NA14 Regresija

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3. prosinca 2018.

1 Regresija

Regresija je provlačenje funkcije f koja ovisi o n parametara kroz točke (x_i, y_i), $i = 1, 2, \ldots, m$, pri čemu je $m > n$, tako da se *minimizira norma odstupanja*:

$$\|f(x_i) - y_i\|_{1,2,\infty} \to \min .$$

Regresija u *smislu najmanjih kvadrata* je

$$\|f(x_i) - y_i\|_2 \to \min .$$

Kada je funkcija f pravac,

$$f(x) = kx + l,$$

radi se o *linearnoj regresiji*. U tom slučaju dobije se sustav linearnih jednadžbi

$$kx_i + l = y_i, \quad i = 1, 2, \ldots, m.$$

Ukoliko sve točke *ne leže na istom pravcu*, sustav nije rješiv pa se računa kvadratična prilagodba.

1.1 Primjer

Provucimo pravac kroz točke (x_i, y_i), $i = 1, \ldots, m$, i izračunajmo kvalitetu prilagodbe.

```
In [1]: n=5
        x=collect([1,2,4,6,7])
        y=collect([1,3,2,4,3])
        A=[x ones(5)]
```

```
Out[1]: 5×2 Array{Float64,2}:
         1.0  1.0
         2.0  1.0
```

```
 4.0  1.0
 6.0  1.0
 7.0  1.0
```

In [2]:
```
# Koeficijenti
kl=A\y
```

Out[2]:
```
2-element Array{Float64,1}:
 0.30769230769230765
 1.3692307692307688
```

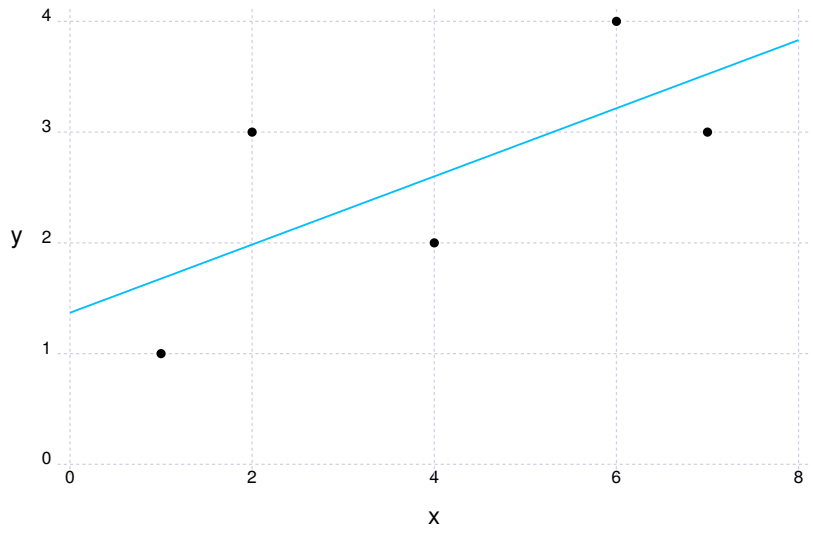
In [3]:
```
# Nacrtajmo točke i pravac
using Gadfly
```

In [4]:
```
xx=range(0,stop=8,length=100)
yy=kl[1].*xx.+kl[2]
```

Out[4]:
```
1.3692307692307688:0.02486402486402486:3.83076923076923
```

In [5]:
```
plot(layer(x=x,y=y),layer(x=xx,y=yy,Geom.line))
```

Out[5]:



In [6]:
```
# Kvaliteta prilagodbe
using LinearAlgebra
q=sqrt(norm(A*kl-y)/norm(y))
```

Out[6]:
```
0.5147667768205637
```

1.2 Primjer - Kvadratična regresija

Kroz točke možemo provući i kvadratni polinom $y = ax^2 + bx + c$. Ukoliko sve točke ne leže na istoj paraboli, sustav linearnih jednadžbi

$$ax_i^2 + bx_i + c = y_i, \quad i = 1, \ldots, m,$$

nije rješiv pa računamo kvadratičnu prilagodbu.

```
In [7]: n=5
        x=collect([1,2,4,5,6])
        y=collect([0,1,4,8,14])
        A=[x.^2 x ones(n)]
```

```
Out[7]: 5×3 Array{Float64,2}:
          1.0  1.0  1.0
          4.0  2.0  1.0
         16.0  4.0  1.0
         25.0  5.0  1.0
         36.0  6.0  1.0
```

```
In [8]: # Koeficijenti polinoma
        abc=A\y
```

```
Out[8]: 3-element Array{Float64,1}:
          0.6899350649350633
         -2.1607142857142745
          1.8636363636363449
```

