Symmetries and Conservation Laws in the Economic Field: Global Phase, Translations, and Coherent Invariants

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Abstract

This document develops the symmetry structure of the economic field and the associated conservation laws. Building on the density–phase description introduced in the foundational chapter, we identify three principal symmetries: global phase shifts, spatial translations, and time translations. These symmetries are interpreted in economic terms as invariance under re-labellings of orientation, homogeneity of the underlying domain, and persistence of internal structure. Each symmetry leads to a conserved quantity: total density, a momentum-like flow invariant, and a Hamiltonian functional. Together, these results provide a geometric backbone for coherent economic dynamics, linking field symmetries to observable macroscopic invariants.

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1 Introduction

The economic field is defined in terms of a density field $\rho(x,t)$ and a phase field $\theta(x,t)$, whose joint dynamics are governed by a Lagrangian functional. Beyond the specific form of the Lagrangian, its *symmetries* encode deep structural properties of the system. They determine which quantities are conserved, how coherence can persist, and which transformations leave the underlying economics unchanged.

This chapter examines three principal symmetries:

- global phase symmetry,
- spatial translation symmetry,
- time translation symmetry.

Each symmetry has a clear economic interpretation and an associated conserved quantity, in direct analogy with Noether's theorem in field theory.

2 Lagrangian Structure of the Economic Field

We recall the Lagrangian density for a multi-component economic field:

$$\mathcal{L} = \sum_{i=1}^{N} \left[\rho_i \frac{\partial \theta_i}{\partial t} - \frac{1}{2} \rho_i \, |\nabla \theta_i|^2 - V_i(\rho_i) \right] - \sum_{1 \le i < j \le N} J_{ij} \, \rho_i \, \rho_j \left[1 - \cos(\theta_i - \theta_j) \right], \tag{1}$$

where:

- $\rho_i(x,t)$ is the density field of subsystem i,
- $\theta_i(x,t)$ is its phase field,
- $V_i(\rho_i)$ is a local potential stabilizing ρ_i,
- $J_{ij} \ge 0$ encodes the strength of interdependence between i and j.

The action functional is

$$S = \int \mathcal{L} \, \mathrm{d}^d x \, \mathrm{d}t, \tag{2}$$

and the dynamics follow from the stationary action principle $\delta S = 0$ under suitable variations of ρ_i and θ_i.

3 Global Phase Symmetry

3.1 Definition of the symmetry

Consider a global shift of all phases:

$$\theta_i(x,t) \;\mapsto\; \theta_i(x,t) + \alpha, \qquad \forall i, \; \forall x, \; \forall t, \tag{3}$$

where α is a constant independent of space, time, and subsystem index.

Under this transformation:

- The phase differences $\theta_i - \theta_j$ remain unchanged.
- The gradients $\nabla\theta_i$ are unaffected.
- The densities ρ_i are unchanged.

Since the Lagrangian (1) depends on θ_i only through $\partial_t\theta_i$, $\nabla\theta_i$, and differences $\theta_i - \theta_j$, a uniform shift (3) leaves $\mathcal{L}$ invariant. Thus, the economic field exhibits a global phase symmetry.

3.2 Economic interpretation

Global phase symmetry reflects the fact that only *relative* orientation matters. If all subsystems rotate their strategic or expectation state by the same constant offset, the economic configuration is unchanged. There is no absolute reference direction in phase space; only differences drive reallocation and tension.

This expresses an economic principle:

Value flows and tensions depend on relative misalignment, not on any absolute orientation.

3.3 Associated conservation law: total density

As in standard field theory, continuous symmetries of the action correspond to conserved quantities. For global phase symmetry, the conserved quantity is the total density.

To see this, consider the continuity equation obtained by varying θ_i:

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \nabla\theta_i) = \sum_{j \neq i} J_{ij}\, \rho_i \rho_j\, \sin(\theta_i - \theta_j), \tag{4}$$

and sum over all i:

$$\frac{\partial}{\partial t}\left(\sum_i \rho_i\right) + \nabla \cdot \left(\sum_i \rho_i \nabla\theta_i\right) = \sum_i \sum_{j \neq i} J_{ij}\, \rho_i \rho_j\, \sin(\theta_i - \theta_j). \tag{5}$$

The double sum on the right-hand side cancels due to antisymmetry:

$$J_{ij}\, \rho_i \rho_j\, \sin(\theta_i - \theta_j) = -J_{ji}\, \rho_j \rho_i\, \sin(\theta_j - \theta_i),$$

and $J_{ij} = J_{ji}$.

Therefore,

$$\frac{\partial}{\partial t}\left(\sum_i \rho_i\right) + \nabla \cdot \left(\sum_i \rho_i \nabla\theta_i\right) = 0. \tag{6}$$

Integrating over the whole domain and assuming appropriate boundary conditions (e.g. vanishing flux at infinity or periodic boundaries), we obtain:

$$\frac{\mathrm{d}}{\mathrm{d}t} \int \sum_i \rho_i(x,t)\, \mathrm{d}^d x = 0. \tag{7}$$

Thus, the total density is conserved.

