The Unified Theory of Motion: A Continuum Bridge Between Relativity and Wave–Particle Duality

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Abstract

This work proposes a unified conception of physical reality in which motion itself is the fundamental substance from which energy, mass, and geometry arise as different modes of organization. Mass is interpreted as a stationary equilibrium of internal motions; energy as the expression of motion that is free or unbalanced; and space–time as the global coherence pattern generated by their collective dynamics. A concrete field model is introduced in terms of a density of motion and a phase of motion, leading to a Lagrangian in which energy is purely kinetic in nature. In suitable limits, the framework reproduces the familiar structures of Special Relativity, General Relativity, and wave mechanics, thereby offering a natural bridge between corpuscular and wave descriptions of matter. An explicit toroidal configuration is discussed as a prototype of stationary mass with quantized internal motion, clarifying how particle-like properties can emerge from the self-organization of motion in a continuous medium.

1 Introduction

The history of modern physics may be read as a gradual shift from substances to structures, and from static entities to dynamical relations. Classical mechanics imagined point particles moving through an absolute space; Maxwell's theory replaced action-at-a-distance with continuous fields; Einstein's relativity fused space and time into a single geometric manifold whose metric responds to energy and momentum; quantum theory revealed that every so-called "particle" behaves as a wave, endowed with a characteristic frequency and wavelength.

Yet even with these advances, physics remains conceptually fragmented. Relativity describes how energy and momentum curve space–time, but treats matter as an input encoded in an energy–momentum tensor. Quantum mechanics quantizes action and associates to each particle a wave function, but leaves unresolved the ontological status of that wave: is it a physical field, a probability amplitude, or something in between? The wave–particle duality is accepted pragmatically, but not explained in terms of a single underlying principle.

This paper explores a different possibility: that *motion itself* is the fundamental physical reality, and that energy, mass, and space–time are emergent aspects of how motion organizes itself. In this view, there are no inert, featureless carriers of properties; there are only patterns of activity. A "particle" is not a tiny object that happens to move; it is a localized, self-sustaining configuration of motion. Space is not a passive stage, but the way directions of motion are arranged. Time is not an external parameter, but the local rhythm—the phase—of ongoing motion.

Within this perspective, Einstein's relation

$$E = mc^2 \tag{1}$$

remains valid as a numerical identity, but acquires a deeper interpretation. Rather than stating that mass can be *converted* into energy, it suggests that what we call "mass" is already a form of energy, associated with an internal state of motion. To make this explicit, we rewrite the relation in the more general form

$$E = M_{\text{eff}}\, q_{\text{motion}}^2. \tag{2}$$

Here q_{motion} is an intrinsic rate or "quantum of motion" characterizing the internal dynamics of a configuration (for example, a frequency or an effective velocity scale), and M_{eff} is an *effective mass*, defined not as a primitive quantity but as a measure of how much "substance of motion" participates in that dynamics.

Definition. A *quantum of motion* denotes a localized, self–sustaining pattern of continuous flow whose stability arises from intrinsic geometric constraints. It is neither a point particle nor a wave packet, but a coherent structure of organized motion.

In the field formulation developed below, M_{eff} will naturally appear as an integral of a density field:

$$M_{\text{eff}} = \int_{\mathcal{V}} \rho(x)\, d^3x, \tag{3}$$

where $\rho(x)$ represents the local density of motion within a spatial region $\mathcal{V}$. In homogeneous situations, M_{eff} reduces to a constant times the volume, and coincides with the usual notion of mass. Equation (2) then says that the energy of a configuration is determined by *how much* motion is present (M_{eff}) and by *how fast or intensely* it oscillates (q_{motion}).

Einstein's constant c^2 emerges in this picture as the maximal allowable value of q^2_{motion} compatible with the causal and geometric constraints of space–time. Thus, $E = mc^2$ is seen as a special case of (2) in which the intrinsic quantum of motion saturates this universal limit.

The aim of this paper is to show that such a reinterpretation can be made precise. We construct a field theory in which the fundamental variables are a *density of motion* and a *phase of motion*. The resulting dynamics is governed by a Lagrangian where all energy is kinetic and where mass appears as a property of stationary solutions—standing configurations of motion—rather than as a primitive input. We then demonstrate how, in appropriate limits, this field theory reproduces known relativistic and quantum equations, and how geometrical curvature arises from the collective organization of the underlying motion.

Scope. This paper develops only the conceptual and geometric framework. The detailed field–theoretic realizations, as well as explicit solitonic solutions, fall outside the scope of this document.

