Algorithm Complexity I

DCA 0209

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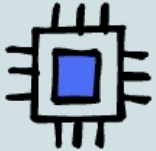
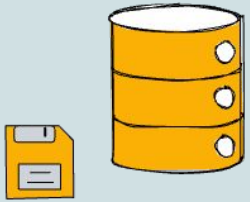
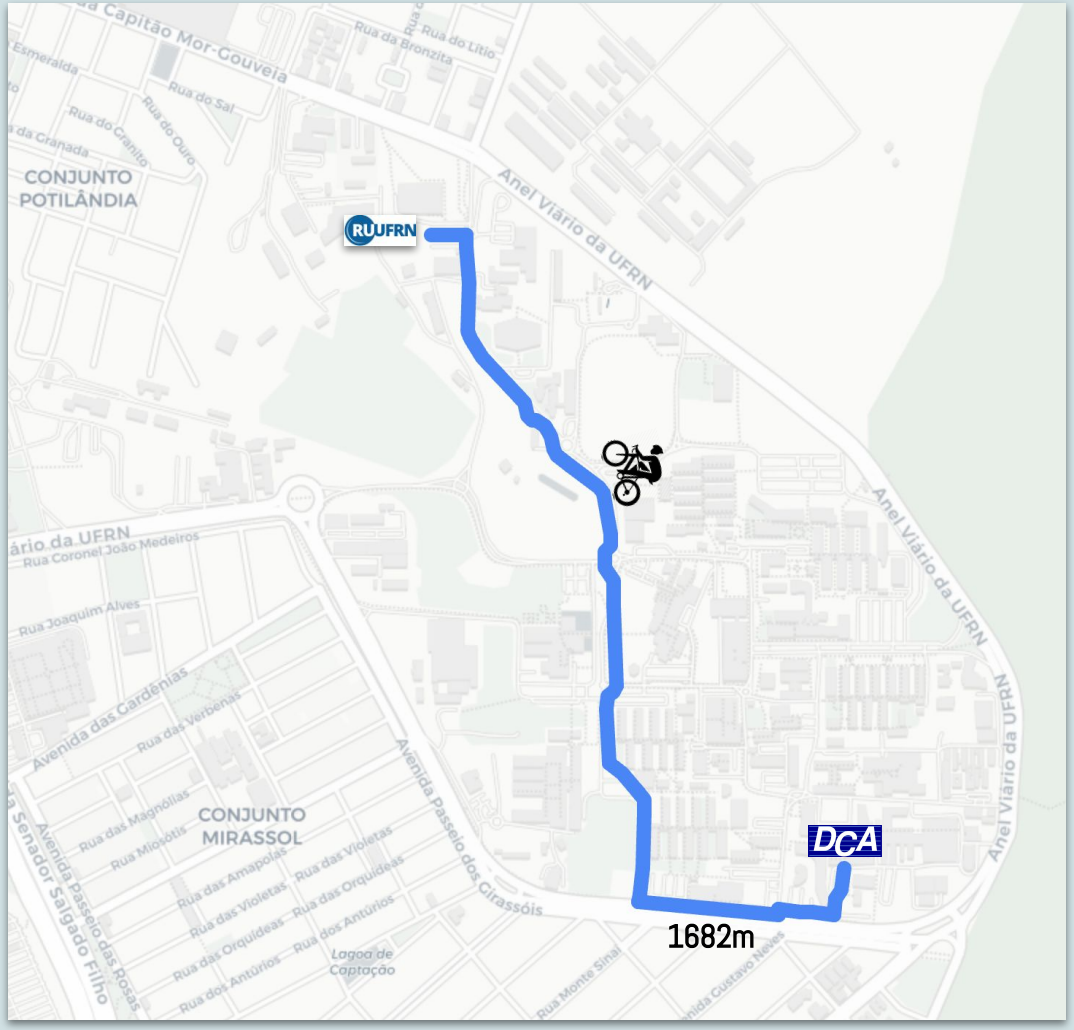
How to compare different algorithms and find the one that will perform **the best**?