Economically, this means that the field dynamics *redistribute* value but do not create or annihilate it internally. The global amount of economic presence is an invariant of the motion.

4 Spatial Translation Symmetry

4.1 Homogeneity of the underlying domain

Consider a spatial translation:

$$x \mapsto x + a, \qquad a \in \mathbb{R}^d, \tag{8}$$

with time unchanged. If the Lagrangian density does not depend explicitly on x (i.e. there are no externally imposed spatial inhomogeneities), then $\mathcal{L}$ is invariant under (8).

This reflects an assumption of *homogeneity*: the local rules governing interaction and evolution are the same across the domain. Spatial structure may still emerge through the configuration of ρ_i and θ_i, but it is not hard-coded into $\mathcal{L}$.

4.2 Momentum-like conserved quantity

Spatial translation symmetry implies the conservation of a momentum-like quantity. Formally, one can define a stress-energy tensor $T^{\mu\nu}$ associated with the Lagrangian, whose spatial components yield conserved currents.

In the purely economic reading, we restrict attention to the continuity of economic flows:

$$\partial_t \rho_i + \nabla \cdot j_i = (\text{interaction terms}), \tag{9}$$

with

$$j_i = \rho_i \nabla \theta_i,$$

the flux of density of subsystem i.

Translation invariance implies that the *integrated* momentum of the field, constructed from ρ_i and $\nabla\theta_i$, is conserved in the absence of external forces or explicit spatial dependence in V_i and J_{ij}.

Intuitively, the system cannot generate a net drift in one direction without breaking the underlying homogeneity. Patterns may propagate and clusters may move, but the aggregate momentum-like quantity remains invariant.

4.3 Economic interpretation

Spatial translation symmetry captures the idea that the dynamics do not privilege any specific location. Regions may differ in realized density and phase, but the underlying rules are uniform.

Economically, this is a natural abstraction for modeling large-scale structures where local differences emerge endogenously, rather than being imposed by external spatial biases.

5 Time Translation Symmetry

5.1 Autonomous dynamics

A key property of the Lagrangian (1) is that it has no explicit dependence on time t: it depends on t only through the fields $\rho_i(x,t)$ and $\theta_i(x,t)$ and their derivatives.

A time translation:

$$t \mapsto t + \tau, \tag{10}$$

with τ constant, leaves the action invariant.

This is the statement that the system is *autonomous*: the rules of evolution are the same at all times. Dynamics may lead to different configurations, but the underlying law does not change.

5.2 Conservation of the Hamiltonian functional

Time translation symmetry implies the conservation of a Hamiltonian functional H, often interpreted as the total energy of the field.

For the economic field, H takes the schematic form:

$$H = \int \left[\sum_i \frac{1}{2} \rho_i \, |\nabla \theta_i|^2 + \sum_i V_i(\rho_i) + \sum_{i<j} J_{ij} \, \rho_i \rho_j \left[1 - \cos(\theta_i - \theta_j) \right] \right] \mathrm{d}^d x. \tag{11}$$

This Hamiltonian collects three contributions:

- a kinetic-like term associated with phase gradients,
- local structural energy from the potentials V_i,
- interaction energy from phase-dependent couplings.

Time translation symmetry then yields:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = 0, \tag{12}$$

under the field equations derived from $\mathcal{L}$.

5.3 Economic meaning of H

The Hamiltonian H measures the global tension and structure of the economic field:

- large gradients of phase correspond to high directional tension,
- deviations of ρ_i from their structural baselines contribute local strain,
- misaligned phases across strongly coupled subsystems raise the interaction energy.

Conservation of H in the autonomous model means that the system can redistribute and reorganize tension, but not eliminate it entirely through internal dynamics alone. Relaxation toward coherent states redistributes energy between kinetic, potential, and interaction contributions, but the total remains fixed.

6 Summary of Symmetry Structure

We summarize the symmetry structure and associated invariants:

- **Global phase symmetry**: invariance under $\theta_i \mapsto \theta_i + \alpha$ for all i, leading to conservation of total density $\int \sum_i \rho_i \, \mathrm{d}^d x$.

- **Spatial translation symmetry**: invariance under $x \mapsto x + a$, implying the existence of a momentum-like conserved quantity associated with the uniformity of the underlying domain.
- **Time translation symmetry**: invariance under $t \mapsto t + \tau$, yielding conservation of the Hamiltonian functional H in (11).

These symmetries define the geometric backbone of the economic field. They constrain the possible dynamics, explain the persistence of certain macroscopic quantities, and identify coherent states as low-energy configurations consistent with the invariants of motion.

Future chapters will build on this structure to analyze coherence clusters, oscillatory regimes, and the response of the field to external perturbations.