これは日本語のタイトルですよ

Ivan Setiawan, イファン　スティアワン, 赤井　英和

February 19, 2020

1 Problem または日本語で、問題

- Try with Japanese Font
    - 日本語フォントで試してみます。
- タイトルはできるでしょうか。
    - サブタイトルは？不可
        - ホゲホゲ
        - Author, 著者は？
- 4 スペースタブ for 箇条書き

2 Solution

Let Prob{**e**} be the probability of an event **e** occurs. Thus $P(a)$ is:

$$P(a) = \mathrm{Prob}\{\sin^2(\pi \mathrm{x}) + \sin^2(\pi \mathrm{y}) > 1\}, \tag{1}$$

where x and y be chosen independently at uniformly random from the interval $[0, a]$ and $[0, 1]$, respectively.

横浜港に停泊したクルーズ船ダイヤモンド・プリンセス号については，厚労省の新着情報から「クルーズ船」で検索して集計した延べ人数は COVID-DP.csv のようになる。発表日ベースで集計した。検体採取の日は不明（NHK の 2 月 18 日のニュースによれば「結果が出るまでにおよそ 3 日かかる」）。どういう人を選んで検査したかによって陽性率は大きく変わるであろうから，要注意。NHK の 2020-02-15 22:48 のニュースによれば，「7 日以前にウイルスに感染し、　7 日に発症した乗客が最も多かった。その後の新たな発症者は、特に今月 10 日以降は急激に少なくなっていて、検疫の効果が出ていると考えている」（厚生労働省の担当者）とのことだが，これはよくわからない。

Because

$$\begin{aligned}
\sin^2(\pi x) + \sin^2(\pi y) > 1 &\Rightarrow \frac{1 - \cos(2\pi x)}{2} + \frac{1 - \cos(2\pi y)}{2} > 1 && \text{(pakai rumus } \cos(2\alpha)\text{)} \\
&\Rightarrow 2 - [\cos(2\pi x) + \cos(2\pi y)] > 2 \\
&\Rightarrow -[\cos(2\pi x) + \cos(2\pi y)] > 0 \\
&\Rightarrow \cos(2\pi x) + \cos(2\pi y) < 0 \\
&\Rightarrow 2\cos(\pi(x+y)) \times \cos(\pi(x-y)) < 0 && \text{(pakai rumus } \cos\alpha + \cos\beta\text{)} \\
&\Rightarrow \cos(\pi(x+y)) \times \cos(\pi(x-y)) < 0,
\end{aligned}$$

横浜港に停泊したクルーズ船ダイヤモンド・プリンセス号については，厚労省の新着情報から「クルーズ船」で検索して集計した延べ人数は COVID-DP.csv のようになる。発表日ベースで集計した。検体採取の日は不明（NHK の 2 月 18 日のニュースによれば「結果が出るまでにおよそ 3 日かかる」）。どういう人を選んで検査したかによって陽性率は大きく変わるであろうから，要注意。NHK の 2020-02-15 22:48 のニュースによれば，「7 日以前にウイルスに感染し、　7 日に発症した乗客が最も多かった。その後の新たな発症者は、特に今月 10 日以降は急激に少なくなっていて、検疫の効果が出ていると考えている」（厚生労働省の担当者）とのことだが，これはよくわからない。

then, the above event probabilty $P(a)$ in equation (1) is identical to:

$$P(a) = \text{Prob}\{\cos(\pi(\text{x}+\text{y})) \times \cos(\pi(\text{x}-\text{y})) < 0\}. \tag{2}$$

As the first step, we need to find the location of points (x, y) inside the given area, that satisfy $\cos(\pi(x+y)) \times \cos(\pi(x-y)) < 0$, that is:

$$\{(x,y) \mid (\cos(\pi(x+y)) \times \cos(\pi(x-y)) < 0) \cap (0 \le x \le a) \cap (0 \le y \le 1)\}. \tag{3}$$

There are 2 cases that satisfy $\cos(\pi(x+y)) \times \cos(\pi(x-y)) < 0$, that is, $\cos(\pi(x+y)) > 0$ AND $\cos(\pi(x-y)) < 0$ as the first case, OR $\cos(\pi(x+y)) < 0$ AND $\cos(\pi(x-y)) > 0$ as the second case. So, equation (3) can be found as the union of the above 2 cases, that is (all inside $((0 \le x \le a) \cap (0 \le y \le 1))$):