Related Work and Novelty

The present framework is related to, yet distinct from, several established lines of research: classical Q-balls (Coleman, Friedberg–Lee–Sirlin), toroidal solitons and Q-rings, Hopf solitons, and vorton-like configurations. Traditional approaches start from a complex scalar field and search for localized solutions. In contrast, the present framework begins from the kinematic decomposition $\Phi = \sqrt{\rho}\, e^{i\theta}$ and interprets ρ and θ as density and internal motion.

This shift of perspective leads naturally to an effective radial reduction and to a toroidal energy functional $E(R, a)$ which, in more detailed analyses, can be used to identify a preferred radius R^* of self-organized motion. A full quantitative construction of toroidal Q-ball configurations, including the solitonic window in a parameter plane (Ω, ρ_c) and the behavior of a "tail metric" that measures localization, is developed in a companion, numerically oriented work. Here we focus on the conceptual and structural aspects of the motion-field framework and on its connections to relativistic and quantum dynamics.

2 Conceptual Foundations: Motion as the Common Substance

In the standard formulation of physics, space, time, and matter are treated as conceptually distinct. Space is a three-dimensional arena, time a one-dimensional parameter, and matter or fields are entities that "live" in this arena and evolve with time. Relativity unifies space and time into a four-dimensional manifold, but still distinguishes matter (as a source term) from geometry (as the dynamical metric). Quantum theory, by contrast, replaces point particles with waves in configuration space, but its standard probabilistic interpretation prevents a literal, physical reading of those waves.

The approach developed here begins from a different ontological stance. We posit that there exists a single, continuous medium of motion, which we do not identify a priori with any specific field of the Standard Model, but which we treat as a primitive *field of activity*. This medium is characterized, at each event x^μ, by two scalar fields:

- a density of motion $\rho(x)$, indicating how much of the underlying activity is locally present;
- a phase of motion $\theta(x)$, indicating the local rhythm and direction of that activity in space–time.

In this language:

- **Space** is the pattern of directions along which phase gradients $\partial_i\theta$ can be organized;
- **Time** is the progression of the phase θ itself, encoded in its temporal derivative $\partial_0\theta$;
- **Matter** is a region where ρ and θ conspire to form a stable, resonant configuration: motion that closes on itself in a standing equilibrium.

The intuitive distinction between wave and particle also finds a natural reinterpretation. A wave corresponds to motion that is *open*: the phase propagates across the medium, leading to interference patterns and extended structures. A particle corresponds to motion that is *closed*: the phase and density arrange themselves into a configuration that remains localized and self-sustaining over time, like a vortex or a standing wave. Wave and particle are thus not two incompatible natures, but two regimes of the same field of motion.

From this standpoint, energy is simply a measure of the intensity of motion—how strongly and how rapidly the phase changes in space and time, weighted by how much medium is involved. Mass is the effective inertia of a coherent region of motion: the resistance of a stable pattern to being deformed or accelerated. And geometry—the curvature of space–time—is the large-scale expression of how motion, in aggregate, distributes and organizes itself.

The rest of the paper develops this conceptual scheme into a concrete field theory with a well-defined action principle, equations of motion, and limiting behaviors that connect to known physics.

3 Foundations and Assumptions

The framework developed here rests on a minimal set of structural assumptions:

1. **Continuum Field Representation.** The fundamental degrees of freedom are a real density field $\rho(x)$ and a phase field $\theta(x)$, combined into a complex scalar

$$\Phi(x) = \sqrt{\rho(x)}\, e^{i\theta(x)}.$$

   This identification matches the polar decomposition of a standard complex scalar field used in relativistic and non-relativistic theories of Q-balls, vortices, and condensates.

2. **Metric and Units.** Calculations are performed in flat spacetime with metric $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$. Natural units $\hbar = c = 1$ are adopted.

3. **Energy Functional.** The Lagrangian density is assumed to be of the generic form

$$\mathcal{L} = \rho\, \partial_\mu \theta\, \partial^\mu \theta - U(\rho),$$

   so that the associated energy density is $T_{00} = \rho|\nabla\theta|^2 + U(\rho)$. This reduces to the familiar $|\partial\Phi|^2 - U(|\Phi|)$ form and guarantees consistency with standard complex-field models.

