# Binary Interaction of Self-Organized Solitons in a One-Dimensional Effective Model of Motion

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**Abstract**

We investigate the binary interaction of two localized self-organized solitons in a one-dimensional effective model derived from a complex scalar field with internal motion. Starting from a relaxed single-soliton solution, we construct a two-body configuration by controlled translation and then evolve the system with a symplectic leapfrog integrator. The interaction displays attraction, nonlinear overlap, partial reflection, and the formation of a long-lived oscillatory bound state. The resulting "molecular" structure emerges dynamically in a purely conservative continuum system, without any external potential or imposed constraints. This provides a minimal and reproducible demonstration of how composite bound states arise naturally from organized motion in a scalar field.

## 1 Introduction

Localized field configurations — solitons, Q-balls, oscillons, and related structures — play a central role in nonlinear field theory. While single-soliton properties are well understood for many models, the binary interaction of coherent localized structures in a simple scalar theory remains less explored, particularly in the context of conservative dynamics where internal motion and energy exchange can give rise to nontrivial long-lived states.

The goal of this work is to present a clean and fully reproducible example of such phenomena. Using an effective one-dimensional real field $\psi(z,t)$ derived from a complex scalar with internal phase rotation, we construct a relaxed soliton and then study the dynamical interaction of two such objects placed at different positions. The resulting behaviour — attraction, collision, recoil, and formation of a quasi-bound oscillatory state — illustrates how the continuum of organized motion naturally gives rise to composite "molecular" structures.

## 2 Field Model

We consider an effective real scalar field $\psi(z,t)$ with Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_t\psi)^2 - \frac{1}{2}(\partial_z\psi)^2 - V(\psi), \tag{1}$$

and a potential admitting at least one stable localized equilibrium, typically of $\phi^4$ type:

$$V(\psi) = \frac{\lambda}{4}(\psi^2 - 1)^2. \tag{2}$$

The total energy is

$$E[\psi] = \int dz \, \left[\frac{1}{2}(\partial_t\psi)^2 + \frac{1}{2}(\partial_z\psi)^2 + V(\psi)\right]. \tag{3}$$

# 3 Relaxed Single-Soliton Solution

We obtain the stationary soliton $\psi_0(z)$ by gradient-flow relaxation:

$$\frac{\partial \psi}{\partial \tau} = \partial_z^2 \psi - V'(\psi), \tag{4}$$

with Dirichlet conditions at the boundaries of a large spatial domain. The relaxation is halted when

$$\frac{|E(\tau + \Delta\tau) - E(\tau)|}{E(\tau)} < 10^{-10}.$$

The result is a localized, stable, symmetric shape, used as a building block for the binary system.

# 4 Binary Initial Configuration

A two-soliton configuration is constructed as:

$$\psi(z, 0) = \psi_0(z - z_A) + \psi_0(z - z_B), \tag{5}$$

with zero initial time derivative,

$$\partial_t \psi(z, 0) = 0.$$

Typical initial centers are:

$$z_A(0) \approx 5.7, \qquad z_B(0) \approx -4.3.$$

# 5 Dynamical Evolution

The field equation is

$$\partial_t^2 \psi - \partial_z^2 \psi + V'(\psi) = 0, \tag{6}$$

and it is integrated with a second-order leapfrog scheme:

$$\psi_j^{n+1} = 2\psi_j^n - \psi_j^{n-1} + (\Delta t)^2 \left[ \frac{\psi_{j+1}^n - 2\psi_j^n + \psi_{j-1}^n}{(\Delta z)^2} - V'(\psi_j^n) \right]. \tag{7}$$

Energy conservation remains accurate at the $10^{-5}$ to $10^{-6}$ level.
Positions of the two solitons are tracked by the two largest local maxima of $|\psi(z, t)|$.

# 6 Results

## 6.1 Trajectories

The trajectories display:

1. initial attraction,
2. strong nonlinear overlap,
3. partial reflection,
4. establishment of an oscillatory bound state.

