

MATH 0120 Business Calculus. Worksheet 1.

University of Pittsburgh, 6W2-Summer 2019

Name: _____

Limits and continuity

1. Evaluate the following limits

(a) $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x^2 + 2x - 3}$

(b) $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$

(c) $\lim_{x \rightarrow -3} \frac{|x-2| - 5}{6 - 2|x|}$

(d) $\lim_{x \rightarrow \infty} \sqrt{x^2 + 4x + 1} - x$

2. Determine the values of a and b that make the function $f(x)$ continuous on the entire real line. Make sure you state and verify all continuity conditions.

$$f(x) = \begin{cases} \frac{ax^2 + 1}{x - 3} & x < 2 \\ b & x = 2 \\ 2ax - 3 & x > 2 \end{cases}$$

Differentiation

3. Use the definition of derivative to find $f'(x)$ for the function $f(x) = \frac{1 - x}{2 + x}$

4. Find the derivative.

(a) $y = e^{2x^2 - 3} \left(\sqrt{5} + \frac{1}{x} \right) + (x^2 + x^3)^4$

(b) $y = \frac{e^{4x^2-x}}{\ln(2-5x)}$

5. Find the equation of the tangent line to the curve $x^5 + y^5 - 2xy = 0$ at $(1, 1)$.

Elasticity of demand

6. The demand function for a newspaper is $D(p) = 80000\sqrt{75-p}$, where p is the price in cents. The publisher currently charges 50 cents, and it plans to raise the price to increase revenues. Will this strategy succeed?

7. An electrical utility asks the Federal Regulatory Commission for permission to raise rates to increase revenues. The utility's demand function is $D(p) = \frac{120}{10+p}$ where p is the price (in cents) of a kilowatt-hour of electricity. If the utility currently charges 6 cents per kilowatt-hour, should the commission grant the request?

Integration

8. Evaluate the following integrals:

(a) $\int_1^9 \frac{t - 2t^2}{\sqrt{t}} dt$

(b) $\int \frac{(3^x - 6^x)^2}{18^x} dx$

(c) $\int \frac{1}{\sqrt{x} (1 + \sqrt{x})^2} dx$

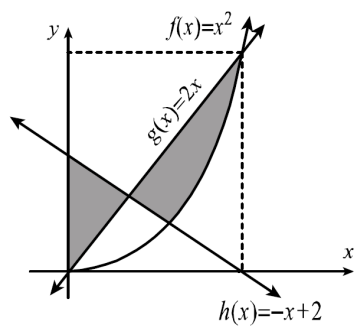
(d) $\int_4^{12} x\sqrt{x-3} dx$

(e) $\int \ln(\sqrt{x}) dx$

(f) $\int_1^2 x^2 e^{3x} dx$

9. Find the area enclosed by the curves $y = x^2 + 1$ and $y = x + 3$.

10. Find the area of the shaded region shown in the picture.



11. Find the area of the shaded region shown in the picture.

