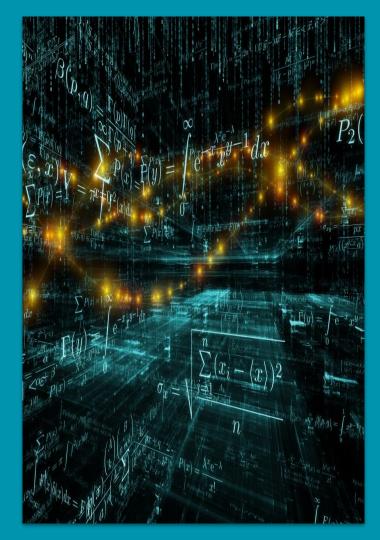
Algorithm Complexity II Big 0, Ω,Θ Notations

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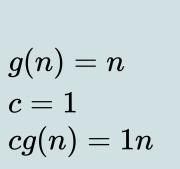


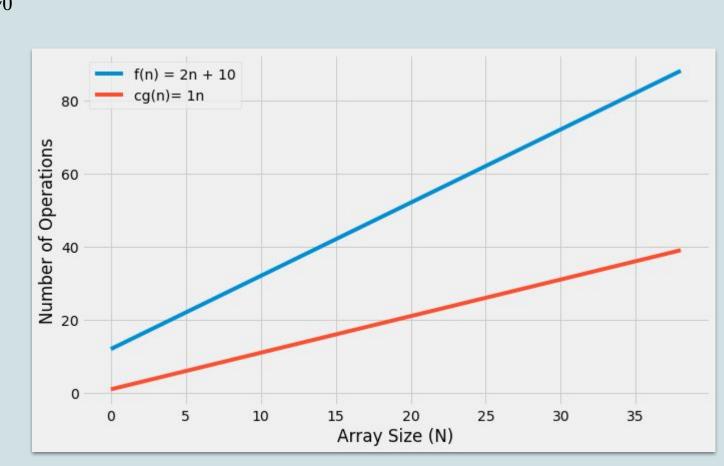
Big O

$$egin{aligned} f(n) &= O(g(n)) \ if \; \exists c, n_0 \; \, orall n \geq n_0 \ f(n) &\leq c g(n) \end{aligned}$$

Meaning: f(n) is O(g(n))if there exist two constants \mathbf{c} and $\mathbf{n}_{\mathbf{n}}$ such that for every n greater than or equal to n_n , f(n)is smaller than or equal to cg(n).

 $egin{aligned} f(n) &= O(g(n)) \ if \ \exists c, n_0 \ \ orall n \geq n_0 \ f(n) \leq c g(n) \end{aligned}$