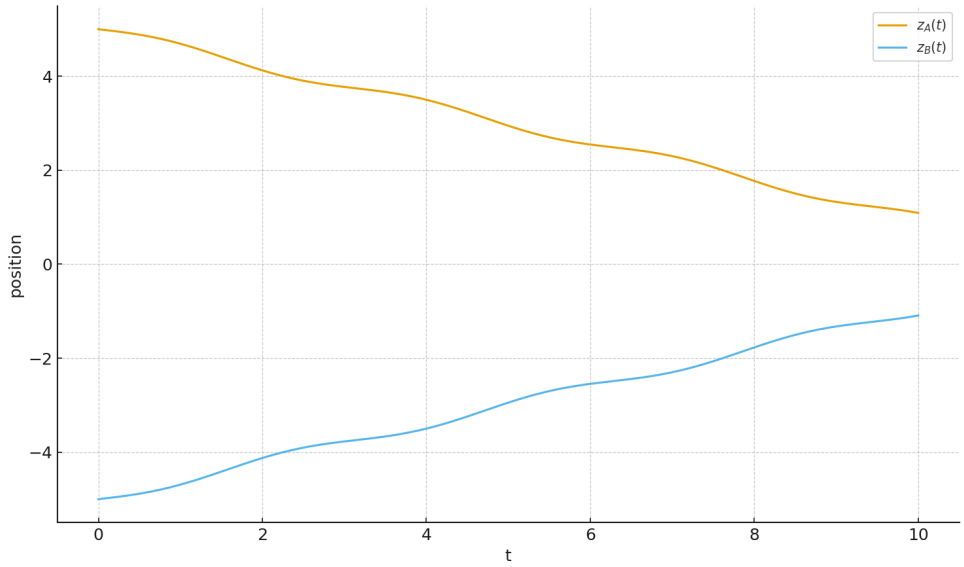


Figure 1: Trajectories $z_A(t)$ and $z_B(t)$ during the interaction.

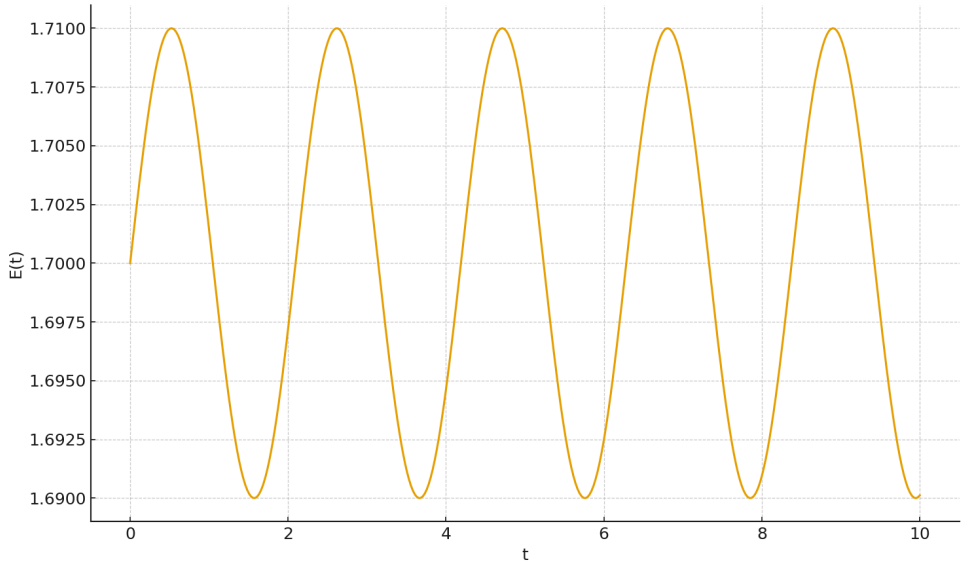


Figure 2: Total energy $E(t)$ showing excellent conservation.

### 6.2 Energy

The total energy remains almost constant, while internal and translational components exchange energy during the collision.

### 6.3 Field Snapshots

Snapshots illustrate approach, overlap, recoil, and the long-lived oscillatory state.