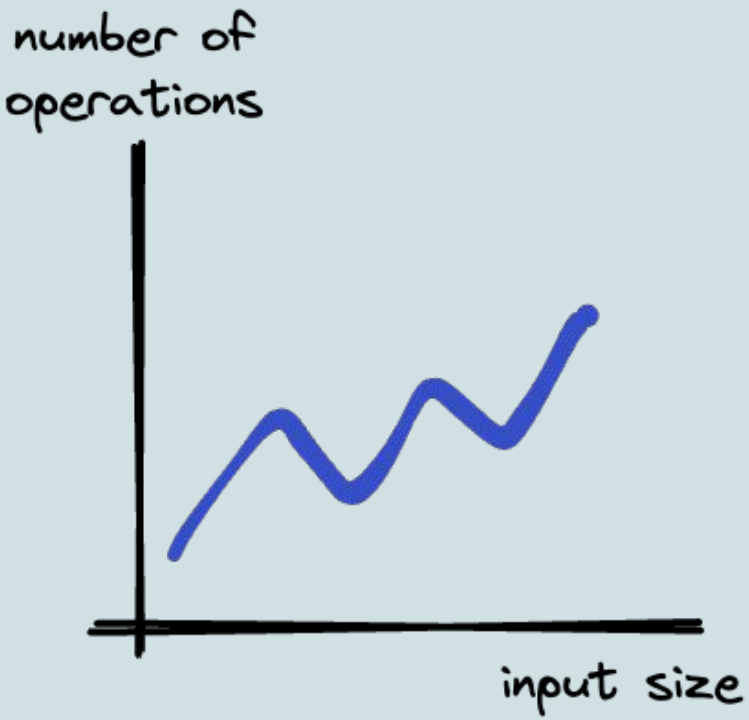
Time

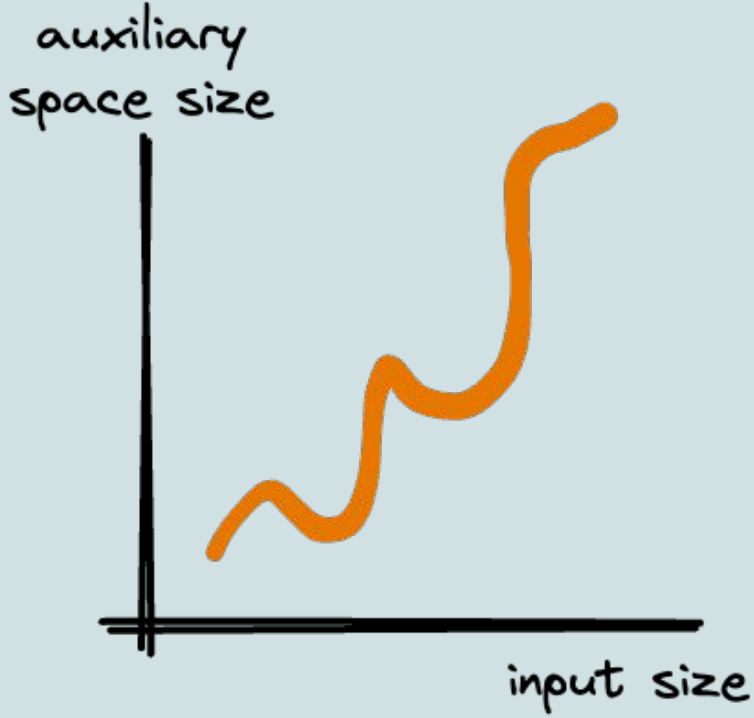




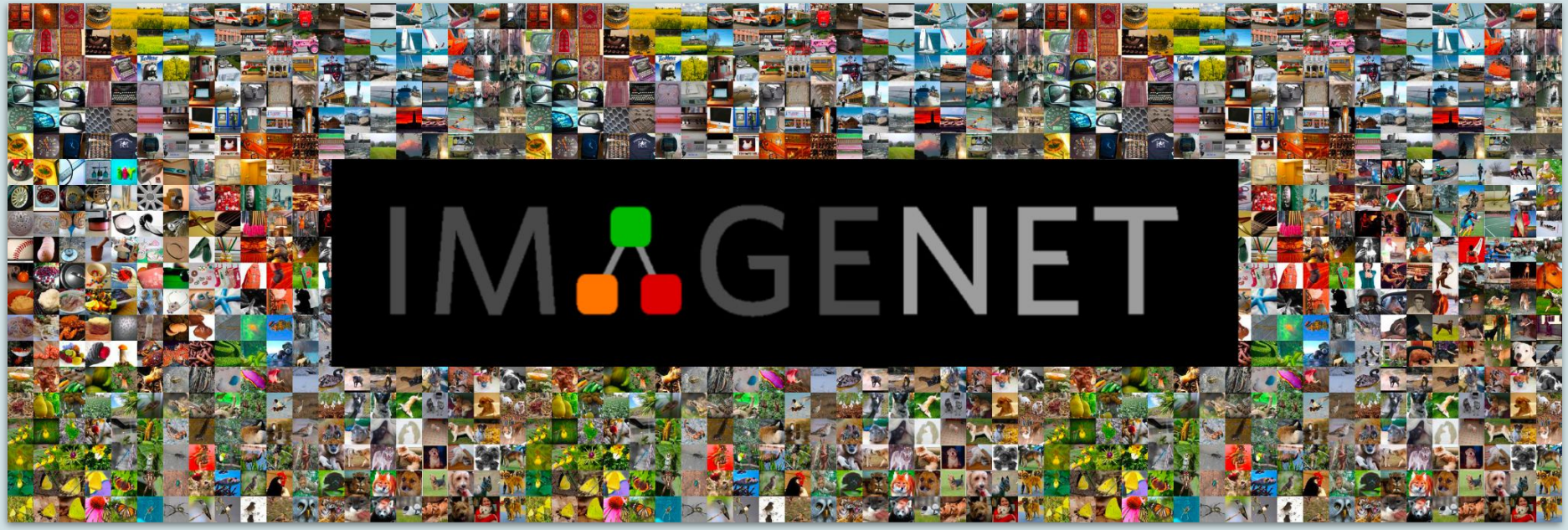
Space


CONJUNTO
POTILÂNDIA
RU UFRN
Anel Viário da UFRN
Rua Coronel João Medeiros
Rua Joaquim Alves
Avenida das Gardênias
Rua das Verbenas
Rua das Magnólias
Rua Miosótis
CONJUNTO
MIRASSOL
Avenida Passeio das Rosas
Avenida Passeio dos Girassóis
Rua das Amapolas
Rua das Violetas
Rua das Orquídeas
Rua dos Antúrios
Lagoa de
Captação
Rua Monte Sinai
DCA
1682m

