Chapter 1

Notation

1.1 Pitch

Definition 1 (Pitch) *Pitch is the property of the sound which allows a relative ordering of perceived sounds on a frequency-related scale.*

On a keyboard, pitch goes up to the right of the keyboard, while it goes down on the left.

Pitches are expressed through **notes**. There are 7 note names[1], which are repeated in **octave registers**, identified by the bottom number.

$$\cdots A_3 B_3 \underbrace{C_4 D_4 E_4 F_4 G_4 A_4 B_4}_{\text{Octave register 4}} C_5 D_5 \cdots$$

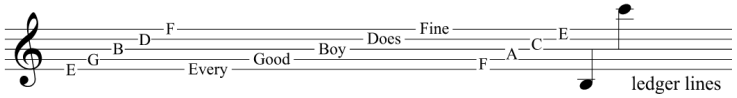

Figure 1.1: Treble clef

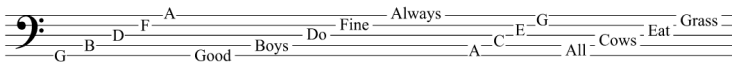

Figure 1.2: Bass clef

Definition 2 (Octave) *The distance / interval between two notes with the same name.*

[1] C-B in anglophone countries, C-H in Germany and Do-Si for the rest of Europe.

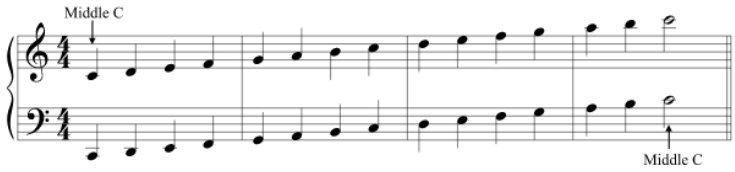

Figure 1.3: The Grand Staff (a specific stave *system*)

Definition 3 (Middle C) *The C_4 pitch, usually located in the middle of a keyboard (on the instrument) and always annotated in the middle of the grand staff, shared by the two staves.*

Definition 4 (Accidental) *A symbol placed before a note to raise / lower its pitch by a given amount.*

An accidental is effective only for a measure. They affect the entire piece if they are placed before the clef in a **key signature**.

♭	Flat	-1 half step
♯	Sharp	$+1$ half step
𝄫	Double flat	-2 half steps / -1 whole step
𝄪	Double sharp	$+2$ half steps / $+1$ whole step
♮	Natural	Cancels preceding accidentals

There exists also **half-accidentals**, whose altered notes cannot be played on a keyboard.

Definition 5 (Half step) *On the keyboard, the distance / interval between one key (either black or white) and the next (either black or white).*

Definition 6 (Whole step) *The interval made up of two half steps.*

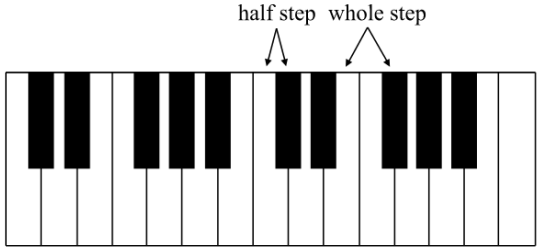

Figure 1.4: Half steps and whole steps

Definition 7 (Enharmonic) *Which has the same sound, but different name.*

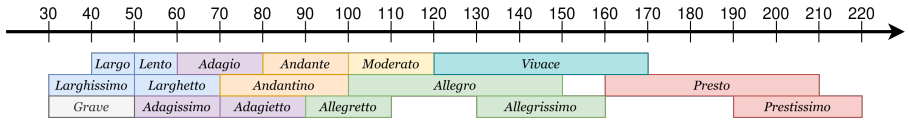

Figure 1.5: Common italian metronome markings

1.2 Rhythm

Definition 8 (Beat / pulse) *The basic pulse underlying measured music and thus the unit by which musical time is reckoned.*

Definition 9 (Tempo) *Speed of the beat.*

The tempo is usually expressed through metronome markings in **BPM / Beats Per Minute**.

1.2.1 Time signatures

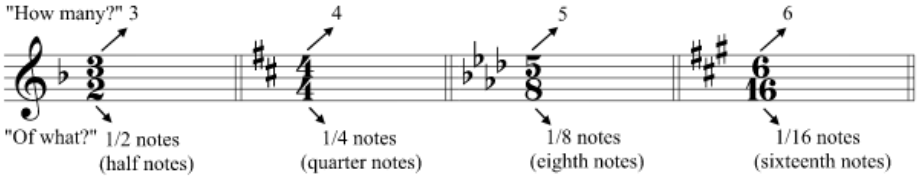

Figure 1.6: Meaning of the time signatures

1.2.2 Note / rests durations

Both notes and rests last for certain duration, which is always a 2^n number of beats, where $n \in \mathbb{Z}$. Common values for 2^n are the following ones:

$$\left\{4, 2, 1, \frac{1}{2}, \frac{1}{4}\right\} \text{ beats}$$

Values different from these ones can be gathered through **ties** and **dots**. A dot adds $\frac{1}{2}$ the value of the note dotted, while a double dot adds $\frac{1}{2} + \frac{1}{4}$ the original value.

