Classical Algorithms

Kruskal
Minimum Spanning Tree (MST)

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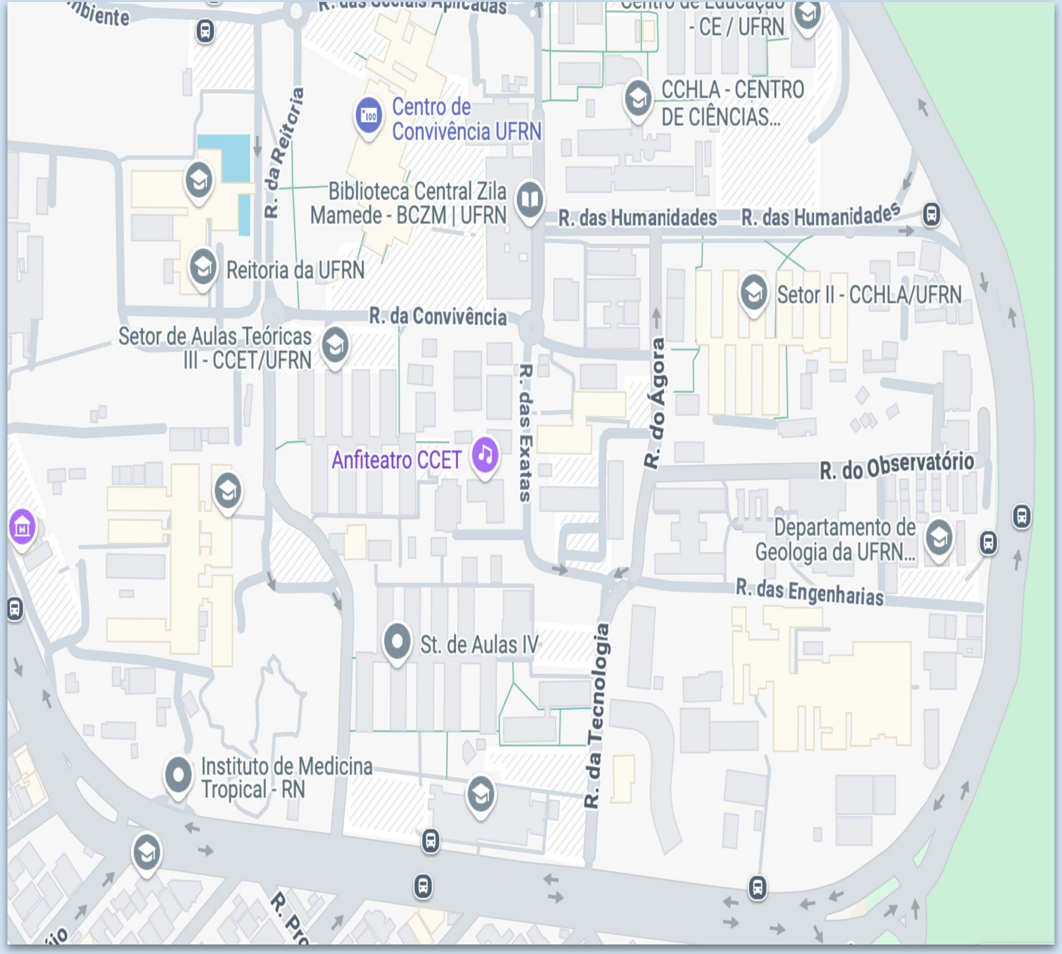
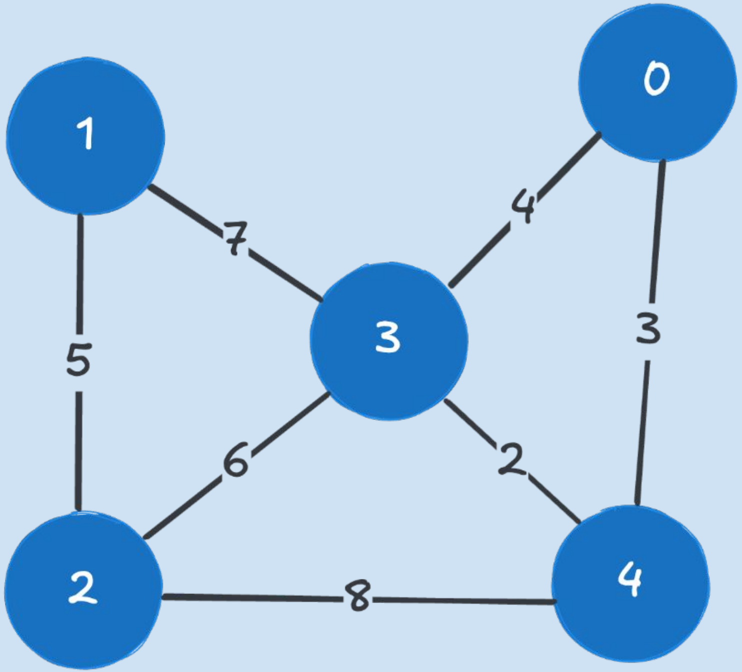
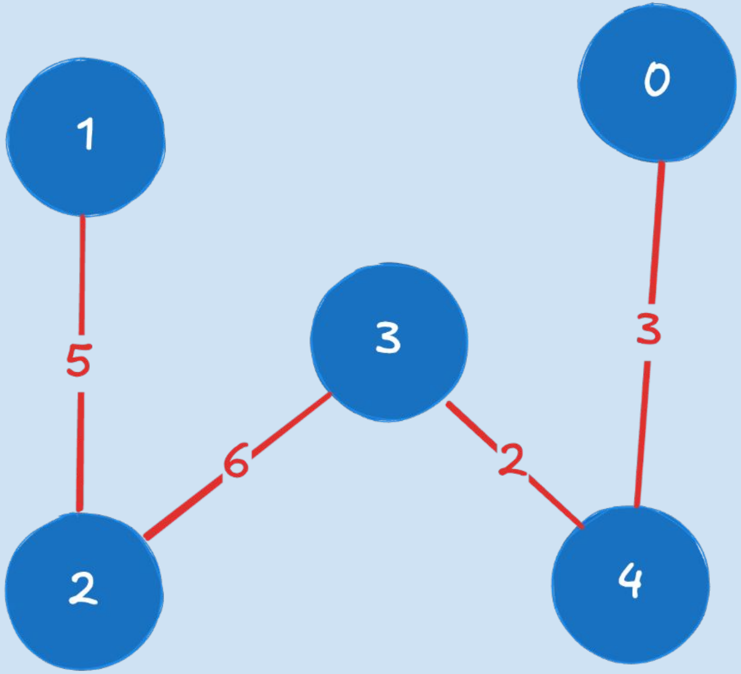
Imagine that you are installing internet cables at **UFRN**. You want to ensure that:

- All buildings are connected.
- You use the shortest total length of cables.
- There are no unnecessary cables going back and forth.

