Optimization Methods - Project Report

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1 Question 1

In developing a linear optimization model to determine the most efficient hub-and-spoke configuration for FreshFarm, we first consider the various elements that contribute to the total distance traveled by packages. The distance between each pair of cities, denoted as d_{ij} plays a critical role in the computation. Given f_{ij}, the number of packages to be shipped from city i to city j, we aim to minimize the overall distance each package travels.

Our proposed model takes into account a network of N cities with a variable number of hubs K that can range from 1 to (N - 1). To calculate the cost-effectiveness of transporting packages through hubs, we apply a discount factor α to the distances between hubs. This discount factor is less than or equal to 1, reflecting economies of scale for large volume inter-hub shipping, which decreases the cost per package.

$$\begin{aligned} \min_{x,h} \quad & \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=0}^{N}\sum_{l=0}^{N} f_{ij}x_{ik}x_{jl}(d_{ik} + \alpha * d_{kl} + d_{jl}) \\ \text{s.t.} \quad & \sum_{k=0}^{N} h_k = K \\ & \sum_{j=0}^{N} x_{ij} = 1 \quad \forall i \in N \\ & x_{ik} <= h_k \quad \forall i,k \in N \end{aligned} \tag{1}$$

- h_k - indicates whether city k is a hub, binary variable
- x_{ik} - indicates whether city i uses hub k, binary variable

The first constraint puts an upper limit on the number of hubs to K. The second constraint ensures that each city is connected to one and only one hub. The third constraint ensures that a city cannot be connected to a city that is not a hub.

Through this formula, we are optimizing the selection of hubs and the paths taken from origin to destination cities, ensuring that the total travel distance, adjusted for volume and discounted for inter-hub economies, is minimized. This optimization allows FreshFarm to leverage both direct city-to-city shipments for high volume routes and consolidated shipments through hubs for lower-volume routes, reducing the overall cost of their delivery network. We will be using the Gurobi library for Python to solve the LP.

2 Question 2

2.1 Part 1

Based on the provided dataset and the criteria set forth, we executed the optimization problem for a network consisting of a specific number of cities, with the goal of identifying the most efficient hub-and-spoke system with $K = 2$ hubs. Utilizing the given coordinates and the estimated numbers of packages to be transported, we applied the Euclidean distance formula for the calculations, and incorporated a discount factor $\alpha = 0.75$.

```
K = 2, alpha = 0.75
Objective value:           13531340.148177857
Point-to-Point Distance: 9501054.112511646

Chosen hubs (in city indices):
2
16

Routes:
City 0 -> Hub 16
City 1 -> Hub 16
City 2 -> Hub 2
City 3 -> Hub 16
City 4 -> Hub 16
City 5 -> Hub 2
City 6 -> Hub 2
City 7 -> Hub 2
City 8 -> Hub 2
City 9 -> Hub 2
City 10 -> Hub 2
City 11 -> Hub 16
City 12 -> Hub 2
City 13 -> Hub 2
City 14 -> Hub 2
City 15 -> Hub 2
City 16 -> Hub 16
```

