Part 1: Simulation Exercise

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Assignment Scope

In this assignment the exponential distribution and averages will be investigated in R and compared it with the Central Limit Theorem. The exponential distribution will be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is 1/lambda and the standard deviation is also 1/lambda. The distribution of averages of n exponentials will be numerically investigated with thousand simulations.

In this assignment, we set

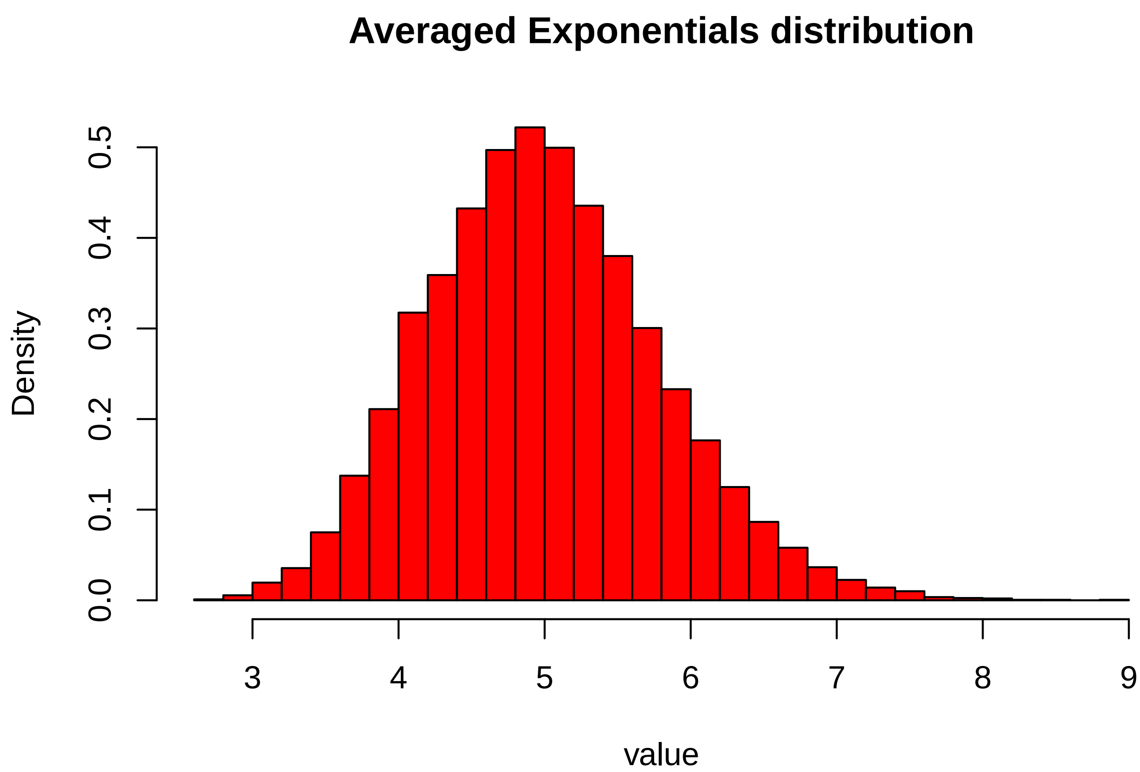
```
set.seed(42)
lambda=0.2
m=40
n=10000
```

where we use lambda = 0.2 for all of the simulations, assume m = 40 averages of exponentials, and each simulation is repeated n = 10000 times

40-averaged Exponential Distribution

Lets first plot an approximation of the distribution of 10^{4} averages of 40 random exponentials. The Central Limit Theorem (CLT) states that given a distribution with a mean μ and variance σ^2, the sampling distribution of the mean approaches a normal distribution with a mean (μ) and a variance σ^2/n as n increases.

```
avg_exps <- replicate(n, mean(rexp(m, lambda)))
hist(avg_exps, breaks=m, col='red', xlab='value',
     main="Averaged Exponentials distribution", freq=FALSE)
```



Sample Mean versus Theoretical Mean

This section compares the mean of the sampling distribution of the sample mean versus the theoretical mean of the exponential distribution, known to be (1/lambda). We will show that, as stated in the CLT, the averaging of exponentials will give us a good estimator of the mean.

```
simulated.mean <- mean(avg_exps)
expected.mean <- 1/lambda
```

Sampled	Expected
4.9971068	5

which seems to agree with the CLT, that states that given a distribution with a mean μ and variance σ^2, the sampling distribution of the mean approaches a normal distribution with a mean (μ).

