Quasi-Monte Carlo in ABC

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Ingredients for ABC

- summary statistic $S(y) : \mathbb{R}^n \to \mathbb{R}^d$
- kernel function $k(u_1, \dots, u_d)$
- bandwidth $h > 0$
- proposal density $g(\theta)$

ABC approximation

$$\pi_{ABC}(\theta|s_{obs}) \propto \pi(\theta) \int \pi(s|\theta) K((s - s_{obs})/h) ds \to \pi(\theta|s_{obs})$$

If $\pi(\theta|s_{obs}) \approx \pi(\theta|y)$, then ABC density is a proper approximation of the posterior $\pi(\theta|y)$.

ABC convergence rates

Consider the estimation of $\mu = E[a(\theta)|s_{obs}]$. The acceptance-rejection (AR) based ABC estimate is given by

$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} a(\theta^{(i)}).$$

Under regular conditions, ABC bias is

$$\text{bias} = |E_{ABC}[a(\theta)|s_{obs}] - \mu| = O(h^2),$$

and the acceptance probability is $R = O(h^{-d})$,

and Monte Carlo variance is

$$\text{variance} = \frac{\sigma^2_{ABC}}{N} = O\left(\frac{h^d}{C}\right),$$

where C is the complexity. For a given $C > 0$,

- the optimal $h^* = 0(C^{-1/(4+d)})$
- the optimal $\text{MSE} = O(C^{-4/(d+4)})$

This reveals that ABC suffers from the **curse of dimensionality**.

Quasi-Monte Carlo: A review