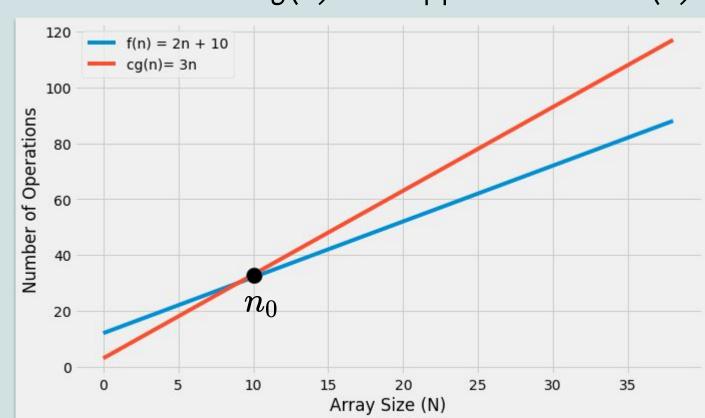


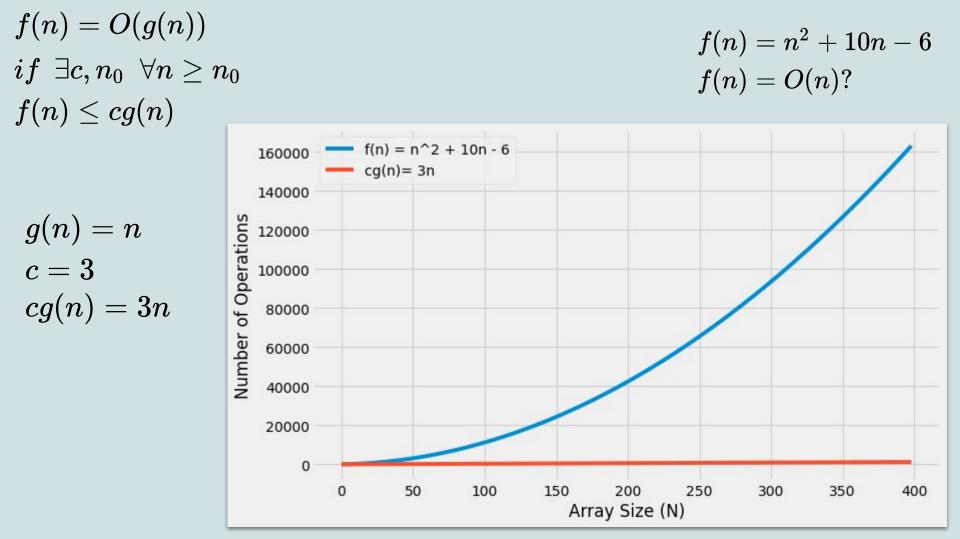


 $egin{aligned} f(n) &= O(g(n)) \ if \ \exists c, n_0 \ \ orall n \geq n_0 \ f(n) \leq c g(n) \end{aligned}$

$$egin{aligned} g(n) &= n \ c &= 3 \ cg(n) &= 3n \end{aligned}$$

g(n) is a upper bound of f(n)





f(n) = O(g(n)) $f(n) = n^2 + 10n - 6$ $if \ \exists c, n_0 \ \ \forall n \geq n_0$ f(n) = O(n)? $f(n) \le cg(n)$ $f(n) = n^2 + 10n - 6$ 160000 cg(n)= 100n 140000 g(n) = nNumber of Operations 120000 c = 100100000 cg(n) = 100n80000 60000

 n_0

100

150

200

Array Size (N)

250

300

350

400

50

40000

20000

0

0

f(n) = O(g(n)) $if \ \exists c, n_0 \ \ orall n \geq n_0$ $f(n) \le cg(n)$

$$n_0$$

200

0

 $g(n) = n^2$ c = 2 $cg(n)=2n^2$

 n^2 is a upper bound of f(n) $f(n) = O(n^2)$? 1200 $f(n) = n^2 + 10n - 6$ $cg(n) = 2n^2$ 1000 Number of Operations 800 600 400

 n_0

10

Array Size (N)

15

 $f(n)=n^2+10n-6$

20

 $egin{aligned} if \ \exists c, n_0 \ \ orall n \geq n_0 \ f(n) \leq cg(n) \end{aligned}$

f(n) = O(g(n))

n! is also a upper bound of f(n)

 $f(n) = n^2 + 10n - 6$ f(n) = O(n!)? $f(n) = n^2 + 10n - 6$ cg(n) = 1n!



 n_{0} 10

20

30

Array Size (N)

40

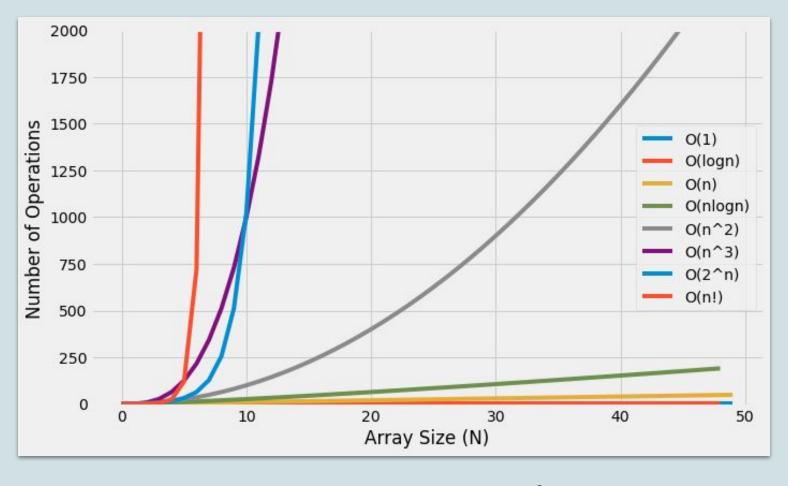
50

60

250

0

n² and n! are both upper bounds of $f(n)=n^2+10n-6$, but the tight one is n²



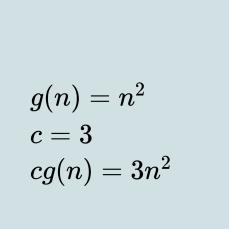
 $O(1) < O(logn) < O(n) < O(nlogn) < O(n^2) < O(2^n) < O(n!)$

