# Gravitational Quantization of Toroidal Solitons

Semiclassical Backreaction and Loop-Like Geometric Structures

Ivan Salines

*Independent Researcher*

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**Abstract**

This seventeenth document addresses the gravitational quantization of toroidal solitons within the unified motion-based theory. Building on the GR+QFT Effective Field Theory (Document 16), we develop a semiclassical scheme where the soliton's quantum stress tensor backreacts on the spacetime metric, and explore a loop-like description in which the torus is represented as a quantized ring of geometric flux. We introduce holonomy and area variables adapted to the toroidal geometry, discuss discrete spectra for effective geometric operators, and outline a bridge between the motion-based framework and loop-inspired approaches to quantum gravity.

## 1 Introduction

Toroidal solitons are extended, stable, and intrinsically geometric quanta of motion. They couple to gravity via their stress tensor $T_{\mu\nu}$ and admit a full quantum description of their internal degrees of freedom (Documents 13 and 16).

The next conceptual step is:

1. to incorporate the *quantum expectation value* $\langle T_{\mu\nu} \rangle$ in Einstein's equations (semi-classical gravity),
2. to identify loop-like geometric variables associated with the toroidal ring, in analogy with holonomies and fluxes in loop quantum gravity (LQG).

This document does not attempt a full quantum gravity theory; rather, it constructs the precise interface between the unified motion theory and gravitational quantization schemes.

## 2 Semiclassical Backreaction: $\langle T_{\mu\nu} \rangle$

In semiclassical gravity, one replaces:

$$G_{\mu\nu} = 8\pi G \, \langle \hat{T}_{\mu\nu} \rangle,$$

where the expectation value is taken in a suitable quantum state containing the toroidal soliton:

$$|\Psi\rangle = |\mathcal{T}_{Q,n}; \{n_k\}\rangle.$$

The stress tensor operator can be split as:

$$\hat{T}_{\mu\nu} = T^{(0)}_{\mu\nu} + \delta \hat{T}_{\mu\nu},$$

where:

- $T^{(0)}_{\mu\nu}$ is the classical soliton stress tensor evaluated on the background profile,
- $\delta \hat{T}_{\mu\nu}$ contains quantum fluctuations from internal modes.

The leading contribution:

$$\langle \hat{T}_{\mu\nu} \rangle \simeq T^{(0)}_{\mu\nu} + \sum_k \langle n_k | \delta \hat{T}^{(k)}_{\mu\nu} | n_k \rangle.$$

This yields a self-consistent system:

$$G_{\mu\nu}[g] = 8\pi G \, \langle \hat{T}_{\mu\nu}[g, \psi] \rangle, \tag{1}$$

$$\hat{\psi} \text{ satisfies the quantum field equations on } g_{\mu\nu}. \tag{2}$$

## 3 Backreacted Geometry of a Quantum Toroidal Soliton

For a stationary, isolated soliton, the metric can be approximated as:

$$ds^2 = -e^{2\Phi(R,z)} dt^2 + e^{2\Lambda(R,z)} (dR^2 + dz^2 + R^2 d\varphi^2),$$

with $(R, \varphi, z)$ cylindrical coordinates.

The Einstein equations with source $\langle \hat{T}_{\mu\nu} \rangle$ determine:

- the "ring mass" contribution,
- the modification of the toroidal cavity,

- quantum corrections from excited internal modes.

In the ground state ($n_k = 0$ for all $k$),

$$\langle \hat{T}_{\mu\nu} \rangle \approx T^{(0)}_{\mu\nu} + \frac{1}{2} \sum_k \langle 0 | \delta \hat{T}^{(k)}_{\mu\nu} | 0 \rangle,$$

producing a small Casimir-type correction.

# 4 Loop-Like Variables for the Toroidal Geometry

The toroidal soliton naturally defines a closed loop $\gamma$:

$$\gamma : \varphi \in [0, 2\pi) \mapsto x^\mu(\varphi),$$

located at radius $R_c$ and height $z = 0$.

In a loop-like approach, one associates:

- a *holonomy* along the loop,
- an *area* of the minimal surface spanned by $\gamma$,
- a *flux* of geometric or matter fields through that surface.