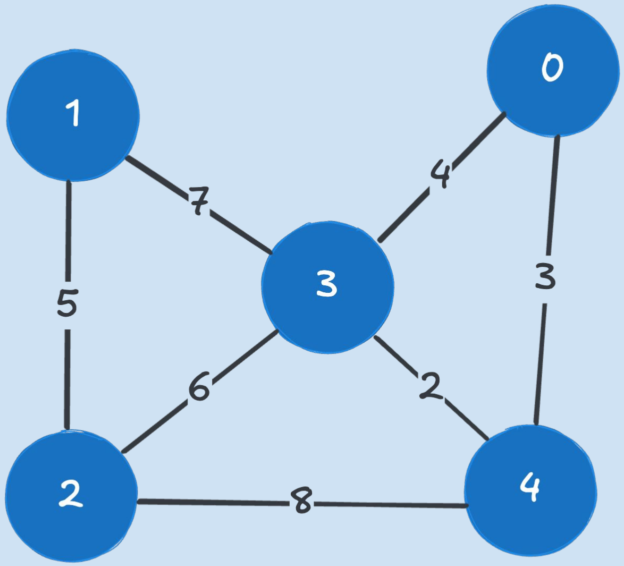




graph - connected, undirected,
weighted (non-negative)

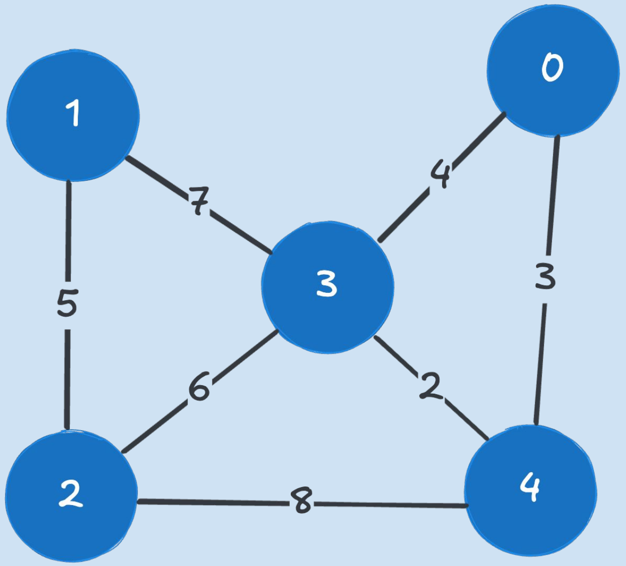


Minimum Spanning Tree (MST)



Graph Representation

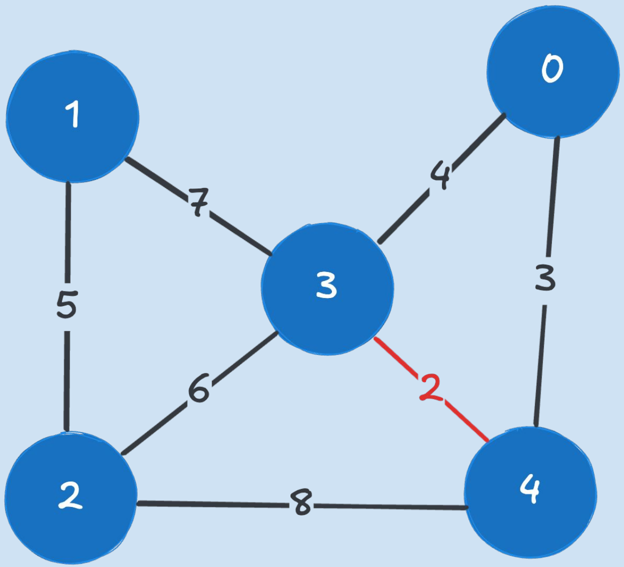
```
edges = [
    [[4, 3], [3, 4]],           # Vertex 0
    [[2, 5], [3, 7]],           # Vertex 1
    [[1, 5], [3, 6], [4, 8]], # Vertex 2
    [[0, 4], [1, 7], [2, 6], [4, 2]], # Vertex 3
    [[3, 2], [0, 3], [2, 8]], # Vertex 4
]
```



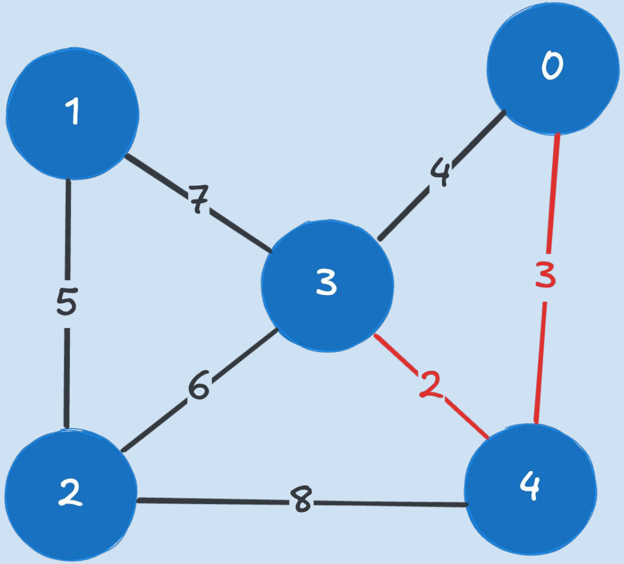

```
edgeList = []
for sourceIndex, vertex in enumerate(edges):
    for edge in vertex:
        if edge[0] > sourceIndex:  # Evita duplicatas
            edgeList.append([sourceIndex, edge[0], edge[1]])
sortedEdges = sorted(edgeList, key=lambda edge: edge[2])
```

```
[
    [3, 4, 2],  # Weight 2
    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```

Sort the Edges by Weight




```
[
    [3, 4, 2],   # Weight 2
    [0, 4, 3],   # Weight 3
    [0, 3, 4],   # Weight 4
    [1, 2, 5],   # Weight 5
    [2, 3, 6],   # Weight 6
    [1, 3, 7],   # Weight 7
    [2, 4, 8],   # Weight 8
]
```




```
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    [3, 4, 2],  # Weight 2
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    [0, 3, 4],  # Weight 4
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    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```

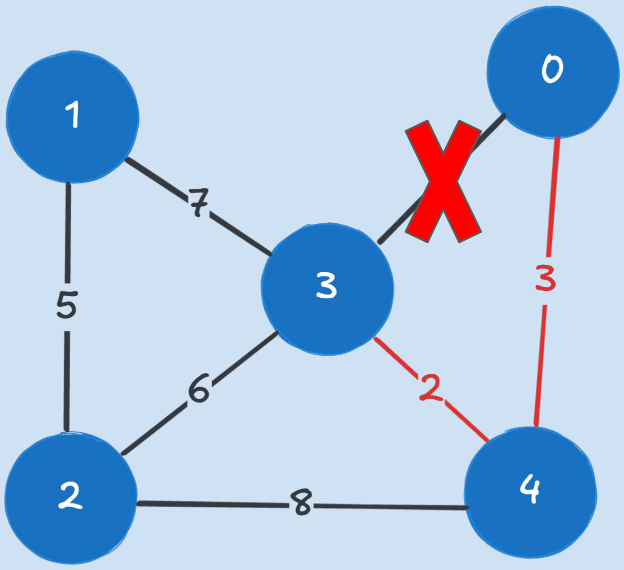


```
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    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```

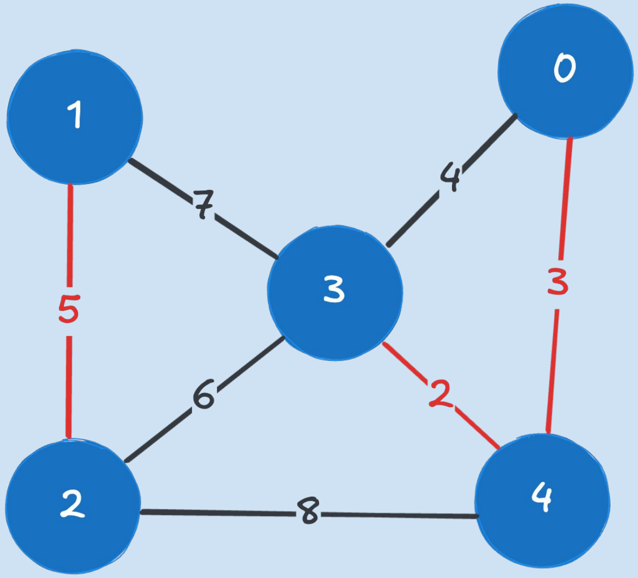