1.2.3 Meters

Definition 10 (Meter) *Describes the number of beats in a measure / bar and how they are divided.*

Simple meters break the beat into 2 parts, while **compound meters** break it into 3 parts.

They can be **double** (2 beats / bar), **triple** (3 beats / bar) or **quadruple** (4 beats / bar).

Simple or Compound?	Duple, Triple, Quadruple?	Beat Grouping	Beat Division	Example Time Signatures
Simple	Duple	2	2	$\frac{2}{4}$ $\frac{2}{8}$ $\frac{2}{2}$ $\frac{2}{16}$
Simple	Triple	3	2	$\frac{3}{4}$ $\frac{3}{8}$ $\frac{3}{2}$ $\frac{3}{16}$
Simple	Quadruple	4	2	$\frac{4}{4}$ $\frac{4}{8}$ $\frac{4}{2}$ $\frac{4}{16}$
Compound	Duple	2	3	$\frac{6}{8}$ $\frac{6}{4}$ $\frac{6}{16}$
Compound	Triple	3	3	$\frac{9}{8}$ $\frac{9}{4}$ $\frac{9}{16}$
Compound	Quadruple	4	3	$\frac{12}{8}$ $\frac{12}{4}$ $\frac{12}{16}$

Figure 1.7: Meters

The meter is traditionally identified by the time signature.

When a piece shifts between time signatures / meters often the composers employ a **metric modulation**.

Definition 11 (Metric modulation) *A change in tempo or subdivision, suggested by a change of meter.*

1.2.4 Tuplets

Definition 12 (Tuplet) *Rhythmic grouping of notes which would typically not occur in the specified meter.*

Definition 13 (Duplet / Triplet / Quadruplet / Quintuplet) *Common tuplet instances.*

Definition 14 (Drag triplet) *A common type of triplet, made up of quarter notes. They are called in this fashion because the rhythm seems to* drag.

A drag triplet is also a common example of **hemiola**.

Definition 15 (Hemiola (rhythm)) *In rhythm, playing a pattern of 3 against a pattern of 2 (e.g. a drag triplet against 2 quarter notes).*

1.2.5 Accents and syncopation

A certain meter / time signature usually implies a certain beat hierarchy. That is, some beats are played with stronger / weaker emphases:

- 4/4: ● · ○ ·

- 12/8: $\bullet \cdot \circ \cdot$ (es. *Nightmare King*)
- 2/4: $\bullet \cdot$
- 6/8: $\bullet \cdot$ (es. *White Palace*, *Tarantella Napoletana*)
- 3/4: $\bullet \cdot \cdot$ (es. *Valse di Fantastica*)
- 9/8: $\bullet \cdot \cdot$
- 3/8: $\bullet$ (feels like 1 beat per measure)
- 2/2: $\bullet\bullet$

This should also explain why some pieces are better written as 2/4 over 4/4: because the beat hierarchy in the measures is different.

Definition 16 (Downbeat) *The first beat in a measure. Usually it is played with a very strong emphasis.*

Through **accents**, **ties** and **rests** it is possible to alter this rhythmic framework, obtaining **syncopation** in the process.

Definition 17 (Syncopation) *Playing music with a stronger emphasis on the weak beats and / or a weaker emphasis on the strong beats.*

Through syncopation some notes can also be played on the *offbeats*.

Definition 18 (Offbeat) *Which is not a beat.*

1.2.6 Irregular meters

These meters can be explained by thinking of normal meters with an uneven beat duration. That is, every measure has a fixed number of beats, but with different beat durations.

- 5/4: 5 uneven beats (es. *Mars, Bringer of War*, *Cinco de Chocobo*)
  - $3 + 2 : \bullet \cdot \cdot \circ \cdot$
  - $2 + 3 : \circ \cdot \bullet \cdot \cdot$
- 7/8: 3 uneven beats (3-2-2, 2-2-3).
- 13/8: 5 uneven beats (3-3-2-2-3, etc.).

1.2.7 Swing

Swing can be conceptualized as a way to write 6/8 in 4/4. The metronome text usually shows whether the 8th or 16th notes should be swung.

The opposite of a swing rhythm is called **straight** rhythm.

1.3 Dynamics

Dynamics hint at the volume of a given music segment. Often they range between ***ppp*** and ***fff***. The intermediate dynamic ***mf*** is often used as a standard base volume.

n stands for *niente*, and it is usually used at the end of a decrescendo.

fp means to play the note as ***f***, but then quickly fade to ***p***.

sfz and ***rfz*** instead indicate to play a single note stronger than the surrounding ones.