For a connection $A_\mu$ (gravitational or effective), define:

$$h_\gamma[A] = \mathcal{P} \exp \left( \oint_\gamma A_\mu dx^\mu \right).$$

In the present context, $A_\mu$ can be:

- a spin connection $\omega^{ab}_\mu$ (in tetrad formulation),
- an effective "motion connection" derived from $\nabla_\mu \theta$,
- a combination of both.

The toroidal soliton state can then be labeled by eigenvalues of holonomy and flux operators associated with $\gamma$.

# 5 Quantized Area and Volume Associated to the Torus

Define:

$$\mathcal{A}_\Sigma = \int_\Sigma \sqrt{h}\, d^2\sigma,$$

where $\Sigma$ is a surface spanning the ring, and $h$ is the induced metric.

Similarly, define the effective volume of the toroidal region:

$$\mathcal{V} = \int_{\mathcal{T}} \sqrt{\gamma}\, d^3x,$$

with $\mathcal{T}$ the region where $\rho$ is significantly nonzero.

In a loop-like quantization, one promotes these to operators $\hat{\mathcal{A}}_\Sigma$, $\hat{\mathcal{V}}$, whose spectra can become discrete when expressed in terms of fundamental geometric quanta.

The presence of a stable toroidal soliton suggests:

- a *preferred* quantum of area linked to $R_c$ and $a$,
- a discrete set of allowed volumes associated with different $Q, n$.

This motivates an interpretation of the soliton as a *geometric eigenstate* of area/volume operators.

# 6 Holonomy of the Internal Motion Connection

The internal phase $\theta$ defines a "motion connection":

$$\mathcal{A}_\mu = \partial_\mu \theta.$$

The holonomy around the torus is:

$$\oint_\gamma \mathcal{A}_\mu dx^\mu = \oint_\gamma \nabla\theta \cdot d\ell = 2\pi n.$$

This quantization condition survives in the quantum theory and is analogous to flux quantization in superconductors and to holonomy quantization in LQG.

Promoting $\theta$ to an operator, the holonomy becomes:

$$\hat{h}_\gamma = \exp\left(i\hat{n}2\pi\right),$$

with integer-valued $\hat{n}$.

Thus, the torus naturally realizes a loop-like quantum number.

# 7 Semiclassical Loop Picture: Hybrid Description

Combining:

- semiclassical gravity ($G_{\mu\nu} = 8\pi G\langle T_{\mu\nu}\rangle$),
- loop-like holonomies and geometric operators,

- quantum internal modes of the torus,

we obtain a hybrid picture:

1. the metric $g_{\mu\nu}$ is classical but corrected by $\langle T_{\mu\nu} \rangle$;
2. the torus is a quantum object with discrete internal spectrum;
3. the ring $\gamma$ carries quantized holonomies and fluxes;
4. area and volume associated to the torus admit discrete spectra.

This hybrid framework is consistent as long as curvature scales are larger than Planckian and the number of solitons is not enormous.

# 8 Towards a Fully Quantum Geometric Interpretation

The toroidal soliton suggests a deeper picture in which:

- *organized motion* is the microscopic origin of geometry,
- toroidal quanta are elementary "chunks" of quantized geometry,
- loop-like variables arise as emergent descriptors of stabilized motion patterns.

A speculative but natural conjecture is:

> Spacetime at the smallest scales may be composed of a dense network of interacting geometric quanta of motion, each similar in structure to the toroidal soliton, with loop-like holonomies encoding their connectivity.

Although beyond the scope of this document, this provides a conceptual bridge between the unified motion theory and loop-inspired quantum gravity.

# 9 Conclusion

We have outlined a two-layer approach to the gravitational quantization of toroidal solitons:

1. a *semiclassical* scheme where $\langle \hat{T}_{\mu\nu} \rangle$ sources Einstein's equations, providing a backreacted geometry of a quantum torus;
2. a *loop-like* geometric description in which the torus is associated with quantized holonomies, areas, volumes, and winding numbers.

This construction does not claim a full quantum gravity theory but establishes a precise and consistent interface between the motion-based framework and gravitational quantization programs. It strengthens the interpretation of the toroidal soliton as a genuine quantum of geometry and motion, with both matter-like and geometry-like attributes.