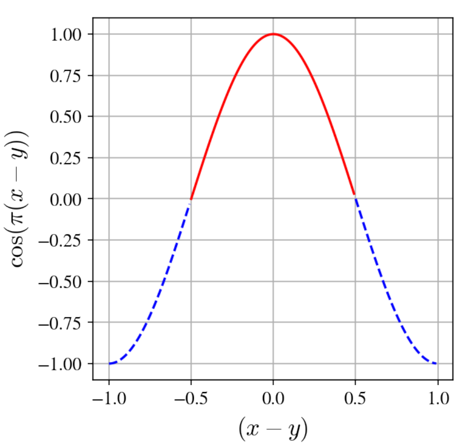
$$\begin{aligned}
\{(x,y) \mid (\cos(\pi(x+y)) \times \cos(\pi(x-y)) < 0)\} = & & (4)\\
\{(x,y) \mid (\cos(\pi(x+y)) > 0) \cap (\cos(\pi(x-y)) < 0)\} & \qquad \text{(case 1)} & (5)\\
\cup\{(x,y) \mid (\cos(\pi(x+y)) < 0) \cap (\cos(\pi(x-y)) > 0)\} & \qquad \text{(case 2)}. & (6)
\end{aligned}$$

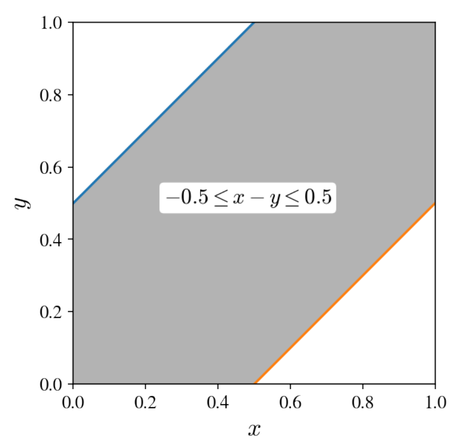
Keeping in mind that $x \in [0, a]$ (with $0 \le a \le 1$) and $y \in [0, 1]$, below we find the regions of (x, y) that satisfy case 1 and case 2.

In case 2, the former term of equation (6), that is, $\cos(\pi(x+y))$, will satisfy $(\cos(\pi(x+y)) < 0$ at the above-left red-solid-line cosine plot. So, the region of (x, y) is:

$$(0.5 \le (x+y) \le 1.5), \tag{7}$$

which is shown in the above-right figure.





Similarly in case 2, the later term of equation (6), that is, $\cos(\pi(x-y))$, will satisfy $(\cos(\pi(x-y)) > 0$ at the above-left red-solid-line cosine plot. So, the region of (x, y) is:

$$(-0.5 \leq (x-y) \leq 0.5), \tag{8}$$

which is shown in the above-right figure.