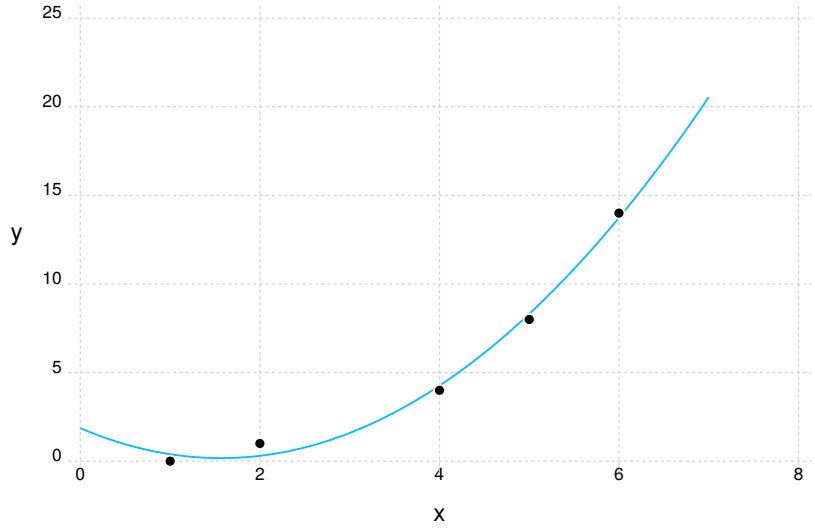
```
In [9]: # Nacrtajmo točke i parabolu
        xx=range(0,stop=7,length=100)
        yy=abc[1]*(xx.^2)+abc[2]*xx.+abc[3]
```

```
Out[9]: 100-element Array{Float64,1}:
          1.8636363636363449
          1.7143079092114726
          1.5718781014924095
          1.4363469404791553
          1.3077144261717106
          1.1859805585700753
          1.071145337674249
          0.963208763484232
          0.8621708360000242
          0.7680315552216255
          0.6807909211490362
          0.6004489337822561
```

```
 0.5270055931212854
 ⋮
15.130751964085285
15.588504419771626
16.053155522163774
16.524705271261727
17.0031536670655
17.488500709575078
17.980746398790455
18.479890734711653
18.985933717338664
19.498875346671472
20.01871562271009
20.545454545454525
```

In [10]:
```
plot(layer(x=x,y=y),layer(x=xx,y=yy,Geom.line))
```

Out[10]:



In [11]:
```
# Kvaliteta prilagodbe
q=sqrt(norm(A*abc-y)/norm(y))
```

Out[11]:
```
0.23696295042459756
```

1.3 Primjer - Rast svjetske populacije

Dosadašnji rast populacije (u milionima) da je u sljedećoj tablici (vidi http://en.wikipedia.org/wiki/World_population).

godina	1750	1800	1850	1900	1950	1999	2008	2010	2012
populacija (milijuni)	791	978	1262	1650	2521	5978	6707	6896	7052

Aproksimirajmo rast populacije eksponencijalnom funkcijom

$$P(t) = Ce^{kt}$$

i predvidimo populaciju 2050. godine.

Sustav jednadžbi

$$Ce^{kt_i} = P_i, \quad i = 1, 2, \ldots, 9$$

logaritmiranjem prelazi u sustav linearnih jednadžbi

$$k\,t_i + \ln C = \ln P_i.$$

Sve točke ne leže na istoj krivulji pa sustav nije rješiv i računamo kvadratičnu prilagodbu.

```
In [12]: n=9
         t=collect([1750,1800,1850,1900,1950,1999,2008,2010,2012])
         P=collect([791,978,1262,1650,2521,5978,6707,6896,7052])
         A= [t ones(9)]
         kC=A\log.(P)
```

