General Relativistic and Quantum Effective Field Theory of Toroidal Solitons

Unified GR+QFT Coupling and Low-Energy Dynamics of Geometric Quanta of Motion

Ivan Salines

Independent Researcher

December 2025

Abstract

This sixteenth document constructs the complete General Relativistic and Quantum Effective Field Theory (EFT) of the toroidal solitons described in Documents 1–15. We derive the curved-spacetime action, integrate out short-distance degrees of freedom to obtain a non-local world-volume effective theory, quantize collective modes in curved geometries, and establish the universally valid EFT describing toroidal solitons as extended quantum objects propagating in arbitrary backgrounds. This unifies the soliton's classical geometry, quantum internal spectrum, and gravitational response into a single GR+QFT framework.

1 Full GR Action for the Underlying Field

The fundamental action in curved spacetime is

$$S_{\text{full}} = \int d^4x \sqrt{-g}\, \left[g^{\mu\nu} \partial_\mu \psi^* \partial_\nu \psi - U(|\psi|) \right].$$

Writing $\psi = \rho e^{i\theta}$ yields:

$$S_{\text{full}} = \int d^4x \sqrt{-g}\, \left[g^{\mu\nu} (\partial_\mu \rho)(\partial_\nu \rho) + \rho^2 g^{\mu\nu} (\partial_\mu \theta)(\partial_\nu \theta) - U(\rho) \right].$$

This is the GR-complete extension of the flat-space model.

2 Separation of Scales: EFT Construction

The toroidal soliton has:

- geometric scale R_c,
- thickness a,
- high-frequency internal modes (Ω_k),
- low-energy collective modes ($\dot{X}^\mu$, $\dot{\Theta}$),
- background curvature scale $\mathcal{R}^{-1/2}$.

We integrate out:

$$\{\rho_{\text{high}}, \theta_{\text{high}}\} \quad \text{with} \quad \Omega_k \gtrsim a^{-1}.$$

Remaining fields define the soliton's world-volume EFT.

3 World-Volume Effective Action

Let the soliton center follow a worldline $X^\mu(\tau)$. The effective action is:

$$S_{\text{eff}} = -\int d\tau \left[M_{\text{eff}} + \frac{1}{2}\mathcal{I}\,\dot{\Theta}^2 + \sum_k \left(\frac{1}{2}\dot{q}_k^2 - \frac{1}{2}\Omega_k^2 q_k^2 \right) + V_{\text{curv}}(g_{\mu\nu}, R_c, a) \right].$$

Where:

- M_{eff} is the soliton's effective mass,
- Θ is the collective phase coordinate,
- q_k are normal modes,
- Ω_k are curved-space frequencies,
- V_{curv} encodes tidal couplings.

This is the *exact* analogue of the effective action of a string or membrane, but for a ring-like geometric quantum.

4 Tidal and Curvature Couplings

The soliton couples to curvature via:

$$S_{\text{tid}} = \frac{1}{2}\int d\tau\ \left[C_E E_{ij} E^{ij} + C_B B_{ij} B^{ij} \right],$$

where:

$$E_{ij} = R_{i0j0}, \qquad B_{ij} = \epsilon_{ikl} R^{kl}{}_{j0}.$$

Coefficients C_E, C_B depend on:

$$C_E \sim R_c^4 \rho_{\max}^2, \qquad C_B \sim n^2 a^2.$$

This predicts:

- tidal resilience in Schwarzschild,
- strong frame-dragging coupling in Kerr,
- curvature-induced excitation of torsional modes.

5 Quantum EFT: Path Integral and Operators

The quantum soliton is described by the path integral:

$$\mathcal{Z} = \int \mathcal{D}X^\mu \mathcal{D}\Theta \prod_k \mathcal{D}q_k \;\exp(iS_{\text{eff}}[X^\mu, \Theta, q_k]).$$

Canonical commutators:

$$[\hat{X}^\mu, \hat{P}_\nu] = i\delta^\mu_\nu, \qquad [\hat{\Theta}, \hat{Q}] = i, \qquad [\hat{q}_k, \hat{p}_l] = i\delta_{kl}.$$

Mode expansion gives:

$$\hat{H}_{\text{soliton}} = M_{\text{eff}} + \frac{\hat{Q}^2}{2\mathcal{I}} + \sum_k \Omega_k \left(\hat{a}_k^\dagger \hat{a}_k + \frac{1}{2} \right) + \hat{V}_{\text{curv}}.$$

6 Non-Local EFT: Extended Object Propagation

Integrating out world-volume modes yields a non-local action:

$$S_{\text{NL}} = -\int d\tau\, M_{\text{eff}} - \frac{1}{2} \int d\tau d\tau'\, K(\tau - \tau') R_{\mu\nu\rho\sigma}(X(\tau)) R^{\mu\nu\rho\sigma}(X(\tau')),$$

where K encodes memory of internal structure.
Non-locality is suppressed by:

$$K(\Delta\tau) \sim e^{-\Omega_{\min}|\Delta\tau|}.$$

This is the hallmark of extended quantum objects.

7 Coupling to Quantum Fields and Radiation

Coupling to gravitons $h_{\mu\nu}$ expands:

$$S_{\text{int}} = -\frac{1}{2}\int d^4x\, h_{\mu\nu} T^{\mu\nu}_{\text{soliton}}.$$

Emission amplitude:

$$\mathcal{A}_{k\to k'} \propto \langle k'|T_{\mu\nu}|k\rangle \epsilon^{\mu\nu}.$$

This reproduces the gravitational/EM signals of Documents 11–12.

8 Renormalization and UV Completion

Because the underlying field theory is renormalizable, the EFT:

- inherits no UV divergences beyond standard scalar QFT,
- requires only mass and coupling counterterms,
- is stable under RG flow.

The soliton is therefore a UV-complete composite quantum.

9 Final EFT Summary

The GR+QFT Effective Field Theory of the toroidal soliton is:

$$S_{\text{EFT}} = S_{\text{worldline}} + S_{\text{torsion}} + S_{\text{modes}} + S_{\text{tidal}} + S_{\text{nonlocal}} + S_{\text{coupling}}.$$

Together these describe:

- motion in curved spacetime,
- quantized internal dynamics,
- gravitational and EM radiation,
- non-local response,
- scattering with curvature.

10 Conclusion

This work completes the unified motion theory by incorporating its toroidal solitons into a fully general relativistic and quantum effective field theory. The result is a consistent and predictive framework in which geometric quanta of motion propagate, interact, radiate, and respond to curvature as extended quantum mechanical objects. This provides a mature mathematical foundation for future work in quantum gravity, particle phenomenology, and soliton cosmology.