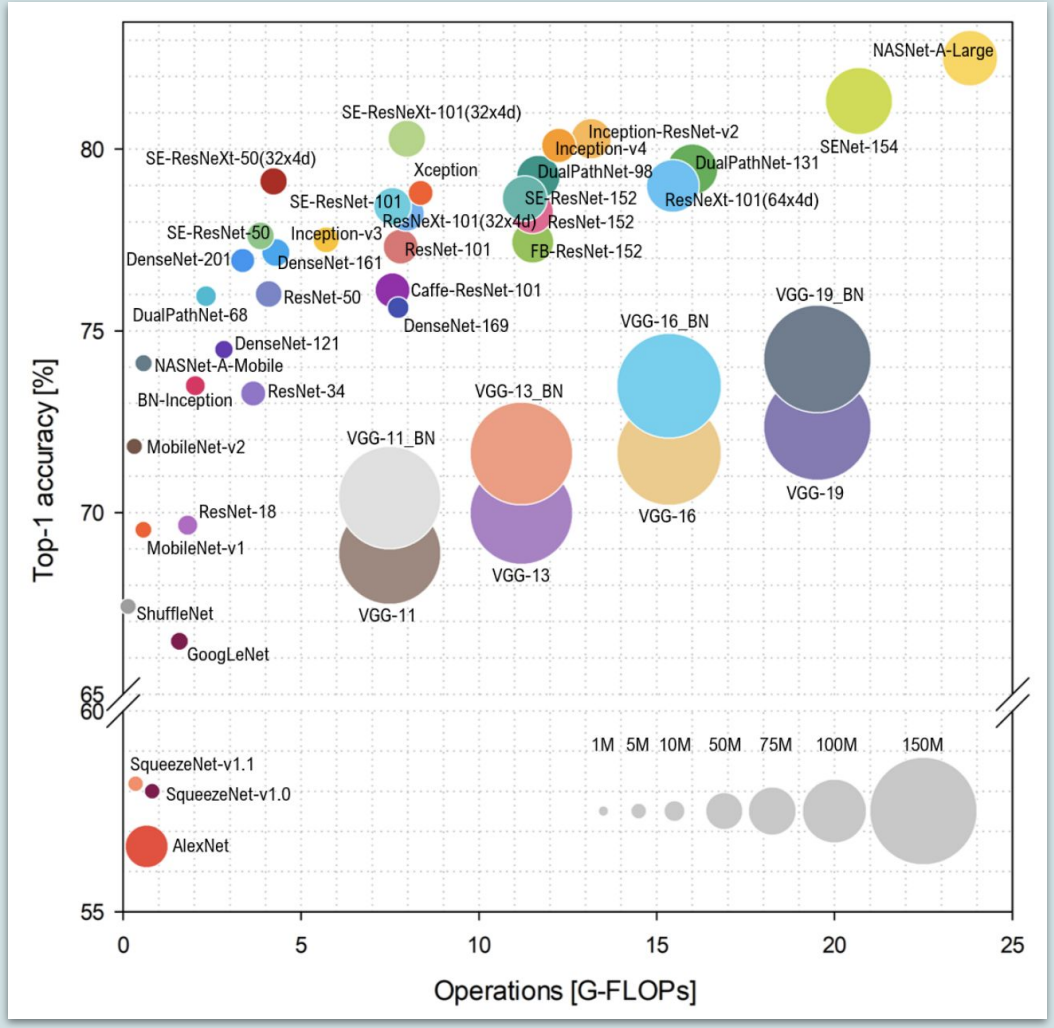

number of
operations
input size


auxiliary
space size
input size

14,197,122 images, 21841 synsets indexed



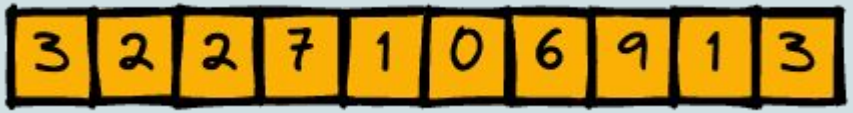
https://www.image-net.org/



Algorithm Complexity

ML Model Evolution ImageNet

Source: S. Bianco, R. Cadene, L. Celona, and P. Napoletano, "Benchmark analysis of representative deep neural network architectures". IEEE Access, vol. 6, 2018.

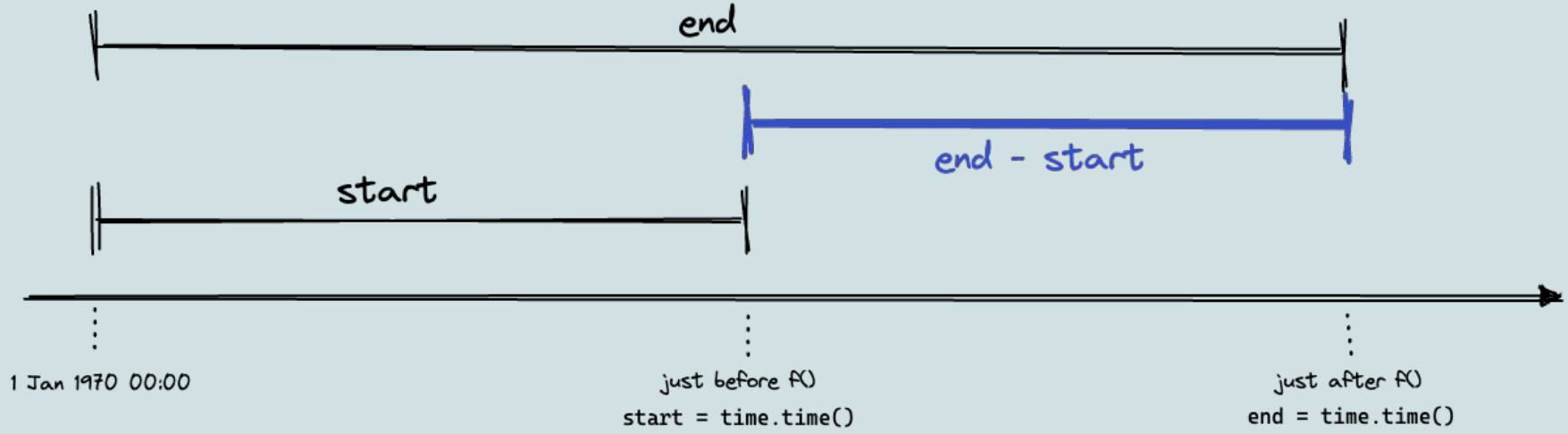



```
def maximum(values):
    answer = None
    for value in values:
        if answer == None or answer < value:
            answer = value
    return answer
```