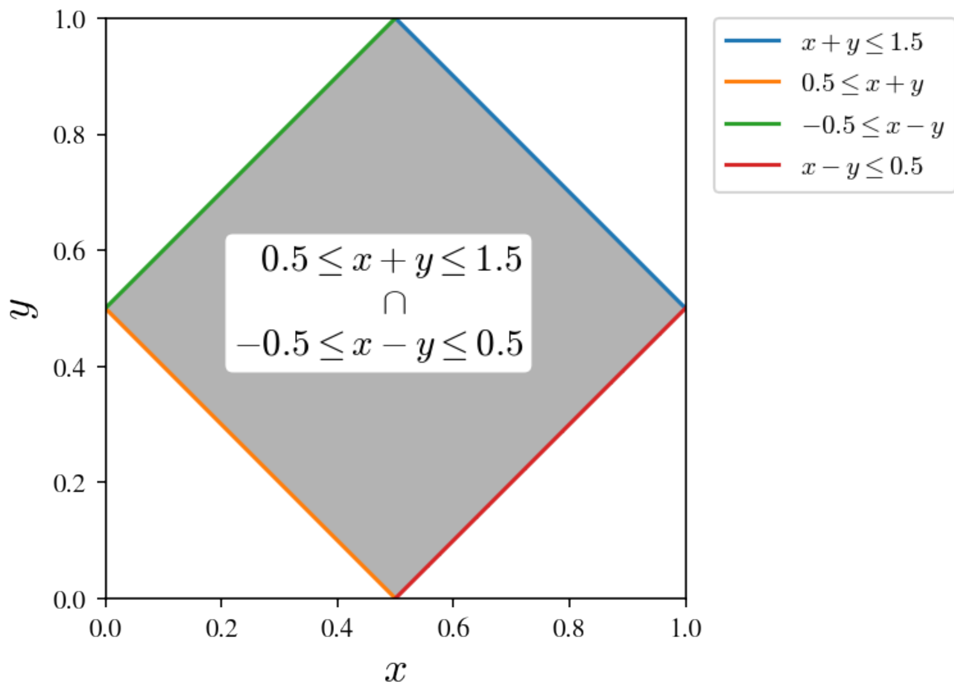
We see from the figures that the region

$$(0.5 \leq (x+y) \leq 1.5),$$

and the region

$$(-0.5 \leq (x-y) \leq 0.5),$$

do overlap, so the region that satisfy case 2 is:



Thus, the above region is the region of (x, y) which satisfy $\{\cos(\pi(x+y)) \times \cos(\pi(x-y)) < 0\}$:

$$\{(x, y) \mid (\cos(\pi(x+y)) \times \cos(\pi(x-y)) < 0) \cap (0 \leq x \leq 1) \cap (0 \leq y \leq 1)\}, \tag{9}$$

which is also the region of (x, y) which satisfy $\{\sin^2(\pi x) + \sin^2(\pi y) > 1\}$:

$$\{(x, y) \mid (\sin^2(\pi x) + \sin^2(\pi y) > 1) \cap (0 \leq x \leq 1) \cap (0 \leq y \leq 1)\}. \tag{10}$$

2.1 Formulation of function $P(a)$

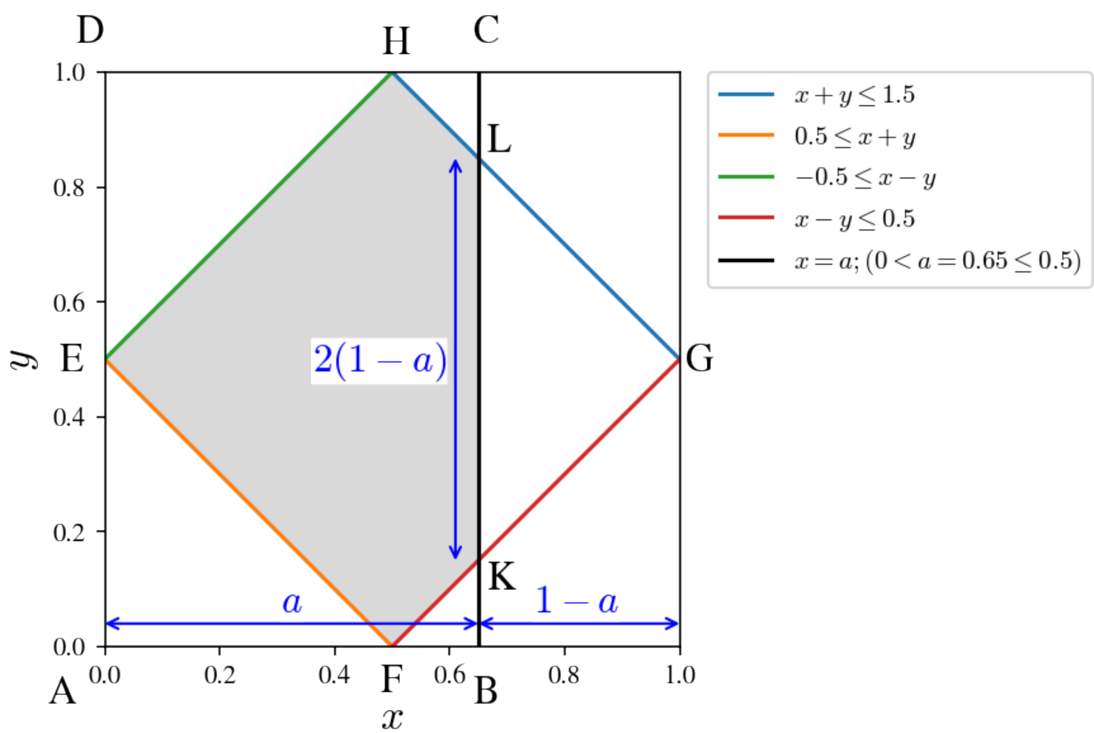
$P(a)$ is the probability of point (x, y) lies inside the above shown region with additional constraint $\{x \in [0, a]\}$, when x and y is chosen uniform-randomly independently from region $\{(x, y) \mid x \in [0, a], y \in [0, 1]\}$. This probability is the ratio of "the area of the above shown region with additional constraint $\{x \in [0, a]\}$" to "the area of the region $\{(x, y) \mid x \in [0, a], y \in [0, 1]\}$." Hence,