1.4 Control structures

In a concert score setting often some parts do not need to play for a long number of measures. This situation is notated through a **multirest**.

1.4.1 Repeats

Repeats are sometimes highlighted with wings-like decorations, with the only purpose of making them stand out more.

Definition 19 (Segno) *Used as a landmark in a* ***D.S.*** *marking.* ***D.S.*** *means to play from the segno.*

Definition 20 (Coda) *Used as a landmark in a* ***Al coda*** *marking.* ***Al coda*** *means to play till the coda, then to continue playing the separate coda.*

Note that during a **D.C** or **D.S.** notation, repeats are *not* performed for a second time.

1.5 Articulations

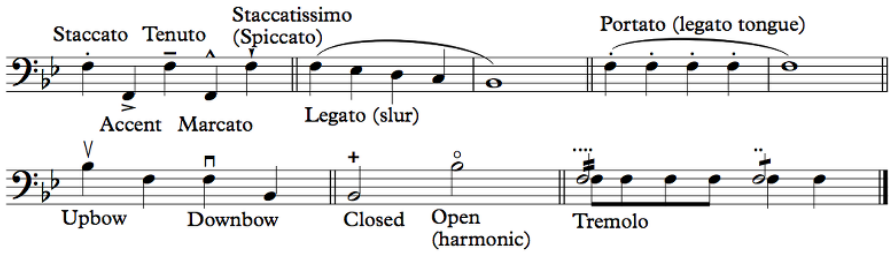

Figure 1.8: Articulations

There a variety of articulations used to tell the player how to produce the sounds. The meaning of these often varies from instrument to instrument:

Definition 21 (Staccato) *Play the note short, lightly and briefly detached from the next and the previous ones.*

Definition 22 (Accent) *Emphasize the note, with a quick attack and a gentle decay / release.*

Definition 23 (Marcato) *Emphasize the note with a strong attack and a quick release / decay.*

Definition 24 (Tenuto) *The player should be careful as to keep the note for its whole duration.*

Definition 25 (Staccatissimo) *A stronger staccato.*

Definition 26 (Spiccato) *Exclusively used in string instruments. Means to lighly bounce the bow upon the strings.*

Definition 27 (Portato) *A legato-staccato. Usually means to play the notes with a light disconnection between them.*

Definition 28 (Upbow & Downbow) *Indicates a corresponding motion of the bow on string instruments. The downbow is usually stronger.*

Definition 29 (Closed / Mute & Open) *Usually used on percussion and brasses. These indicate whether the sound should be muted (through the* sordino*, the hand, etc.) or left open to ring.*

Definition 30 (Tremolo (single-note)) *Repeat the note 2^n times, where n is the number of strips on the stem.*

Definition 31 (Tremolo (two-note)) *Quickly alternate between the notated pitches. The actual speed of the tremolo is usually derived from context (usually: one strip $\Rightarrow$ 8th notes).*

Definition 32 (Arpeggio) *Play a series of notes in a quick sequence, but not simultaneously.*

Definition 33 (Glissando) *A quick run through all the notes between the notated ones. On piano, usually only the white notes are played.*

Often a glissando may be actually notated note per note, in which case it is called a **run**. Notes in a run should not be played too carefully; instead, the player should focus on the whole sequence speed.

A glissando is a *discrete* change of pitch, but some instruments are able to produce a *continuous* change of pitch (e.g. trombone, timpani, strings, voice).

Definition 34 (Portamento) *A continuous glissando.*

Definition 35 (Vibrato) *Periodic variation of a sound pitch. It is not notated.*

Definition 36 (Scoop / Doit / Plop / Fall) *Jazz articulations. A note is started a little*[2] *lower / higher or is terminated a little lower / higher.*

[2]Subjective.