```
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    [0, 3, 4],  # Weight 4
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    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```
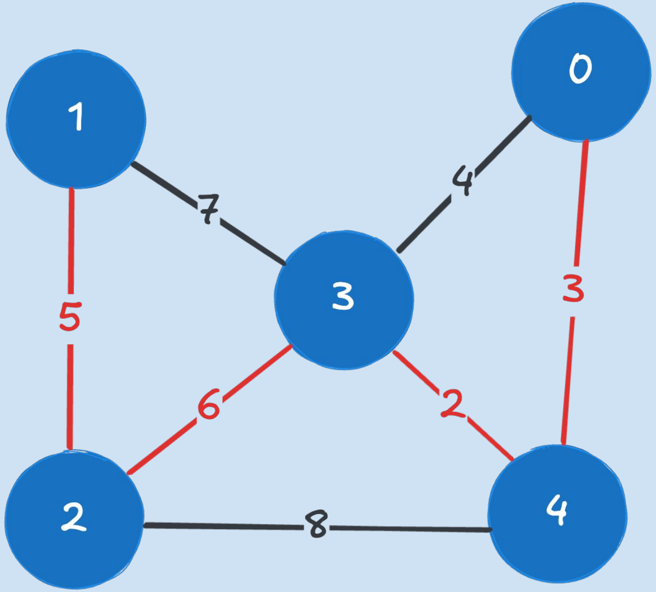
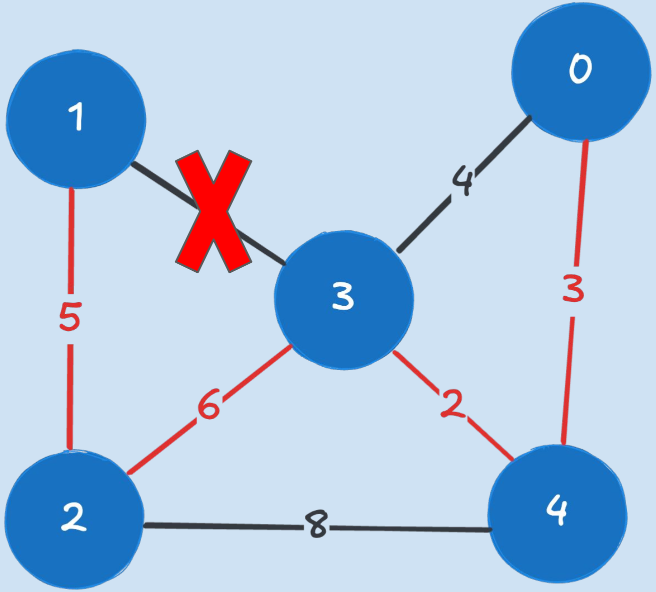
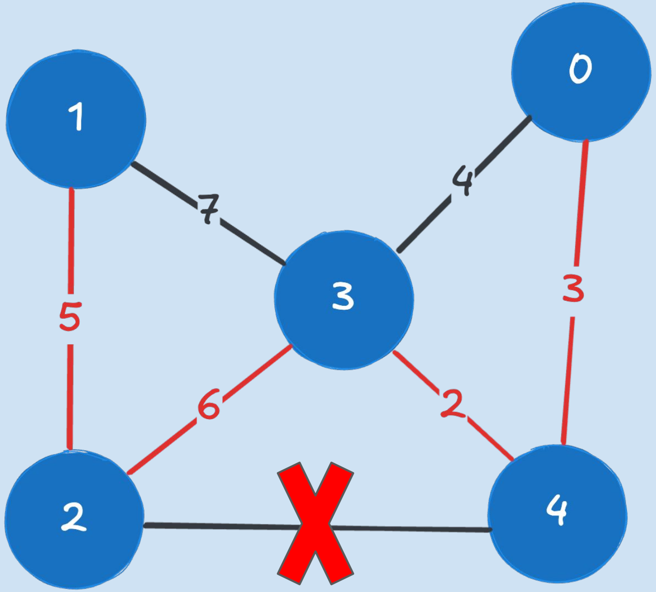



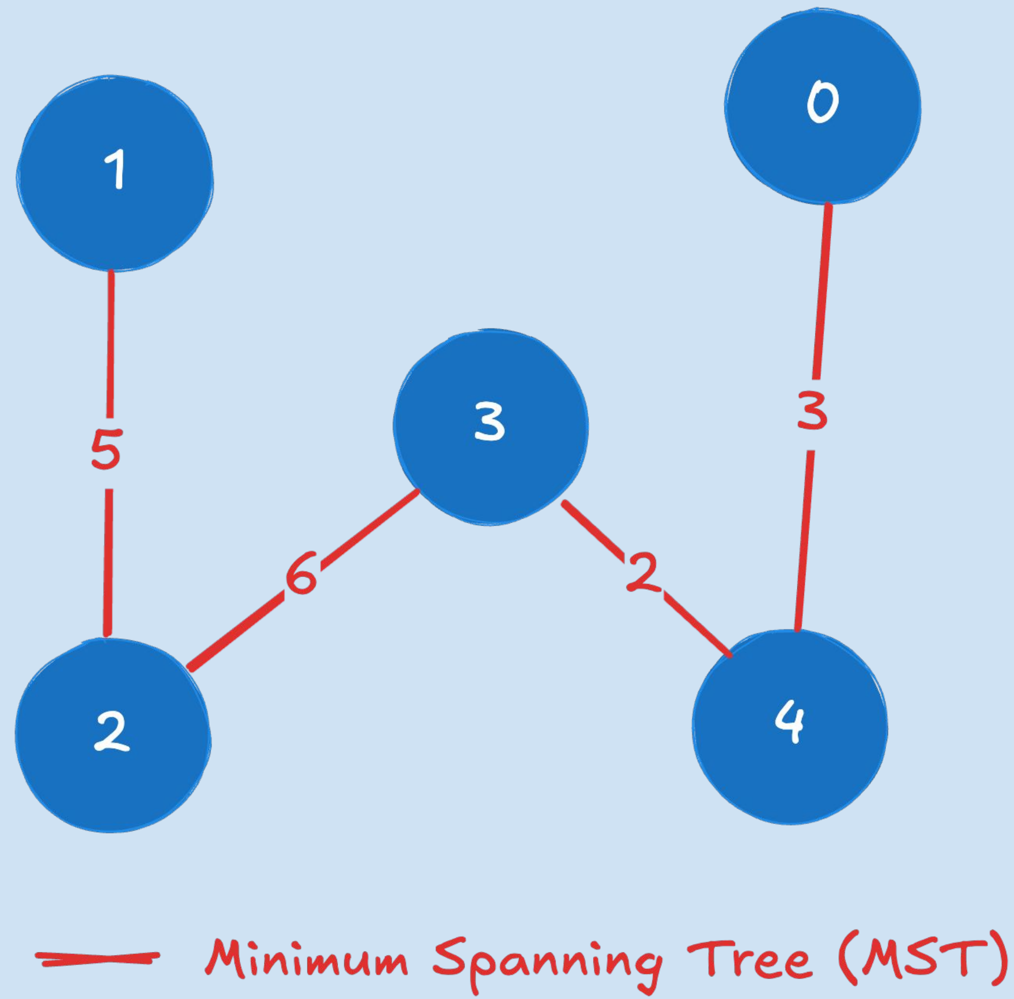

```
[
    [3, 4, 2],  # Weight 2
    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```