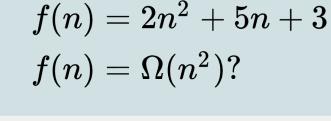
Big Ω

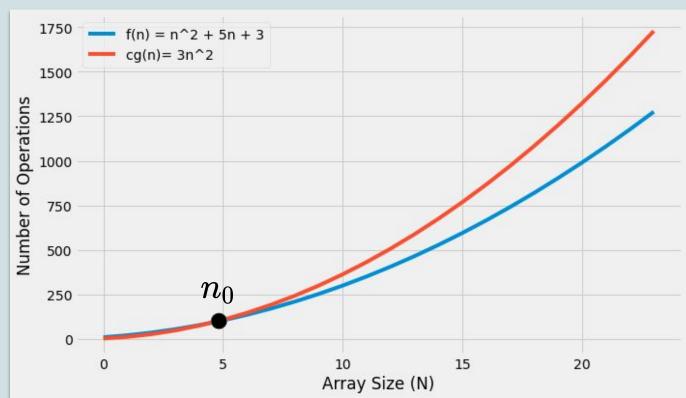
$$egin{aligned} f(n) &= \Omega(g(n)) \ if \; \exists c, n_0 \; \, orall n \geq n_0 \ f(n) &\geq c g(n) \end{aligned}$$

Meaning: f(n) is O(g(n))if there exist two constants ${\bf c}$ and ${\bf n_n}$ such that for every n greater than or equal to n_n , f(n)is greater than or equal **to** cg(n).

 $egin{aligned} f(n) &= \Omega(g(n)) \ if \;\; \exists c, n_0 \;\; orall n \geq n_0 \ f(n) &\geq c g(n) \end{aligned}$

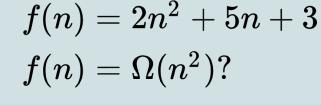


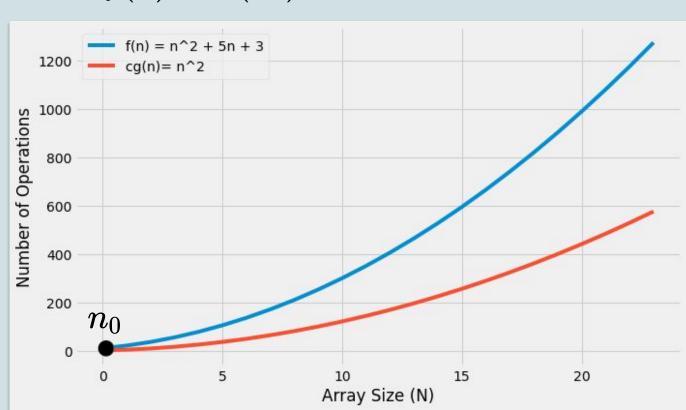




 $egin{aligned} f(n) &= \Omega(g(n)) \ if \;\; \exists c, n_0 \;\; orall n \geq n_0 \ f(n) &\geq c g(n) \end{aligned}$

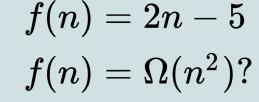
$$egin{aligned} g(n) &= n^2 \ c &= 1 \ cg(n) &= 1n^2 \end{aligned}$$