2.2 Part 2

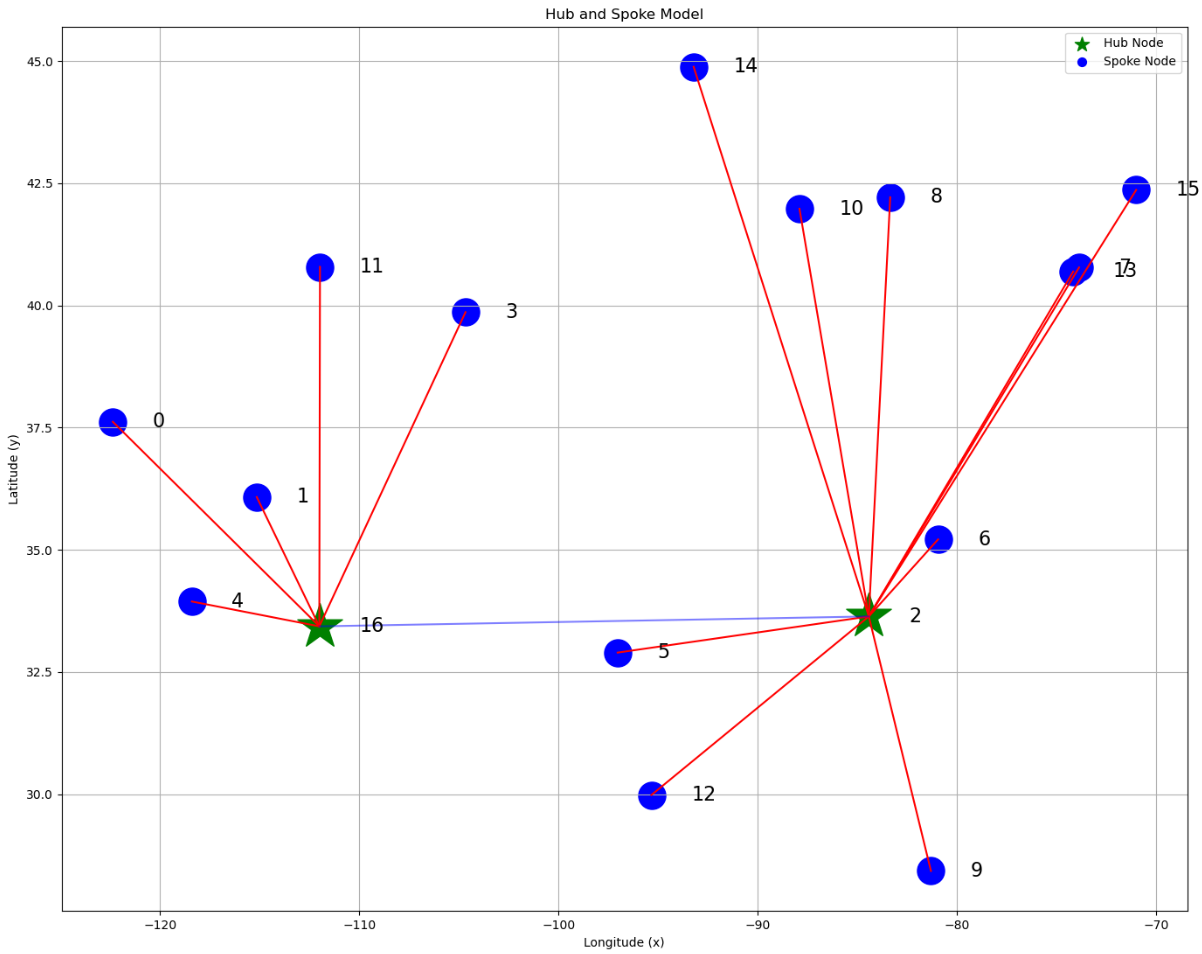

Figure 1: K=2

2.3 Part 3

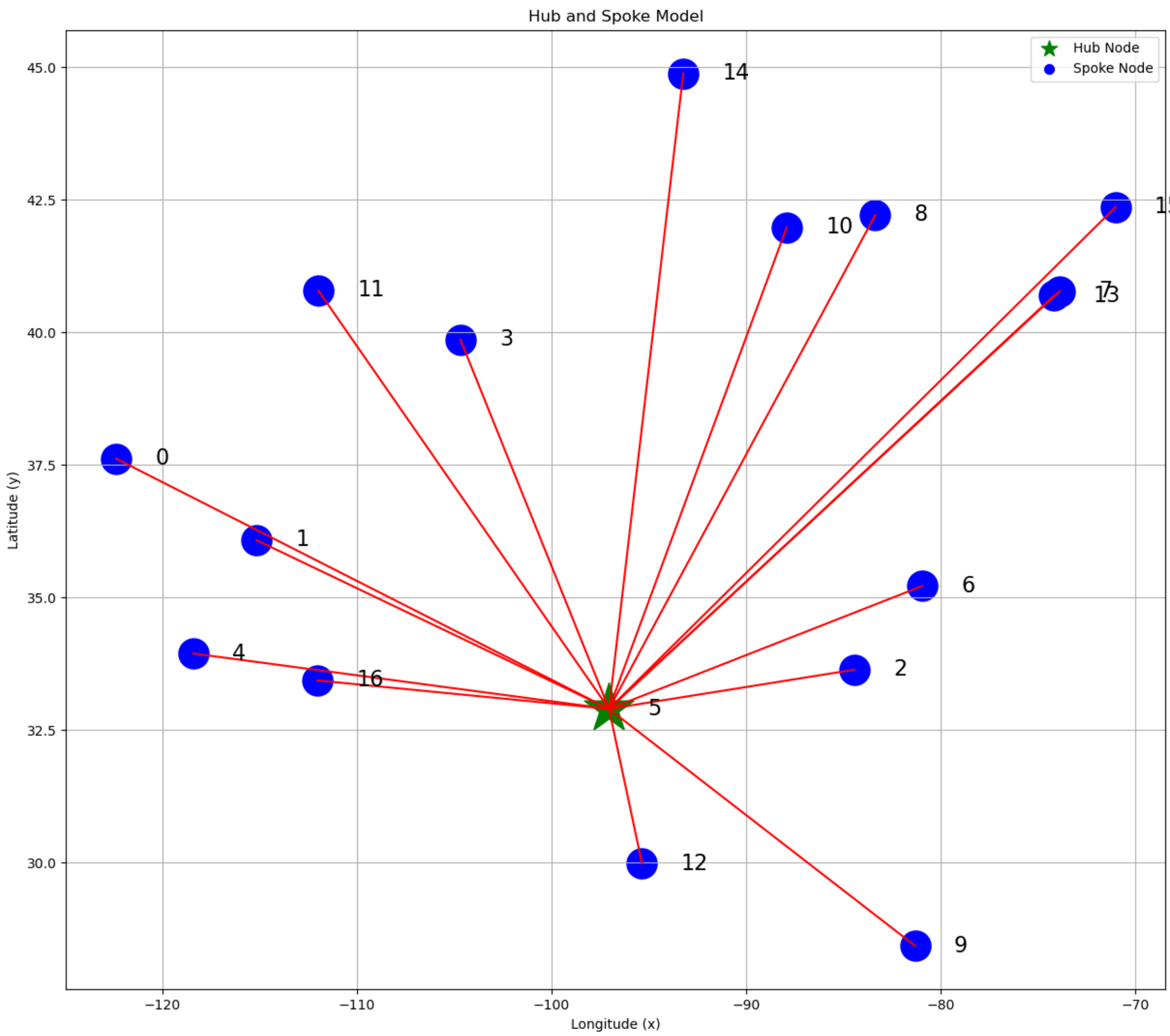

Figure 2: K=1

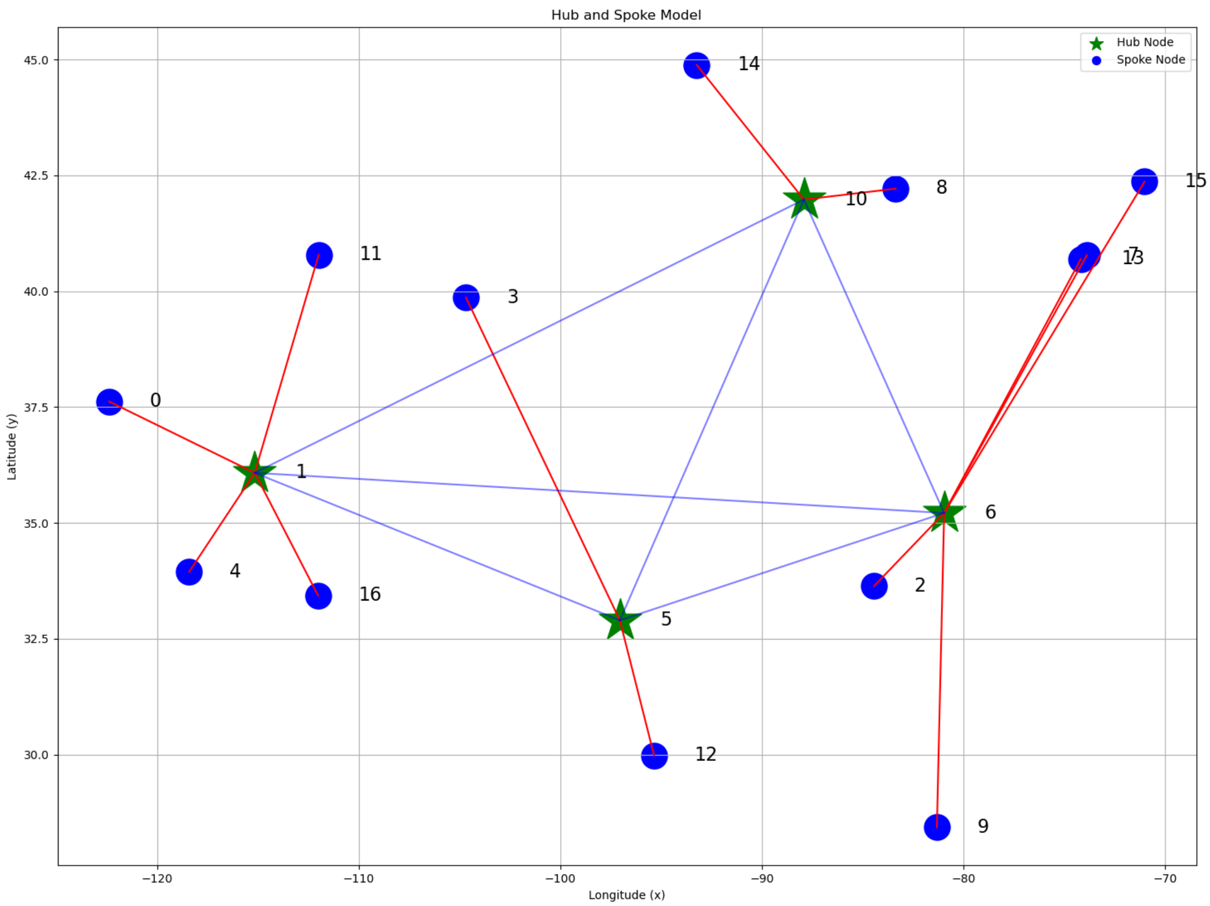
Hub and Spoke Model
Hub Node
Spoke Node
Latitude (y)
Longitude (x)
45.0
42.5
40.0
37.5
35.0
32.5
30.0
−120
−110
−100
−90
−80
−70
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