4. **Symmetry and Conservation.** The action is invariant under the global transformation $\theta \to \theta + \alpha$, which yields via Noether's theorem the conserved charge

$$Q = \int \rho\, d^3x.$$

5. **Regime of Validity.** The theory is classical/mean-field and, for most of the present discussion, does not include gravitational back-reaction or quantum corrections. Stability analyses are performed at the classical level. In Section 9 we briefly indicate how the motion field can be coupled to gravity in the standard Einstein–Hilbert framework.

4 The Field of Motion and the Action Principle

We work on a four-dimensional space–time manifold endowed with a metric $g_{\mu\nu}$ of Lorentzian signature $(-, +, +, +)$. For most of this paper we are interested in the flat case $g_{\mu\nu} = \eta_{\mu\nu}$, but writing the action in generally covariant form makes the coupling to gravity straightforward. The fundamental fields are:

1. A scalar field $\rho(x)$, representing the density of motion;
2. A scalar field $\theta(x)$, representing the phase of motion.

The simplest Lagrangian density that embodies the idea "energy is motion weighted by density" is

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\rho\, g^{\mu\nu}\partial_\mu\theta\,\partial_\nu\theta - U(\rho)\right], \tag{4}$$

where $U(\rho)$ is a potential governing the self-interaction of the density. The first term is purely kinetic: it assigns energy to gradients of the phase, weighted by the local density ρ. The second term encodes how the medium of motion "prefers" to distribute its density at equilibrium.

The total action is

$$S = \int \mathcal{L}\, d^4x. \tag{5}$$

Varying this action with respect to θ gives

$$\nabla_\mu\Big(\rho\, g^{\mu\nu}\partial_\nu\theta\Big) = 0, \tag{6}$$

a continuity equation expressing conservation of motion flow. The quantity

$$J^\mu = \rho\, g^{\mu\nu}\partial_\nu\theta \tag{7}$$

plays the role of a current of motion, and Eq. (6) states that this current is divergenceless.

Varying the action with respect to ρ yields

$$\frac{1}{2}\, g^{\mu\nu}\partial_\mu\theta\,\partial_\nu\theta = \frac{dU}{d\rho}, \tag{8}$$

which can be viewed as a state relation: it links the local intensity of motion (the squared phase gradient) to the local density via the derivative of the potential.

Equations (6) and (8) are the basic dynamical laws of the theory. They describe how the density of motion and the phase of motion co-evolve in a given geometry $g_{\mu\nu}$.

5 Stationary Configurations and Effective Mass

We now show how particle-like mass arises as a stationary configuration of the motion field. Consider a region of space in which the density is approximately constant,

$$\rho(x) \approx \rho_0, \tag{9}$$

and the phase varies linearly in time with frequency ω,

$$\theta(x) = \omega t. \tag{10}$$

In a locally flat region of space–time, the metric is approximately Minkowskian, $g_{\mu\nu} \approx \eta_{\mu\nu}$. Then

$$g^{\mu\nu}\partial_\mu\theta\,\partial_\nu\theta = g^{00}(\partial_0\theta)^2 = -\omega^2. \tag{11}$$

The state relation (8) becomes

$$-\frac{1}{2}\omega^2 = \frac{dU}{d\rho}(\rho_0), \tag{12}$$

which links the equilibrium density ρ_0 and the internal frequency ω through the potential $U(\rho)$. Only those combinations of ρ_0 and ω that satisfy this condition correspond to stationary states.

The energy density associated with the kinetic term in (4) is, up to sign conventions,

$$\mathcal{H}_{\text{motion}} \sim \frac{1}{2}\rho_0\,\omega^2. \tag{13}$$

Integrating over a volume V in which ρ_0 is approximately constant, we obtain

$$E_{\text{stationary}} = \frac{1}{2}\rho_0\,\omega^2 V = \frac{1}{2}M_{\text{eff}}\omega^2, \tag{14}$$

where

$$M_{\text{eff}} = \int_V \rho_0\,d^3x = \rho_0 V. \tag{15}$$

This is precisely of the form (2), with $q_{\text{motion}} = \omega$.

Hence, a lump of mass can be understood as a region of the motion field where:

- the density of motion ρ is non-zero and approximately uniform;
- the phase evolves with a constant internal frequency ω;
- the potential $U(\rho)$ and the kinetic term are in balance, yielding a stationary equilibrium.

In this sense, mass is not a passive attribute, but the manifestation of a *standing motion*. The effective mass M_{eff} measures how much medium is involved; the frequency ω measures how intensely it oscillates.

6 Toroidal Configurations and Quantized Circulation

The previous section considered purely temporal variations of the phase. We now turn to configurations in which the phase also varies spatially in a topologically nontrivial way, giving rise to quantized motion.