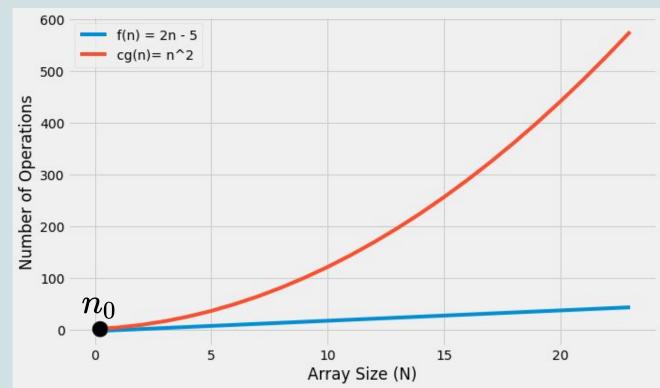




$$egin{aligned} f(n) &= \Omega(g(n)) \ if \;\; \exists c, n_0 \;\; orall n \geq n_0 \ f(n) \geq c g(n) \end{aligned}$$

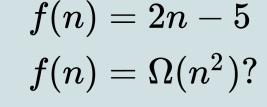
$$egin{aligned} g(n) &= n^2 \ c &= 1 \ cg(n) &= 1n^2 \end{aligned}$$

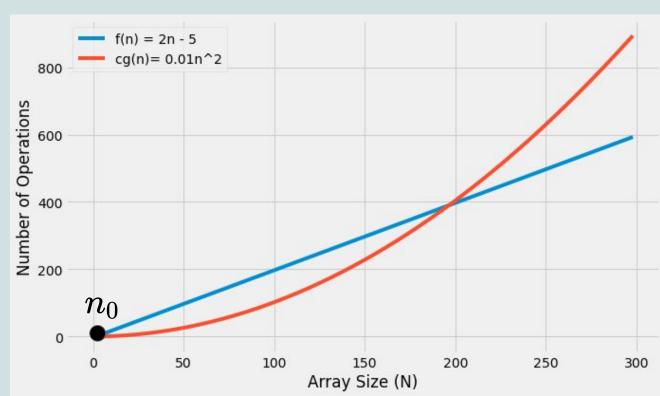




$$egin{aligned} f(n) &= \Omega(g(n)) \ if \;\; \exists c, n_0 \;\; orall n \geq n_0 \ f(n) \geq c g(n) \end{aligned}$$

$$g(n)=n^2 \ c=0.01 \ cg(n)=0.01n^2$$

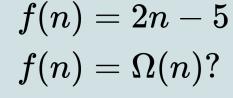


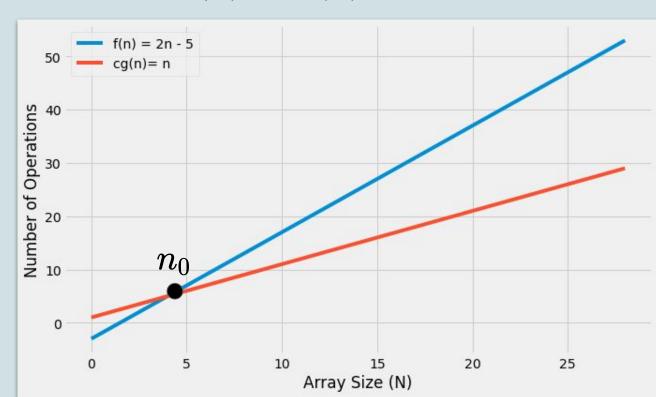


 $egin{aligned} f(n) &= \Omega(g(n)) \ if \;\; \exists c, n_0 \;\; orall n \geq n_0 \ f(n) \geq c g(n) \end{aligned}$

$$egin{aligned} g(n) &= n \ c &= 1 \ cg(n) &= n \end{aligned}$$

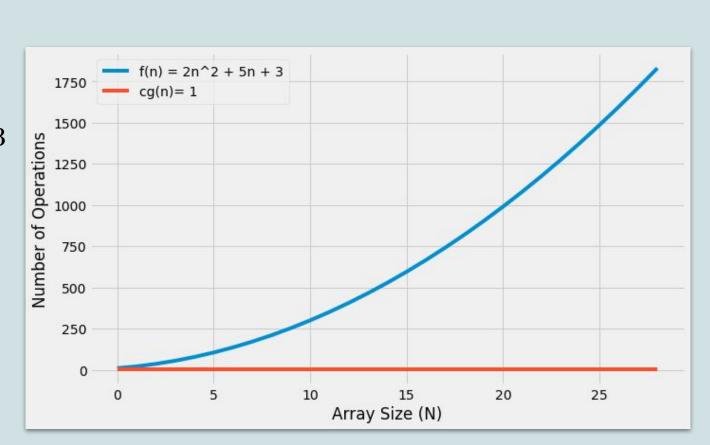
n is a lower bound
of f(n)