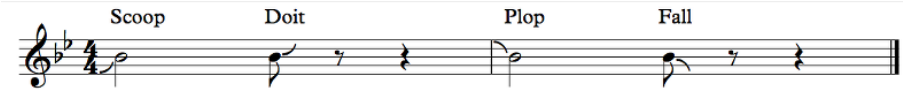

Figure 1.9: Scoop / Doit / Plop / Fall

1.6 Ornaments

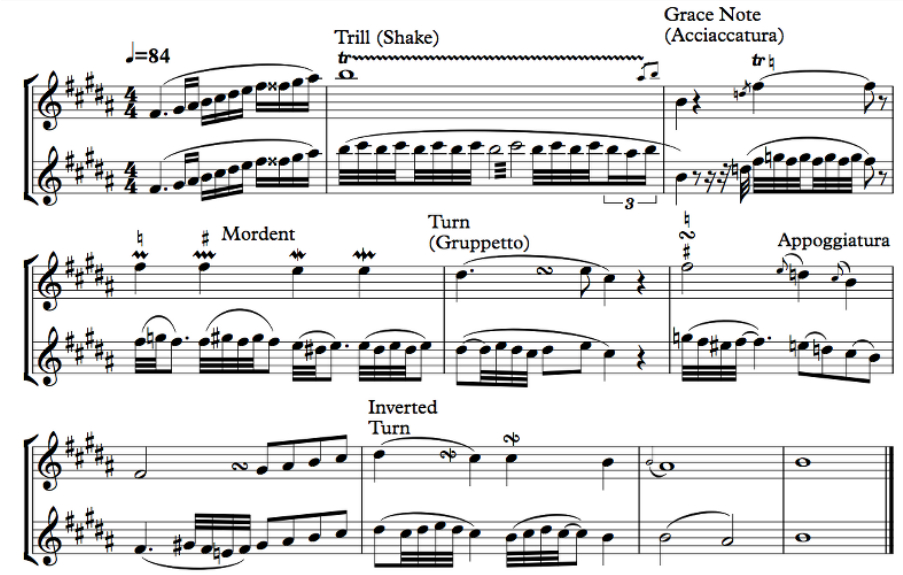

Figure 1.10: Ornaments

Ornaments differ from articulations in the sense that they are non-essential additions of notes used to make the melody prettier.

Definition 37 (Trill) *Play the note, then rapidly alternate between it and the note above it,* in the key. *The note above can be changed through an accidental above the trill marking.*

A wavy line on the trill allows for a clear definition of the trill extent / duration, but it is entirely optional.

Definition 38 (Grace note) *A little note that steals time from the surrounding notes.*

Definition 39 (Acciaccatura) *A grace note played very quickly before the attached note it is attached to.*

Definition 40 (Appoggiatura) *A grace note which takes time from the note which it is attached to.*

They are rarely used nowadays, as they can be expressed through conventional notation.

Definition 41 (Mordent) *Quickly goes up a note (in the key) then goes down (or viceversa, indicated by a line through).*

The actual note of the mordent can be changed through accidentals. A longer mordent instead can be used to hint a double duration.

Definition 42 (Turn / Gruppetto) *Goes up a note, then down to it, then down again, then up back at it.*

It is rarely used nowadays, as it can be expressed through conventional notes or grace notes.

A turn can be inverted (down, up, up, down) with a line through.

1.7 Stopping the time / tempo alterations

Definition 43 (Fermata / Corona) *Hold the note for as long as you want. In an ensemble, you have to look to your conductor.*

When on a rest, it just means to take a free pause.

As an heuristic, most of the times you just hold the note with a fermata for two times its duration.

Definition 44 (Breath mark) *In voice or wind instruments parts, used to mark suitable times for breathing.*

Definition 45 (Caesura) *An arbitrary time of silence.*

Definition 46 (Grand Pause) *A whole measure of silence.*

Compared to the caesura, the grand pause is a rest, but with a planned and well-defined time duration. It is seldom used to mark measures when the whole ensemble should stay silent.

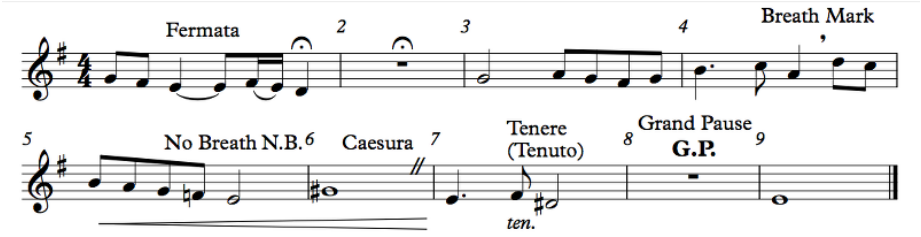

Figure 1.11: Examples of tempo modifications

Chapter 2

Scales

2.1 Major scale

Definition 47 (Tetrachord) *A 4-note scale segment with the following steps:* $W - W - H$.

Definition 48 (Major scale) *A 8-note scale made up of 2 tetrachords, joined by a whole step.*

$$\underbrace{W - W - H}_{T1} - W - \underbrace{W - W - H}_{T2}$$

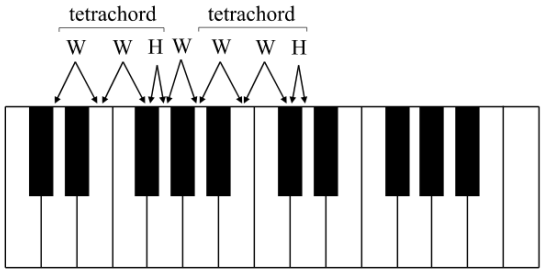

Figure 2.1: Tetrachords in a (D) major scale

A major scale uses all the 7 notes in order. No one is skipped and there are no duplicates.

2.1.1 Key signatures

There are 15 major key signatures:

- 1 with no accidentals: C Major.
- 7 with 1 to 7 flats.
- 7 with 1 to 7 sharps.

