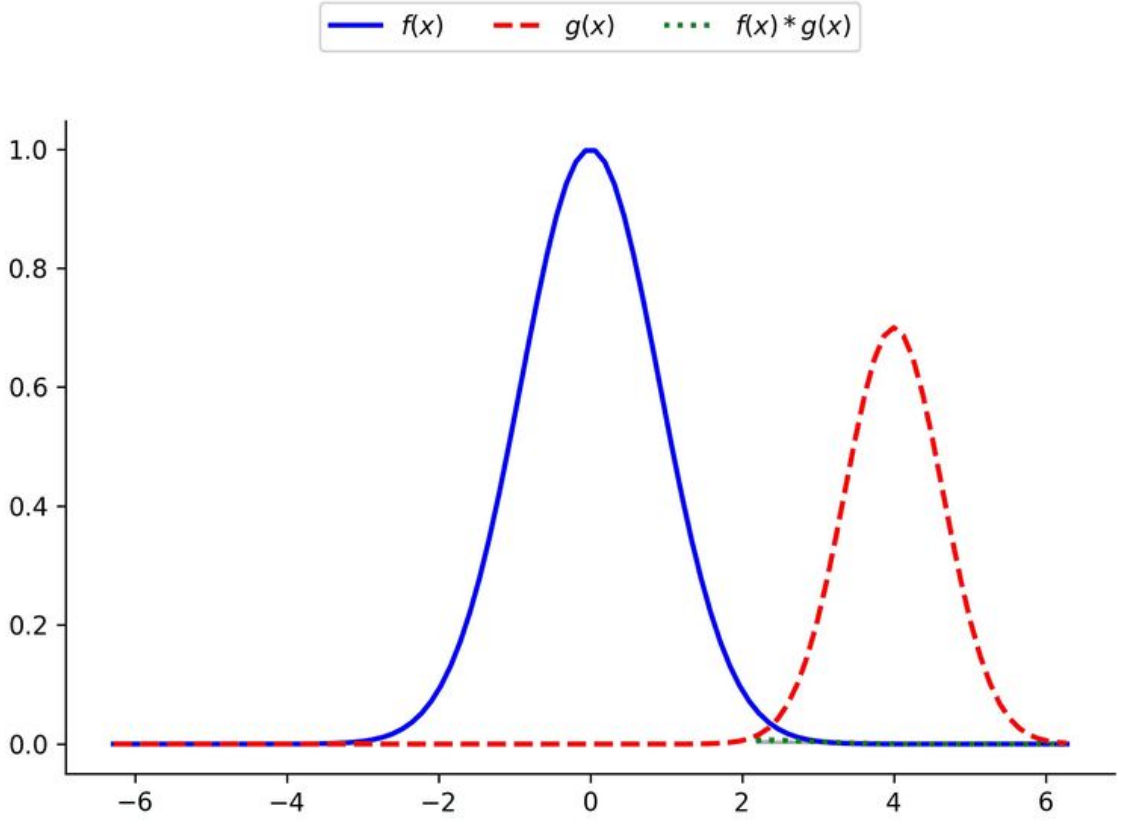
PPGEEC2318

Machine Learning

Convolutions



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Convolutions

Agenda

In this lesson, we will

1. **Understand**
   Arithmetic of Convolution layers

2. **Build**
   A model for multiclass classification

3. **Understand**
   The role of the softmax function

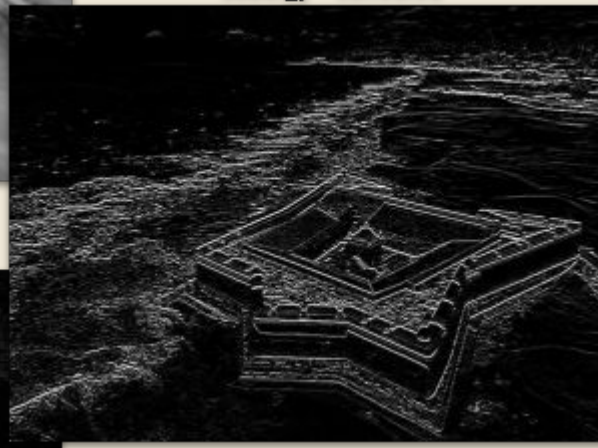
4. **Use**
   Cross-entropy losses

5. **Visualize**
   Filters learned by Conv. Layers

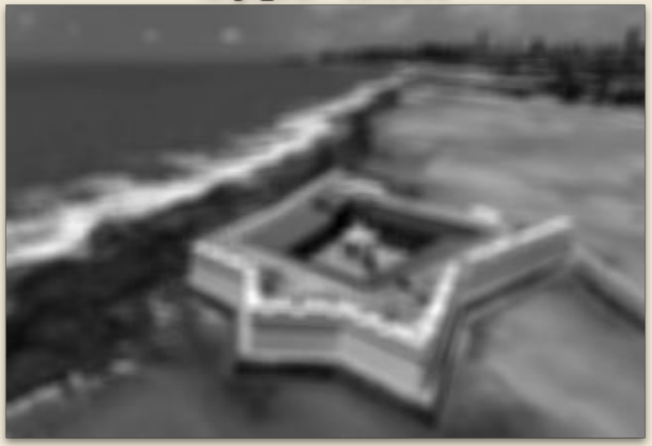
Original

small_blur - convolve

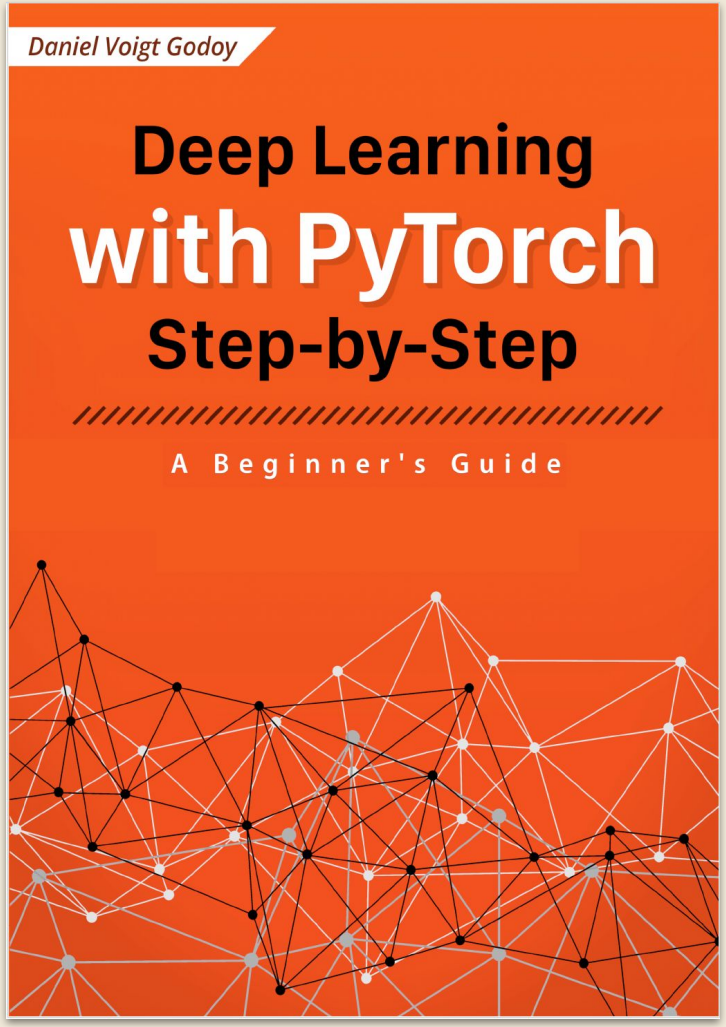
sobel_y - convolve



large_blur - convolve

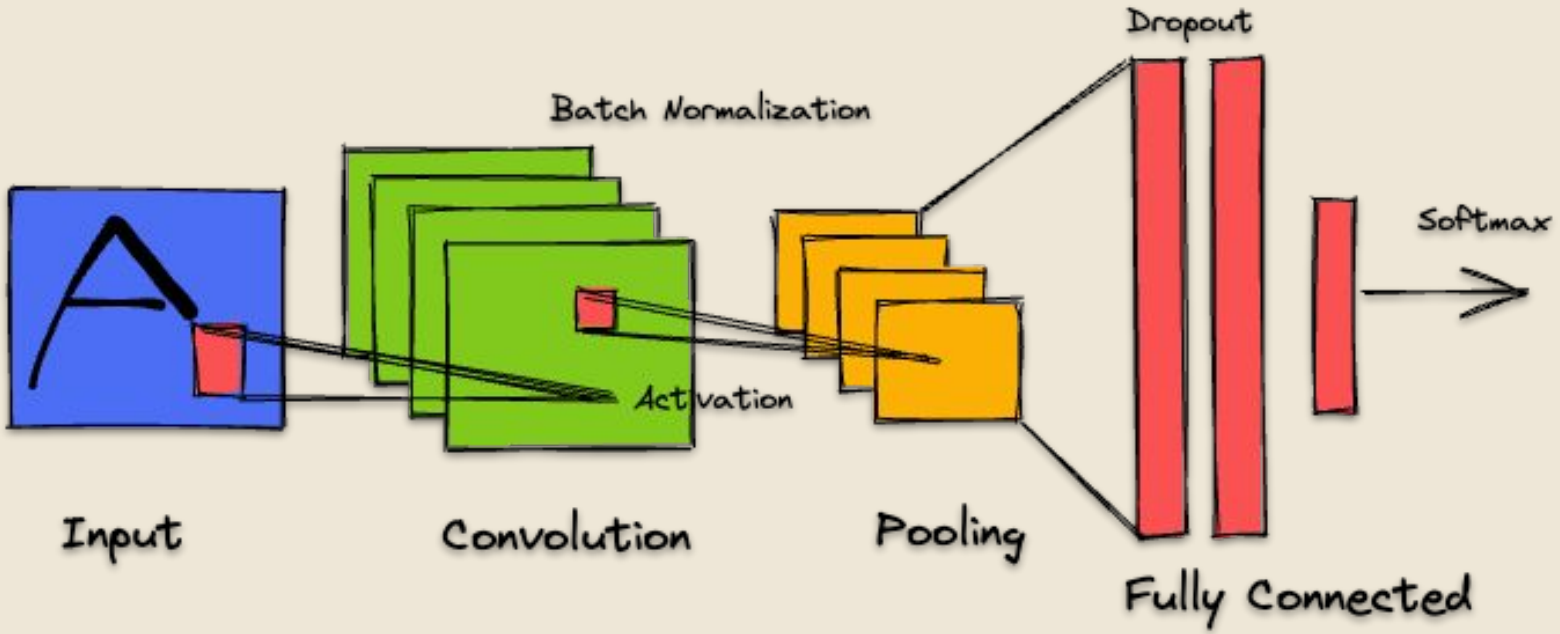


laplacian - convolve



~70 pag.