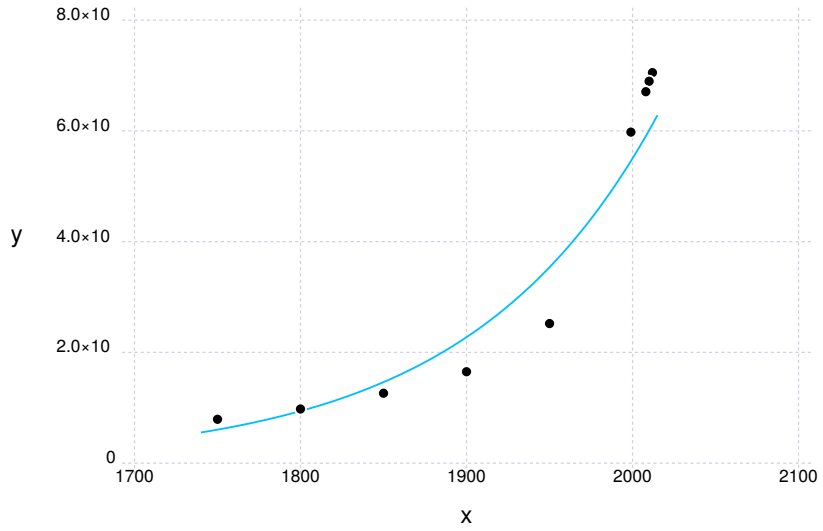
```
Out[12]: 2-element Array{Float64,1}:
           0.008834419929650809
          -9.055914228293918
```

```
In [13]: # Vrijednosti na krivulji
         Pv=exp.(kC[2]).*exp.(kC[1].*t)
```

```
Out[13]: 9-element Array{Float64,1}:
           604.4510245115797
           940.152075924994
          1462.2953556581206
          2274.427469700024
          3537.6029164763945
          5453.927176892081
          5905.273182327283
          6010.539738912149
          6117.682762104567
```

In [14]:
```
# Nacrtajmo točke i krivulju
tt=range(1740,stop=2015,length=200)
PP=exp.(kC[2]).*exp.(kC[1].*tt)
plot(layer(x=t,y=P),layer(x=tt,y=PP,Geom.line))
```

Out[14]:



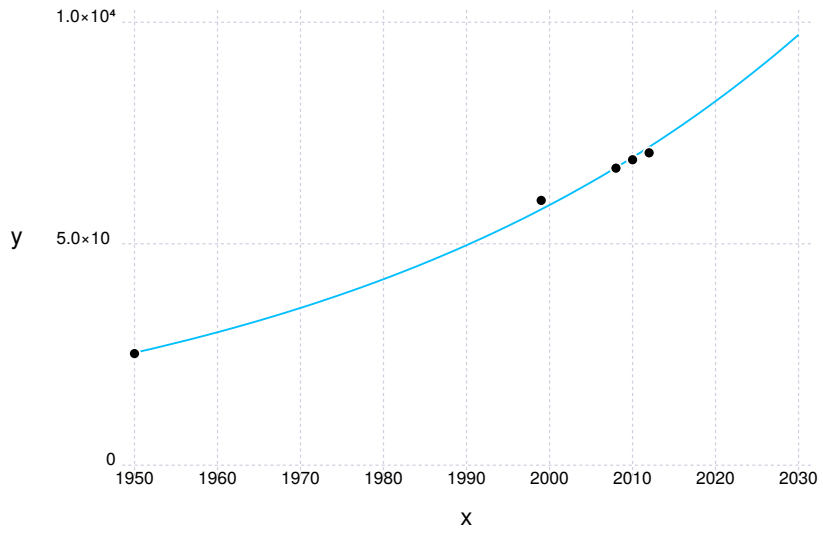
In [15]:
```
# predvidimo populaciju 2050 godine
P2050=exp(kC[2])*exp(kC[1]*2050)
```

Out[15]:
```
8558.212502172595
```

Izračunata predikcija je manja od one u tablici. Ako se ograničimo na razdoblje od 1950 godine imamo:

In [16]:
```
A= [t[5:end] ones(5)]
kC=A\log.(P[5:end])
tt=range(1950,stop=2030,200)
PP=exp.(kC[2]).*exp.(kC[1].*tt)
plot(layer(x=t[5:end],y=P[5:end]),layer(x=tt,y=PP,Geom.line))
```

Out[16]:




```
In [17]: P2050=exp(kC[2])*exp(kC[1]*2050)
```

```
Out[17]: 13592.0770645014
```