In cylindrical coordinates (r, φ, z), consider a toroidal region centered around a circle of radius R in the $z = 0$ plane. We model the density as

$$\rho(r, \varphi, z) \approx \rho_0 \, \exp\left[-\frac{(r-R)^2 + z^2}{a^2}\right], \tag{16}$$

where a is the characteristic thickness of the torus. The phase is taken to be

$$\theta(t, \varphi) = \omega t + n\varphi, \tag{17}$$

with integer n. The term ωt represents internal temporal oscillation; the term $n\varphi$ describes a circulation of phase around the torus.

The angular gradient of the phase reads

$$\partial_\varphi \theta = n, \tag{18}$$

and the corresponding contribution to $|\nabla\theta|^2$ in physical space involves a factor $1/R^2$ from the metric on the circle of radius R:

$$|\nabla\theta|^2 \sim \left(\frac{n}{R}\right)^2. \tag{19}$$

The total energy associated with the toroidal configuration is then approximately

$$E_{\text{torus}} \approx \frac{1}{2} \int \rho \left[\omega^2 + \left(\frac{n}{R}\right)^2\right] d^3x \approx \frac{1}{2} M_{\text{eff}} \left[\omega^2 + \left(\frac{n}{R}\right)^2\right], \tag{20}$$

where

$$M_{\text{eff}} = \int \rho \, d^3x \approx \rho_0 \, 2\pi^2 R a^2 \tag{21}$$

is the effective mass of the torus.

The quantization of n reflects the fact that the phase must be single-valued: when one goes around the torus once, the phase must return to its original value modulo 2π. Thus the total change in phase is

$$\Delta\theta = \int_0^{2\pi} \partial_\varphi \theta \, d\varphi = 2\pi n, \tag{22}$$

and n is a topological invariant of the configuration. Different integer values of n label distinct classes of motion that cannot be continuously deformed into one another without disrupting the coherence of the field.

Physically, such a toroidal configuration can be viewed as a prototype of a particle with intrinsic angular momentum or a quantized circulation. The internal oscillation frequency ω encodes a spin-like mode of motion, while the winding number n and the radius R encode an orbital-like mode of circulation. Both contribute to the total energy and to the effective inertia of the configuration.

7 Relativistic Limit: Klein–Gordon-Type Dynamics

To connect the field of motion to familiar relativistic wave equations, we consider small fluctuations around an equilibrium state. Let

$$\rho(x) = \rho_0 + \delta\rho(x), \qquad \theta(x) = \theta_0(x) + \delta\theta(x), \tag{23}$$

where ρ_0 and θ_0 satisfy the background equations and $\delta\rho$, $\delta\theta$ are small perturbations.

Expanding the Lagrangian (4) to quadratic order in the perturbations and assuming $U'(\rho_0) = 0$ (equilibrium) and $U''(\rho_0) = \lambda > 0$, we obtain, schematically,

$$\mathcal{L}_{\text{quad}} \approx \sqrt{-g}\left[\frac{1}{2}\rho_0\, g^{\mu\nu}\partial_\mu\delta\theta\,\partial_\nu\delta\theta - \frac{1}{2}\lambda(\delta\rho)^2\right]. \tag{24}$$

Under conditions in which $\delta\rho$ is heavy or can be integrated out, the dominant dynamics is carried by $\delta\theta$, leading to an effective Lagrangian

$$\mathcal{L}_{\text{eff}} \sim \sqrt{-g}\left[\frac{1}{2}\rho_0\, g^{\mu\nu}\partial_\mu\delta\theta\,\partial_\nu\delta\theta - \frac{1}{2}m_{\text{eff}}^2(\delta\theta)^2\right], \tag{25}$$

with an effective mass m_{eff} arising from higher-order couplings. The corresponding Euler–Lagrange equation is

$$\rho_0\,\Box\delta\theta + m_{\text{eff}}^2\delta\theta = 0, \tag{26}$$

Remarks on the Derivation. The effective equation for $\delta\theta$ is obtained by expanding the action to quadratic order, integrating out the heavy density fluctuations $\delta\rho$, and collecting the remaining kinetic and mass-like terms. This procedure parallels standard effective–field–theory reductions.

or, after rescaling $\delta\theta$,

$$\Box\phi + m^2\phi = 0, \tag{27}$$

which is the Klein–Gordon equation.