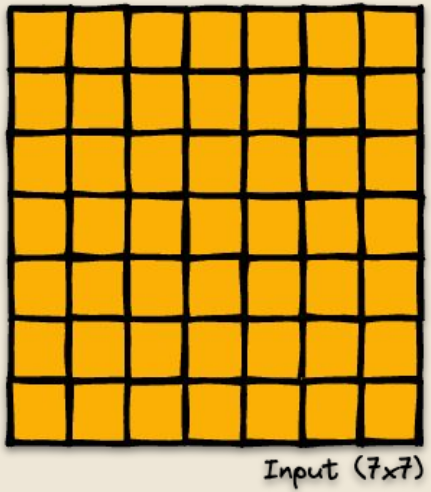
CNN Building Blocks



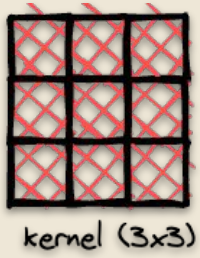
Convolution Explained in Code

```
import numpy as np
image = np.array((
    [1,2,3],
    [4,5,6],
    [7,8,9]
))
kernel = np.array((
    [-1,0,1],
    [-2,0,2],
    [-1,0,1]
))

conv = image*kernel
conv
array([[-1,  0,  3],
       [-8,  0, 12],
       [-7,  0,  9]])
```

```
np.sum(conv)
8
```



Input (7x7)

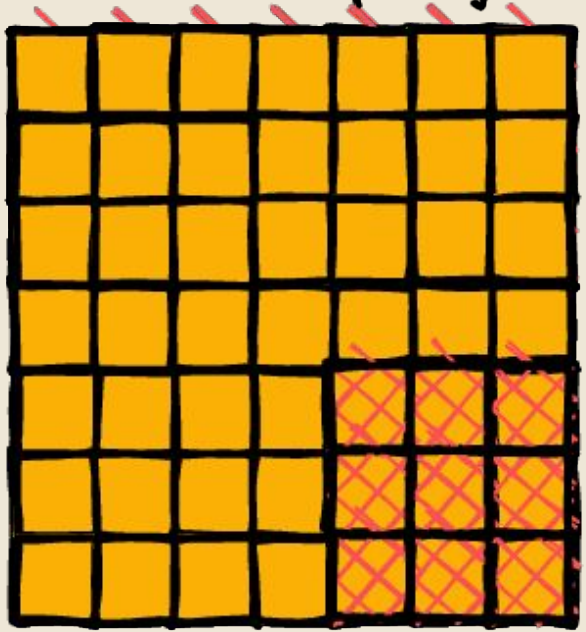


kernel (3x3)

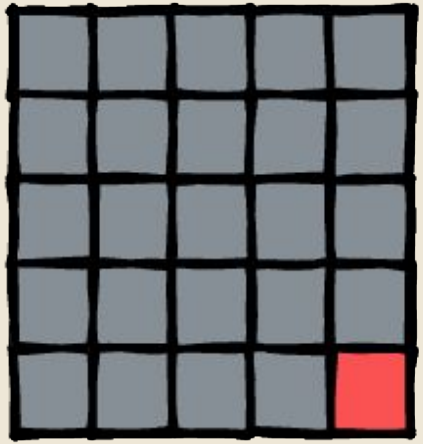
```
conv.weight
Parameter containing:
tensor([[[[ 0.1872,  0.1167,  0.1626],
          [-0.2925,  0.1007, -0.0899],
          [-0.0626, -0.0032,  0.2826]]]],
           requires_grad=True)
conv.bias
Parameter containing:
tensor([-0.0145], requires_grad=True)
```

```
import torch.nn as nn

conv = nn.Conv2d(in_channels=1,
                 out_channels=1,
                 kernel_size=3,
                 stride=1, padding=0)

fm = conv(image)
fm.shape
torch.Size([1, 1, 5, 5])
```

Stride = 1
padding = 0



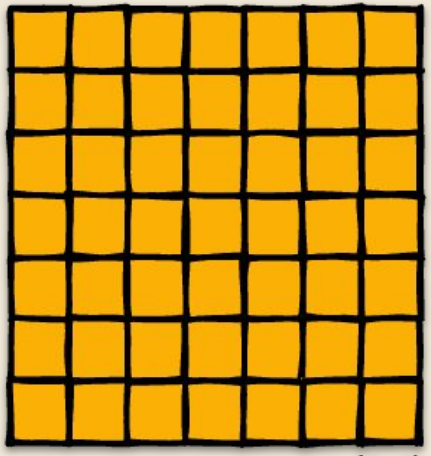
Convolution (input * kernel)



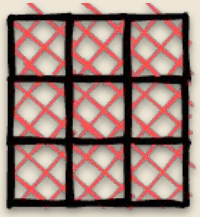
feature map (5x5)

Summary of Convolutions

n x n input
k x k kernel
padding p
stride s

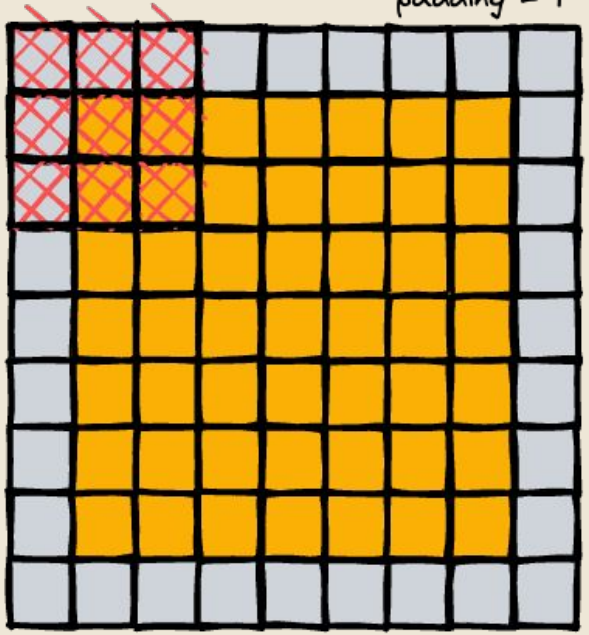


Input (7x7)

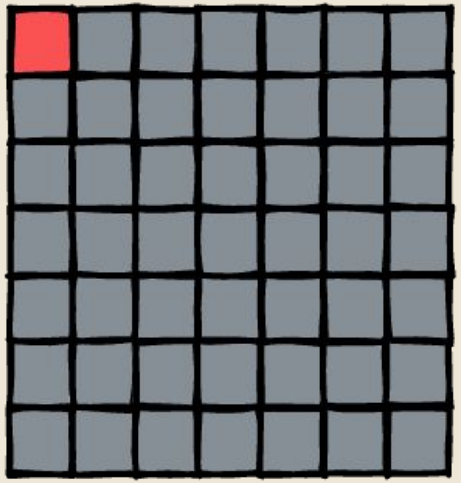


kernel (3x3)

Stride = 1
padding = 1



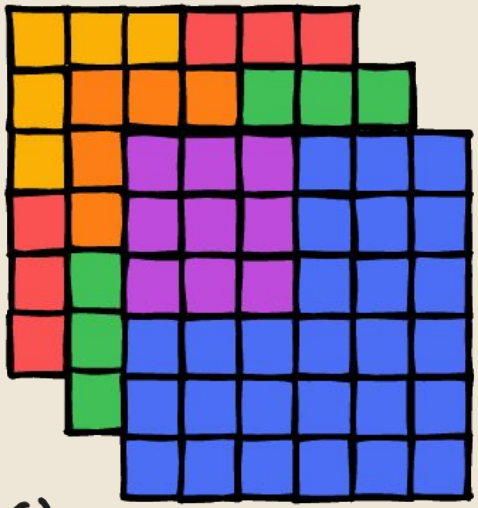
Convolution (input * kernel)



feature map (7x7)

$$\left\lfloor \frac{n + 2p - k}{s} + 1 \right\rfloor$$

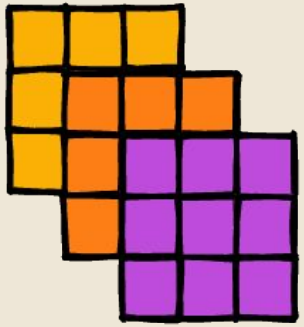
$$\left\lfloor \frac{n + 2p - k}{s} + 1 \right\rfloor$$



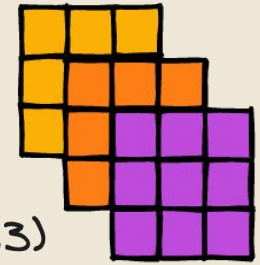
(1,3,6,6)

RGB image

*



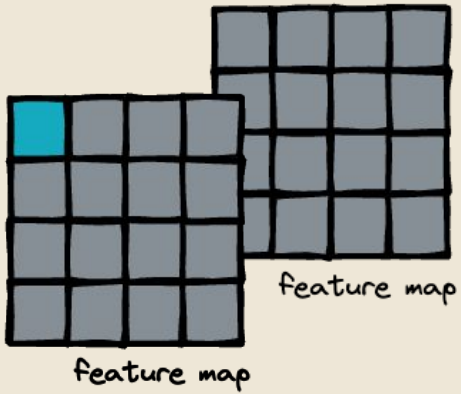
Kernel



(2,3,3,3)