Sample Variance versus Theoretical Variance

This section compares the variance of the sampling distribution of the sample mean versus the theoretical variance of the exponential distribution, known to be (1/lambda)^2/m

```
simulated.var <- var(avg_exps)
expected.var <- (1/lambda)^2/m
```

Sampled	Expected
0.6279625	0.625

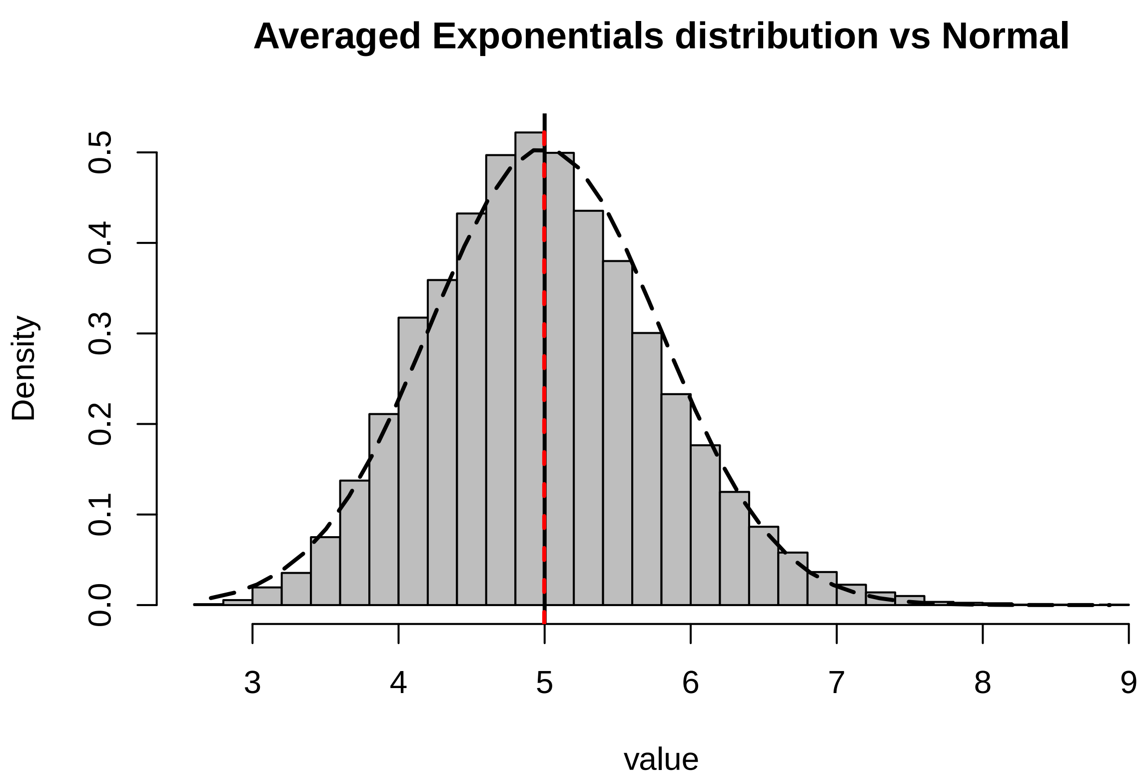
which seems to agree with the CLT, that states that given a distribution with a mean μ and variance σ^2, the sampling distribution of the mean approaches a normal distribution with variance σ^2/n as n increases.

Distribution

This section will show that the averaged exponentials distribution is approximately normal.

