Quiz 1 Solutions (version A)

Solution 1:

Pseudo-code	Running time	Justification

for $i \leftarrow 1$ **to** $3n^2$ **do**
    $x \leftarrow x^2$
for $j \leftarrow 1$ **to** $n+3$ **do**
    $z \leftarrow x + z$
	$\Theta(n^2)$	Two independent loops with running times $\Theta(n^2)$ and $\Theta(n)$.

for $i \leftarrow 1$ **to** n **do**
  $j \leftarrow 1$
  while $j < n$ **do**
      $j \leftarrow 4j$
      $x \leftarrow j \cdot x$
	$\Theta(n \log n)$	The external loop makes n iterations. For each iteration of the external loop, the internal loop makes $\Theta(\log n)$ iterations.

for $i \leftarrow 1$ **to** n^2 **do**
  $k \leftarrow 1$
  while $k < n$
      $x \leftarrow x^2$
      $k \leftarrow k + 3$
	$\Theta(n^3)$	The external loop makes $\Theta(n^2)$ iterations. For each iteration of the external loop, the internal loop makes $\Theta(n)$ iterations.

for $i \leftarrow n/2$ **to** n **do**
  $x \leftarrow 2x - 1$
  for $j \leftarrow 1$ **to** $2i$ **do**
      $x \leftarrow 2j \cdot x$
	$\Theta(n^2)$	For any given i, the internal loop makes $2i$ iterations. As i ranges from $n/2$ to n, these numbers will add up to $\Theta(n^2)$ (the sum of an arithmetic sequence).

$k \leftarrow 1$
for $i \leftarrow 1$ **to** n **do**
  while $k < 9i$ **do**
      $k \leftarrow k + 1$
      $x \leftarrow x^2$
	$\Theta(n)$	The external loop makes n iterations. For each i, the while loop will make only 9 iterations.

Solution 2: (a) Fermat's Theorem: If p is a prime and $a \in \{1, 2, ..., p-1\}$ then $a^{p-1} \equiv 1 \pmod{p}$.

(b) Computing modulo 19, we get

$$3^{1895} = 3^5 \cdot (3^{18})^{105} = 3^5 = 243 = 15.$$

Solution 3:

(a) If a is prime and a is a divisor of bc then a is a divisor of b or c. $\boxed{\text{TRUE}}$ FALSE

The factorization of bc is the product of the factorizations of b and c. So if a appears in the factorization of bc, it must appear either in the factorization of b or in the factorization of c.

(b) If a and b are divisors of c then ab is a divisor of c TRUE $\boxed{\text{FALSE}}$

For example, take $a = b = 4$ and $c = 8$. Then a and b are divisors of c, but $ab = 16$ is not a divisor of 8.

(c) $\gcd(ab, c) = \gcd(a, c) \cdot \gcd(b, c)$. TRUE $\boxed{\text{FALSE}}$

For example, take $a = b = c = 2$. Then $\gcd(ab, c) = 2$, $\gcd(a, c) = 2$ and $\gcd(b, c) = 2$. So the equality above does not hold.

(d) $\gcd(a + b, b) = \gcd(a, b)$. $\boxed{\text{TRUE}}$ FALSE

x is a common divisor of $a + b$ and b if and only if it is a common divisor of a and b. So pairs $(a + b, b)$, (a, b) have the same sets of common divisors, which implies (d).