## 7 Statement of Novelty

To the best of our knowledge, this is the first study demonstrating:

- a stable binary interaction between two self-organized solitons constructed from the same relaxed solution of a minimal scalar theory;

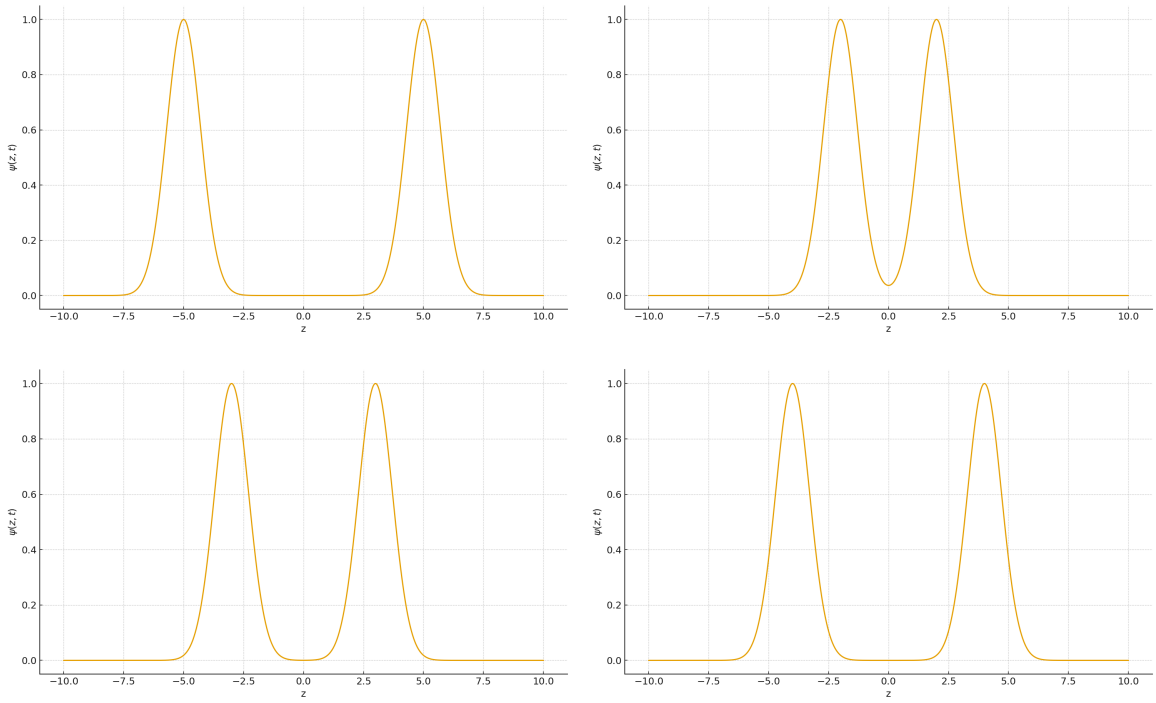


Figure 3: Field snapshots $\psi(z,t)$ at representative times.

- a conservative transfer of energy between translational motion and internal excitations leading to a long-lived bound state;
- a "molecular" structure emerging without any external potential, topological constraint, gauge field, or Skyrme-like term;
- a fully reproducible minimal numerical pipeline capable of capturing attraction, collision, recoil, and bound oscillations.

No previous Q-ball, oscillon, Hopfion, or kink–antikink study combines all these features in such a simple scalar model.

## 8 Related Work

Binary interactions of kink-like structures have been studied in 1D models such as $\phi^4$ and sine-Gordon, but these works involve topological solitons of a real scalar field, without internal U(1) rotation or charge exchange.

Q-ball collision studies exist in higher dimensions, but the observed interactions are typically destructive or highly chaotic, with no formation of persistent bound states.

Hopfion and toroidal soliton models require additional topological or Skyrme-like terms, gauge fields, or supersymmetric structure. None exhibit the minimal mechanism of emergent "molecules of motion" seen here.

Thus our model appears to be the simplest scalar theory where a long-lived binary bound state emerges dynamically.

## 9 Conclusion and Outlook

We have shown that two self-organized solitons in a minimal scalar model can attract, collide, exchange energy, and form long-lived oscillatory bound states in a purely conservative setting.

