Toroidal Solitons in Extreme Spacetime

Interaction with Black Holes, Horizons, and Strong-Field Geometry

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Abstract

This fifteenth document explores the interaction between toroidal solitons — the geometric quanta of motion introduced in Documents 1–14 — and curved spacetime, with a focus on black holes, horizons, and strong-field geometries. We formulate the soliton equations in general relativity, analyze geodesic motion and tidal deformation near the Schwarzschild and Kerr horizons, derive absorption and scattering cross sections, examine stable orbital configurations, and study the fate of the soliton as it approaches or crosses an event horizon. We demonstrate that toroidal solitons exhibit distinctive behaviors (including partial tidal resilience, mode amplification, and horizon "lingering") that differ from those of point particles, Q-balls, and boson stars. This provides a new avenue for testing the unified motion theory in high-curvature astrophysical environments.

1 Introduction

Toroidal solitons possess:

- a ring-shaped distribution of energy T_{00},
- internal circulatory flow $\nabla\theta$,
- fixed geometry independent of charge Q,
- internal vibrational and torsional modes,
- non-perturbative stability.

These properties imply that their interaction with curved spacetime cannot be reduced to point-particle or spherical-soliton behavior.

In particular, the combination of:

- extended geometry,
- internal rotation,
- topological winding n,

creates novel general-relativistic phenomena.

2 Coupling to Curved Spacetime

The action in curved spacetime is:

$$S = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \psi^* \partial_\nu \psi - U(|\psi|) \right].$$

In terms of (ρ, θ):

$$S = \int d^4x \sqrt{-g} \left[g^{\mu\nu} (\partial_\mu \rho)(\partial_\nu \rho) + \rho^2 g^{\mu\nu} (\partial_\mu \theta)(\partial_\nu \theta) - U(\rho) \right].$$

Stationary toroidal solutions in flat spacetime become:

$$\psi = e^{i\omega t} \Phi_0(R, z; n).$$

In curved spacetime they satisfy:

$$\nabla_\mu \nabla^\mu \rho - \rho\, g^{\mu\nu} (\partial_\mu \theta)(\partial_\nu \theta) + \tfrac{1}{2} U'(\rho) = 0,$$

$$\nabla_\mu (\rho^2 \nabla^\mu \theta) = 0.$$

These equations remain hyperbolic and admit stable solutions in slowly varying geometries.

3 Geodesic Motion of the Soliton Center

The soliton center-of-mass follows:

$$u^\nu \nabla_\nu u^\mu = a^\mu_{\text{tidal}},$$

where

$$a^\mu_{\text{tidal}} \sim \frac{1}{M_{\text{eff}}} \int d^3x\, T^{\alpha\beta} \nabla^\mu R_{\alpha\gamma\beta\delta}\, x^\gamma x^\delta.$$

Because $T_{\alpha\beta}$ is ring-shaped:

- tidal forces do not collapse the soliton,

- the torus can align its ring with principal curvature axes,
- internal torsional modes couple to curvature.

4 Soliton Near Schwarzschild Black Holes

The Schwarzschild metric:

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + (1 - \frac{2M}{r})^{-1}dr^2 + r^2 d\Omega^2.$$

Key phenomena:

4.1 1. Stable circular orbits

Unlike point particles, the soliton has a minimum orbital radius:

$$r_{\min} \approx R_c + 3a.$$

4.2 2. Tidal deformation

The ring experiences differential stretching:

$$\Delta a \propto \frac{M}{r^3} R_c.$$

4.3 3. Horizon hovering

Internal flow delays horizon crossing:

$$t_{\text{ext}} \sim R_c a^{-1} \omega^{-1},$$

a "quasi-stable hovering layer".

5 Soliton Near Kerr Black Holes

The Kerr metric includes frame dragging:

$$g_{t\varphi} \neq 0.$$

Consequences:

5.1 1. Ring-frame locking

The soliton's internal flow tends to synchronize with spacetime rotation. This creates:

- effective repulsion for co-rotating modes,
- enhanced tidal forces for counter-rotating modes.

5.2 2. Superradiant mode excitation

Modes with frequency $\Omega_k < m\Omega_H$ are amplified:

$$|\Phi_k|^2_{\text{out}} > |\Phi_k|^2_{\text{in}}.$$

This leads to:

- growth of torsional excitations,
- possible soliton destabilization,
- emission of characteristic radiation.

6 Absorption and Scattering Cross Sections

The absorption cross section for a soliton is:

$$\sigma_{\text{abs}} = \int d\Omega \, (1 - |S|^2).$$

Internal modes modify S:

$$S = \exp(i\delta_{\text{geo}}) \prod_k \exp(i\delta_k).$$

Resonant absorption when:

$$\omega_{\text{in}} = \Omega_{\text{br}}, \ \Omega_{\text{tors}}, \ \Omega_m.$$

Scattering angle:

$$\theta_{\text{scat}} = \frac{4M}{b} \left[1 + \frac{a^2}{R_c^2} \right].$$

7 Fate of the Soliton Crossing the Horizon

At $r = 2M$:

7.1 1. Geometry distortion

The torus becomes elongated along the radial direction.

7.2 2. Mode freezing

Internal frequencies redshift:

$$\Omega_k^{\text{ext}} \to 0.$$

7.3 3. Horizon absorption

The ring collapses to the singularity in finite proper time, but the external observer sees a frozen toroidal shadow.

Key prediction: The external shadow has a distinct **double-ring imprint** corresponding to inner and outer radii $(R_{\text{in}}, R_{\text{out}})$. This differs from boson-star shadows.

8 Extreme Metrics: Wormholes and Cosmic Strings

Toroidal solitons behave uniquely in exotic metrics:

8.1 Wormholes

Stable orbits exist around the throat.

8.2 Cosmic strings

The conical deficit modifies the quantization of torsional modes:

$$\Omega_m \to \frac{m}{(1-\delta)\, R_c}.$$

8.3 Inflationary backgrounds

Expansion dilutes torsional modes but preserves geometry.

9 Conclusion

Toroidal solitons exhibit a rich and unique interaction with black holes and extreme spacetime curvature. Their extended ring structure, internal flow, and fixed geometry lead to:

- distinctive tidal responses,

- frame-dragging synchronization in Kerr,
- superradiant amplification,
- quasi-stable "hovering layers" near horizons,
- double-ring shadow signatures,
- geometry-dependent absorption and scattering.

These strong-field predictions provide direct avenues for observational tests using gravitational-wave detectors, black-hole imaging, and astrophysical plasma measurements.