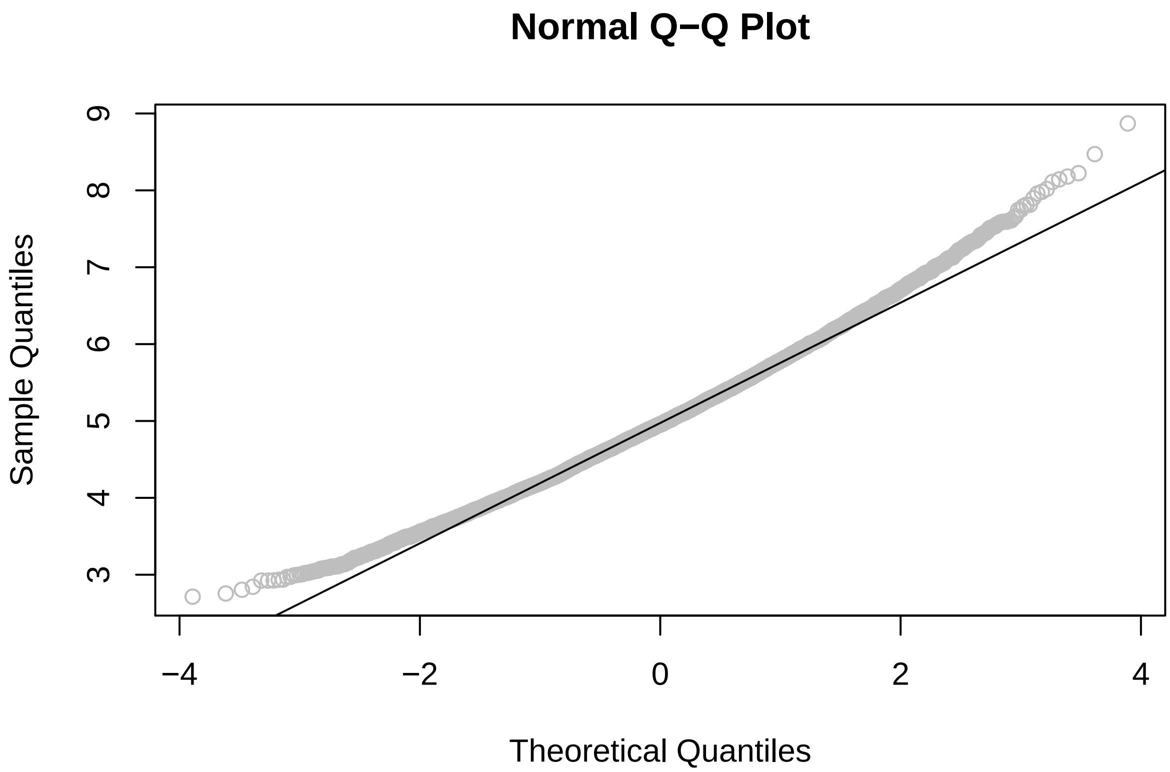
First we will show graphically that the sample distribution closely resembles a Normal distribution, centered at 5 with an standard deviation of 0.7905694

```
rx <- range(avg_exps)
x <- seq(rx[1], rx[2], length=m)
y <- dnorm(x, mean=expected.mean, sd=sqrt(expected.var))
hist(avg_exps,breaks=m,freq=FALSE,col="grey",xlab="value",
     main="Averaged Exponentials distribution vs Normal")
lines(x, y, pch=22, col="black", lty=5, lwd=2)
# plots the expected mean in a solid black line
abline(v = expected.mean, col = "black", lwd=2)
# plots the populaton mean in a dashed red line
abline(v = simulated.mean, col = "red", lty=2,lwd=2)
```



Moreover, we can plot a quantile-to-quantile comparision plot, also known as Q-Q plot, in order to compare the quantiles between the averaged exponential sample distribution and the normal distribuition

```r
qqnorm(avg_exps, main="Normal Q-Q Plot", xlab="Theoretical Quantiles",
       ylab="Sample Quantiles", col="grey")
qqline(avg_exps, col="black")
```



It can be seen that all sample distribution quantiles lie close to the line representing the quantiles of the Normal distribution, as stated by the theory.