Coherence Clusters in Multi–Subsystem Networks: Emergent Domains, Interfaces, and Collective Organization

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Abstract

This chapter extends the economic field theory to networks of many interacting subsystems. Building on the density–phase formulation and the two-subsystem dynamics, we introduce the concept of a *coherence cluster*: a stable group of subsystems whose phases evolve in unison and whose density flows organize into internally efficient patterns. We provide a precise operational definition, analyze the energy structure that drives cluster formation, distinguish strong-link and weak-link regimes, and show how networks naturally decompose into coherent economic domains. Interfaces between clusters give rise to tension, persistence of oscillatory modes, and long-lived metastable states. This framework reveals the geometry underlying economic macro-organization.

Contents

1 Introduction

Real economic systems are composed of many interacting subsystems: regions, sectors, supply chains, institutions, and strategic blocs. These subsystems do not evolve independently;

they influence one another through dense, heterogeneous networks of interdependence.

A central prediction of the economic field formulation is that such networks organize into *coherence clusters*—groups of nodes whose phases lock together due to strong internal couplings. Within a cluster, tensions vanish, flows stabilize, and the group behaves as a single macro-entity. Between clusters, weaker couplings generate interfaces that support oscillations, misalignment, and slow relaxation.

This chapter formalizes these ideas and establishes the geometry of coherent organization in multi-agent economic fields.

2 Network Structure and Interaction Geometry

We consider a system of N subsystems indexed by $i = 1, \dots, N$, connected by a symmetric coupling matrix

$$J_{ij} = J_{ji} \geq 0.$$

Each subsystem has:

- a density $\rho_i(t)$,
- a phase $\theta_i(t)$,

and interacts with others through the phase-dependent term in the Lagrangian:

$$E_{\text{int}} = \sum_{i<j} J_{ij}\, \rho_i \rho_j \left[1 - \cos(\theta_i - \theta_j)\right]. \tag{1}$$

The energetic cost of misalignment between subsystems i and j is proportional to:

$$J_{ij}\, \rho_i \rho_j. \tag{2}$$

Thus:

- pairs with *large* J_{ij} experience strong alignment pressure,
- pairs with *small* J_{ij} tolerate persistent misalignment.

This heterogeneity is the seed of cluster formation.

3 Definition of a Coherence Cluster

Let $\epsilon > 0$ be a small phase tolerance. A set $C \subset \{1, \dots, N\}$ is a *coherence cluster* if:

1. For all $i, j \in C$,
$$|\theta_i(t) - \theta_j(t)| < \epsilon \quad \text{for } t \text{ large},$$
2. For any $i \in C$ and $k \notin C$,
$$J_{ik} < J_{\text{int}},$$
where J_{int} is a threshold separating strong from weak links.

In words:

- phases within the cluster lock together,
- the cluster is insulated from external nodes by weak links.

The threshold J_{int} may be defined through spectral or graph-theoretic properties, but at an intuitive level:

$$J_{ij} \gg J_{ik}, J_{jk} \quad \text{for } i, j \in C,\ k \notin C.$$

4 Energy Minimization and Cluster Formation

The interaction energy (1) can be decomposed into intra-cluster and inter-cluster contributions.

Let $C_1, \dots, C_m$ be a partition of the nodes. Then:

$$E_{\text{int}} = \sum_{a=1}^{m} \sum_{\substack{i<j \\ i,j \in C_a}} J_{ij}\rho_i\rho_j \left[1 - \cos(\theta_i - \theta_j)\right] \quad + \quad \sum_{\substack{i \in C_a,\, j \in C_b \\ a \neq b}} J_{ij}\rho_i\rho_j \left[1 - \cos(\theta_i - \theta_j)\right]. \tag{3}$$

If intra-cluster couplings satisfy:

$$J_{ij} \gg J_{kl} \quad \text{whenever } i, j \in C_a,\ k \in C_a,\ l \notin C_a,$$

then minimizing E_{int} forces phase alignment *within* each cluster:

$$\theta_i \approx \theta_j \quad \text{for all } i, j \in C_a.$$

Inter-cluster phases satisfy weaker constraints, allowing for:

- slow drifts of relative phases,
- interfaces supporting oscillatory regimes,
- metastable non-aligned configurations.

5 Interfaces and Weak Link Geometry

Suppose $i \in C_a$ and $k \in C_b$ with $a \neq b$. If J_{ik} is small, then the interaction contributes:

$$E_{ik} \approx \tfrac{1}{2} J_{ik}\rho_i\rho_k(\Delta\theta_{ik})^2$$

near alignment.

Thus:

- misalignment costs little energy,
- θ_i and θ_k may oscillate relative to each other,
- the interface becomes a location of *soft* tension.

Interfaces act as shock absorbers in the economic field.

6 Dynamics in a Three-Cluster System

Consider three clusters C_1, C_2, C_3. The reduced dynamics for cluster phases are approximately:

$$\dot{\Theta}_a = \bar{V}'_a + \sum_{b \neq a} \overline{J}_{ab} \left[1 - \cos(\Theta_a - \Theta_b)\right], \tag{4}$$

where:

- Θ_a is the common phase of cluster C_a,

- $\overline{J}_{ab}$ is the effective inter-cluster coupling,
- $\bar{V}_a'$ contains structural forces internal to the cluster.

This is a coarse-grained field theory on the cluster graph.
Depending on $\overline{J}_{ab}$, several regimes appear:

- full global coherence ($\Theta_1 = \Theta_2 = \Theta_3$),
- partial coherence: two clusters lock, one oscillates,
- frustrated cycles: cyclic drift of $(\Theta_1, \Theta_2, \Theta_3)$,
- metastable patterns with slow reorientation.

7 TikZ Illustration

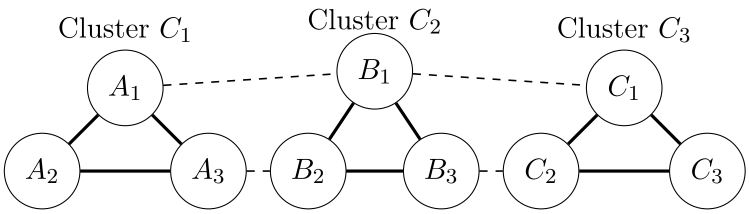

Figure 1: Three coherence clusters with strong internal couplings (solid lines) and weak inter-cluster links (dashed lines).

8 Summary

Multi-subsystem networks naturally organize into coherence clusters due to the heterogeneous structure of interdependence. Within each cluster, strong couplings enforce phase alignment and stable density flows. Between clusters, weak links create interfaces that support oscillatory modes, metastability, and slow relaxation.

Cluster formation is an emergent geometric phenomenon: it arises from the minimization of interaction energy and the dynamics of the density–phase field, not from external constraints or imposed structure.

This chapter prepares the ground for the analysis of oscillatory regimes, global coherence, and economic phase transitions in the next chapters.