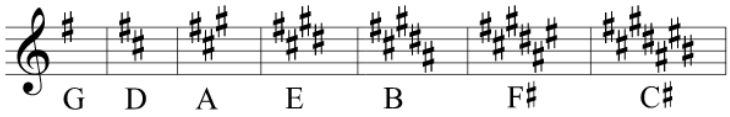

Figure 2.2: Major key signatures (sharps)

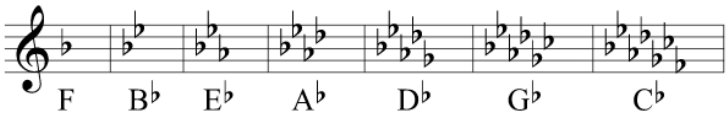

Figure 2.3: Major key signatures (flats)

A key signature can be quickly identified with the following mnemonic:

- With *sharps*: +1 half step from the last "sharped note".
- With *flats*: the second to last flat is the key (along with the flat).

2.2 Minor scales

In contrast to major scales, there are 3 different minor scale forms. They all follow the following formulas, while the melodic minor is only used as an *ascending* scale (the *descending* part is the same as the natural minor scale).

2.2.1 Harmonic minor scale

The harmonic minor scale was born as a way to obtain a stronger resolution to the tonic. In fact, the lowered $\hat{7}$ scale degree in the natural minor scale leads to a steep resolution to the tonic (a whole step).

So, the harmonic minor scale *raises* the $\hat{7}$ scale degree so as to obtain a so-called **leading tone**. In this way we are able to use a smoother resolution to the tonic from the $\hat{7}$ scale degree.

The scale is called *harmonic* minor, because the chords usually come from this harmonic scale (if this is the case, the dominant triad is the same as its major scale counterpart, and they fulfill the same role; the other chords are different between the two modes).

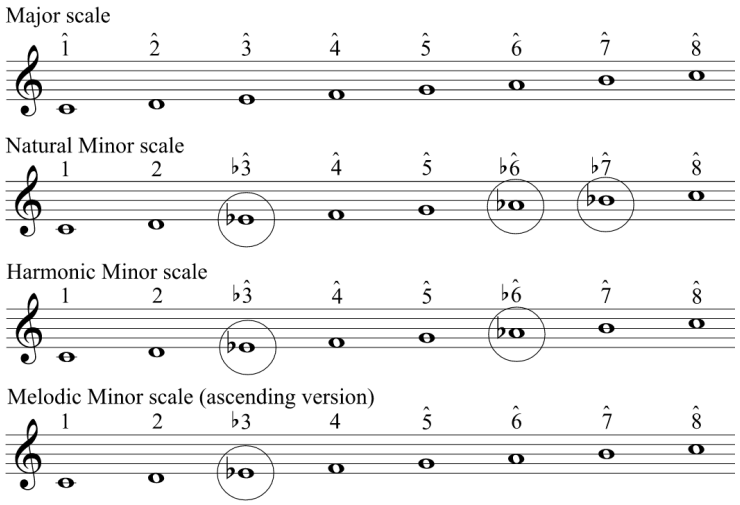

Figure 2.4: Minor scales

2.2.2 Melodic minor scale

The melodic minor scale shows its purpose in a similar manner as the harmonic minor scale. If we were to use an harmonic minor scale for a resolution then we probably want a passage with ends with $\hat{6}\hat{7}\hat{8}$. Note however that in this case we have the same problem as before: the distance between $\hat{6}$ and $\hat{7}$ is too steep in the harmonic minor scale.

Therefore, we *raise* also the $\hat{6}$ scale degree, obtaining the melodic minor scale.

The scale is called *melodic* minor because it shows its purpose during melodic passages which lead to a tonic resolution. In the Common Practice period (1600-1900), minor melodies often followed this scale pattern.

2.2.3 Key signatures

In respect to the major keys, minor keys can be derived by adding 3 flats (or subtracting sharps and adding flats if needed).

In doing so, the corresponding major scale will also have three of its scale degrees lowered, resulting in what is called a **parallel** minor scale.

Definition 49 (Parallel scale relationship) *Two major / minor scales with the same* 1^{st} *scale degree.*

On the other hand, if it is the key signature to be shared, then we call it a **relative** minor key.

Definition 50 (Relative key relationship) *Two major / minor key signatures with the same key signature.*

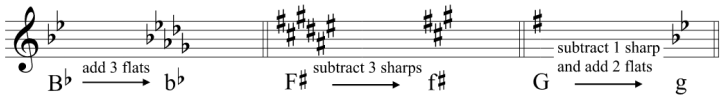

Figure 2.5: Parallel relationship

2.3 Circle of fifths

The circle of fifths is a convenient aid for the visualization of both minor and major keys and scales:

- To the right, we add sharps / remove flats and we go up a 5^{th}.
- To the left, we remove sharps / add flats and we go down a 5^{th}.

