This is the new title

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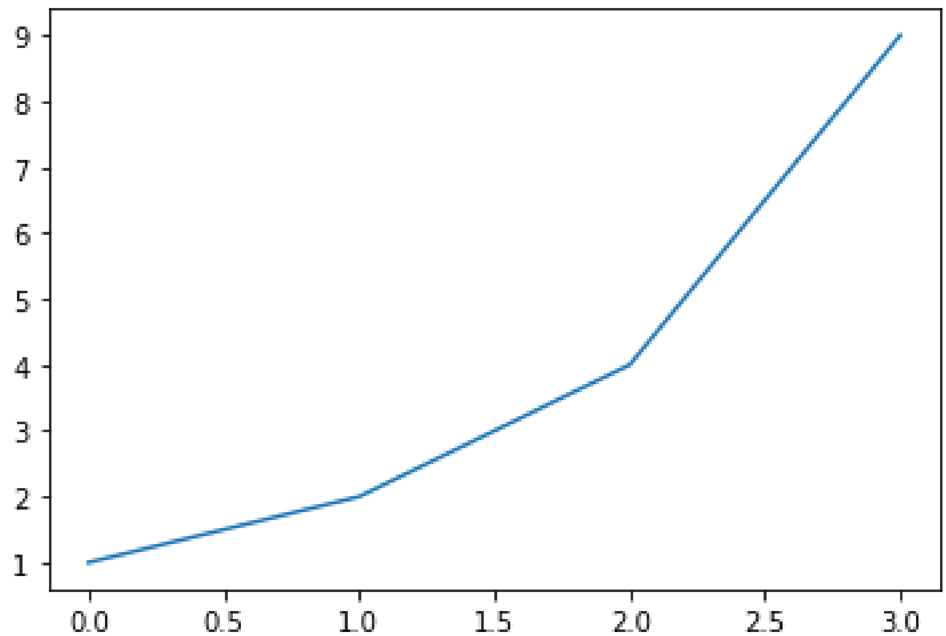
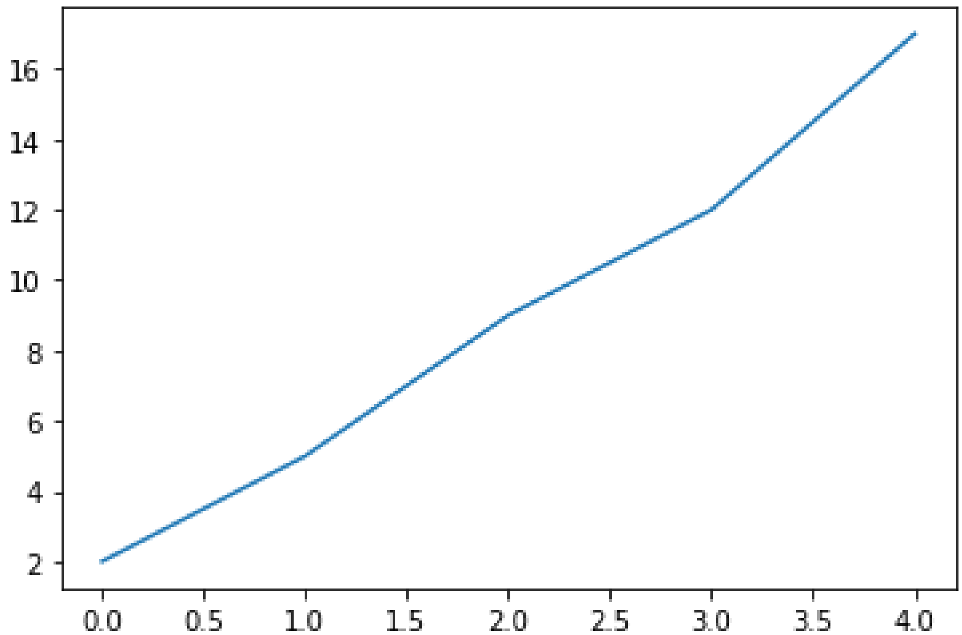