Kernel

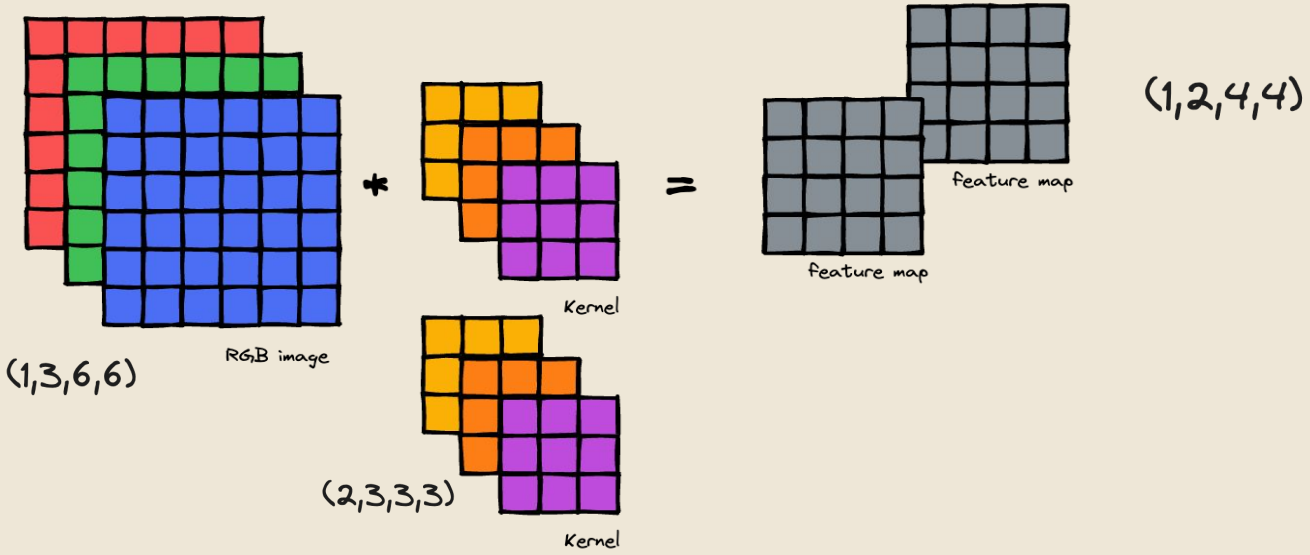
=



feature map

feature map

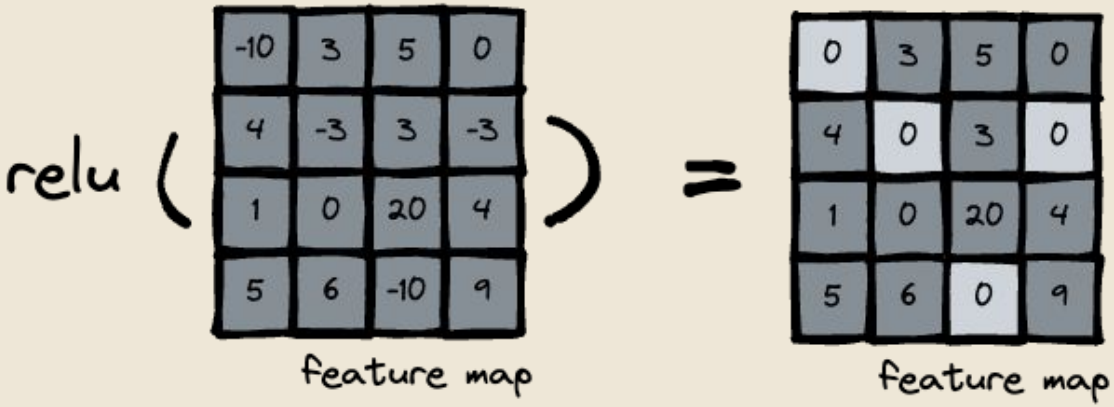
(1,2,4,4)




```
import numpy as np

random_array = np.random.rand(1, 3, 6, 6)
image = torch.as_tensor(random_array).float()

conv_multiple = nn.Conv2d(in_channels=3, out_channels=2,
                          kernel_size=3, stride=1, padding=0)

fm = conv_multiple(image)
```

```
fm.shape
torch.Size([1, 2, 4, 4])

conv_multiple.weight.shape
torch.Size([2, 3, 3, 3])

conv_multiple.bias.shape
torch.Size([2])
```

Activation Function

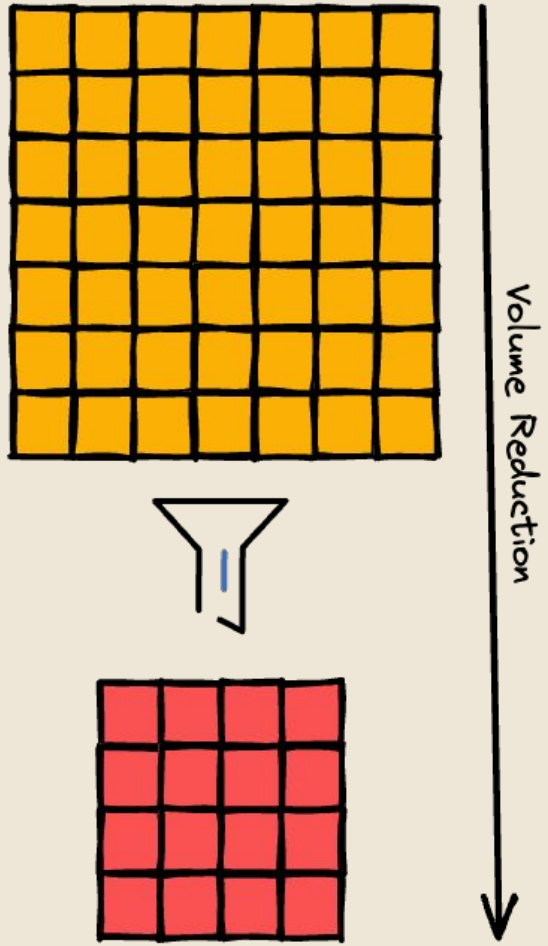



```
fm = np.array([[[[-10,3,5,0],
                 [4,-3,3,-3],
                 [1,0,20,4],
                 [5,6,-10,9]]]])

tensor = torch.tensor(fm).float()
# Apply ReLU function
relu = nn.ReLU()
relu(tensor)

tensor([[[[ 0.,  3.,  5.,  0.],
          [ 4.,  0.,  3.,  0.],
          [ 1.,  0., 20.,  4.],
          [ 5.,  6.,  0.,  9.]]]])
```

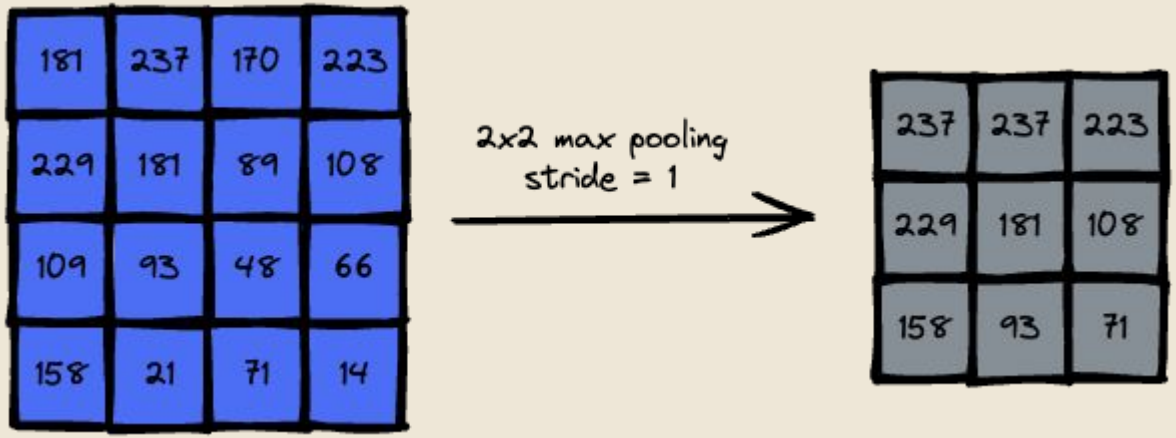
Pooling Layers

Two methods to reduce size of volume in CNN

1. CONV layers, typically with stride S > 1
2. Pooling layers
    a. Kernel (only a function and without weights, S > 2)



Pooling Layers

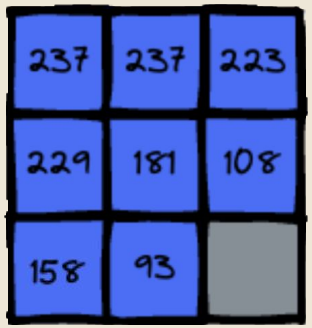


$$\left\lfloor \frac{n + 2p - k}{s} + 1 \right\rfloor$$

Pooling Layers

```
pooled
tensor([[[[237., 237., 223.],
          [229., 181., 108.],
          [158.,  93.,  71.]]]])
```

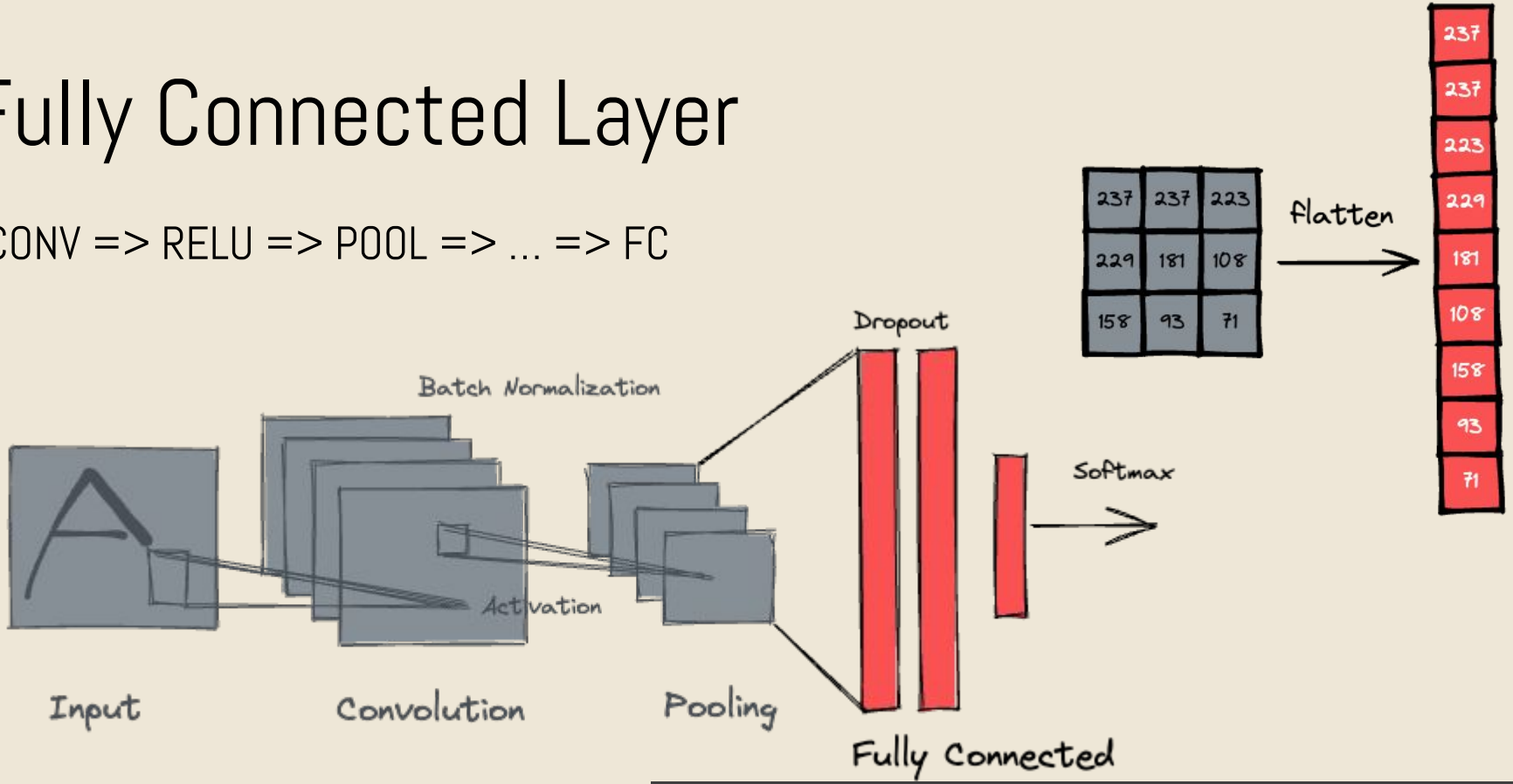
```
pool_data = np.array([[[[181,237,170,223],
                        [229,181,89,108],
                        [109,93,48,66],
                        [158,21,71,14]]]])

tensor = torch.tensor(pool_data).float()
tensor.shape
torch.Size([1, 1, 4, 4])
```




```
maxpool = nn.MaxPool2d(kernel_size=2,stride=1)
pooled = maxpool(tensor)
pooled.shape
torch.Size([1, 1, 3, 3])
```

