Motion-Based Field Theory: Coupling to General Relativity and Relation to Quantum Field Theory

Foundations Beyond the Flat-Space Toroidal Soliton Framework

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November 2025

Abstract

This eighth document establishes the explicit connection between the motion-based scalar field theory developed in Documents 1–7 and the frameworks of General Relativity (GR) and Quantum Field Theory (QFT). We derive the full curved-space action, compute the energy–momentum tensor, discuss the semiclassical and quantum interpretations of the motion field, examine renormalizability, and identify the conditions under which toroidal solitons can persist in curved spacetime. This bridges the unified motion theory with the established pillars of modern physics.

1 Introduction

Documents 1–7 introduced a unified theory in which mass, energy and geometry arise from organized motion encoded in a complex scalar field

$$\psi = \rho e^{i\theta}.$$

The theory was formulated primarily in flat spacetime and within a classical field-theoretic context.

This document extends the framework by:

1. Formulating the motion field in a curved background.

2. Deriving the energy–momentum tensor sourcing Einstein's equations.

3. Establishing the semiclassical and quantum interpretation of the field.

4. Analyzing renormalizability and the UV structure.
5. Identifying the conditions for the survival of toroidal solitons in GR.

2 Curved-Space Action and Field Equations

The natural generalization of the action is

$$S = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \psi^* \, \partial_\nu \psi - U(|\psi|) \right], \tag{1}$$

with $U(\rho)$ as in Documents 3–7.

Using the decomposition $\psi = \rho e^{i\theta}$,

$$\mathcal{L} = g^{\mu\nu}(\partial_\mu \rho)(\partial_\nu \rho) + \rho^2 g^{\mu\nu}(\partial_\mu \theta)(\partial_\nu \theta) - U(\rho). \tag{2}$$

Variation gives the field equations:

$$\nabla_\mu \nabla^\mu \rho - \rho \, g^{\mu\nu} \partial_\mu \theta \, \partial_\nu \theta + \tfrac{1}{2} U'(\rho) = 0, \tag{3}$$

$$\nabla_\mu (\rho^2 \, \partial^\mu \theta) = 0. \tag{4}$$

The second equation expresses charge conservation in curved spacetime.

3 Energy–Momentum Tensor

The stress tensor is obtained by

$$T_{\mu\nu} = \partial_\mu \psi^* \partial_\nu \psi + \partial_\mu \psi \partial_\nu \psi^* - g_{\mu\nu} \mathcal{L}. \tag{5}$$

In (ρ, θ) variables:

$$T_{\mu\nu} = (\partial_\mu \rho)(\partial_\nu \rho) + \rho^2 (\partial_\mu \theta)(\partial_\nu \theta) \tag{6}$$

$$- g_{\mu\nu} \left[g^{\alpha\beta}(\partial_\alpha \rho)(\partial_\beta \rho) + \rho^2 g^{\alpha\beta}(\partial_\alpha \theta)(\partial_\beta \theta) - U(\rho) \right]. \tag{7}$$

This tensor is the source of gravity through Einstein's equations:

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}. \tag{8}$$

4 Toroidal Solitons in Curved Spacetime

The flat-space toroidal soliton solutions remain valid approximations whenever the geometry varies slowly over the soliton thickness.

This requires:

$$a \ll R_{\text{curv}}, \tag{9}$$

where R_{curv} is the curvature radius of the background spacetime.

Thus the tori survive:

- in weak gravitational fields,
- in cosmological backgrounds with slowly varying scale factor,
- near massive bodies if not too close to horizons.

The dominant curvature correction arises from redshift of the frequency ω in the phase $\theta = \omega t + n\varphi$.

5 Relation to Quantum Field Theory

5.1 Semiclassical interpretation

The field ψ can be quantized by promoting it to an operator $\hat{\psi}$ with canonical commutation relations:

$$[\hat{\psi}(\mathbf{x}), \hat{\pi}(\mathbf{y})] = i\delta(\mathbf{x} - \mathbf{y}). \tag{10}$$

Toroidal solitons correspond semiclassically to nonperturbative coherent states:

$$|\text{torus}(Q)\rangle \approx \exp\left(\int d^3x\, \Phi_0(x)\hat{\psi}^\dagger(x)\right)|0\rangle, \tag{11}$$

with charge Q.

5.2 Spectrum and excitations

Linear stability implies a tower of vibrational modes with frequencies Ω_k, representing quantized excitations around the torus.

5.3 Renormalizability

The Lagrangian with quartic potential is renormalizable in $3+1$ dimensions. The cubic term is super-renormalizable. Thus the model is well-defined as a quantum field theory.

The decomposition $\psi = \rho e^{i\theta}$ remains valid, though care is required near $\rho = 0$; this is analogous to polar decomposition in the Standard Model Higgs sector.

6 Motion-Based Interpretation and GR/QFT Synthesis

In the unified motion theory:

- $T_{\mu\nu}$ encodes the distribution of organized motion.
- Gravity is sourced by the geometry of internal flow.
- The toroidal soliton contributes an effective mass:

$$M_{\text{eff}} = \int d^3x \, T_{00}.$$

- Quantum fluctuations correspond to oscillations of $\delta\rho$ and $\delta\theta$ around the organized motion pattern.

This yields a fully integrated picture: **organized motion produces energy, mass, and gravitational effects, while remaining compatible with both GR and QFT.**

7 Conclusion

The unified theory of motion extends naturally into the frameworks of General Relativity and Quantum Field Theory. The curved-space action and stress tensor identify organized motion as a source of gravity. The semiclassical quantization of the toroidal soliton yields a consistent description of quanta of geometry. The model is power-counting renormalizable and fits within standard QFT.

This document completes the theoretical integration required for the framework to be positioned as a credible extension of modern physics.