It also provides some interesting insights into the key structure:

- Given a note, the next note clockwise is its dominant.
- Given two adjacent keys, their scales share $\frac{6}{7}$ of the notes (e.g. C Major and G Major differ only by the F#).
- In 18th century music often a key modulated itself only to the its 5 other neighboring keys (because they shared $\frac{6}{7}$ notes). These keys and its relative minor are the 6 most *closely related keys*, while others are said to be *distantly related*. In fact, the key on the opposite side of the circle of fifths is said to be the most distantly related.

  Beethoven was one of the first composer to deviate from this trend.

2.4 Key identification

Definition 51 (Key (≈)) *The **tonic** and the **mode** the piece "is in".*

What does it mean for a piece to "be in a key" (say, the key of C Major)? It means[1]:

- The most important note in the piece (the one that feels like "home") is the first scale degree of our scale.
- Notes from the key scale (say, the C Major scale) seem like to belong to the piece, while notes from other scales (say, the C minor scale) seem to be extraneous.

Given a piece of sheet music we can devise its key as follows:

[1] Assuming we are talking about *tonal music*.

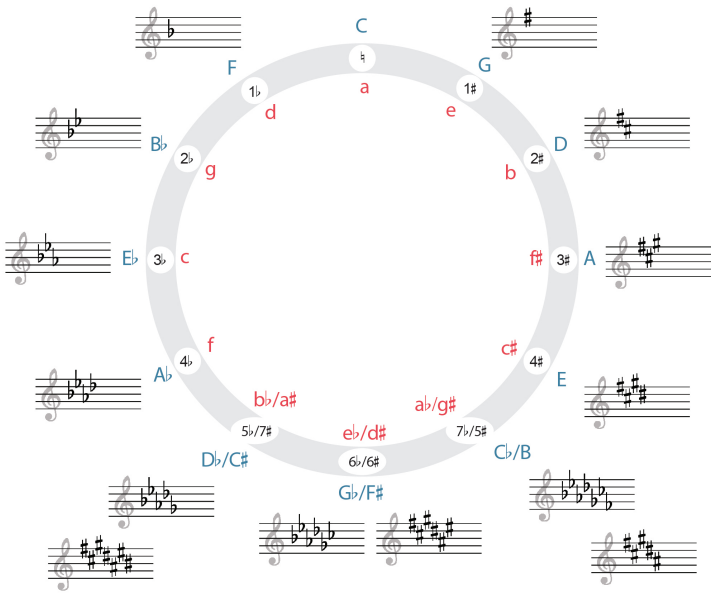

Figure 2.6: Circle of fifths

1. Through the number of flats / sharps we restrict ourselves to 2 key signatures: a major one and a minor one.
2. The tonic can help us do the final discrimination. Usually the tonic note is located at the beginning / end of the piece either in the lower or upper parts (as we want to look for the piece*resolution*, often the tonic can be devised from the last note).

2.4.1 Key modulation

Definition 52 (Key modulation) *A temporary change of keys.*

A key modulation can be notated with a text marking like $G : 5$, which indicates a modulation to the G as the tonic for the following measures.

A key modulation may or may not be highlighted by a key signature change in the sheet music. When this happens, usually a note or chord is used as a *pivot* for the key modulation; said note or chord is usually in common between the two keys (albeit maybe with different roles).

Definition 53 (Pivot) *A common note / chord between two keys, used as for pivoting between the two keys.*

A weaker form of modulation is **tonicization**, in which a different note is briefly used in place of the actual key's root.

Through leading tones or secondary dominants it is easy to *tonicize* other notes.

Definition 54 (Tonicization) *Very briefly treat a different note as the tonic for a piece in a given key.*

2.5 Diatonic scale / modes

Definition 55 (Diatonic scale) *A collection of all the seven note pitches in order.*

The diatonic scale can be thought as a *meta-scale*. Its general formula is any shift of the major scale formula, thus we have 7 possible diatonic scale **modes**.

Definition 56 (Diatonic mode) *A specific instance of a diatonic scale.*

In fact, both the major scale and the (natural) minor scale are diatonic modes. The diatonic modes are called through greek names, where Ionian and Aeolian are the major and (natural) minor scales, respectively.

Mode	Intervals sequence	Example with white keys
Ionian	$W-W-H-W-W-W-H$	$C-D-E-F-G-A-B$
Dorian	$W-H-W-W-W-H-W$	$D-E-F-G-A-B-C$
Phrygian	$W-W-H-W-W-W-H$	$E-F-G-A-B-C-D$
Lydian	$W-H-W-W-W-H-W$	$F-G-A-B-C-D-E$
Mixolydian	$H-W-W-W-H-W-W$	$G-A-B-C-D-E-F$
Aeolian	$W-W-W-H-W-W-H$	$A-B-C-D-E-F-G$
Locrian	$W-W-H-W-W-H-W$	$B-C-D-E-F-G-A$

2.6 Scale degrees and functions

In a scale there are seven scale degrees, each of them with a different name.