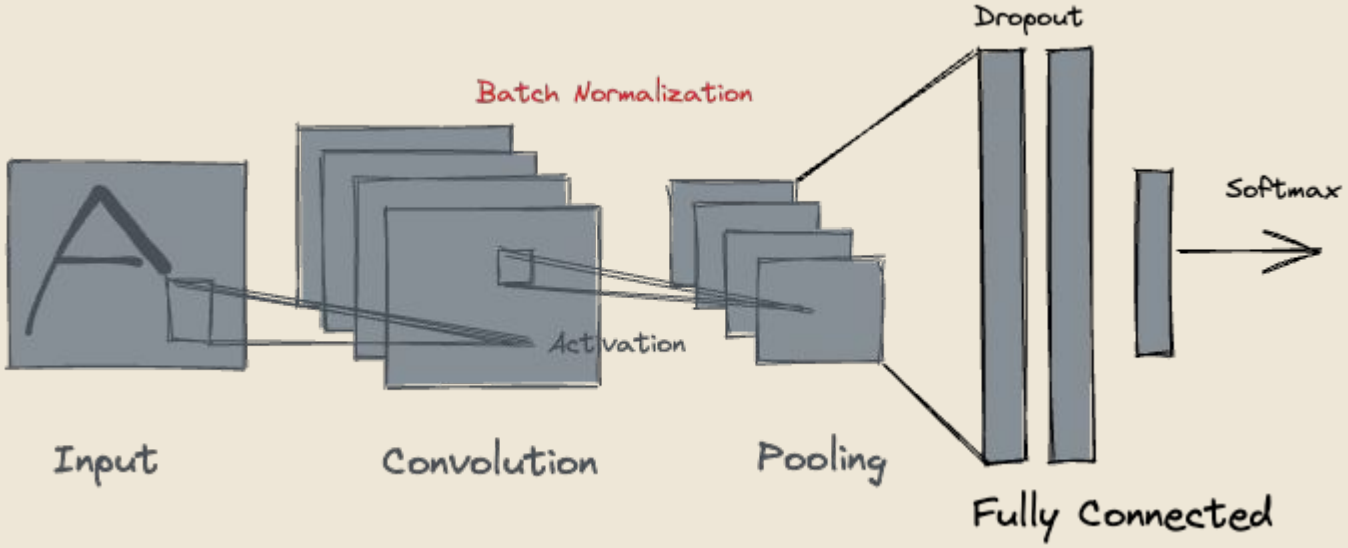
Fully Connected Layer

CONV => RELU => POOL => ... => FC




```
flattened = nn.Flatten()
flattened(pooled)
tensor([[237., 237., 223., 229., 181., 108., 158.,  93.,  71.]])
```

Batch Normalization

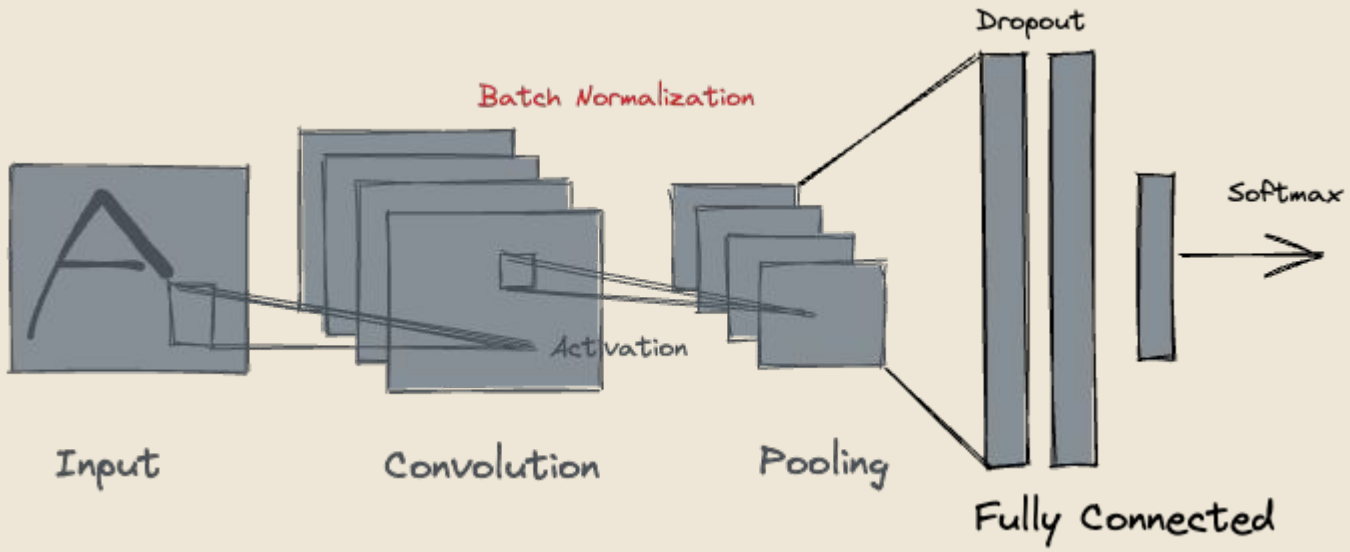


Accelerate the training process
Lightweight regularization

```
z = np.random.rand(3,3)
mean, std = np.mean(z), np.std(z)
norm = (z - mean)/(std + 1e-7)
```

IOFFE, Sergey; SZEGEDY, Christian. *Batch normalization: accelerating deep network training by reducing internal covariate shift.* ICML, 2015.

Batch Normalization

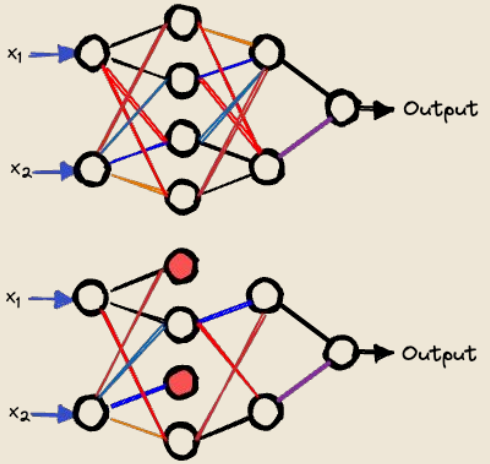


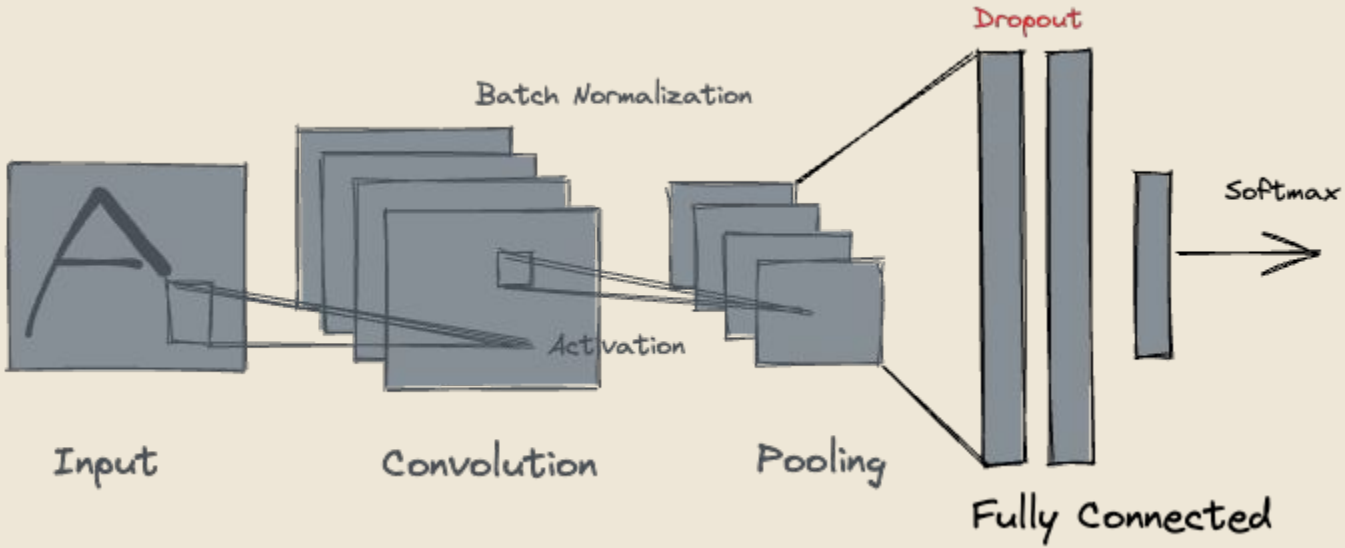
CONV => RELU => **BN** => POOL => ... => FC

CONV => **BN** => RELU => POOL => ... => FC

SRIVASTAVA, Nitish; HINTON, Geoffrey; KRIZHEVSKY, Alex; SUTSKEVER, Ilya; SALAKHUTDINOV, Ruslan. *Dropout: A simple way to prevent neural networks from overfitting*. Journal of Machine Learning Research, 2014.