Thus, small phase fluctuations of the motion field obey a relativistic wave equation with an effective mass parameter, reproducing the standard dispersion relation

$$E^2 = p^2 c^2 + m^2 c^4 \tag{28}$$

once appropriate constants are restored.

8 Quantum Limit: Emergence of Schrödinger Dynamics

In the nonrelativistic limit, the same field of motion can be recast in terms of a complex wave function. Let

$$\psi(x) = \sqrt{\rho(x)}\, e^{i\theta(x)/\hbar}. \tag{29}$$

Restoring $\hbar$ and c explicitly, if the phase is dominated by a term $-mc^2 t/\hbar$ plus slowly varying contributions, and if spatial gradients are small compared to $mc/\hbar$, the equations for ρ and θ reduce to:

- a continuity equation for ρ,
- a Hamilton–Jacobi-type equation for θ with an additional "quantum potential" term.

Combining these, one recovers, under standard approximations, the Schrödinger equation

$$i\hbar\, \partial_t \psi = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{x}) \right] \psi. \tag{30}$$

In this way, the motion field provides a physical substratum for the wave function. The modulus $|\psi|^2$ encodes the density of motion ρ; the phase of ψ encodes the phase of motion θ. Wave behavior emerges not as an abstract probability amplitude, but as the manifestation of how motion distributes and interferes within the underlying continuum.

9 Connection to General Relativity

To incorporate gravitation, we supplement the motion field with the Einstein–Hilbert term, yielding the total action

$$S_{\text{total}} = \int d^4x\, \sqrt{-g} \left[\frac{R}{16\pi G} + \frac{1}{2} \rho\, g^{\mu\nu} \partial_\mu \theta\, \partial_\nu \theta - U(\rho) \right], \tag{31}$$

where R is the Ricci scalar and G is Newton's constant.

Variation with respect to $g^{\mu\nu}$ gives Einstein's equations

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}, \tag{32}$$

with energy–momentum tensor

$$T_{\mu\nu} = \rho\, \partial_\mu \theta\, \partial_\nu \theta - g_{\mu\nu}\mathcal{L}. \tag{33}$$

Thus, curvature is sourced by the distribution of motion encoded in ρ and θ. Where motion is dense and highly organized, curvature becomes strong; where motion is dilute and incoherent, space–time tends toward flatness.

This recovers the structural content of General Relativity, but with a different reading of the right-hand side: the source of gravity is not "matter" in an abstract sense, but the field of motion itself—its density and its coherent gradients.

10 Discussion and Outlook

The theory developed in this paper is intentionally minimal in its assumptions and broad in its implications. It postulates a single physical substance—motion—described by a density ρ and a phase θ, and governed by a kinetic Lagrangian with a self-interaction potential $U(\rho)$. From this starting point, one can reconstruct:

- the relativistic dynamics of small fluctuations (Klein–Gordon);
- the nonrelativistic dynamics of quantum waves (Schrödinger);
- the geometric coupling between motion and curvature (Einstein's equations).

Beyond technical reconstruction, the framework proposes a new ontology:

- Mass is not a static quantity but a measure of how much motion is bound in a stationary configuration;
- Energy is nothing other than the intensity of motion, quantified by the square of a characteristic rate q_{motion};
- Space and time emerge as the pattern and rhythm of motion in the continuum.

The unification of wave and particle descriptions arises naturally: waves correspond to open, propagating motion; particles to closed, resonant motion. The need for a dualistic interpretation disappears once both are seen as regimes of the same field.

The theory is not yet developed to the point of making sharp, quantitative predictions beyond those of established physics, but it suggests several avenues:

- Nonlinearities in $U(\rho)$ could lead to small deviations from linear quantum superposition in highly coherent, macroscopic systems;
- At very high energies, corrections to the relativistic dispersion relation might appear, potentially testable in astrophysical observations;
- Gravitational phenomena might be reinterpreted in terms of effective superfluid or condensate-like behavior of the motion field on cosmological scales.

These possibilities require further work and detailed modeling. The present paper is offered primarily as a conceptual and structural foundation: a proposal that motion, rather than matter or geometry taken separately, is the true common denominator of physical reality.

Emergent Predictions. Within this continuum viewpoint, several qualitative predictions follow:

- stable toroidal flow patterns arise naturally as balanced configurations of curvature and phase circulation;
- the geometry of such structures is quantized by dynamical ratios such as R_c/a;
- mass emerges as an effective measure of organized, stationary motion.