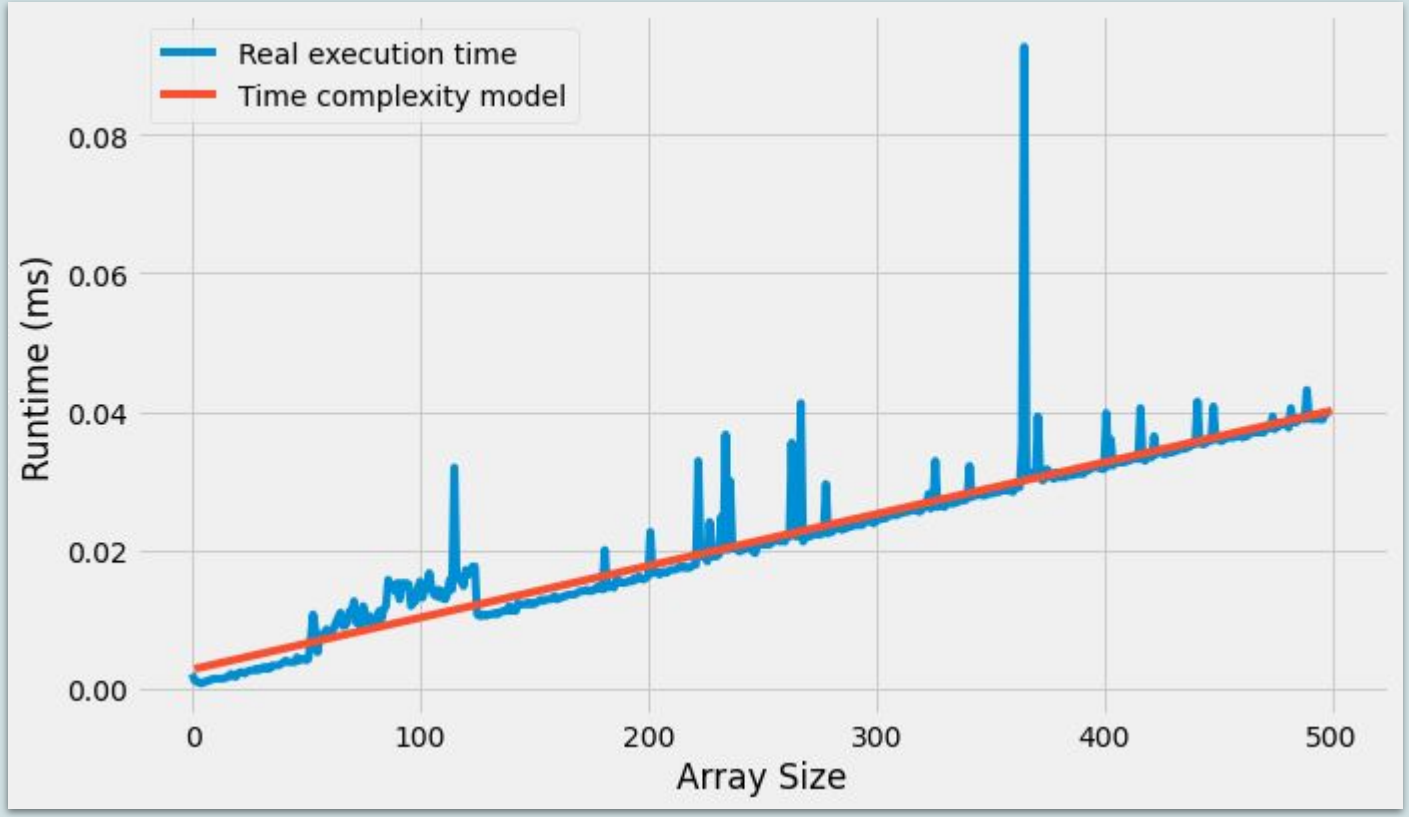
If we double the input, do we double the execution time, do we quadruple it, or something else entirely?

```
import time
start = time.time()
f()
end = time.time()
runtime = end - start
```