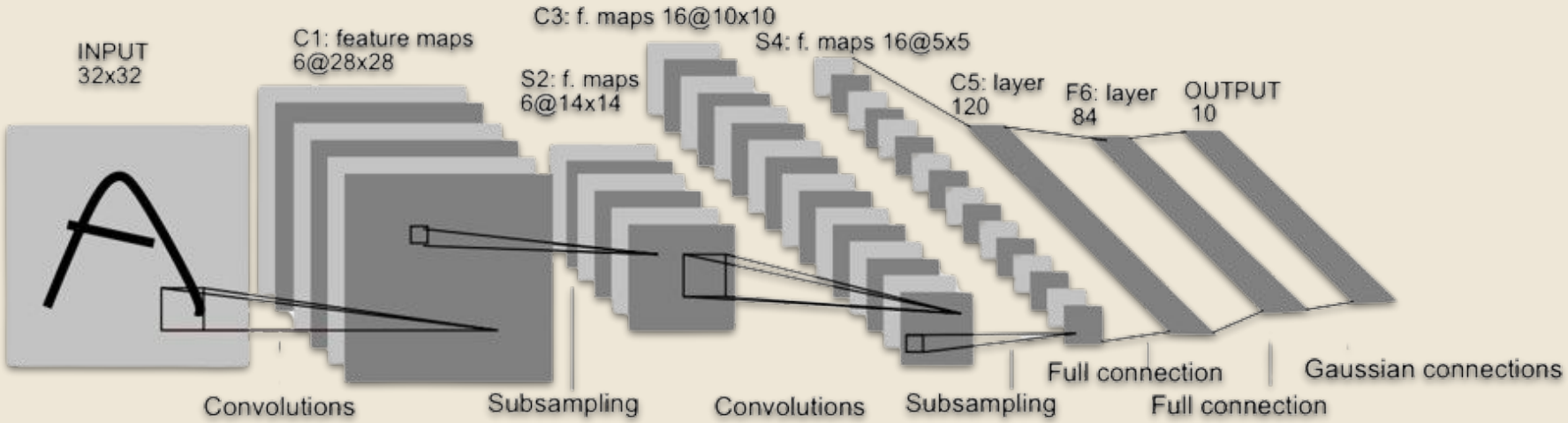
Dropout





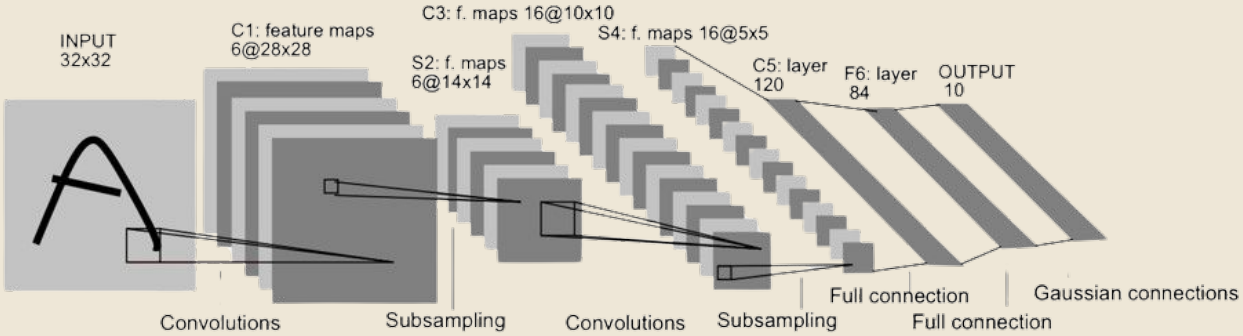
It is a form of regularization
Reduces overfitting
Increases test/validation accuracy (sometimes at expense of training accuracy)
Randomly disconnects node from current layers to next layer with probability, p

LeNet-5 (MNIST)



LECUN, Yann; BOTTOU, Léon; BENGIO, Yoshua; HAFFNER, Patrick. *Gradient-based learning applied to document recognition*. Proceedings of the IEEE, 1998.

LeNet-5



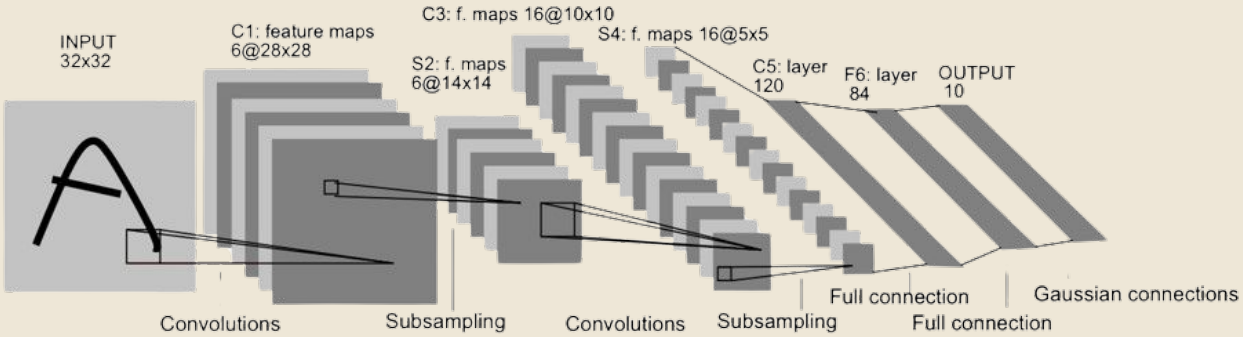

```
lenet = nn.Sequential()

# Featurizer
# Block 1: 1@28x28 -> 6@28x28 -> 6@14x14
lenet.add_module('C1', nn.Conv2d(in_channels=1,
                                 out_channels=6, kernel_size=5, padding=2))
lenet.add_module('func1', nn.ReLU())
lenet.add_module('S2', nn.MaxPool2d(kernel_size=2))
```

$$\left\lfloor \frac{n + 2p - k}{s} + 1 \right\rfloor$$

C1 = 6 x (1 x 5 x 5) = 150 + 6 bias = 156 parameters

NAIR, Vinod; HINTON, Geoffrey E. *Rectified linear units improve restricted Boltzmann machines.* ICML, 2010

LeNet-5

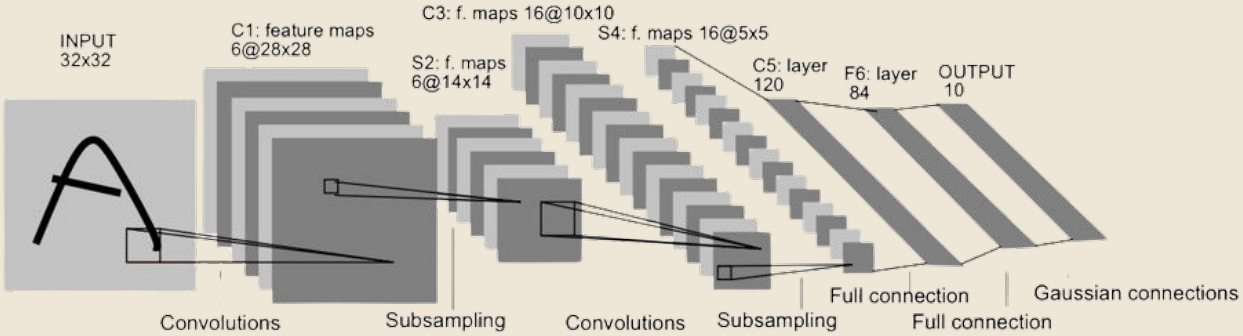



```
# Block 2: 6@14x14 -> 16@10x10 -> 16@5x5
lenet.add_module('C3', nn.Conv2d(in_channels=6, out_channels=16, kernel_size=5))
lenet.add_module('func2', nn.ReLU())
lenet.add_module('S4', nn.MaxPool2d(kernel_size=2))
```

$$\left\lfloor \frac{n + 2p - k}{s} + 1 \right\rfloor$$

C3 = 16 x (6 x 5 x 5) = 2400 + 16 bias = 2416 parameters

LeNet-5

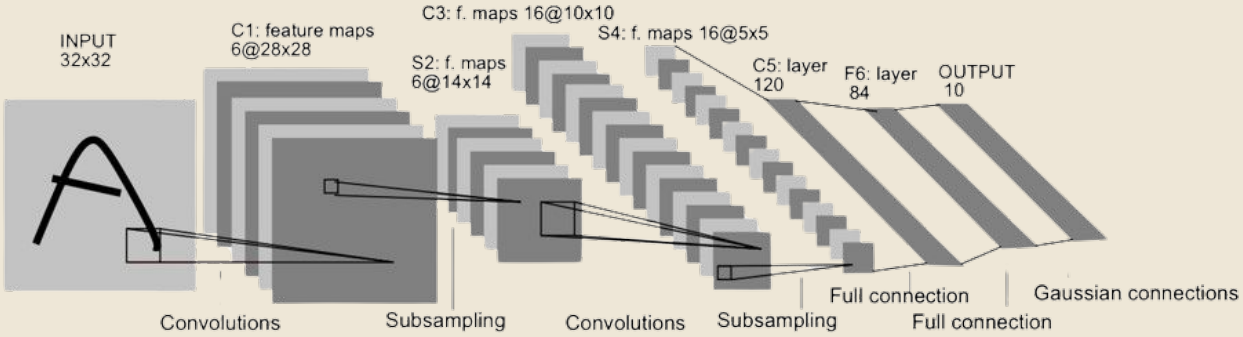



```
# Block 3: 16@5x5 -> 120@1x1
lenet.add_module('C5', nn.Conv2d(in_channels=16, out_channels=120, kernel_size=5))
lenet.add_module('func2', nn.ReLU())
# Flattening
lenet.add_module('flatten', nn.Flatten())
```

$$\left\lfloor \frac{n + 2p - k}{s} + 1 \right\rfloor$$

C5 = 120 x (16 x 5 x 5) = 48.000 + 120 bias = 48.120 parameters

LeNet-5



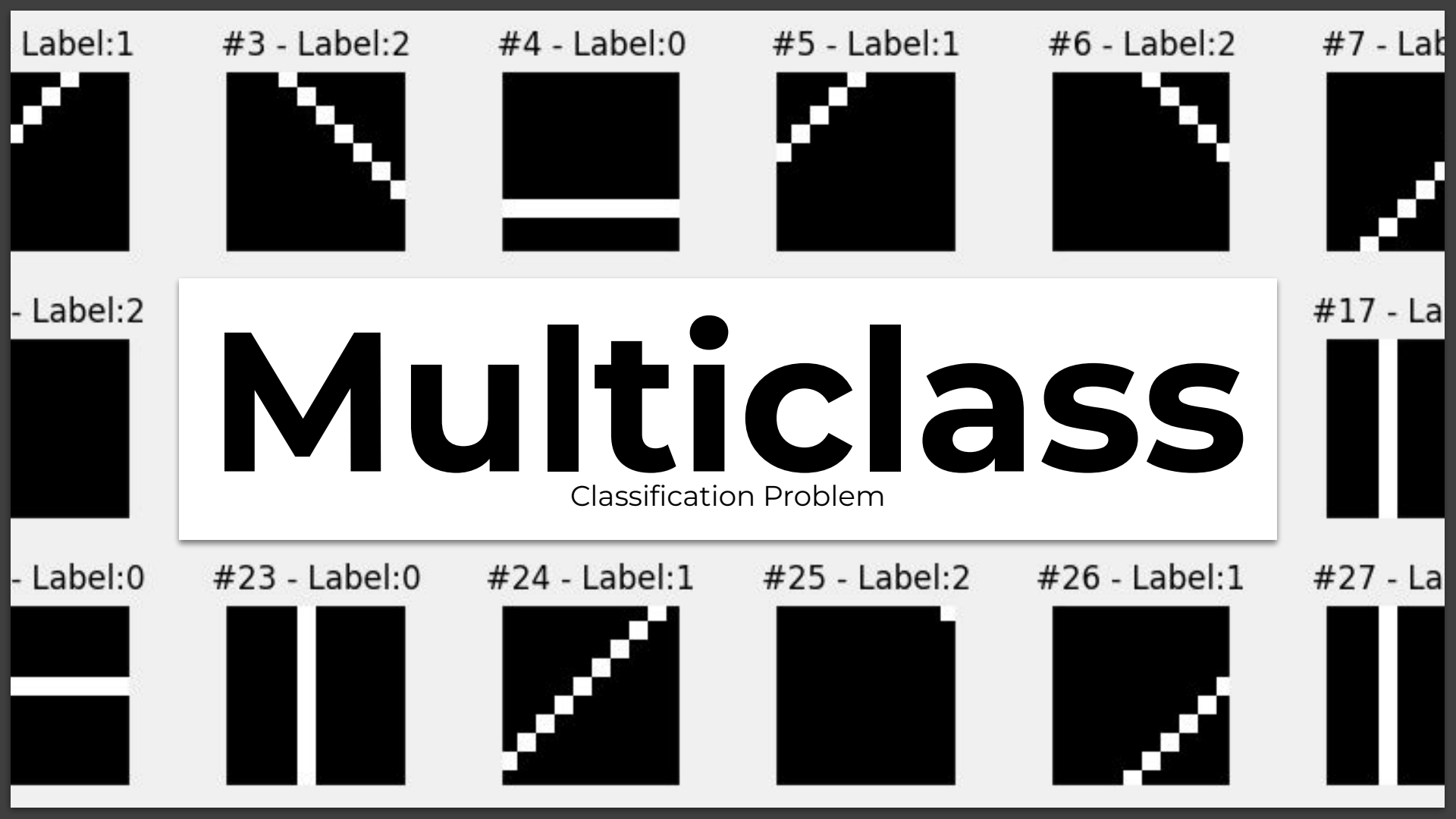
61706
params

```
# Classification
# Hidden Layer
lenet.add_module('F6', nn.Linear(in_features=120, out_features=84))
lenet.add_module('func3', nn.ReLU())
# Output Layer
lenet.add_module('OUTPUT', nn.Linear(in_features=84, out_features=10))
```