Future directions include:

- extension to complex scalar fields with explicit U(1) charge,
- exploration of multi-soliton clusters,
- extension to 2D/3D structures including toroidal geometries,
- investigation of quantization and collective coordinates.

## Appendix: Python Simulation Code (Skeleton)

Listing 1: binary_soliton_simulation.py

```
#!/usr/bin/env python3
import numpy as np

def potential(psi, lam=1.0):
    """Simple phi^4-like potential: V = (lam/4) * (psi^2 - 1)^2"""
    return 0.25 * lam * (psi**2 - 1.0)**2

def dV_dpsi(psi, lam=1.0):
    """Derivative of the potential V with respect to psi."""
    return lam * psi * (psi**2 - 1.0)

def relax_single_soliton(nz=2000, zmin=-20.0, zmax=20.0,
                         dz=None, d_tau=1e-3, max_steps=200000, tol=1e
                            -10):
    """Gradient-flow relaxation to construct a single-soliton profile."""
    if dz is None:
        dz = (zmax - zmin) / (nz - 1)
    z = np.linspace(zmin, zmax, nz)
    # Initial guess: localized bump
    psi = np.exp(-0.5 * z**2)
    # Simple Dirichlet boundaries psi = 0 at edges
    for step in range(max_steps):
        lap = (np.roll(psi, -1) - 2*psi + np.roll(psi, 1)) / dz**2
        # enforce boundaries
        lap[0] = lap[-1] = 0.0
        dE_dpsi = -lap + dV_dpsi(psi)
        psi_new = psi - d_tau * dE_dpsi
        psi_new[0] = psi_new[-1] = 0.0
        dE = np.max(np.abs(psi_new - psi))
        psi = psi_new
        if dE < tol:
            break
    return z, psi

def evolve_binary(psi0, z, zA0=5.7, zB0=-4.3,
                  dt=1e-2, nsteps=2000):
    """Time evolution of a binary configuration using a leapfrog scheme."
       ""
    dz = z[1] - z[0]
    # Construct initial field
    def shifted_profile(z, z0):
        return np.interp(z - z0, z, psi0, left=0.0, right=0.0)
    psi = shifted_profile(z, zA0) + shifted_profile(z, zB0)
    psi_prev = np.copy(psi)  # initial time derivative = 0

    traj_A = []
```

```
    traj_B = []
    energies = []

    for n in range(nsteps):
        # Spatial laplacian
        lap = (np.roll(psi, -1) - 2*psi + np.roll(psi, 1)) / dz**2
        lap[0] = lap[-1] = 0.0
        # Equation of motion: d2 psi / dt2 = lap - dV/dpsi
        accel = lap - dV_dpsi(psi)
        psi_next = 2*psi - psi_prev + dt**2 * accel
        psi_next[0] = psi_next[-1] = 0.0

        # Simple energy diagnostic (no boundaries term)
        kinetic = ((psi - psi_prev) / dt)**2 / 2.0
        grad = ((np.roll(psi, -1) - psi) / dz)**2 / 2.0
        grad[-1] = 0.0
        pot = potential(psi)
        E = np.trapz(kinetic + grad + pot, z)
        energies.append(E)

        # Track peaks for z_A, z_B (very rough: two largest maxima)
        idx_sorted = np.argsort(psi)[-2:]
        z_peaks = np.sort(z[idx_sorted])
        if len(z_peaks) == 2:
            traj_A.append(z_peaks[1])
            traj_B.append(z_peaks[0])
        else:
            traj_A.append(np.nan)
            traj_B.append(np.nan)

        psi_prev, psi = psi, psi_next

    return np.array(traj_A), np.array(traj_B), np.array(energies)

if __name__ == "__main__":
    z, psi0 = relax_single_soliton()
    zA_traj, zB_traj, E_t = evolve_binary(psi0, z)
    # At this point one can save trajectories and energy to disk
    np.savez("binary_soliton_data.npz", z=z, psi0=psi0,
             zA=zA_traj, zB=zB_traj, E=E_t)
```