How to measure the execution times ?



Exec **maximum**(*values*) 500 times change the *values* size



Recall that we don't want to know
the exact execution time, just how fast it is growing

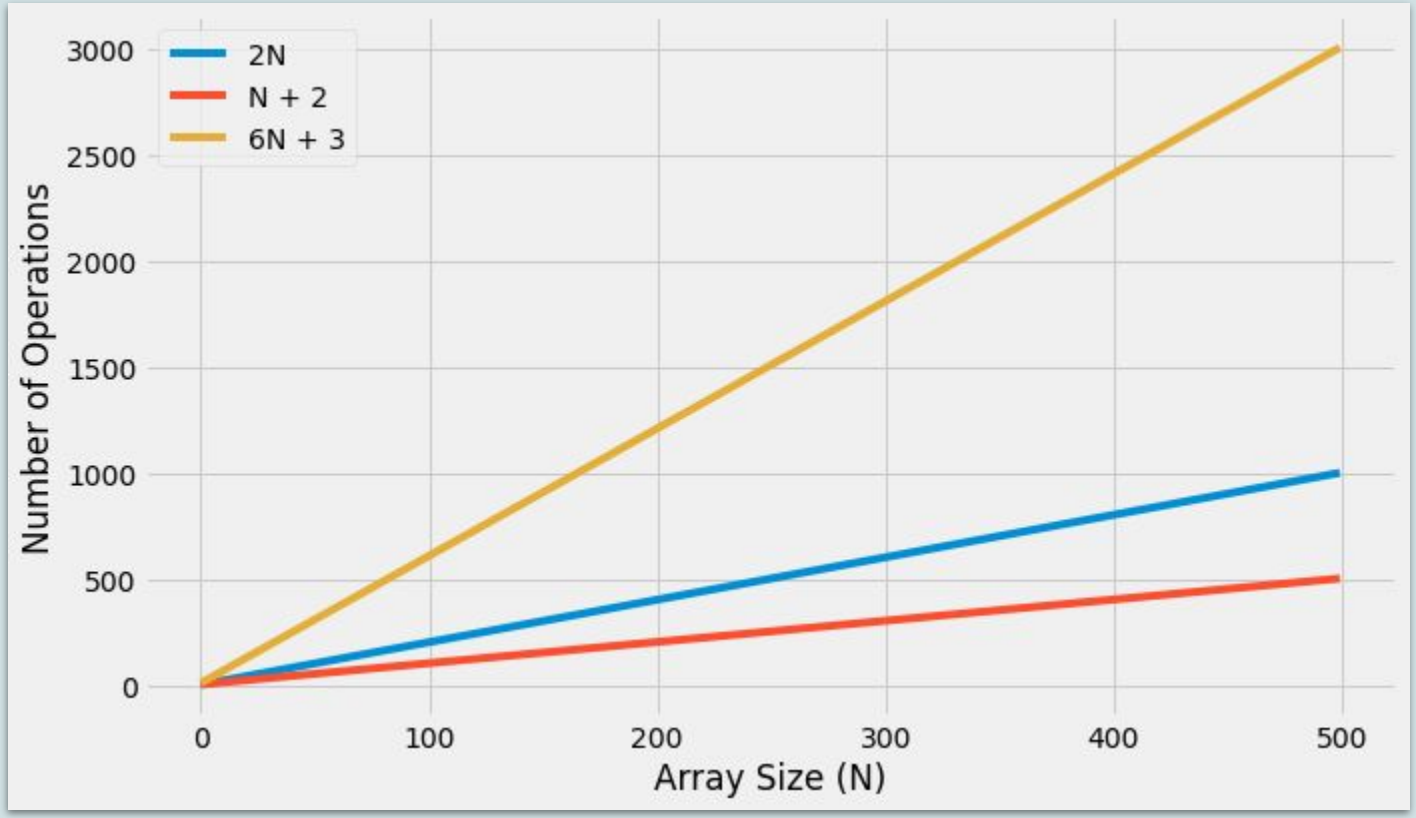
```
def maximum(values):
    answer = None                                  # c1
    for value in values:                           # c2
        if answer == None or answer < value:       # c3
            answer = value                         # c4
    return answer                                  # c5
```