$$\left\lfloor \frac{n + 2p - k}{s} + 1 \right\rfloor$$

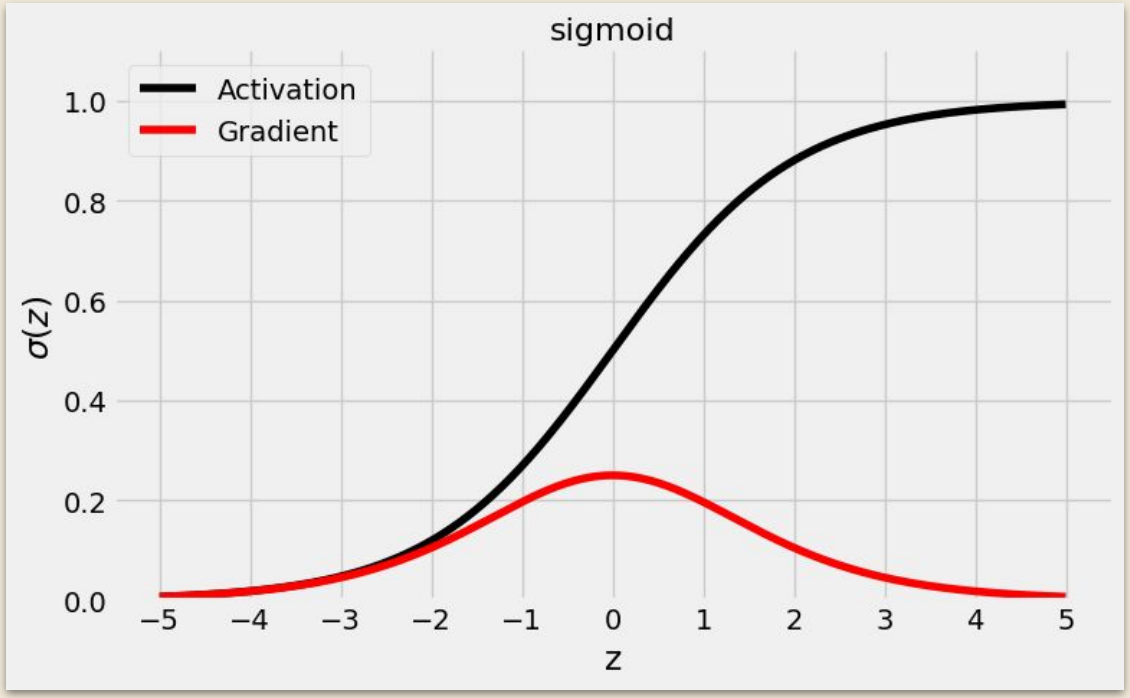
F6 = 120 x 84 = 10.080 + 84 bias = 10.164 parameters

OUTPUT = 84 x 10 = 840 + 10 bias = 850 parameters



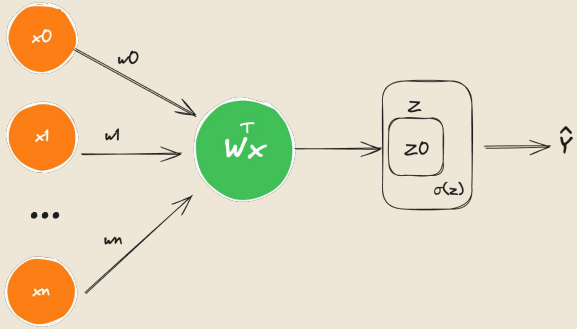
Multiclass

Classification Problem



In **binary classification** problems, the model would **produce one logit** , and one logit only, for each data point. It makes sense, as binary classification is about answering a simple question: "**Does a given data point belong to the positive class?** "

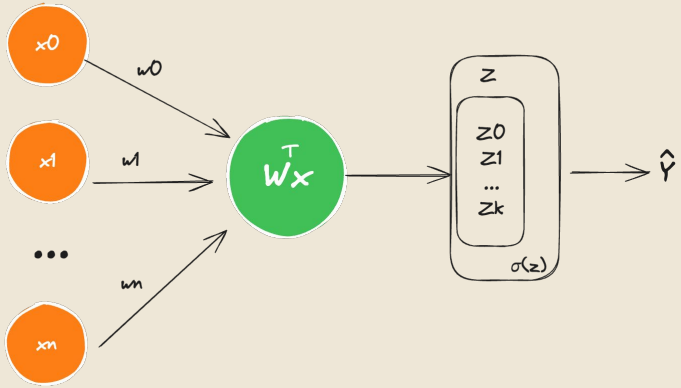
$$\sigma(z) = \frac{1}{1 + e^{-z}}$$



```
nn.Softmax(dim=-1)(logits)
tensor([0.7273, 0.1818, 0.0909])
```

$$\text{softmax}(z_i) = \frac{e^{z_i}}{\sum_{c=0}^{C-1} e^{z_c}}$$

$$\text{softmax}(z) = \left[\frac{e^{z_0}}{e^{z_0} + e^{z_1} + e^{z_2}}, \frac{e^{z_1}}{e^{z_0} + e^{z_1} + e^{z_2}}, \frac{e^{z_2}}{e^{z_0} + e^{z_1} + e^{z_2}} \right]$$




```
logits = torch.tensor([ 1.3863,  0.0000, -0.6931])
odds_ratios = torch.exp(logits)
softmaxed = odds_ratios / odds_ratios.sum()
tensor([0.7273, 0.1818, 0.0909])
```

Loss Function for Multiclass Classification

$$\mathbf{BCE}(y) = -\frac{1}{(N_{\text{pos}} + N_{\text{neg}})}\left[\sum_{i=1}^{N_{\text{pos}}} \log(\mathrm{P}(y_i = 1)) + \sum_{i=1}^{N_{\text{neg}}} \log(1 - \mathrm{P}(y_i = 1))\right]$$

For a binary classification

Cross-entropy measures the discrepancy between the true class distribution (p) and the predicted distribution (q).

$$H(p, q) = -\sum_{i=0}^{C-1} p_i \log(q_i)$$

$$q_i = \frac{e^{z_i}}{\sum_{i=0}^{C-1} e^{z_i}}$$

$$H(p,q) = -\sum_{i=0}^{C-1} p_i \log(q_i) \qquad q_i = \frac{e^{z_i}}{\sum_{i=0}^{C-1} e^{z_i}}$$

```python
import torch

# Seed definition, create logits and labels
torch.manual_seed(11)
dummy_logits = torch.randn((5, 3))
dummy_labels = torch.tensor([0, 0, 1, 2, 1])

# Softmax function
def softmax(logits):
   exp_logits = torch.exp(logits)
   sum_exp_logits = torch.sum(exp_logits, dim=1, keepdim=True)
   return exp_logits / sum_exp_logits

# Apply Softmax
probabilities = softmax(dummy_logits)
```

```
dummy_logits
tensor([[ 0.7376,  1.9459, -0.6995],
        [-1.3023, -0.5133, -0.2696],
        [ 0.2462,  0.4839,  0.4504],
        [-0.9568,  1.5012, -0.3136],
        [-0.2343, -1.0713,  0.1648]])
```

```
probabilities
tensor([[0.2181, 0.7301, 0.0518],
        [0.1664, 0.3663, 0.4673],
        [0.2861, 0.3629, 0.3510],
        [0.0686, 0.8010, 0.1305],
        [0.3421, 0.1481, 0.5098]])
```

$$H(p,q) = -\sum_{i=0}^{C-1} p_i \log(q_i) \qquad q_i = \frac{e^{z_i}}{\sum_{i=0}^{C-1} e^{z_i}}$$

```
# Calculating cross-entropy manually
def cross_entropy_loss(probabilities, labels):
    # Getting the probabilities of the correct classes
    correct_class_probs = probabilities[range(len(labels)), labels]
    # Calculating the cross-entropy loss
    loss = -torch.log(correct_class_probs)
    return torch.mean(loss)
```

```
dummy_logits
tensor([[ 0.7376,  1.9459, -0.6995],
        [-1.3023, -0.5133, -0.2696],
        [ 0.2462,  0.4839,  0.4504],
        [-0.9568,  1.5012, -0.3136],
        [-0.2343, -1.0713,  0.1648]])
```

```
# Calculating the loss
loss = cross_entropy_loss(probabilities, dummy_labels)

print("\nLoss:")
print(loss.item())

Loss:
1.6552671194076538
```