```
[
    [3, 4, 2],  # Weight 2
    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```



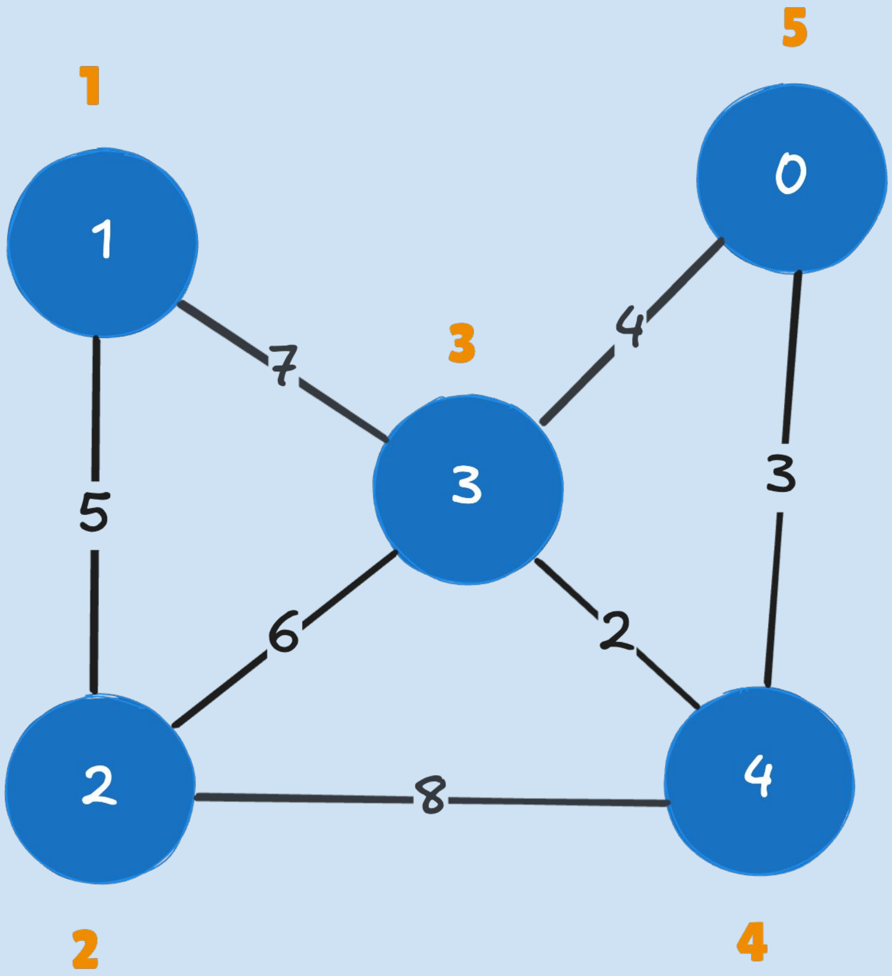

```
[
    [3, 4, 2],  # Weight 2
    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```



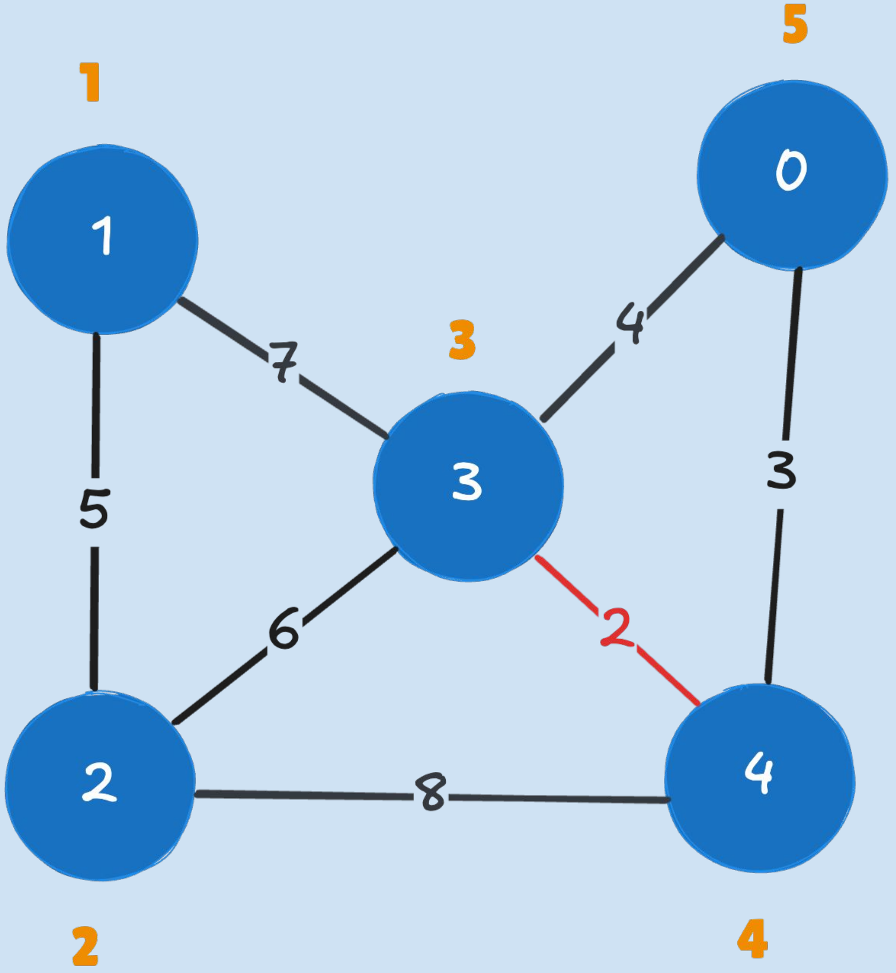
0
1
2
3
4
5
6
2
3
Minimum Spanning Tree (MST)

How can we identify a cycle?

Ans: Union–Find (Disjoint Set Union - DSU) data structure

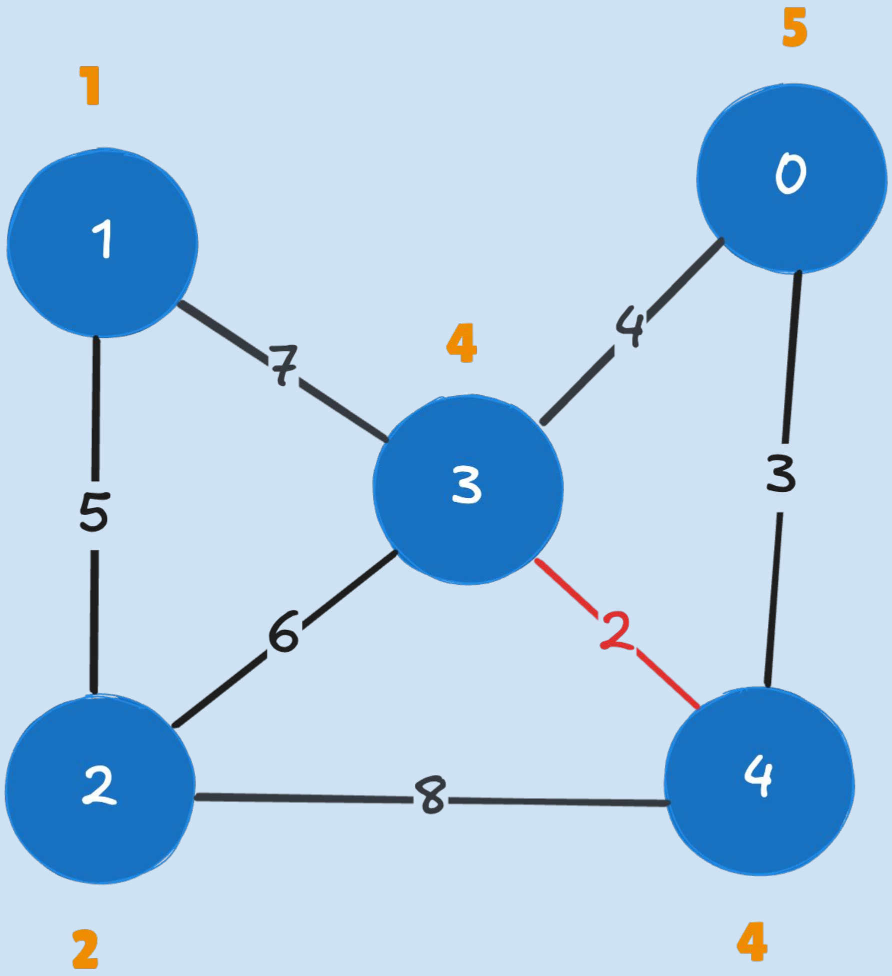