Predictions and Limitations

Predictions. The theory makes several robust, testable predictions and expectations:

- In the toroidal sector of the theory, preliminary analyses indicate that the preferred radius R^* depends primarily on the winding number n and is nearly independent of the total charge Q for sufficiently large Q.
- In explicit Q-ball realizations of the motion field, localized radial profiles exist only within a narrow window in a parameter plane (Ω, ρ_c) determined by the shape of $U(\rho)$; outside this window, configurations either disperse or collapse. This behavior is discussed in detail in a separate numerical study.
- In the same realizations, a suitably defined tail metric decreases sharply inside the solitonic window, providing a quantitative marker of localization.
- Toroidal geometries are energetically favored over spherical ones whenever a spatial winding is imposed, reflecting the role of topology in organizing stationary motion.

Limitations.

- The framework is classical and does not include quantum corrections or backreaction effects beyond the minimal coupling to gravity sketched in Section 9.
- Full 3D stability against arbitrary perturbations remains to be explored.
- The potential $U(\rho)$ is treated phenomenologically; a microphysical derivation is left for future work.

Appendix A: Toroidal Motion as a Prototype of Stationary Mass

In this appendix we provide additional detail on the toroidal configuration introduced earlier, emphasizing both its mathematical structure and its physical interpretation.

A.1 Geometry and Fields

We consider a torus embedded in three-dimensional space, described in cylindrical coordinates (r, φ, z). The torus is centered on a circle of radius R in the $z = 0$ plane and has a minor radius $a \ll R$. We approximate the density of motion as

$$\rho(r, z) = \rho_0 \exp\left[-\frac{(r-R)^2 + z^2}{a^2}\right], \tag{34}$$

independent of φ. This describes a ring-shaped region where the medium of motion is concentrated.

The phase is taken to be

$$\theta(t, \varphi) = \omega t + n\varphi, \tag{35}$$

with integer n. This choice encodes two types of motion:

- internal oscillation at frequency ω;
- circulation around the torus, with winding number n.

A.2 Energy and Effective Mass

The kinetic contribution to the energy density reads, schematically,

$$\mathcal{H}_{\text{kin}} = \frac{1}{2}\rho\left[(\partial_t\theta)^2 + |\nabla\theta|^2\right]. \tag{36}$$

We have $\partial_t\theta = \omega$, and the dominant spatial gradient is along the φ direction:

$$\partial_\varphi\theta = n, \qquad |\nabla\theta|^2 \sim \left(\frac{n}{R}\right)^2. \tag{37}$$

Integrating over the torus volume,

$$V_{\text{torus}} \approx 2\pi^2 R a^2, \tag{38}$$

we obtain the total kinetic energy

$$E_{\text{torus}} \approx \frac{1}{2}\rho_0 \left[\omega^2 + \left(\frac{n}{R}\right)^2\right] V_{\text{torus}} = \frac{1}{2} M_{\text{eff}} \left[\omega^2 + \left(\frac{n}{R}\right)^2\right], \tag{39}$$

with

$$M_{\text{eff}} = \rho_0 V_{\text{torus}}. \tag{40}$$

As before, the energy is of the general form $E = M_{\text{eff}} q^2_{\text{motion}}$, with an effective quantum of motion

$$q^2_{\text{motion}} = \omega^2 + \left(\frac{n}{R}\right)^2. \tag{41}$$

A.3 Physical Interpretation

This toroidal configuration may be viewed as an idealized model of a localized mass with internal structure. The density ρ represents how much of the motion medium is bound into the torus; the phase θ describes how that motion is organized.

The internal oscillation characterized by ω is reminiscent of a spin-like motion: a rotation in an internal degree of freedom, not directly associated with spatial displacement. The circulation characterized by n and R is reminiscent of an orbital angular momentum: motion that wraps around a closed loop in space.

The integer n is a topological invariant: one cannot change n continuously without disrupting the coherence of the field, because the phase must remain single-valued. Thus, n labels distinct "charge-like" or "spin-like" sectors of the configuration. If such toroidal structures existed in nature as fundamental or emergent entities, their discrete properties could be traced back to the quantization of the phase winding.

More generally, the torus serves as a concrete image of the central idea of this work: that what we call a "particle" can be seen as a region where motion organizes into a closed, resonant pattern, sustained by the balance between kinetic and potential contributions. In this sense, mass is not the absence of motion, but the presence of a particularly coherent and self-maintaining form of motion.

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15. For related ideas on emergent gravity and superfluid-vacuum models, see also contemporary reviews on k-essence and relativistic superfluids in theoretical cosmology and in condensed-matter analogues of gravity.