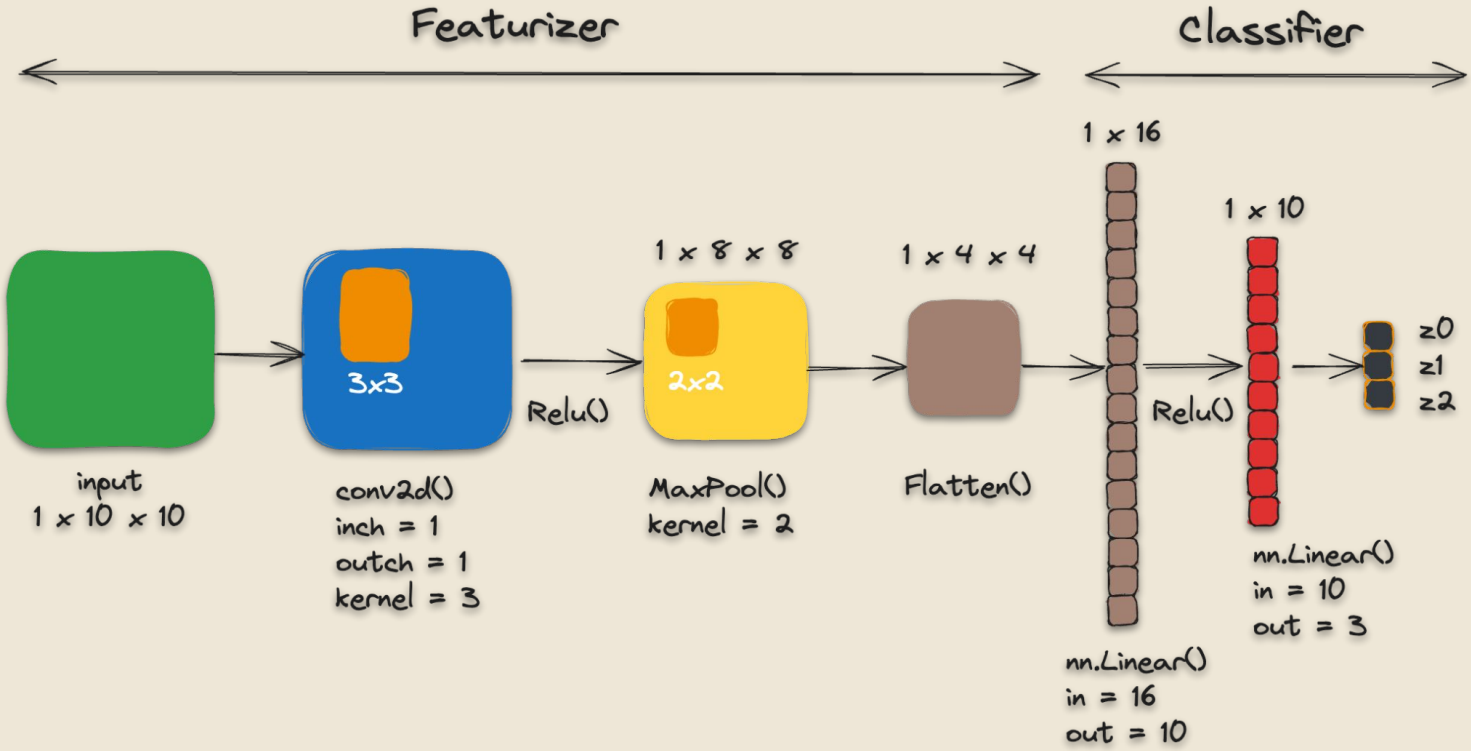
```
probabilities
tensor([[0.2181, 0.7301, 0.0518],
        [0.1664, 0.3663, 0.4673],
        [0.2861, 0.3629, 0.3510],
        [0.0686, 0.8010, 0.1305],
        [0.3421, 0.1481, 0.5098]])
```

$$H(p,q) = -\sum_{i=0}^{C-1} p_i \log(q_i) \qquad q_i = \frac{e^{z_i}}{\sum_{i=0}^{C-1} e^{z_i}}$$

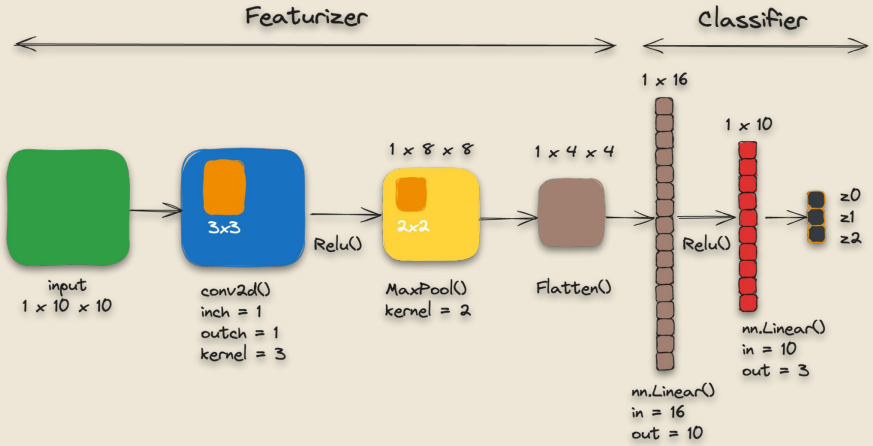
```
torch.manual_seed(11)
dummy_logits = torch.randn((5, 3))
dummy_labels = torch.tensor([0, 0, 1, 2, 1])

loss_fn = nn.CrossEntropyLoss()
loss_fn(dummy_logits, dummy_labels)
tensor(1.6553)
```

```
dummy_logits
tensor([[ 0.7376,  1.9459, -0.6995],
        [-1.3023, -0.5133, -0.2696],
        [ 0.2462,  0.4839,  0.4504],
        [-0.9568,  1.5012, -0.3136],
        [-0.2343, -1.0713,  0.1648]])
```

Case Study


Featurizer
Classifier
1 x 16
1 x 10
1 x 8 x 8
1 x 4 x 4
3x3
2x2
Relu()
Relu()
z0
z1
z2
input
1 x 10 x 10
conv2d()
inch = 1
outch = 1
kernel = 3
MaxPool()
kernel = 2
Flatten()
nn.Linear()
in = 10
out = 3
nn.Linear()
in = 16
out = 10

Case Study



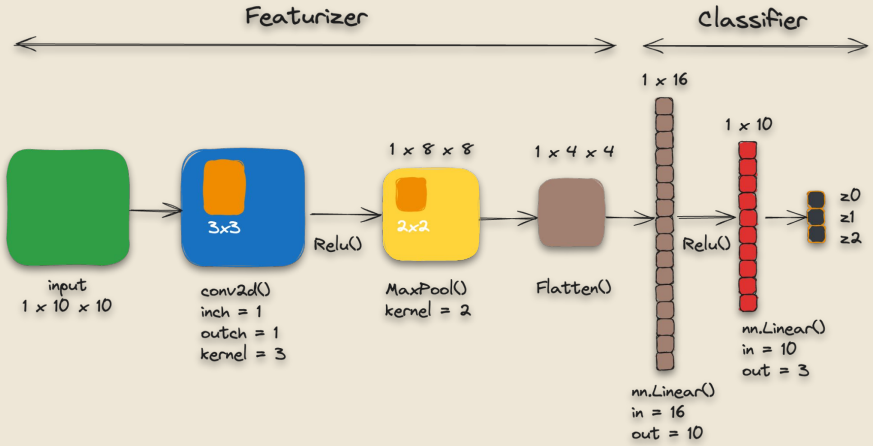

```
torch.manual_seed(13)
model_cnn1 = nn.Sequential()

# Featurizer
# Block 1: 1@10x10 -> n_channels@8x8 -> n_channels@4x4
n_channels = 1
model_cnn1.add_module('conv1', nn.Conv2d(in_channels=1, out_channels=n_channels, kernel_size=3))
model_cnn1.add_module('relu1', nn.ReLU())
model_cnn1.add_module('maxp1', nn.MaxPool2d(kernel_size=2))
# Flattening: n_channels * 4 * 4
model_cnn1.add_module('flatten', nn.Flatten())
```

Case Study

Before : nn.flatten()
(batch_size, n_channels, 4, 4)

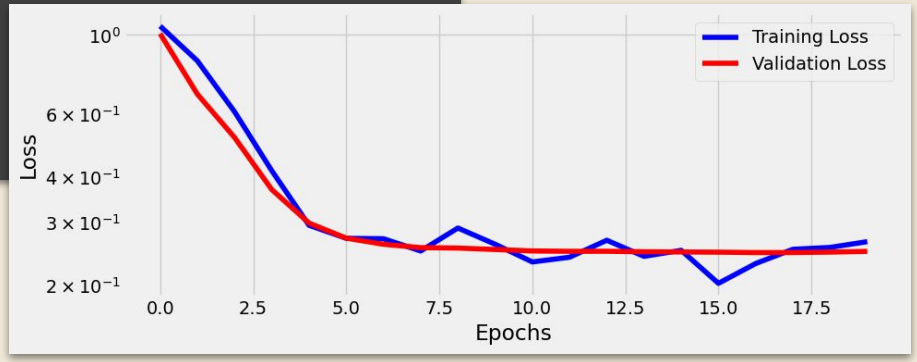
After : nn.flatten()
(batch_size,n_channels x 4 x 4)

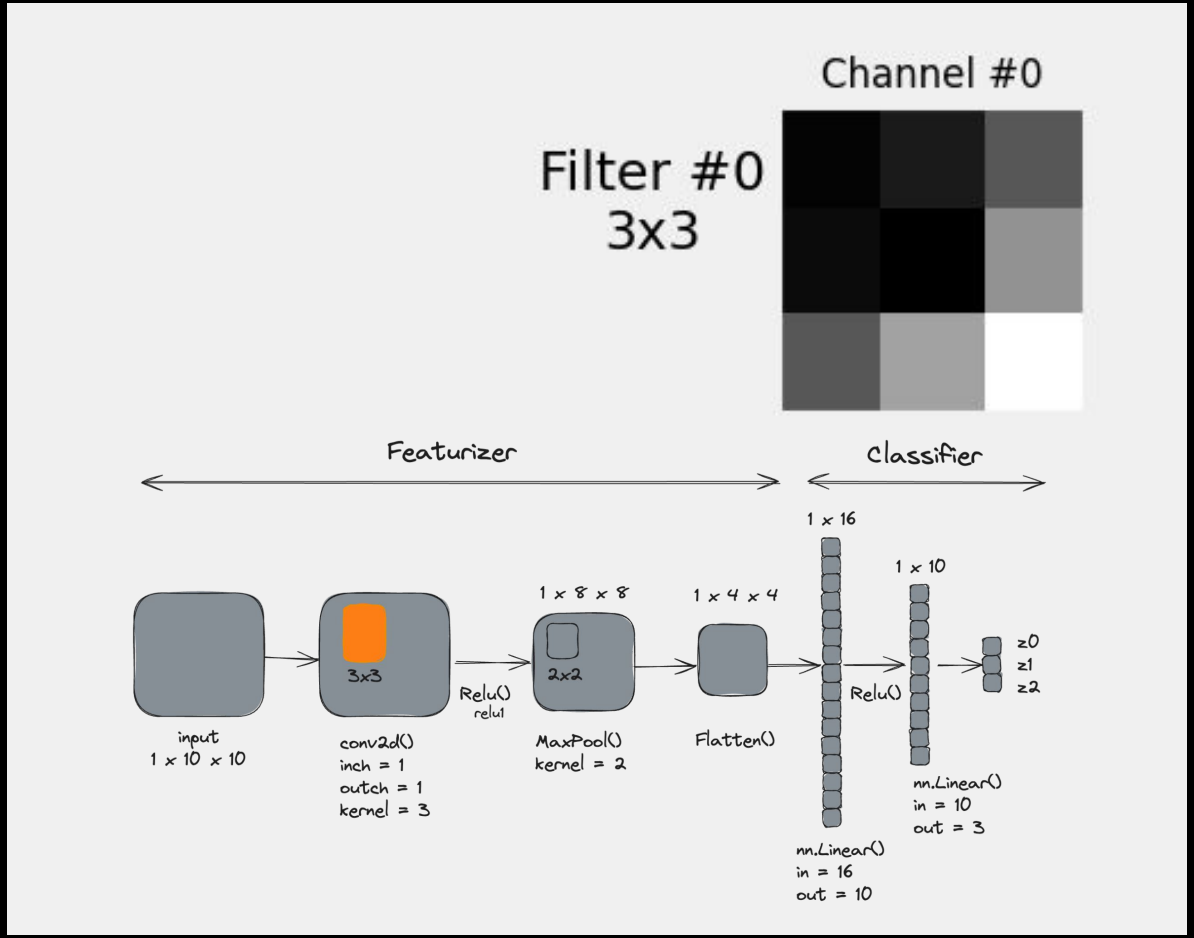
Linear.in_features : *n_channels x 4 x 4*




```
# Classification
# Hidden Layer
model_cnn1.add_module('fc1', nn.Linear(in_features=n_channels*4*4, out_features=10))
model_cnn1.add_module('relu2', nn.ReLU())
# Output Layer
model_cnn1.add_module('fc2', nn.Linear(in_features=10, out_features=3))
```

```python
# Set the learning rate
lr = 0.1

# Define the multi-class loss function with mean reduction
multi_loss_fn = nn.CrossEntropyLoss(reduction='mean')

# Initialize the optimizer (Stochastic Gradient Descent)
optimizer_cnn1 = optim.SGD(model_cnn1.parameters(), lr=lr)

# Wrap the model, loss function, and optimizer in an Architecture object
arch_cnn1 = Architecture(model_cnn1, multi_loss_fn, optimizer_cnn1)

# Set the data loaders for training and validation
arch_cnn1.set_loaders(train_loader, val_loader)

# Train the model for 20 epochs
arch_cnn1.train(20)
```