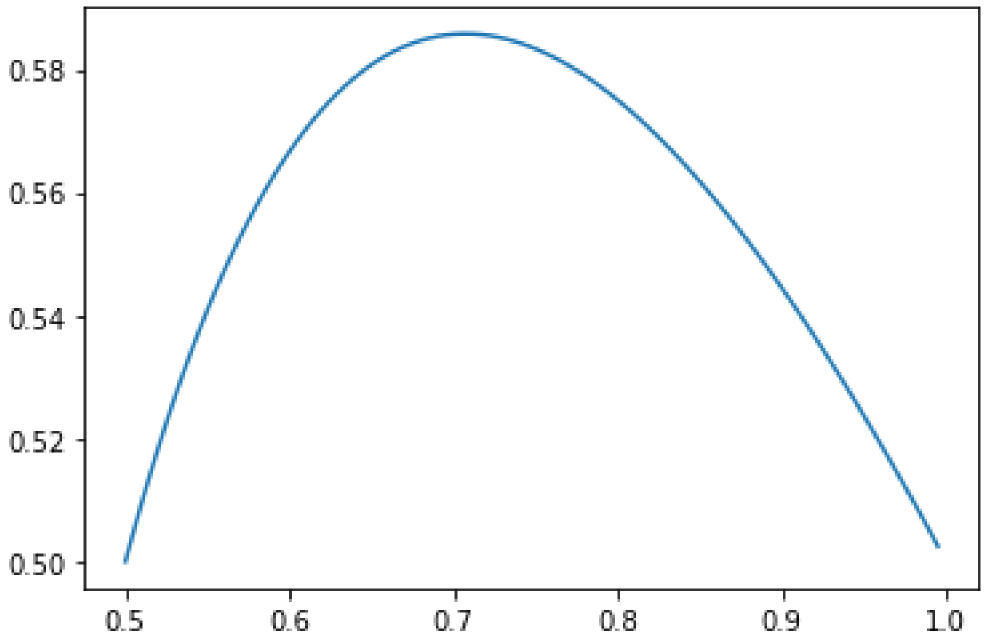
$$c = \sqrt{a^2 + b^2} \tag{1}$$

$$\nabla \times \vec{\mathbf{B}} - \frac{1}{c}\frac{\partial \vec{\mathbf{E}}}{\partial t} = \frac{4\pi}{c}\vec{\mathbf{j}} \tag{2}$$

$$\nabla \cdot \vec{\mathbf{E}} = 4\pi\rho \tag{3}$$

$$\nabla \times \vec{\mathbf{E}} + \frac{1}{c}\frac{\partial \vec{\mathbf{B}}}{\partial t} = \vec{\mathbf{0}} \tag{4}$$

$$\nabla \cdot \vec{\mathbf{B}} = 0 \tag{5}$$



Problem

25. For each real number a with $0 \leq a \leq 1$, let numbers x and y be chosen independently at random from the intervals $[0, a]$ and $[0, 1]$, respectively, and let $P(a)$ be the probability that

$$\sin^2(\pi x) + \sin^2(\pi y) > 1.$$

What is the maximum value of $P(a)$?

(A) $\frac{7}{12}$ **(B)** $2 - \sqrt{2}$ **(C)** $\frac{1+\sqrt{2}}{4}$ **(D)** $\frac{\sqrt{5}-1}{2}$ **(E)** $\frac{5}{8}$ $\boxed{}$

Answer

Let Prob{**e**} be the probability of an event **e** occurs. Thus $P(a)$ is:

$$P(a) = \mathrm{Prob}\{\sin^2(\pi \mathrm{x}) + \sin^2(\pi \mathrm{y}) > 1\}, \tag{6}$$

where x and y be chosen independently at uniformly random from the interval $[0, a]$ and $[0, 1]$, respectively.

Because,

$$\begin{aligned}
\sin^2(\pi x) + \sin^2(\pi y) > 1 &\Rightarrow \frac{1-\cos(2\pi x)}{2} + \frac{1-\cos(2\pi y)}{2} > 1 && \text{(pakai rumus } \cos(2\alpha)\text{)}\\
&\Rightarrow 2 - [\cos(2\pi x) + \cos(2\pi y)] > 2\\
&\Rightarrow -[\cos(2\pi x) + \cos(2\pi y)] > 0\\
&\Rightarrow \cos(2\pi x) + \cos(2\pi y) < 0\\
&\Rightarrow 2\cos(\pi(x+y)) \times \cos(\pi(x-y)) < 0 && \text{(pakai rumus } \cos\alpha + \cos\beta\text{)}\\
&\Rightarrow \cos(\pi(x+y)) \times \cos(\pi(x-y)) < 0
\end{aligned}$$

so, the above event is identical with:

$$P(a) = \mathrm{Prob}\{\cos(\pi(\mathrm{x}+\mathrm{y})) \times \cos(\pi(\mathrm{x}-\mathrm{y})) < 0\}. \tag{7}$$