Algorithm Complexity I

Exercise

ivanovitch.silva@ufrn.br
@ivanovitchm

Example #01

```
def int_sum(n):
    sum = 0          #C1, 1
    while n > 0:     #C2, N
        sum += n     #C3, N
        n -= 1       #C4, N
    return sum       #C5, 1
```

```
int_sum(2)
3

int_sum(3)
6

int_sum(4)
10
```

$$C_1 + N(C_2 + C_3 + C_4) + C_5$$

$$C_1 + C_5 + N(C_2 + C_3 + C_4)$$

$$\alpha + \beta N \gg O(N)$$

Example #02

```
def int_sum(n):
 return n*(n+1)/2  #C1,C2,C3, 1
```

```
int_sum2(2)
```

```
3.0
```

```
int_sum2(3)
```

```
6.0
```

```
int_sum2(4)
```

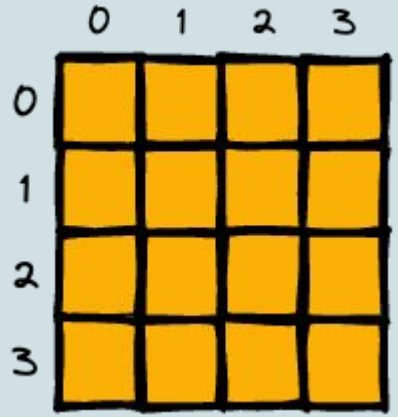
```
10.0
```

$$C_1 + C_2 + C_3$$

$$\alpha \gg O(1)$$

Example #03

```
def print_matrix(mat):
    for row in mat:          #C1, N
        for cell in row:     #C2, N*N
            print(cell)      #C3, N*N
```



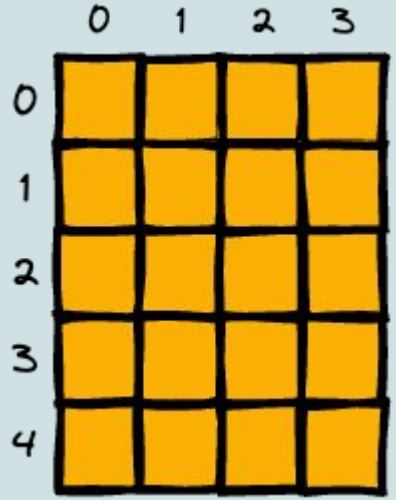
$$C_1 N + C_2 N^2 + C_3 N^2$$

$$N^2(C_2 + C_3) + C_1 N$$

$$O(N^2)$$

Example #04

```
def print_matrix(mat):
    for row in mat:          #C1, N
        for cell in row:     #C2, M*N
            print(cell)      #C3, M*N
```



$$C_1N + C_2MN + C_3MN$$

$$MN(C_2 + C_3) + C_1N$$

$$O(MN)$$

Example #05

```
def foo(n):
    for i in range(10000):    #C1, 10000
        print(i)              #C2, 10000
```

$$C_1 10000 + C_2 10000$$

$$10000(C_1 + C_2)$$

$$O(1)$$

Example #06

```
def foo(n):
 sum = 0                          #C1, 1
 for i in range(n):               #C2, N
   for j in range(2*n):           #C3, 2N^2
     sum += j                     #C4, 2N^2
 for i in range(100*n):           #C5, 100N
   sum += i                       #C6, 100N
 for i in range(4):               #C7, 4
   for j in range(3):             #C8, 12
     sum += j                     #C9, 12
 return sum                       #C10, 1
```

$$4N^2 + 201N + 30$$

$$4N^2$$

$$O(N^2)$$

Example #07

```
def foo(n):
 for i in range(1, n+1):  #C1, n
   for j in range(i):     #C2, n*[i]
     print(j)             #C3, n*[i]
```

How many times "print(j)" will be executed?

$$1 + 2 + 3 + 4 + \ldots N$$

$S_N = \frac{N(a_1+a_N)}{2}$

$S_N = \frac{N(1+N)}{2}$

$S_N = \frac{N^2}{2} + \frac{N}{2}$

$O(N^2)$

Example #08

```
def intersection(arr1, arr2):
 for elem in arr1:          #C1, n
   if elem in arr2:         #C2, n*m
     print(elem)            #C3, n*m
```

$$(C_2 + C_3)NM + C_1N$$

$$O(NM)$$

$$O(N + M)$$

```
def both_contain(arr1, arr2, k):
 return k in arr1 and k in arr2   #C1,n  #C2,m
```

Example #09

```
def foo(n):
 i = 1              #C1, 1
 while i < n:       #C2, logn
   i *= 2           #C3, logn
```

$$2 \log N + 1$$

$$O(\log N)$$

i = [1, 2, 4, 8, 16, … N]

i = [1, 2, 2*2, 2*2*2, 2*2*2*2, … N]

$2^k = N$ $\log 2^k = \log N$ $k = \log N$

```
def where_equal(str1,str2):
 indexes = []                                          #C1, 1
 i = 0                                                 #C2, 1
 while i < len(str1) and i < len(str2):                #C3,min(n,m)
   if str1[i] == str2[i]:                              #C4,min(n,m)
     indexes.append(i)                                 #C5,min(n,m)
   i += 1                                              #C6,min(n,m)
 return indexes                                        #C7,1
```

$4min(n,m)+3$

$O(min(n,m))$

Example #10