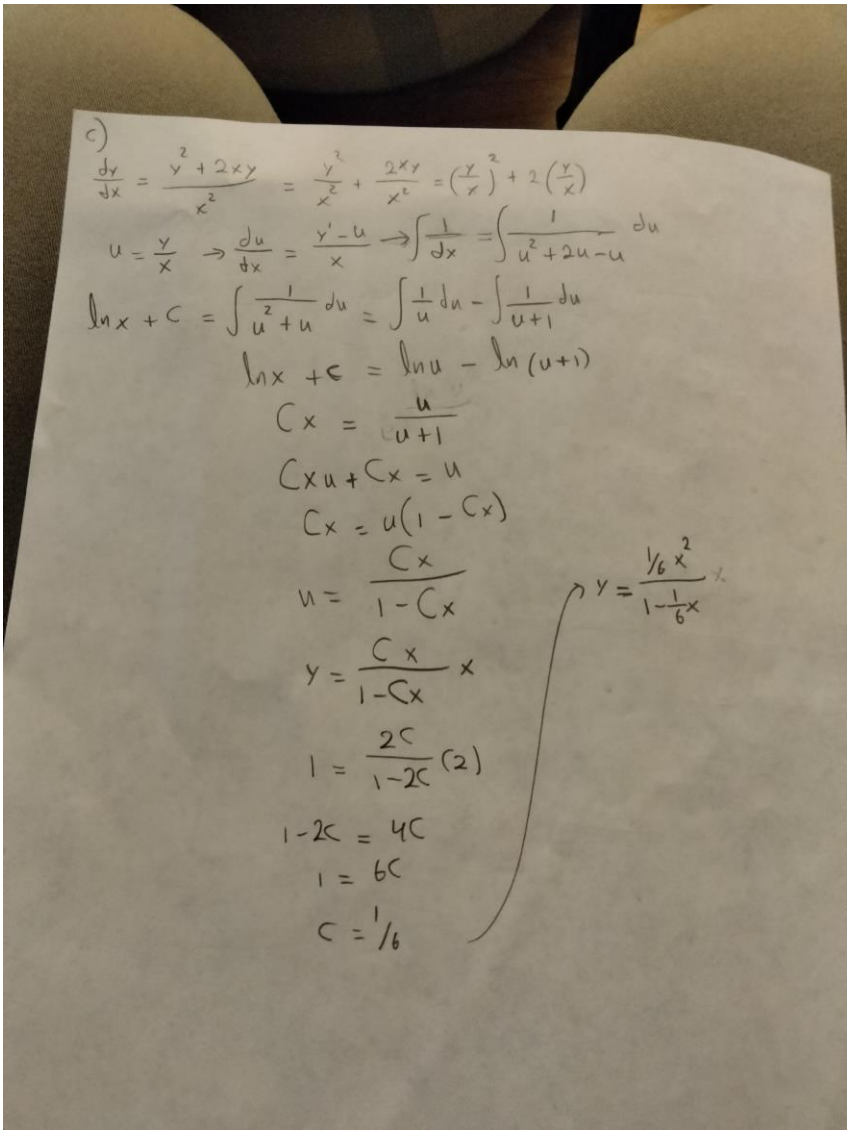
$$9 = -C + 1$$

$$8 = -C$$

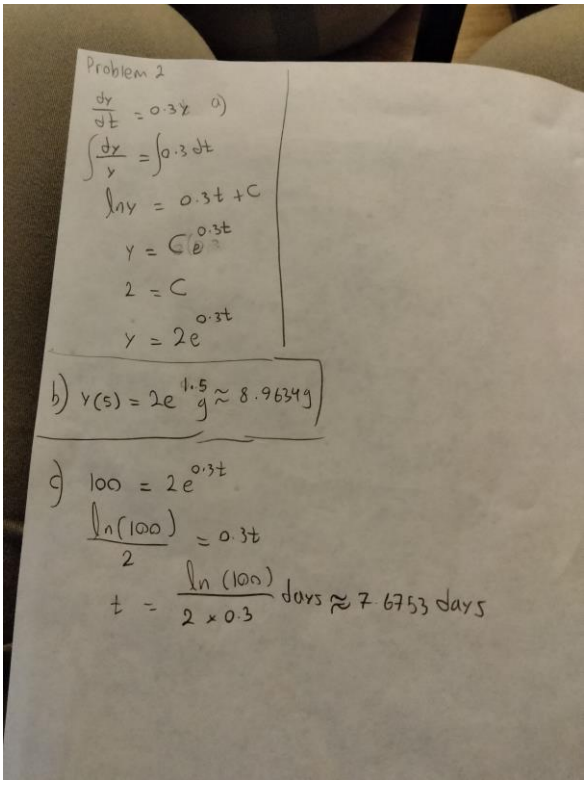
$$C = -8$$

$$C = -8$$

$$y = \sqrt{-8x^2 - 1}\ (x)$$


c)
$\frac{dy}{dx} = \frac{y^2 + 2xy}{x^2} = \frac{y^2}{x^2} + \frac{2xy}{x^2} = \left(\frac{y}{x}\right)^2 + 2\left(\frac{y}{x}\right)$
$u = \frac{y}{x} \rightarrow \frac{du}{dx} = \frac{y' - u}{x} \rightarrow \int \frac{1}{dx} = \int \frac{1}{u^2 + 2u - u} du$
$\ln x + C = \int \frac{1}{u^2 + u} du = \int \frac{1}{u} du - \int \frac{1}{u+1} du$
$\ln x + C = \ln u - \ln(u+1)$
$Cx = \frac{u}{u+1}$
$Cxu + Cx = u$
$Cx = u(1 - Cx)$
$u = \frac{Cx}{1 - Cx}$
$y = \frac{Cx}{1 - Cx} x$
$1 = \frac{2C}{1 - 2C}(2)$
$1 - 2C = 4C$
$1 = 6C$
$C = 1/6$
$y = \frac{1/6\, x^2}{1 - \frac{1}{6}x}$

Problem 2



Problem 2

$\frac{dy}{dt} = 0.3y$ a)

$\int \frac{dy}{y} = \int 0.3\,dt$

$\ln y = 0.3t + C$

$y = Ce^{0.3t}$

$2 = C$

$y = 2e^{0.3t}$

b) $y(5) = 2e^{1.5} g \approx 8.9634 g$

c) $100 = 2e^{0.3t}$

$\frac{\ln(100)}{2} = 0.3t$

$t = \frac{\ln(100)}{2 \times 0.3}$ days ≈ 7.6753 days