1 time
N times
N times
N times
1 time

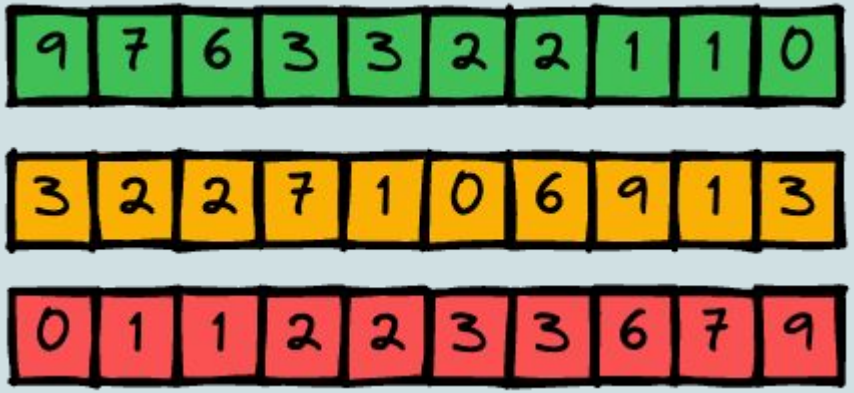
$$C_1 + C_2 \times N + C_3 \times N + C_4 \times N + C_5$$

$$(C_1 + C_5) + N(C_2 + C_3 + C_4)$$

$$\alpha + N\beta$$



Regardless of the values of α and β, the function αN + β is a straight line. We call an algorithm whose time complexity is a straight line a **linear time algorithm.**




```
def maximum(values):
    answer = None
    for value in values:
        if answer == None or answer < value:
            answer = value
    return answer
```

How many times is this line executed?