Figure 3: K=4

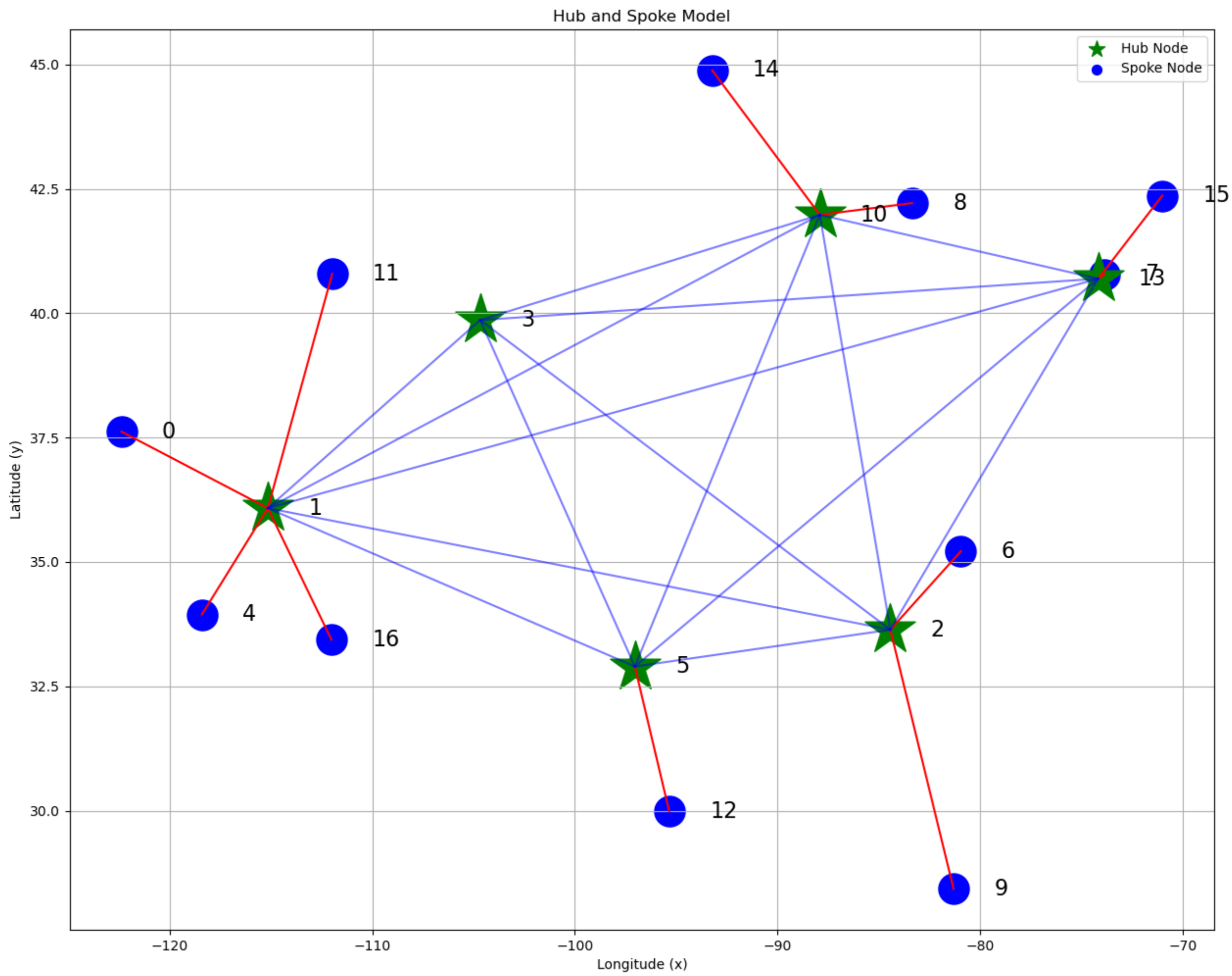
Hub and Spoke Model
Hub Node
Spoke Node
45.0
42.5
40.0
37.5
35.0
32.5
30.0
Latitude (y)
−120
−110
−100
−90
−80
−70
Longitude (x)
14
10
8
15
7
13
11
3
0
1
6
4
16
2
5
12
9

Figure 4: K=6

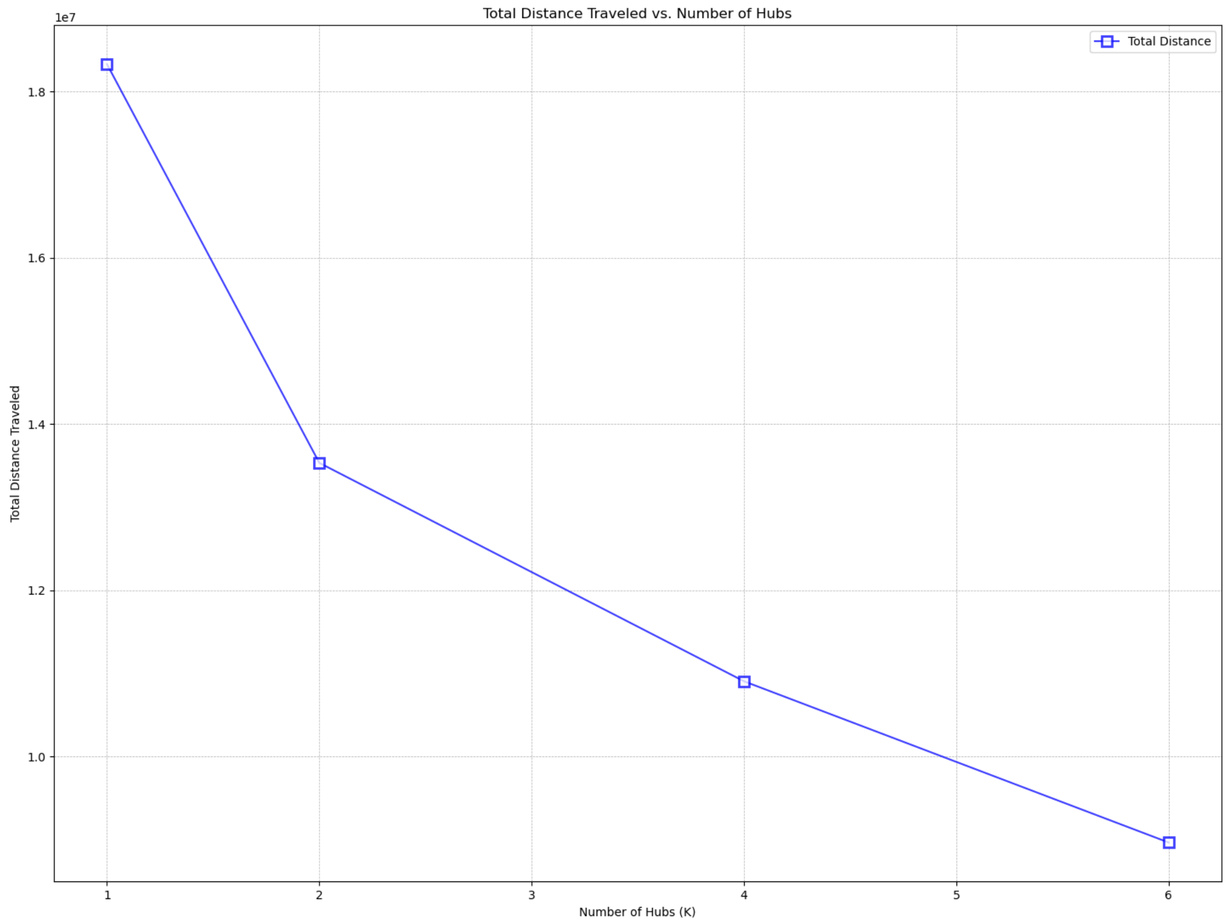

Figure 5: Relation between Number of Hubs and Total Distance Covered

number of hubs = [1, 2, 4, 6]
total distance = [1.8327569109521568e+07, 1.3531340148177857e+07, 1.0904054497059487e+07, 8964540.32239324]