```
[
    [3, 4, 2],  # Weight 2
    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```



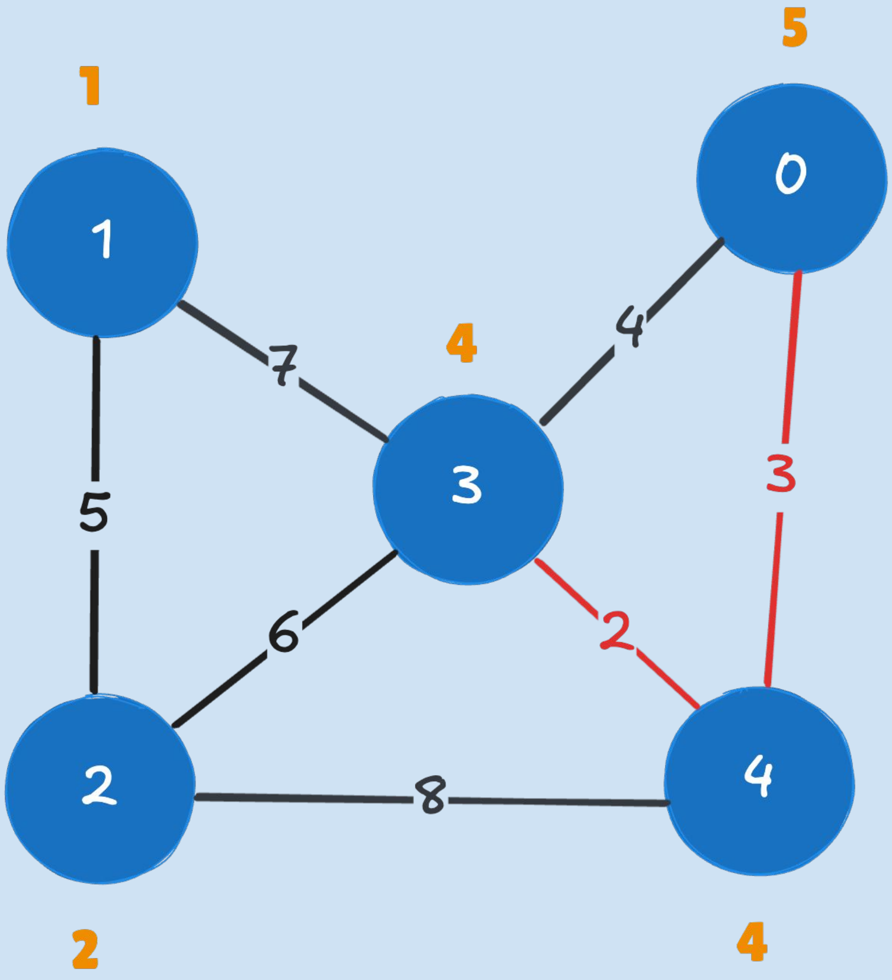
Edge [3,4,2] connects vertices 3 and 4, which belongs to different set.

```
[
    [3, 4, 2],  # Weight 2
    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```



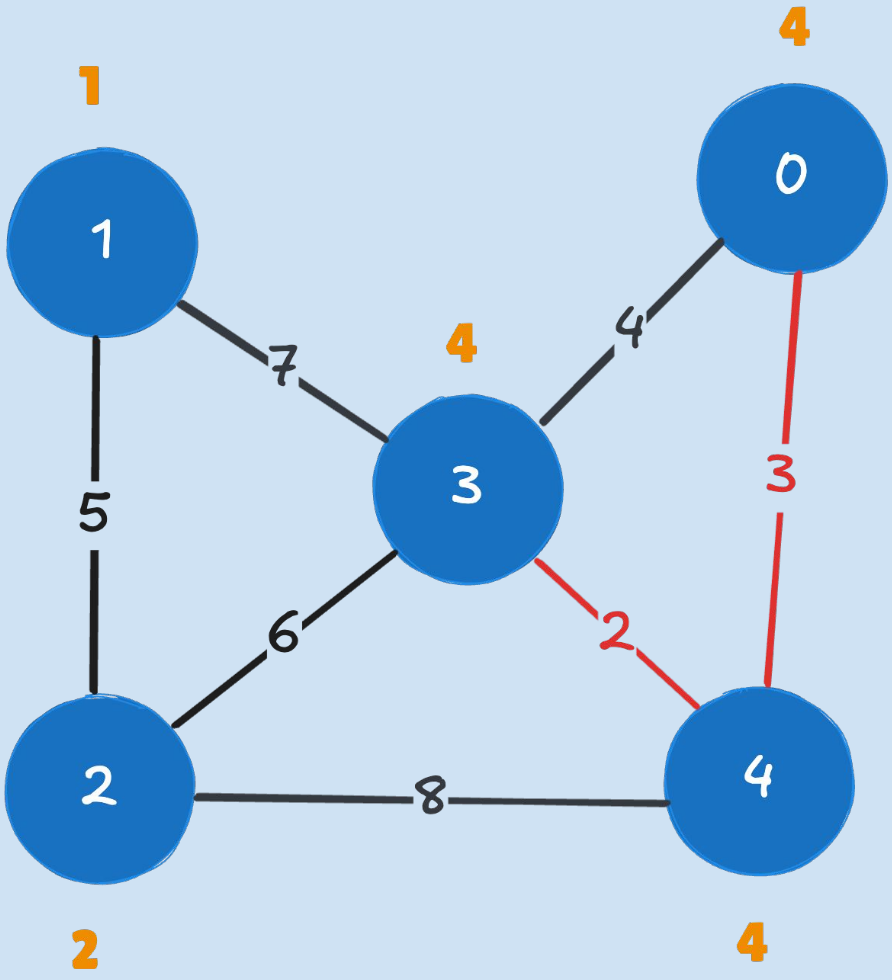
An union between the sets 3 and 4 is done.