These are the three most important degrees:

- $\hat{1}$ / **Tonic**: the center of gravity of a tonal piece of music. Often it ends a piece of music in a given key, which is identified exactly by that tonic.
- $\hat{5}$ / **Dominant**: mainly used as a springboard to the tonic.
- $\hat{3}$ / **Mediant**: it was used as an helper towards the dominant.
- $\hat{2}$ **Supertonic**: can also be used as a *dominant to a dominant*, also called a **secondary dominant**.
- $\hat{7}$ / **Leading tone**: is the most unstable scale degree, which has a strong tendency to resolve to the tonic.

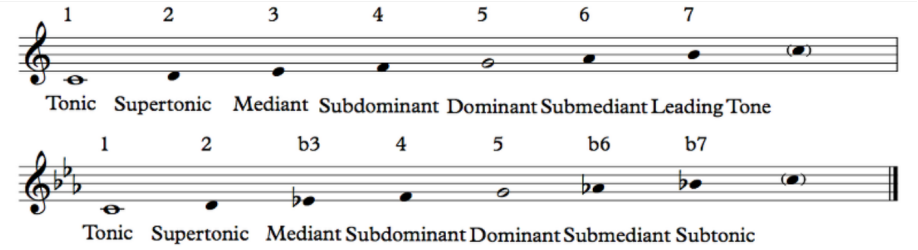

Figure 2.7: Scale degree names

- $\hat{4}$ / **Subdominant**: can be both stable or unstable.
  - Dominant context $\Rightarrow$ unstable (wants to resolve to $\hat{3}$).
  - Tonic context $\Rightarrow$ stable ($\hat{3}$ is its leading tone).

Together, these 3 scale degrees form the **tonic triad**, which is the center of the *harmony* (while the tonic is the center of the *melody*).

Definition 57 (Tonic triad) *The triad formed by the* $\hat{1}, \hat{3}, \hat{5}$ *scale degrees.*

Instead, if we build the triad on the *dominant*, we gather the **dominant triad**, which strongly wants to resolve to the tonic (triad). The **mediant triad** instead often sounds pretty stable.

Definition 58 (Dominant triad) *The triad built on the* $\hat{5}$ *scale degree.*

2.7 Chromaticism

When we use notes which are outside of our scale we are doing chromatic alterations. This may have two different purposes:

- **Modal mixture**. In modern music theory this is often referred just as *extended tonality.*
- Introducing different leading tones (even for scale degrees different than $\hat{7} \to \hat{1}$).

  Examples: # 1 resolves to 2, b2 resolves to 1, etc. Notice how the accidental disambiguates between enharmonic notes, building up an expectation to where the note is supposed to resolve[2].

Definition 59 (Modal mixture) *Using a note from a different mode.*

Definition 60 (Sublimation) *When a note resolves to a different note than the one it is intended to resolve to, while mainaining a pleasuring transition.*

[2]This does not hold for b3: it is stable.

Chapter 3

Intervals

3.1 Quantity and quality

Definition 61 (Harmonic interval) *Distance between two notes played* simulatenously.

Definition 62 (Melodic interval) *Distance between two notes played* one after the other.

Definition 63 (Interval quantity) *The number of notes on the diatonic scale (consecutive unaltered different pitches) from the starting note to the end note.*

Definition 64 (Simple intervals) *Intervals with a quantity less than an octave.*

Definition 65 (Compound intervals) *Intervals with a quantity higher than an octave.*

If we talk about *simple intervals*, we have some predefined quantities. Each of them has some accepted **qualities**:

Quantity	Compatible **quantities**
Unison	Perfect
Second	Major, minor
Third	Major, minor
Fourth	Perfect, augmented, diminished
Fifth	Perfect, augmented, diminished
Sixth	Major, minor
Seventh	Major, minor
Octave	Perfect

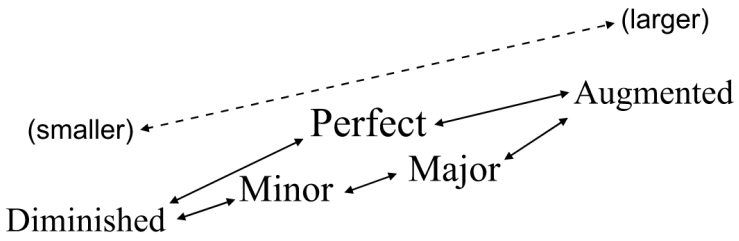

Figure 3.1: Interval quality chart

3.1.1 Multiple augmentation / diminution

An interval can be altered through additions of subtractions of half tones.

In music notation, this is usually carried out through sharps and flats (or double sharps and double flats). From these accidentals we gather augmented / diminished intervals, but also double augmented and double augmented intervals[1]. In practice, double augmented / double diminished intervals are rare in music, not to mention more altered quantities.

3.1.2 Tritone

Both between fifths and fourths there is a *single* interval which is characterized by a much harsher sound than the others: while the others are *consonant* that one is *dissonant*. This interval is called the **tritone**.