3 Question 3

In the context of FreshFarm's hub-and-spoke network, the introduction of a flat handling cost c per package at each intermediate hub brings a new dimension to the optimization of the network. These handling costs are the operational overheads associated with the processes of unloading, transferring, and reloading packages at each hub. The overheads encompass labor costs, administrative duties, and time management, which are crucial when ensuring the efficiency of package handling, especially for a business dealing with perishable goods.

As the number of hubs increases, it is expected that more packages will pass through these intermediate points, thus incurring additional handling costs. However, as the network becomes more complex with more hubs, the total travel distance can decrease because packages can be routed more directly to their final destinations.

Here are the costs per package for different K and the graphical representation of the total:

```
1.72326047125996 * c per package for K = 1
1.49425028761344 * c per package for K = 2
1.30665644041075 * c per package for K = 4
0.99649068984618 * c per package for K = 6
```

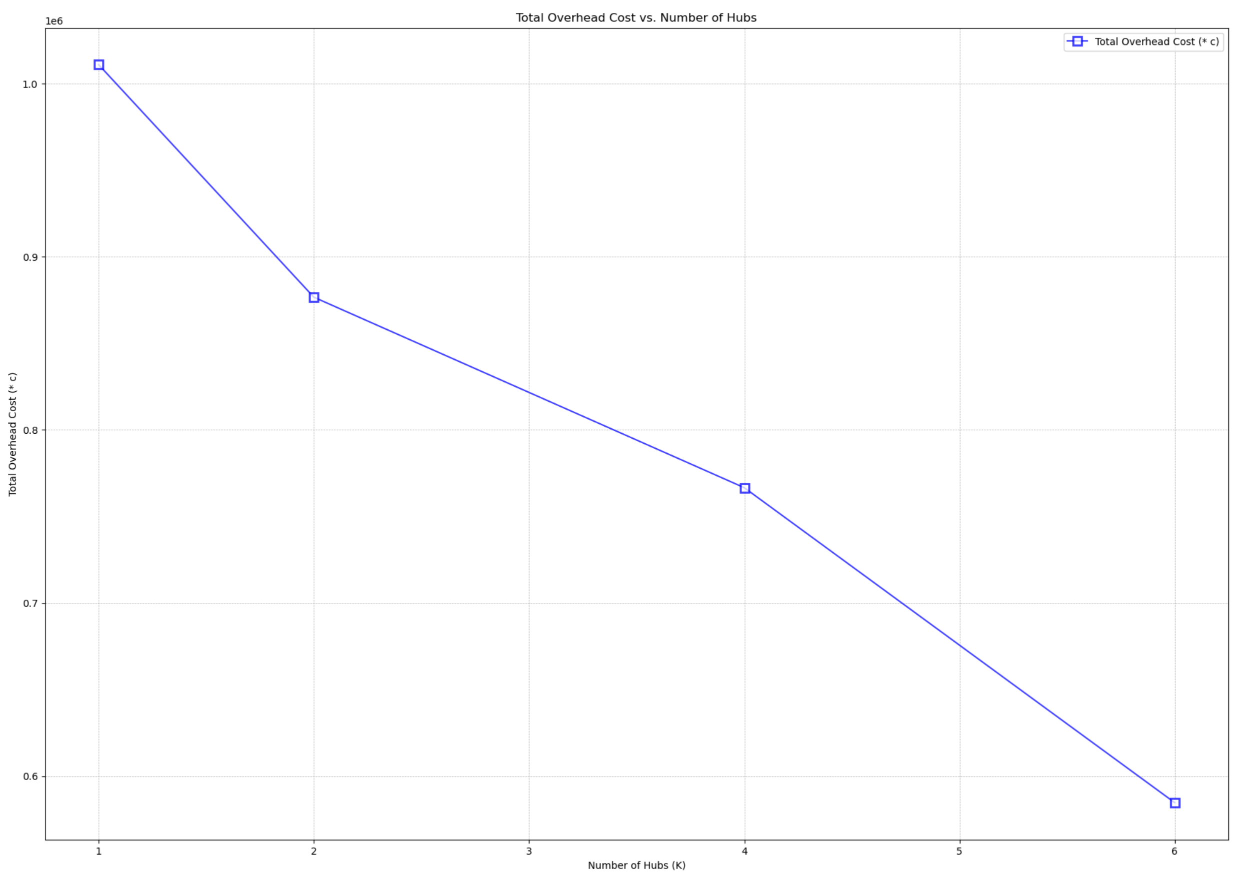

Figure 6: Relation Between Number of Hubs and Total Overhead Cost

4 Part 4

4.1 Part 1

$$
\begin{aligned}
\min_{x,h} \quad & \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=0}^{N}\sum_{l=0}^{N} f_{ij}x_{ik}x_{jl}(d_{ik}+\alpha * d_{kl}+d_{jl}) + \sum_{i\neq k}^{N}\sum_{j\neq l}^{N}\sum_{k\neq l}^{N}\sum_{l\neq k}^{N} f_{ij}x_{ik}x_{jl}c \\
\text{s.t.} \quad & \sum_{k=0}^{N} h_k = K \\
& \sum_{j=0}^{N} x_{ij} = 1 \quad \forall i \in N \\
& x_{ik} <= h_k \quad \forall i,k \in N
\end{aligned}
\tag{2}
$$

