NAME: SID:

Problem 1: For each question (a), (b), (c) below, give the formula for the number of lottery draws and a brief justification (at most 15 words). You do not need to compute the numerical value.

(a) A draw of MegaLoser lottery consists of 5 different numbers from the range $1, 2, ..., 49$. The ordering of the numbers does not matter. (For example, draw $9, 1, 29, 41, 3$ is the same as $3, 1, 9, 41, 29$.) Give the formula for the number of different draws of MegaLoser lottery.

Answer: $\binom{49}{5} = \frac{49!}{5!\,44!}$

Justification: This is the number of 5-element subsets from 49 elements.

(b) A draw of HopelessLotto lottery consists of 5 different numbers from the range $1, 2, ..., 49$. The ordering of the numbers matters. (For example, draw $9, 1, 29, 41, 3$ is different from $3, 1, 9, 41, 29$.) Give the formula for the number of different draws of HopelessLotto lottery.

Answer: $\frac{49!}{44!} = 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45$

Justification: This is the number of 1-1 functions mapping 5 elements to 49 elements.

(c) A draw of BetYourPaycheck lottery consists of 5 numbers (not necessarily different) from the range $1, 2, ..., 49$. The ordering of the numbers matters. (For example, draw $9, 1, 29, 41, 1$ is different from $1, 1, 9, 41, 29$.) Give the formula for the number of different draws of BetYourPaycheck lottery.

Answer: 49^5

Justification: This is the number of all functions mapping 5 elements to 49 elements.

Problem 2: Determine the numerical values of the expressions below. You need to show your work to get credit.

$$1 + 2 + 3 + ... + 29 + 30 = \tfrac{1}{2} \cdot 30 \cdot (30+1) = 465.$$

$$\sum_{i=0}^{5} 10^i = \frac{10^6 - 1}{10 - 1} = \frac{999,999}{9} = 111,111.$$

$$\log_3 81^5 = 5 \cdot \log_3 81 = 5 \cdot 4 = 20.$$

$$\binom{13}{10} = \frac{13!}{10!\,3!} = \frac{11 \cdot 12 \cdot 13}{6} = 286.$$

$$\sum_{i=0}^{\infty} (1/4)^i = \frac{1}{1 - 1/4} = 4/3.$$

Problem 3: For each of the statements below, determine whether it is true or false. Give a brief justification of your answer (at most 10 words). *Note:* to discourage guessing, incorrect T/F answers will receive negative credit.

statement	T/F	justification
$\exists\, x \in \mathbb{Z} \,:\, x^4 + x^3 - 2x^2 - 1 = 0$	F	Only candidates for integral roots are $1, -1$, and neither works
$\exists\, x \in \mathbb{R} \,:\, 2^{x^2} < 1$	F	$x^2 \geq 0$, so $2^{x^2} \geq 2^0 = 1$
$\forall\, x \in \mathbb{R} \,:\, x^2 - x + 1 > 0$	T	The discriminant is -3 (negative)
$\forall\, x \in \mathbb{R}\ \exists\, y \in \mathbb{R} \,:\, y^2 + y = x^2$	T	This is a quadratic equation in y. The discriminant is $1 + 4x^2$, which is always positive
$\exists\, x \in \mathbb{R}\ \forall\, y \in \mathbb{R} \,:\, x^2 y - 2y = 0$	T	Taking $x = \sqrt{2}$, the left-hand side is 0 for all y

Reminders:

- $\mathbb{R}$ denotes the set of all real numbers.
- $\mathbb{Z}$ denotes the set of all integers.
- $\forall$ denotes the universal quantifier ("for all") and $\exists$ denotes the existential quantifier ("there exists").