Two Number Sum

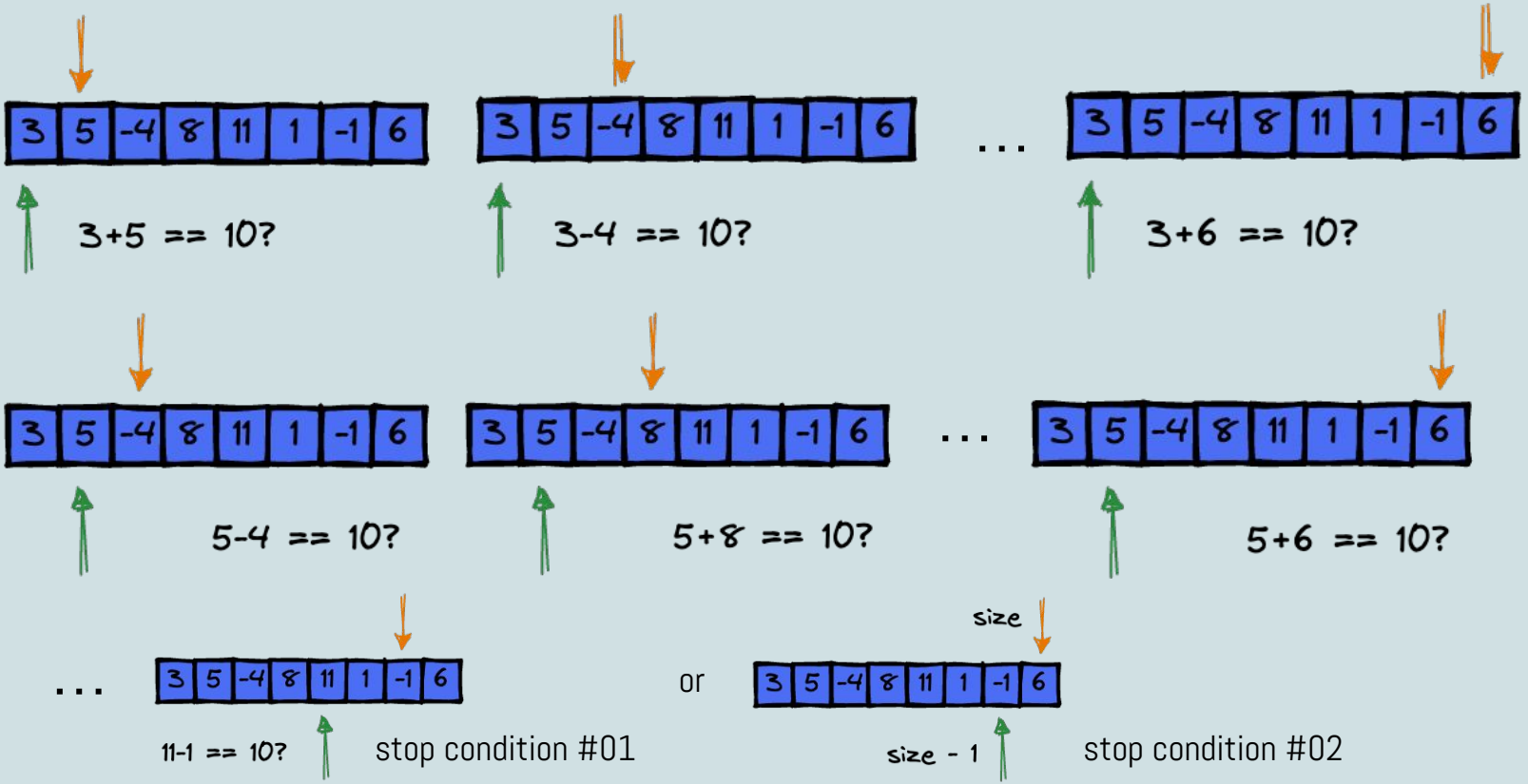
```
# sample input
array = [3,5,-4,8,11,1,-1,6]
targetSum = 10

# sample output
[-1,11] or [11,-1]
```

Write a function that take in a non-empty array of distinct integers and an integer representing a target sum. If any two numbers in the input array sum up to the target sum, the function should return then in an array, in any order. If no two numbers sum up to the target sum, the function should return an empty array.

Note that the target sum has to be obtained by summing two different integers in the array, you can't add a single integer to itself in order to obtain the target sum.

Solution


3 5 -4 8 11 1 -1 6
3+5 == 10?
3 5 -4 8 11 1 -1 6
3-4 == 10?
...
3 5 -4 8 11 1 -1 6
3+6 == 10?
3 5 -4 8 11 1 -1 6
5-4 == 10?
3 5 -4 8 11 1 -1 6
5+8 == 10?
...
3 5 -4 8 11 1 -1 6
5+6 == 10?
...
3 5 -4 8 11 1 -1 6
11-1 == 10?
stop condition #01
or
size
3 5 -4 8 11 1 -1 6
size - 1
stop condition #02

Solution

$$2(N-1) + 3(N-1)^2 + 2$$
$$2(N-1) + 2(N^2 - 2N + 1) + 2$$
$$2N^2 - 2N + 2 \gg N^2$$

```
def twoNumberSum(array, targetSum):
 for i in range(len(array) - 1):                    #C1  N-1
   firstNum = array[i]                              #C2  N-1
   for j in range (i+1, len(array)):                #C3 (N-1)^2
     secondNum = array[j]                           #C4 (N-1)^2
     if firstNum + secondNum == targetSum:          #C5 (N-1)^2
       return [firstNum, secondNum]                 #C6   1
 return []                                          #C7   1
```