$$\begin{aligned} P(a) &= \frac{\text{Area of the above region } \cap \{0 \leq x \leq a\}}{\text{Area of the region } \{(x, y) \mid x \in [0, a], y \in [0, 1]\}} \\ &= \frac{\text{Area of the above region } \cap \{0 \leq x \leq a\}}{a}. \end{aligned}$$

From the above figure, when $0 < a \leq 0.5$, then $P(a)$ is:

$$\begin{aligned} P(a) &= \frac{\text{Area of the shaded region, that is, area of } \triangle\text{EFG}}{\text{Area of rectangle ABCD}} \\ &= \frac{\frac{1}{2} \times 2a \times a}{a \times 1.0} \\ &= a. \end{aligned} \tag{11}$$

2.1.1 Function $P(a)$ when $0.5 < a \leq 1.0$



From the above figure, when $0.5 < a \leq 1$, then $P(a)$ is:

$$\begin{aligned}
P(a) &= \frac{\text{Area of the shaded region, that is, area of } \square\text{EFGH} - \text{area of } \triangle\text{GLK}}{\text{Area of rectangle ABCD}} \\
&= \frac{2 \times (\text{area of } \triangle\text{EGH}) - \text{area of } \triangle\text{GLK}}{\text{Area of rectangle ABCD}} \\
&= \frac{2 \times (\frac{1}{2} \times 1.0 \times 0.5) - \frac{1}{2} \times 2(1-a) \times (1-a)}{a \times 1.0} \\
&= \frac{\frac{1}{2} - (1 - 2a + a^2)}{a} \\
&= \frac{-\frac{1}{2} + 2a - a^2}{a} \\
&= -a + 2 - \frac{1}{2a}. \qquad (12)
\end{aligned}$$

2.1.2 Final form of function $P(a)$

Noticing that $P(a = 0) = 0$ is satisfied in equation (11), and noticing that $P(a = 0.5) = 0.5$ is satisfied in both equation (11) and (12), then $P(a)$ can be expressed as a piecewise-defined function as follows:

$$P(a) = \begin{cases} a & 0 \leq a \leq 0.5 \\ -a + 2 - \frac{1}{2a} & 0.5 \leq a \leq 1 \end{cases} \qquad (13)$$

2.2 Maximum value of function $P(a)$ when $a \in [0, 1]$

Function $P(a)$ has following properties:

1. In the domain $\{a \mid 0 \leq a \leq 0.5\}$, $P(a)$ is linearly increasing with maximum value of 0.5 at $a = 0.5$.
2. At $a = 0.5$, the first derivative values of $P(a)$ from left and right are the same and take a positive value.
$$\lim_{a \to 0.5^-} P'(a) = \lim_{a \to 0.5^+} P'(a) = 1 > 0$$
3. In the domain $\{a \mid 0.5 \leq a \leq 1.0\}$, $P(0.5) = P(1.0) = 0.5$.

So, piecewise-defined function $P(a)$ takes maximum at the second piece (domain), i.e., $\{a \mid 0.5 \leq a \leq 1.0\}$.