```
[
    [3, 4, 2],  # Weight 2
    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```



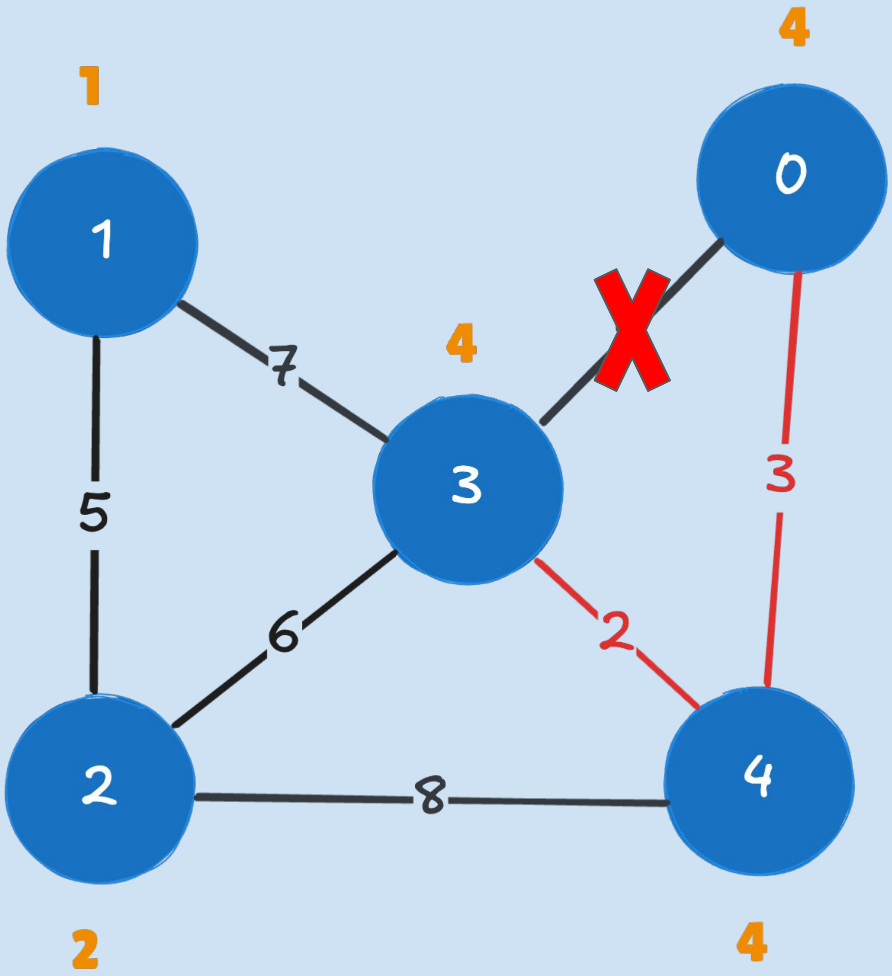
Edge [0,4,3] connects vertices 0 and 4, which belong to different sets.

```
[
    [3, 4, 2],  # Weight 2
    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```



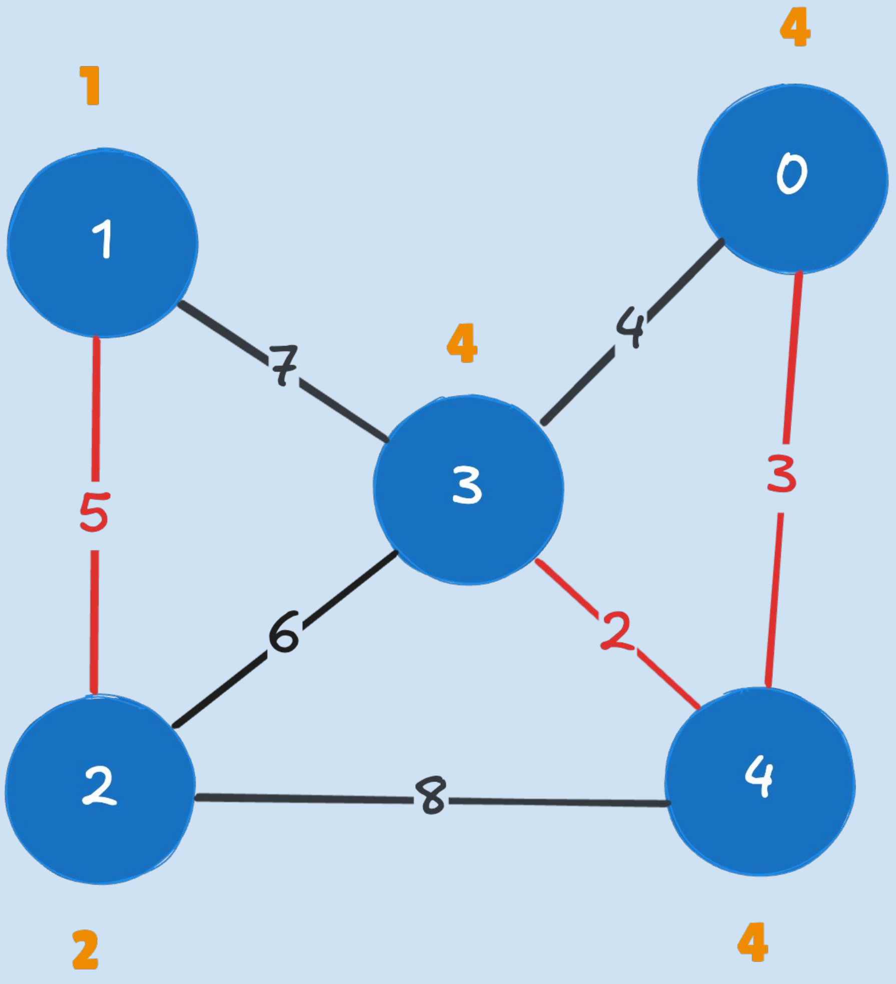
An union between the sets 4 and 5 is done.

