MATH 0240 Midterm Examination I Sample 1

This test consists of 10 problems, each worth 10 points. All work must be shown in order to get credit. Please write legibly and explain your logic by words whenever appropriate. **No calculators, notes, books etc. are allowed.**

Problem 1. Find the angle between two vectors.
$\bar{u} =< 1, -1, 0 >$, $\bar{v} =< 0, 1, -1 >$

Problem 2. Find the directional derivative of

$$f(x, y, z) = x^2 + xy + 2yz + e^{2xz}$$

at the point $(2, -1, 0)$ in the direction of the vector $\vec{v} =< 2, 1, -2 >$

Problem 3. Find the length of the curve

$$r(t) =< \cos t, \sin t, t >, \qquad 0 \leq t \leq 4\pi$$

Problem 4. Find all points on the curve $r(t) = (2t, t^2, t^3)$, where the tangent line is parallel to the plane

$$-6x + 3y + 2z = 15.$$

Problem 5. Find the unit normal vector at the point $(1, \frac{2}{3}, 3)$ for the curve $r(t) =< t^2, \frac{2}{3}t^3, t + 2 >$.

Problem 6. Find the curvature at the point $(0, 0, 1)$ of the curve

$$r(t) =< t, t^2, e^t > .$$

Problem 7. Find the linear approximation at $(1, 0)$ to

$$f(x, y) = x \ln(x + y) + y + 3.$$

Problem 8. Find all the moments t, when the acceleration vector is perpendicular to the velocity vector for

$$r(t) =< t^2, \ln t, 2t + 1 >$$

Problem 9. Find f_{xy} for the function

$$f(x, y) = \sin(\frac{x}{y}).$$

Problem 10. Find the limit or show that it does not exist

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$$