The first derivative function (w.r.t a) of $P(a)$ in the domain $\{a \mid 0.5 \leq a \leq 1.0\}$ is:

$$P'(a) = -1 + \frac{1}{2a^2}. \qquad (14)$$

$P(a)$ takes extreme value when $P'(a) = 0$:

$$\begin{aligned} P'(a) = 0 &\Rightarrow -1 + \frac{1}{2a^2} = 0 \\ &\Rightarrow \frac{1}{2a^2} = 1 \Rightarrow a^2 = \frac{1}{2} \\ &\Rightarrow a = \frac{1}{\sqrt{2}}. \end{aligned}$$

We confirm that $a = \frac{1}{\sqrt{2}} \approx 0.71 \in [0.5, 1.0]$, and also:

$$P''(a) = -\frac{1}{a^3} \Rightarrow P''(a)\big|_{a=\frac{1}{\sqrt{2}}} = -2\sqrt{2} < 0,$$

so $P(a = \frac{1}{\sqrt{2}})$ is a maximum extreme value, with the maximum value of $P(a)$ is (from equation (12)):

$$P(a = \frac{1}{\sqrt{2}}) = -\frac{1}{\sqrt{2}} + 2 - \frac{1}{2 \times \frac{1}{\sqrt{2}}} = -\frac{\sqrt{2}}{2} + 2 - \frac{\sqrt{2}}{2} = 2 - \sqrt{2}, \tag{15}$$

which is B. ■

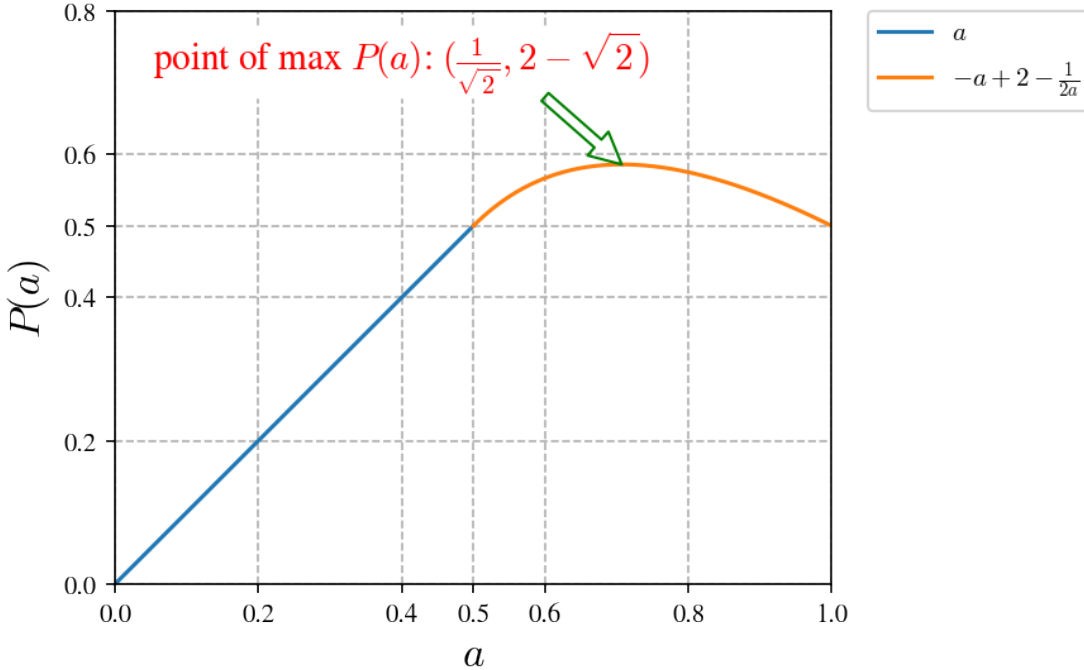
3 Bonus

3.1 $P(a)$ function plot

Plot of function $P(a)$ as in equation (13):

$$P(a) = \begin{cases} a & 0 \leq a \leq 0.5 \\ -a + 2 - \frac{1}{2a} & 0.5 \leq a \leq 1 \end{cases}$$

日本語はできるかな。できるみたいね。



3.2 Function $P(a)$ animated plot

[2]: <IPython.core.display.HTML object>