The enhanced linear optimization model now integrates the overhead costs associated with handling packages at intermediate hubs into the objective function. Our approach accounts for the distance packages travel through the network and the costs incurred at each stop. The objective function, as depicted in the provided solution, consists of the original distance component from our initial model and the overhead cost component, which represents the new aspect of our model. Here, we include the handling cost per package c at each intermediate hub for all packages. We sum the handling costs over all possible routes and hubs by taking the number of packages f_{ij} that are routed through any two hubs that are not equal (k and l) into account, multiplied by the binary variables x_{ik}, x_{jl} that dictate whether a hub is used in the route of a package.

By minimizing the sum of these two components, we aim to find the most cost-efficient configuration for FreshFarm's distribution network. The total cost to minimize is not just the distance traveled but also the handling costs at each hub. The final output will dictate the optimal number of hubs and routing of packages that result in the least possible total cost, aligning with the company's goal to minimize expenses while maintaining an efficient delivery system.

4.2 Part 2

```
Total Distance Traveled:
14839540.15724741 for c = 0
14839540.15724741 for c = 10
14839540.15724741 for c = 20
14839540.15724741 for c = 30
18327569.10952155 for c = 50
18327569.10952155 for c = 100
```

When incorporating the flat cost per package, c, for handling at each intermediate stop into the optimization model for a network with $K = 2$ hubs, the total distance traveled by packages remains constant at 14839540.157 when c varies from 0 to 30. This suggests that within this range, the optimal network design does not change; the routing is such that the additional handling costs do not influence the selection of intermediate hubs or the paths of package delivery. The model is likely finding a configuration where packages are either not passing through intermediate hubs or the number of packages passing through these hubs is not sufficient to influence the overall cost to a point where a different network design is more optimal.

However, when c is increased to 50 or beyond, we observe an increase in the total distance traveled to 18327569.1096. This significant jump indicates that the cost of handling at intermediate stops has become large enough to influence the network configuration, likely leading to a design that minimizes the use of intermediate hubs, despite increasing the point-to-point travel distance. In this case, the network design shifts to avoid incurring the higher handling costs, even though it results in a longer overall travel distance for the packages.

The insights from this exercise highlight the trade-offs between minimizing travel distances and managing the costs associated with intermediate stops. For lower values of c, the network's priority can be to minimize distance, which could imply a more centralized use of hubs. However, as c increases, the cost of intermediate handling becomes a more significant factor, and the network design may favor more direct routes, even at the expense of increased travel distances. This adjustment reflects a balance between the logistical efficiency of hub use and the operational costs associated with package handling at these hubs. This is visually clear in the following figures.

