NAME: SID:

Problem 1: (a) Give a complete statement of the Master Theorem.

(b) Give asymptotic solutions for the following recurrences:

$$f(n) \;=\; 3f(n/3) + n$$

$$f(n) \;=\; 2f(n/3) + n$$

$$f(n) \;=\; 4f(n/3) + n$$

Problem 2: We have a group of people, each of which is a citizen of either US or Mexico or Canada. Half of the people in this group are US citizens, 10 are Mexican citizens, 17 are Canadian citizens, 4 people have dual US-Mexican citizenship, 5 have US-Canadian citizenship, 6 have Canadian-Mexican, and 2 are citizens of all three countries. How many people are in this group? Show your work.

Problem 3: In (a) and (c) draw a 5-vertex graph with given degrees or justify that such graph does not exist.

(a) Degrees: $3, 4, 2, 2, 1$

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