NA06 Pozitivno definitne matrice i rastav Choleskog

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1 Pozitivno definitne matrice i rastav Choleskog

Matrica A je *pozitivno definitna* ako je simetrična i ako su sve njene svojstvene vrijednosti pozitivne. Pozitivno definitnu matricu možemo rastaviti (bez pivotiranja) kao

$$A = LL^T$$

pri čemu je L donje trokutasta matrica s pozitivnim dijagonalnim elementima. Taj rastav se zove *rastav Choleskog* (vidi Numerička matematika, poglavlje 3.6).

```
In [1]: function mychol{T}(A1::Array{T})
            A=deepcopy(A1)
            n,m=size(A)
            for k=1:n
                A[k,k]=sqrt(A[k,k])
                for j=k+1:n
                    A[k,j]=A[k,j]/A[k,k]
                end
                for j=k+1:n
                    for i=k+1:n
                        A[i,j]=A[i,j]-A[k,i]*A[k,j]
                    end
                end
            end
            triu(A)
        end
```

```
Out[1]: mychol (generic function with 1 method)
```

```
In [2]: A=rand(6,6)
        A=A*A'
```

```
Out[2]: 6×6 Array{Float64,2}:
         1.0053    0.564041  0.155442  0.47471   0.920182  0.228271
```

```
 0.564041  1.96055   1.21705   1.38418   1.65399   1.57079
 0.155442  1.21705   1.16517   0.790639  1.06097   1.21644
 0.47471   1.38418   0.790639  1.27492   1.27403   1.06542
 0.920182  1.65399   1.06097   1.27403   1.73067   1.23123
 0.228271  1.57079   1.21644   1.06542   1.23123   1.52464
```

```
In [3]: L=chol(A)
```

```
Out[3]: 6×6 UpperTriangular{Float64,Array{Float64,2}}:
 1.00265  0.562553  0.155031   0.473457   0.917754   0.227669
  ·       1.28222   0.881156   0.871801   0.887292   1.12517
  ·        ·        0.6039    -0.0843774  0.226597   0.314122
  ·        ·         ·         0.532547   0.159782   0.00601989
  ·        ·         ·          ·         0.155671  -0.309648
  ·        ·         ·          ·          ·         0.110502
```

```
In [4]: L'*L-A
```

```
Out[4]: 6×6 Array{Float64,2}:
 0.0  0.0  0.0   0.0           0.0   0.0
 0.0  0.0  0.0   0.0           0.0   0.0
 0.0  0.0  0.0   0.0           0.0   0.0
 0.0  0.0  0.0   0.0           0.0  -2.22045e-16
 0.0  0.0  0.0   0.0           0.0   0.0
 0.0  0.0  0.0  -2.22045e-16   0.0   0.0
```

```
In [5]: L1=mychol(A)
```

```
Out[5]: 6×6 Array{Float64,2}:
 1.00265  0.562553  0.155031   0.473457   0.917754   0.227669
 0.0      1.28222   0.881156   0.871801   0.887292   1.12517
 0.0      0.0       0.6039    -0.0843774  0.226597   0.314122
 0.0      0.0       0.0        0.532547   0.159782   0.00601989
 0.0      0.0       0.0        0.0        0.155671  -0.309648
 0.0      0.0       0.0        0.0        0.0        0.110502
```

```
In [6]: L1'*L1-A
```

```
Out[6]: 6×6 Array{Float64,2}:
 0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0
```