```
[
    [3, 4, 2],   # Weight 2
    [0, 4, 3],   # Weight 3
    [0, 3, 4],   # Weight 4
    [1, 2, 5],   # Weight 5
    [2, 3, 6],   # Weight 6
    [1, 3, 7],   # Weight 7
    [2, 4, 8],   # Weight 8
]
```



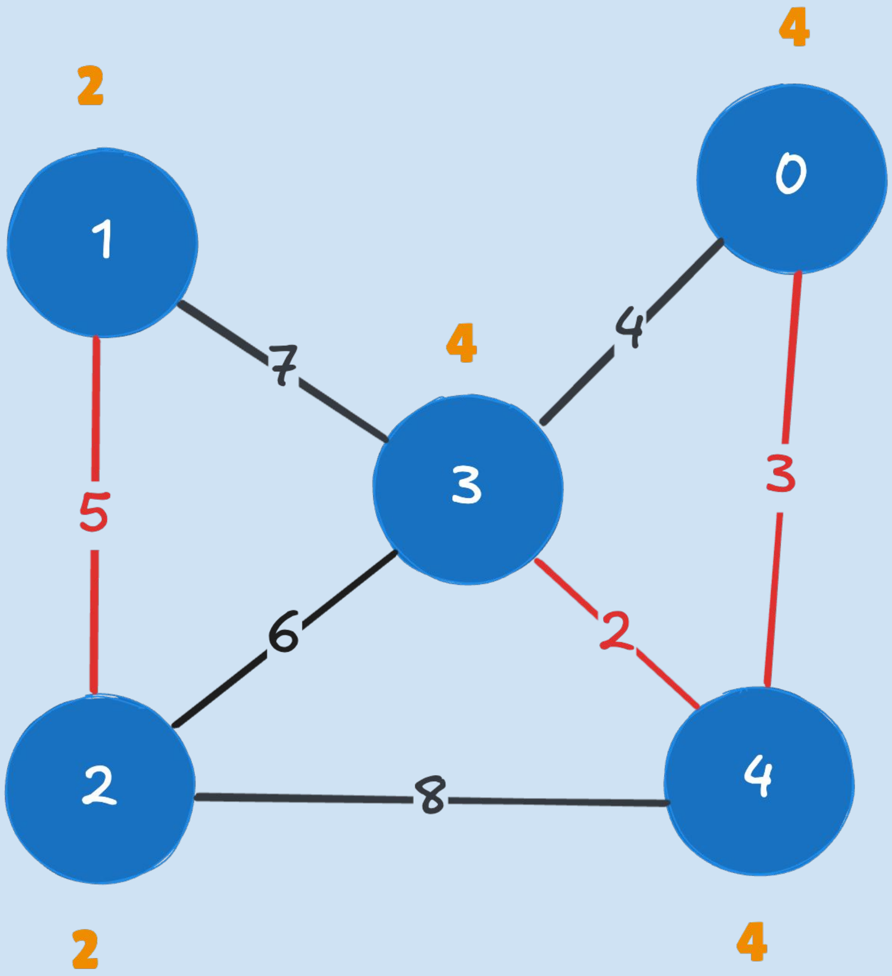
Edge [0,3,4] connects vertices 0 and 3, which belong to same sets.

```
[
    [3, 4, 2],  # Weight 2
    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```



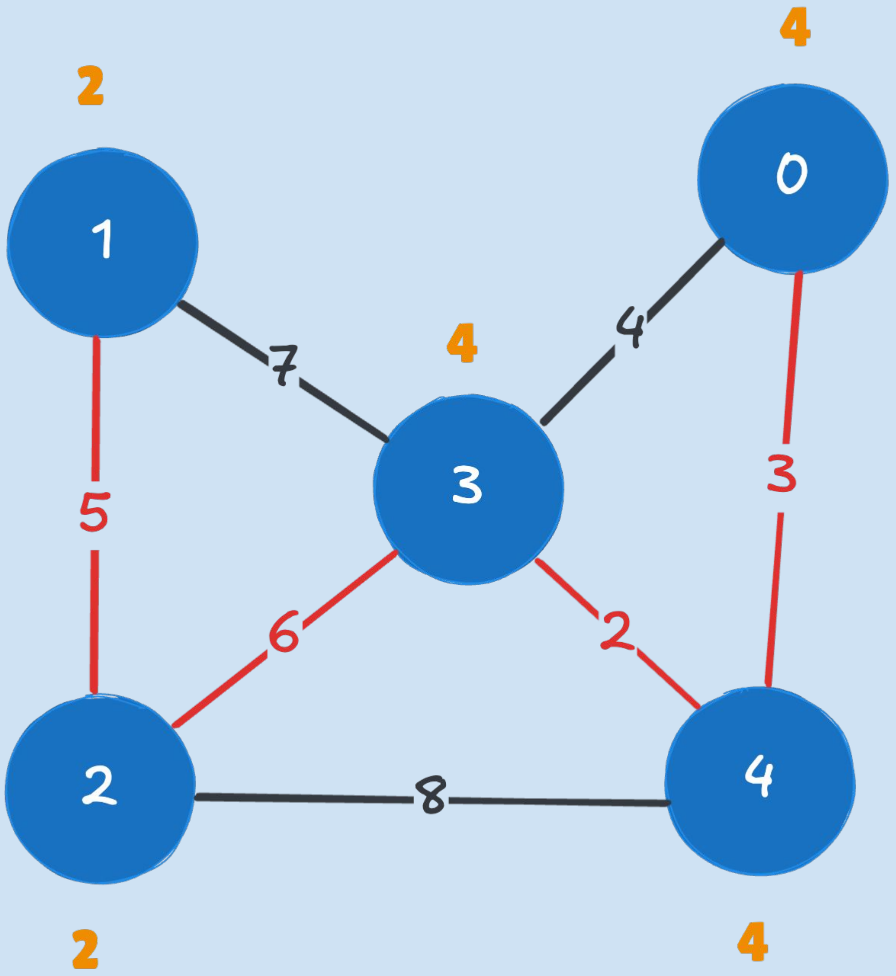
Edge [1,2,5] connects vertices 1 and 2, which belong to different sets.

```
[
    [3, 4, 2],  # Weight 2
    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```



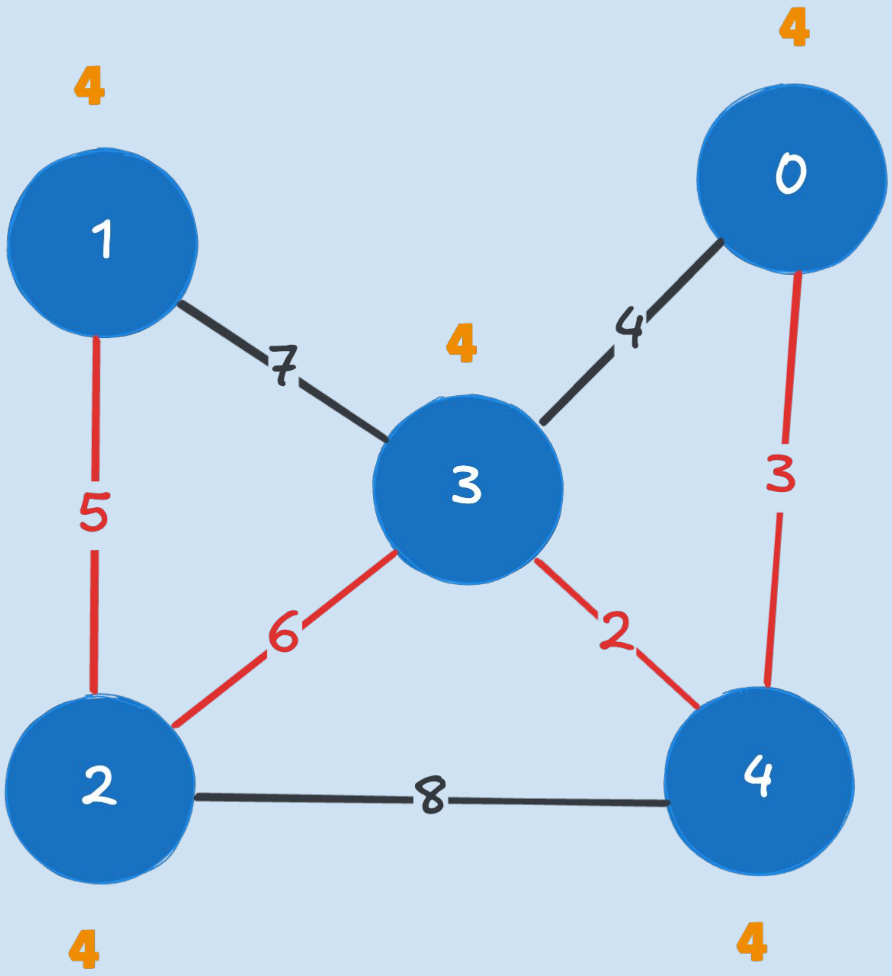
An union between the sets 1 and 2 is done.