Definition 66 (Tritone) *An augmented fourth / diminished fifth (which is always between F and B and always made up of 6 half tones or 3 whole tones - hence the name).*

This interval is also popularly called the *diabolus in musica*, because it is the only dissonant exception to the consonant set of fourths and fifths.

Moreover, the tritone *is half the width of an octave.*

3.1.3 Compound intervals

Compound intervals can be obtained by adding a 7th to an existing simple interval. Therefore, a two-octave interval is a 15th.

3.1.4 Diatonic interval

There is also a more vague type of interval, the **diatonic interval**, which essentially means: *go from this note to the next one in the current diatonic scale.*

[1] It is theoretically possible to reach a maximum quantity of a *quintuply augmented fourth* (or *quintuply diminished fifth*) by altering a tritone through a double flat - double sharp pair.

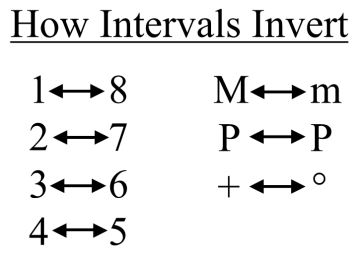

Figure 3.2: Interval inversion chart

Definition 67 (Diatonic interval) *The distance between one note and another in the context of a specific diatonic scale.*

3.2 Interval inversion

An **interval inversion** can be performed by inverting the relative position of two notes, while keeping the pitch classes constant (this can be done through a single-note octave transposition).

As we can see, the sum of an interval quantity with the quantity of its inverted counterpart is always 9. Moreover:

- Perfect intervals invert to perfect intervals.
- Augmented invervals invert to diminished intervals.
- Major intervals invert to minor intervals.

3.3 Sound

Definition 68 (Cent) $\frac{1}{100}$ *of a semitone.*
Therefore, 1 semitone = 100 cents.

Chapter 4

Pitch

4.1 Sound properties

A **sound** is just a bunch of air particles which vibrate really quickly into thin air. They do so on *longitudinal* waves. The vibration in the air is produced by some sort of vibrating device (e.g. the membrane of a PC speaker) and captured by some other device (like the membranes into our ears). In the case of our ears, the brain processes the captured vibrations into sound perceptions.

The number of vibrations per second is expressed through the **frequency** f, expressed in *Hertz*. Higher frequencies produce higher sounds. Conversely, if the signal has a bigger **amplitude** it is perceived as a louder sound. The timbre is instead related to the overall waveform shape and therefore to the Fourier representation of the signal.

Therefore, to summarise:

- **Frequency** translates to **pitch**.
- **Amplitude** translates to **volume**.
- **Waveform shape** translates to **timbre**.

4.2 Pitch

Unfortunately, the relationship between frequency and pitch is nonlinear. In fact, it is *exponential*.

Moving in octaves (in pitch space) to us feels like a linear increase, while in frequency the same movement is exponential (or also, the frequency increase is linear, but the pitch increase is logarithmic). As an example moving a sound to the next note involves *doubling* its frequency.

Therefore, if we express as s the multiplicative factor used to scale the frequency so as to obtain the corresponding pitch raised by a semitone:

$$s^{12} = 2 \Rightarrow s = 2^{\frac{1}{12}}$$

For translating a given pitch into a frequency we can use the following:

$$f = f_{\mathbf{C0}} \cdot 2^{o + \frac{p}{12}}$$

Where p is the number of semitones from the C pitch class and o is the number of octaves.

The reverse can be computed through logarithms and fixed / fractional part functions.

Nevertheless, from this we gather a pretty interesting result: **the pitch we perceive as a linear increase translates in reality as an exponential increase in frequency**.

This means that the most interesting octaves in music (C0 - C10) are effectively translated to a very narrow set of frequencies, up to 1000 Hz.

4.2.1 Pitch class and octave equivalence

The set of C pitches is called the *C pitch class*. This can be generalized.

Definition 69 (Pitch class) *A pitch without its octave.*

We recognize pitches which share the same pitch class as different instances of the same note. In fact, these pitches are n octaves apart so we should say that we recognize pitches *which are n octaves apart* as different instances of the same note.

Why is this so? This is because the waveforms of two notes from an octave interval perfectly line up together.

If we make an equivalence relation out of the notion of *pitch space modulo octave equivalence* we are able to obtain the **pitch class space**.

4.2.2 Temperament

The modern western musical system is said to be a **12-tone equal-tempered** one.

Definition 70 (12-tone equal-temperament) *A musical system in which an octave is subdivided into a set of 12 equal semitones (equal = there is a constant amount between every semitone in pitch space).*

However, more than two centuries ago, composers actually used systems which were not equally-tempered. The semitones had different sizes across the octave, therefore leading to uneven intervals, be it for the better or for the worse. In fact, some of these intervals sound *better* than their equally-tempered counterparts; however, these intervals lead to inconsistencies into other intervals / parts of the octave, which sound drastically worse than their equally-tempered counterparts (these intervals were called **wolf intervals**).