To really understanding the effect this filter has on each image, we need to visualize the intermediate values produced by our model.

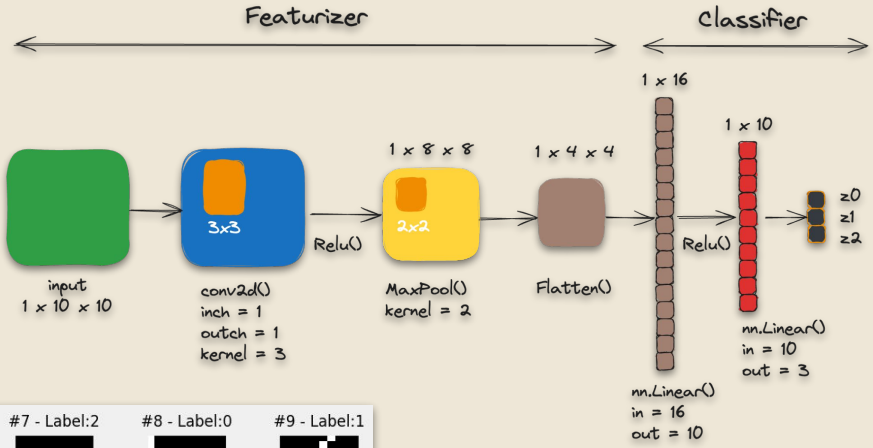
VISUALIZING FILTERS

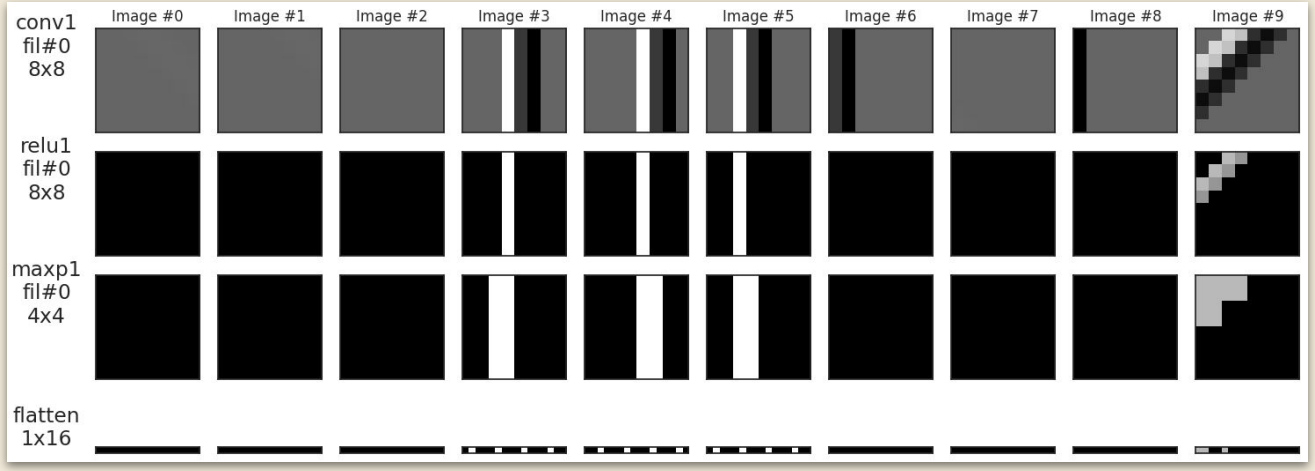
*A **hook** is simply a way to force a model to execute a function (callback) either after its **forward pass** or after its **backward pass**. Hence, there are forward hooks and backward hooks.*

```
# Create a simple linear model with one input and one output
dummy_model = nn.Linear(1, 1)

# Create an empty list to store hook information
dummy_list = []

# Define the hook function
# This function will be called during the forward pass of the network
# The function receives three parameters:
# - layer: the current layer where the hook is registered
# - inputs: the input to the layer
# - outputs: the output from the layer
def dummy_hook(layer, inputs, outputs):
    # Append a tuple containing the layer, the input, and the output to the dummy_list
    dummy_list.append((layer, inputs, outputs))

# Register the hook to the model's layer
# During the model's execution, the hook function will be called
handle = dummy_model.register_forward_hook(dummy_hook)

# Now, whenever the dummy_model layer is executed, the dummy_hook function will be called
# and the layer, input, and output will be added to dummy_list
```

```
# Create a dummy input tensor with a value of 0.3
dummy_x = torch.tensor([0.3])

# Pass the dummy input through the dummy_model
dummy_model(dummy_x)
# Output of the model, which is a tensor with a value and a gradient function
# tensor([-0.2450], grad_fn=<ViewBackward0>)

# Check the dummy_list to see the stored hook information
dummy_list
# The list contains a tuple with:
# - The Linear layer used in the dummy_model
# - The input tensor passed to the layer (dummy_x)
# - The output tensor from the layer with a gradient function
# [(Linear(in_features=1, out_features=1, bias=True),
#   (tensor([0.3000]),),
#   tensor([-0.2450], grad_fn=<ViewBackward0>))]

# Remove the hook from the model
handle.remove()
```

Visualizing Feature Maps

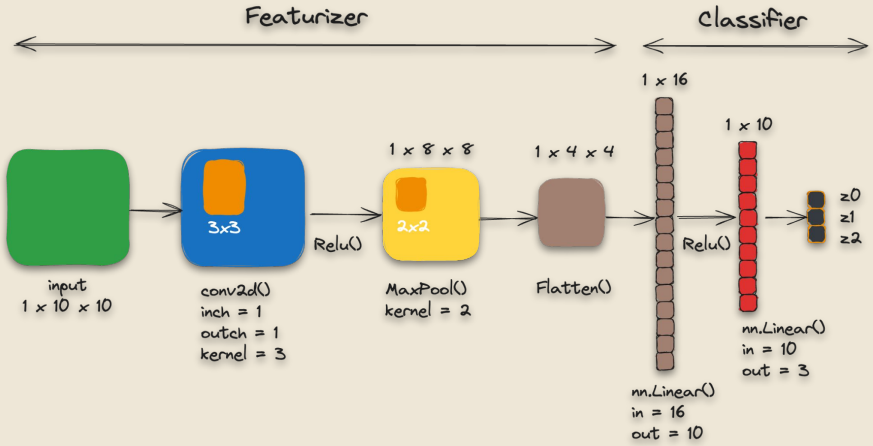


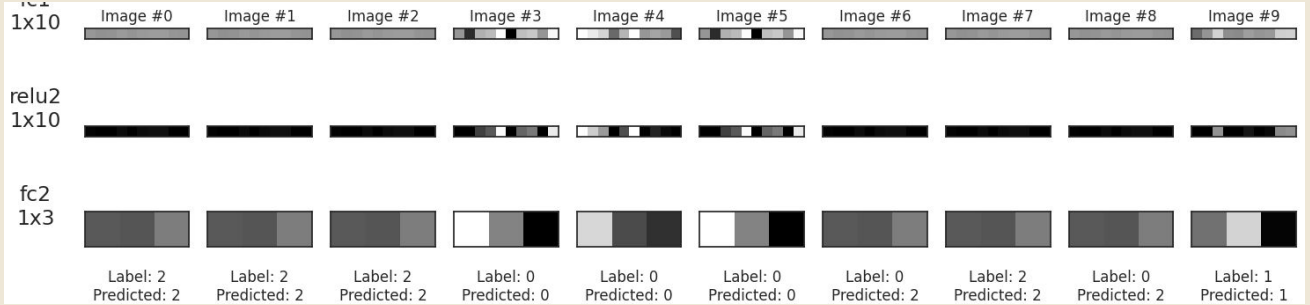



```
images_batch, labels_batch = next(iter(val_loader))
logits = arch_cnn1.predict(images_batch)
fig = plot_images(images_batch.squeeze(),
                  labels_batch.squeeze(),
                  n_plot=10)
```

```
featurizer_layers = ['conv1', 'relu1',
                     'maxp1', 'flatten']

with plt.style.context('seaborn-white'):
   fig = arch_cnn1.visualize_outputs(featurizer_layers)
```

Visualizing Feature Maps





#accuracy in this batch

```
arch_cnn1.correct(images_batch,
                  labels_batch)
tensor([[5, 7],
        [3, 3],
        [6, 6]])
```

```
classifier_layers = ['fc1', 'relu2',
                     'fc2']

with plt.style.context('seaborn-white'):
   fig = arch_cnn1.visualize_outputs(classifier_layers)
```

Accuracy

What if I want to compute the accuracy for all mini-batches in a data loader?

```
    @staticmethod
    def loader_apply(loader, func, reduce='sum'):
        results = [func(x, y) for i, (x, y) in enumerate(loader)]
        results = torch.stack(results, axis=0)

        if reduce == 'sum':
            results = results.sum(axis=0)
        elif reduce == 'mean':
            results = results.float().mean(axis=0)

        return results
```

```
## Loader Apply
Architecture.loader_apply(arch_cnn1.val_loader,
                          arch_cnn1.correct)
tensor([[59, 67],
        [55, 62],
        [71, 71]])
```