```
[
    [3, 4, 2],  # Weight 2
    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```



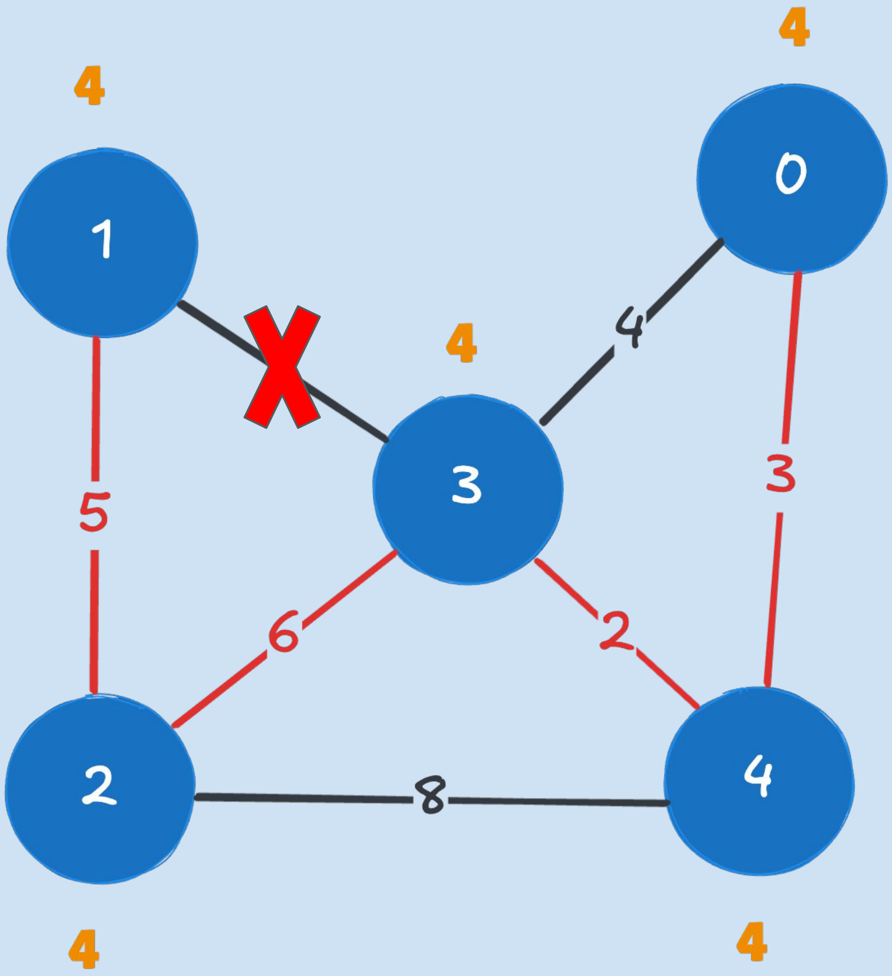
Edge [2,3,6] connects vertices 2 and 3, which belong to different sets.

```
[
    [3, 4, 2],  # Weight 2
    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```



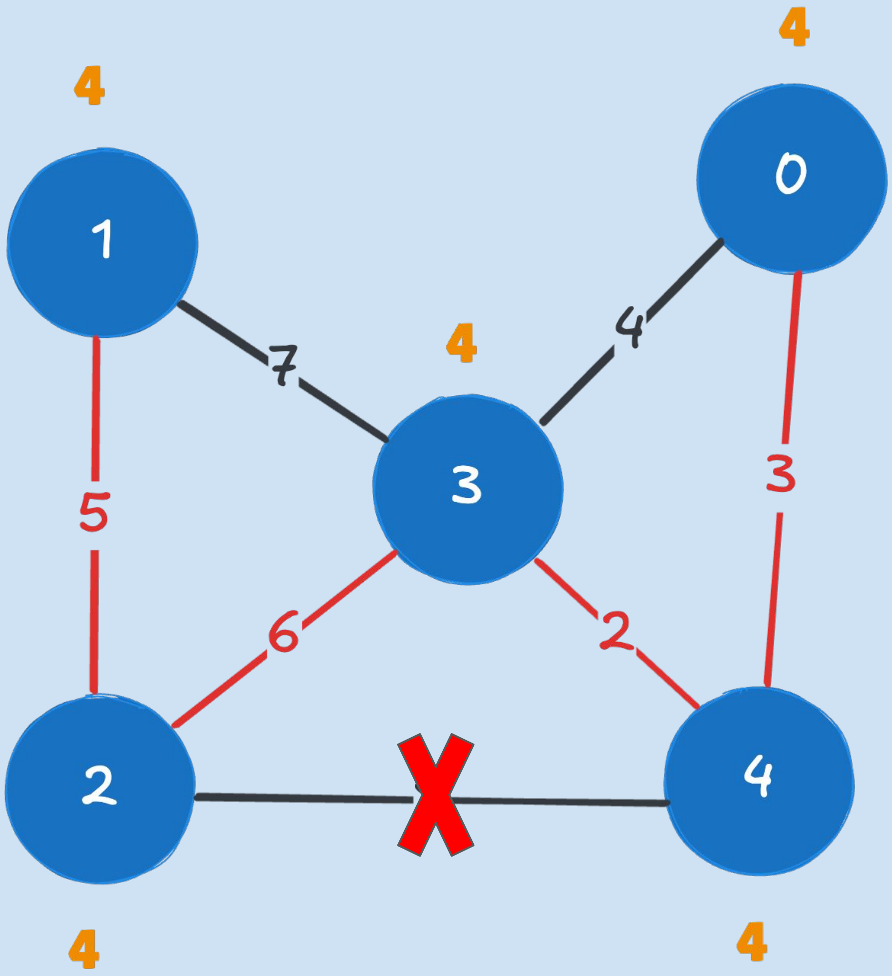
An union between the sets 2 and 4 is done.

```
[
    [3, 4, 2],  # Weight 2
    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```



Edge [1,3,7] connects vertices 1 and 3, which belong to same sets.

```
[
    [3, 4, 2],  # Weight 2
    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```



Edge [2,4,8] connects vertices 2 and 4, which belong to same sets.

```
[
    [3, 4, 2],  # Weight 2
    [0, 4, 3],  # Weight 3
    [0, 3, 4],  # Weight 4
    [1, 2, 5],  # Weight 5
    [2, 3, 6],  # Weight 6
    [1, 3, 7],  # Weight 7
    [2, 4, 8],  # Weight 8
]
```

How about the complexity?

O(E log (E)) time
O(E+V) space

Tirol neighborhoods, Natal-RN

Infrastructure Optimization:
Use Kruskal's Algorithm to design the most cost-effective layout for infrastructure like fiber optics, water pipelines, or power grids by minimizing the total length of required connections.

Transportation Planning:
Connect major transportation hubs (e.g., bus terminals, train stations, and airports) using a Minimum Spanning Tree (MST) to identify the shortest routes and reduce travel distances.

Tourism Route Optimization:
Generate an MST connecting key tourist attractions (e.g., museums, landmarks, beaches) to design efficient sightseeing routes that minimize travel distance.

Urban Expansion Planning:
Use an MST to connect developing neighborhoods or areas under construction to the existing city network with minimal infrastructure costs.

Critical Infrastructure Analysis:
Identify critical roads or intersections by comparing the MST with the original road network, highlighting streets essential for maintaining connectivity.