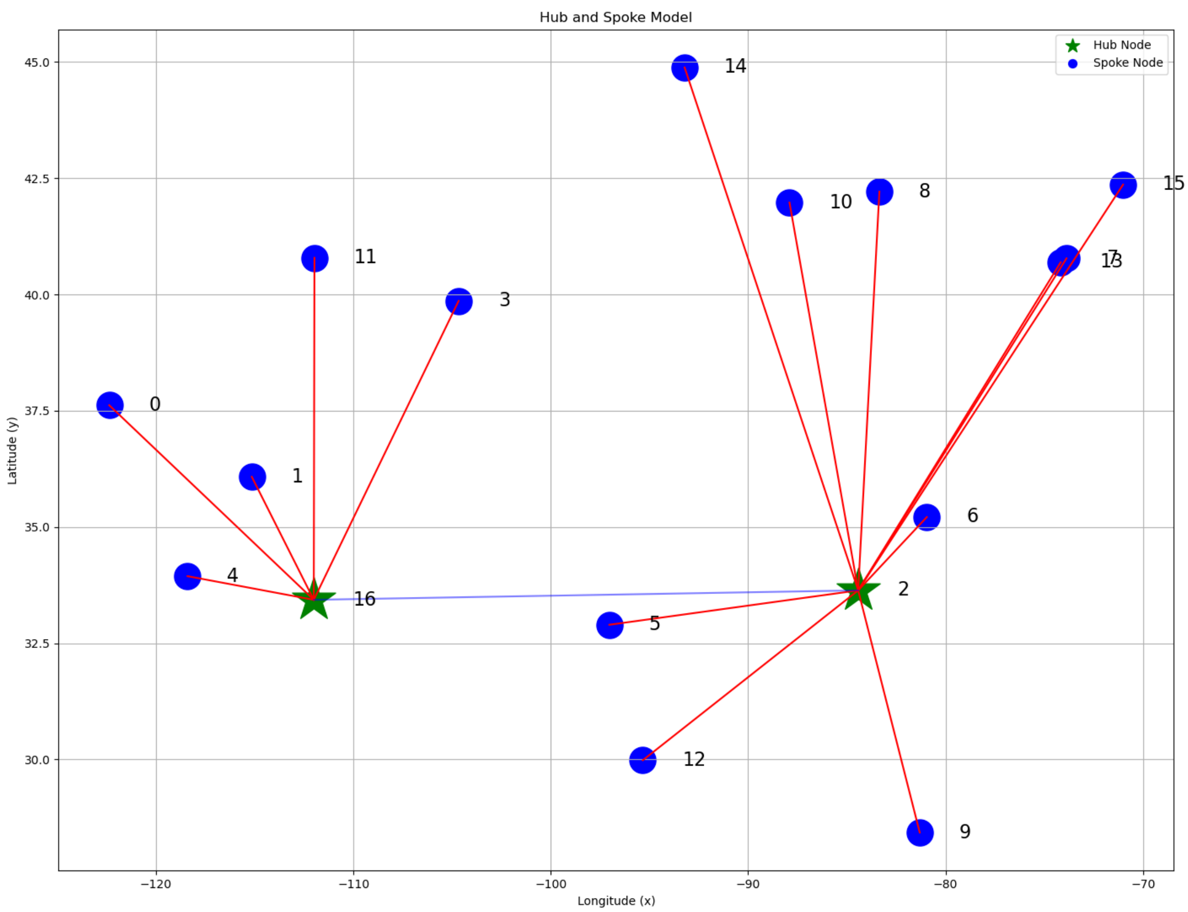

Figure 7: Resulting Configuration for $c = 0, 10, 20, 30$

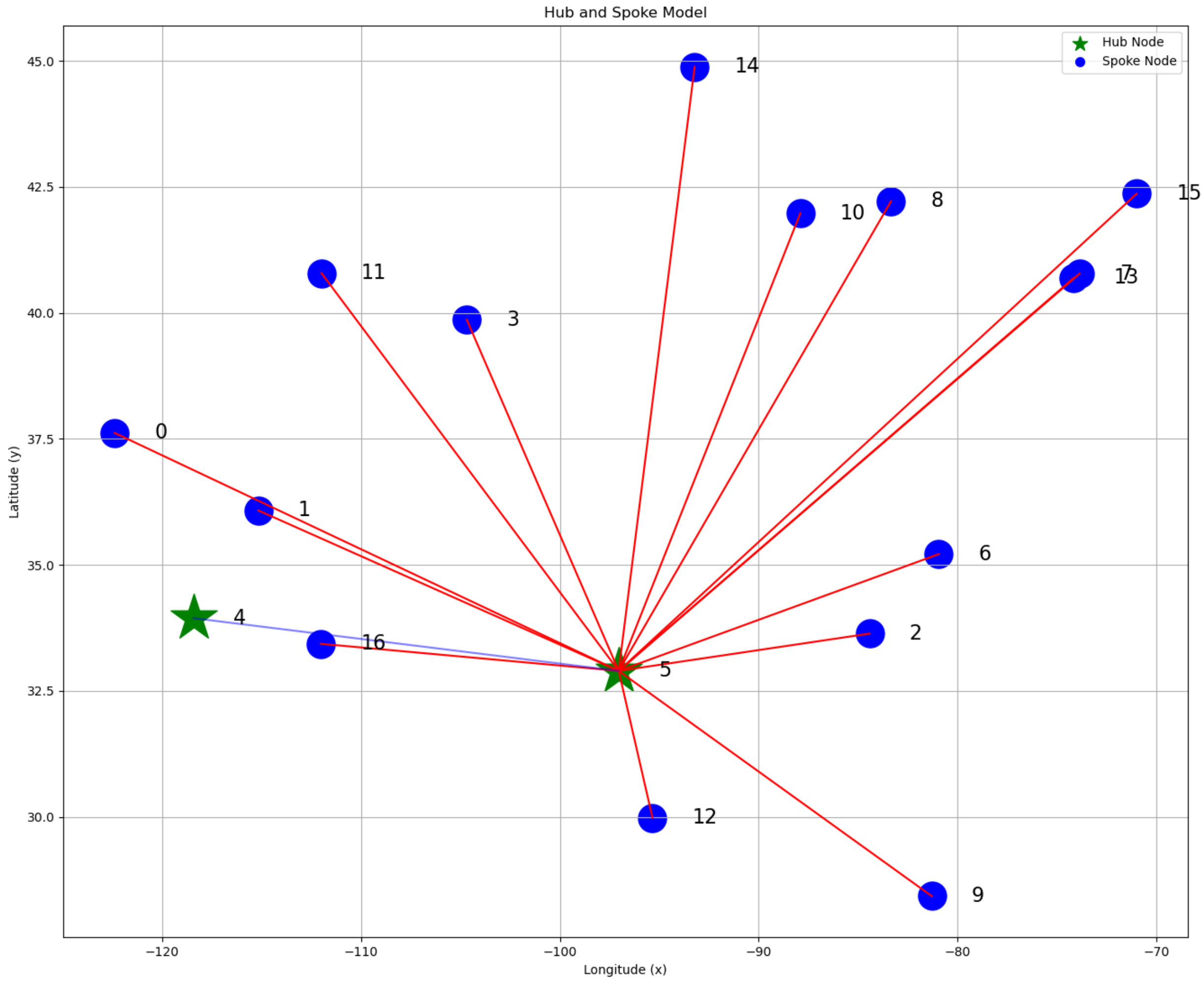

Figure 8: Resulting Configuration for $c = 50, 100$

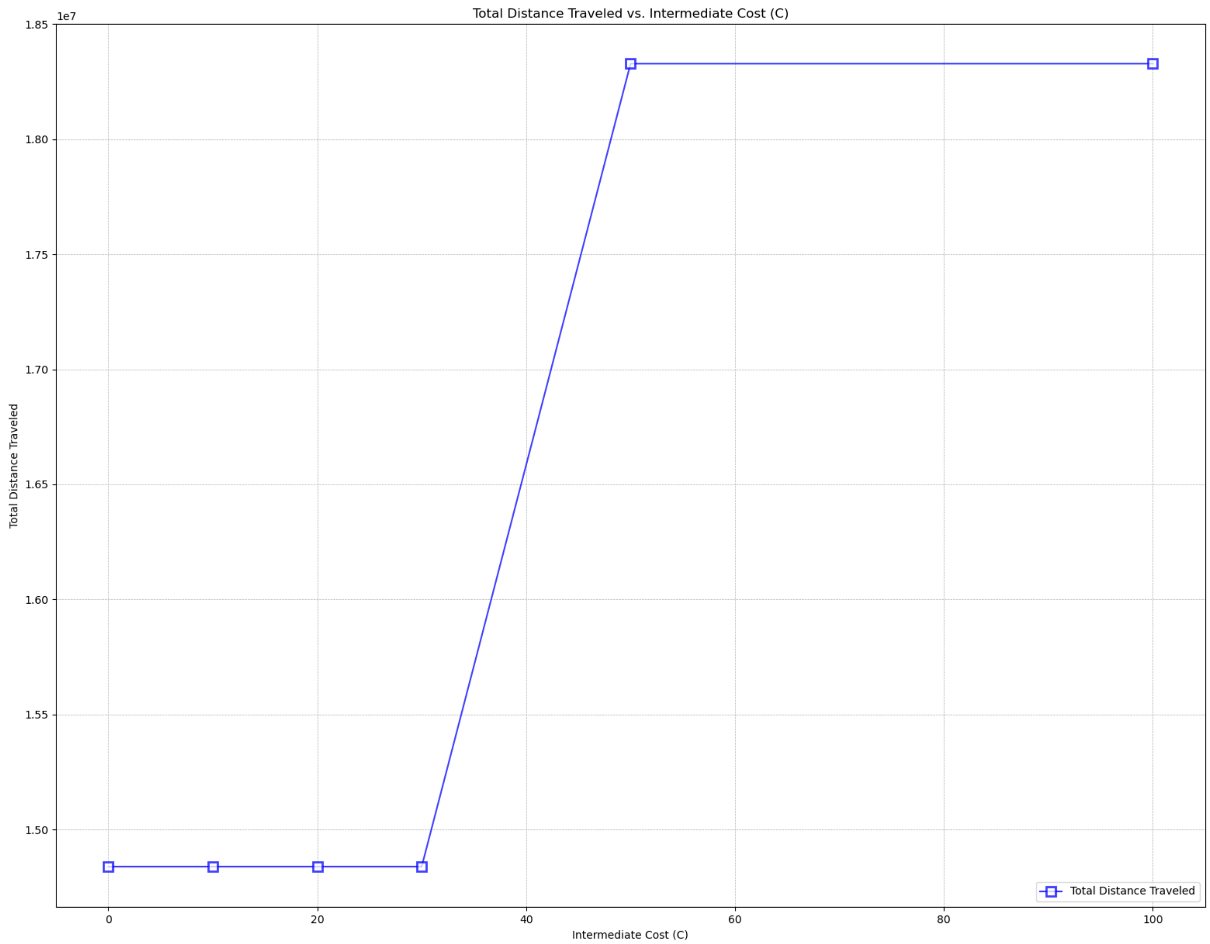

Figure 9: Relation Between Intermediate Cost and Total Distance Traveled

5 Question 5

Given our findings so far and without any further context, the hub-and-spoke model seems to offer potential advantages in terms of cost savings. However, it's essential to also consider other operational, logistical, and service-related aspects before making a final decision. Implementing a hub-and-spoke model might bring in economies of scale and operational efficiencies, but the specifics of FreshFarm's operations, distribution needs, and customer expectations would also play a significant role in determining the best approach.

The hub-and-spoke model offers several advantages, particularly for large-scale operations like FreshFarm, which needs to manage the logistical complexities of delivering perishable goods across a vast network of cities:

1. Economies of Scale: Centralizing operations in a hub allows for bulk handling of goods, leading to cost efficiencies. It's often cheaper to transport large quantities of goods to a central location and then distribute them, rather than having multiple direct routes.

2. Consolidation and Deconsolidation: Goods coming from various source cities can be consolidated at the hub, sorted, and then sent to their respective destination cities. This facilitates efficient packing and reduces the number of half-empty shipments.