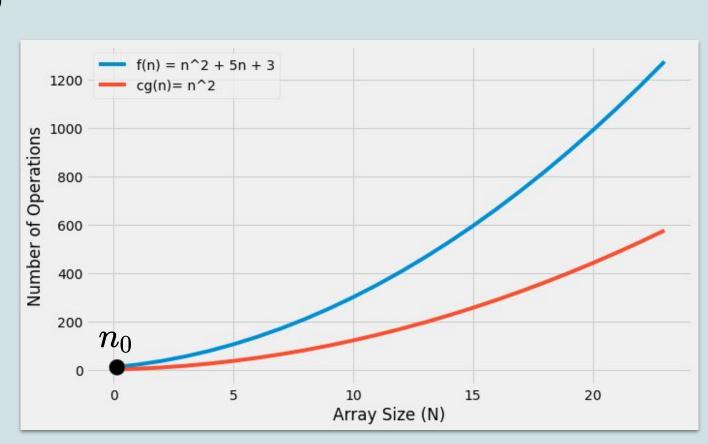
 $egin{aligned} f(n) &= \Omega(g(n)) \ if \ \exists c, n_0 \ \ orall n \geq n_0 \ f(n) \geq c g(n) \end{aligned}$

 $f(n)=2n^2+5n+3$ is $\Omega(1)$, but it is not the tight lower bound.



 $egin{aligned} f(n) &= \Omega(g(n)) \ if \ \exists c, n_0 \ \ orall n \geq n_0 \ f(n) \geq c g(n) \end{aligned}$

$$f(n) = 2n^2 + 5n + 3$$
 $g(n) = n^2$ is the tight lower bound.



Big Θ

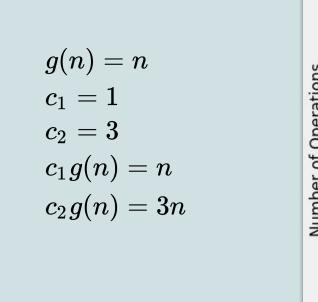
$$egin{aligned} f(n) &= \Theta(g(n)) \ if \ \exists c_1, c_2, n_0 \ \ orall n \geq n_0 \ c_1 g(n) \leq f(n) \leq c_2 g(n) \end{aligned}$$

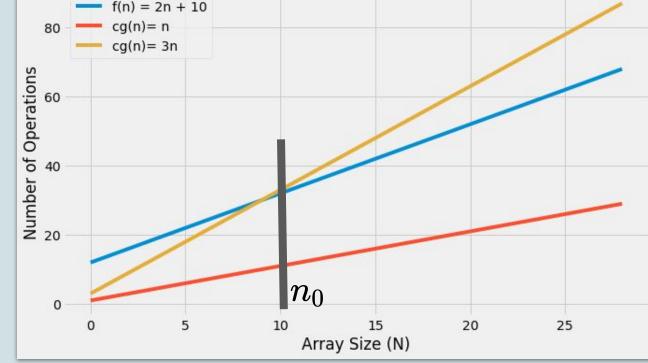
Meaning: f(n) is $\Theta(g(n))$ if there exist three constants c_1, c_2 , and n_0 such that for every n greater or equal to to n_o, f(n) is greater than or equal to $c_1g(n)$ and smaller than or equal to $c_2g(n)$.

 $f(n) = \Theta(g(n))$ $if \ \exists c_1, c_2, n_0 \ \ \forall n \geq n_0$ $c_1g(n) \leq f(n) \leq c_2g(n)$

 $f(n) = \Theta(n)$? f(n) = 2n + 10cg(n) = ncg(n) = 3n

f(n) = 2n + 10





$f(n) = \Theta(n) \Rightarrow f(n) = O(n)$

$$f(n) = O(n) \not\Rightarrow f(n) = \Theta(n)$$

$f(n) = \Theta(n) \Rightarrow f(n) = \Omega(n)$

$$J(n) = \Theta(n) \Rightarrow J(n) = \Omega(n)$$

 $f(n) = \Omega(n) \Leftrightarrow f(n) = \Theta(n)$