3. Reduction in Route Complexity: Without a hub-and-spoke system, the number of routes can increase exponentially with the addition of each new city. With a hub, the number of routes is linear, reducing complexity.

4. Inventory Management: With a centralized hub, it becomes easier to manage inventory, as goods can be stored temporarily in the hub if needed. This is particularly useful for perishable goods, where cold storage can be maintained.

5. Flexibility and Responsiveness: If there's an issue in one of the spokes, rerouting through the hub can be a solution. Similarly, if there's a sudden demand in one city, resources from the hub can be quickly redirected.

6. Reduced Operational Costs: Even if the total distance traveled might be longer, the operational costs (like fuel, labor, and maintenance) could be lower with a hub-and-spoke model, given the consolidated and optimized shipments.

7. Risk Mitigation: If a direct route faces disruptions (due to weather, labor strikes, etc.), the hub can act as a buffer, ensuring that goods are still delivered, albeit with some delay.

8. Facilitates Cross-Shipping: Goods can be transferred from one vehicle to another at the hub, reducing the need for warehousing.

9. Efficient Resource Utilization: Vehicle and crew scheduling can be better managed with the predictability that a hub-and-spoke system offers.

10. Improved Service Levels: With predictable routes and schedules, service levels to customers can be improved, leading to higher customer satisfaction.

In summary, while the direct model might seem efficient in terms of distance, the hub-and-spoke model brings in operational efficiency, cost savings, and better service levels, especially for large-scale operations delivering perishable goods.