SINGAPORE POLYTECHNIC
2021/2022 SEMESTER ONE EXAMINATION

Common Infocomm Technology Programme (CITP)
Diploma in Applied AI & Analytics (DAAA)
Diploma in Infocomm Security Management (DISM)
Diploma in Information Technology (DIT)

MS0105 – Mathematics

Time allowed: 2 hours

Instructions to Candidates

1. The SP examination rules are to be complied with.
Any candidate who cheats or attempts to cheat will face disciplinary action.

2. This paper consists of **8** printed pages (including the cover page and formula sheet).

3. This paper consists of three sections (100 marks in total):

Section A: 5 multiple-choice questions (10 marks)
Answer all questions behind the cover page of the answer booklet.

Section B: 7 structured questions (50 marks)
The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

Section C: 3 structured questions (40 marks)
Answer all questions.

4. Unless otherwise stated, all **non-exact** decimal answers should be rounded to **at least two** decimal places.

5. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to comply may result in loss of marks.

SECTION A (10 marks)

Answer ALL **FIVE** questions. Each question carries 2 marks.
Tick the choice of answer for each question in the box of the MCQ answer sheet provided in the answer booklet.

A1. If $\mathbf{R} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $\mathbf{I}$ is the 2×2 identity matrix, which of the following statements is always true for all $n \in \mathbb{N}$?

(a) $\mathbf{R}^n = \mathbf{I}$ (b) $\mathbf{R}^{2n} = \mathbf{I}$ (c) $\mathbf{R}^{4n} = \mathbf{I}$ (d) $\mathbf{R}^n \neq \mathbf{I}$

A2. What is the minimum number of bits needed to express the largest 6-digit octal number in its binary representation?

(Note: Octal numbers are also known as base-8 numbers.)

(a) 12 (b) 14 (c) 16 (d) 18

A3. Let p, q and r be different propositions.

A compound proposition P contains three types of logical operators – conjunction ($\wedge$), disjunction ($\vee$) and negation ($\neg$), as follows:

$$P = \neg\left(\neg\left(p \vee q\right) \wedge \neg r\right)$$

What is the minimum number of types of logical operators (among those stated above) needed to form another compound proposition Q that is logically equivalent to P?

(a) 0 (b) 1 (c) 2 (d) 3

A4. Two students are to be chosen as representatives from a class of 20 students. If there are 124 different ways to choose at least one female student as a representative, how many male students are there in the class?

(a) 8 (b) 10 (c) 12 (d) 14

A5. Jacob is known to tell the truth half of the time. He rolls a fair six-sided die and reports that the outcome is a ‘1’.

What is the probability that the outcome of the die roll is actually a ‘1’?

(Note: If Jacob were to lie, he would randomly pick one of the incorrect die roll outcomes with equal probability and report that outcome.)

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

<u>SECTION B</u> (50 marks)

Each question carries 10 marks. The total mark of all the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

B1. Let $\mathbf{A} = \begin{bmatrix} -3 & 2 \\ 4 & 7 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} -1 & 4 \\ 2 & 5 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 2 & k \\ -k & -1 \end{bmatrix}$.

(a) Compute the following:

(i) $2\mathbf{A} - 3\mathbf{B}$

(ii) $\mathbf{A}^T\mathbf{B}$

(6 marks)

(b) Find the value of k such that $\mathbf{BC}$ is a symmetric matrix.

(4 marks)

B2. **Solve this question using homogeneous coordinates.**

A square $\mathbf{S}$ with coordinates $(-1,1)$, $(1,1)$, $(1,3)$ and $(-1,3)$ undergoes the following sequence of transformations:

$\mathbf{T}_1$: shearing in the *y*-direction by a factor of –2, followed by

$\mathbf{T}_2$: translation 2 units to the right and 1 unit downwards.

(a) Write down the transformation matrices $\mathbf{T}_1$ and $\mathbf{T}_2$. Hence, compute the composite matrix $\mathbf{C}$ for the above sequence of transformations.

(4 marks)

(b) Find $\mathbf{S}'$, the image matrix of square $\mathbf{S}$ after undergoing the above sequence of transformations.

(2 marks)

(c) Write down the inverse transformation matrices $\mathbf{T}_1^{-1}$ and $\mathbf{T}_2^{-1}$. Hence, compute the composite matrix $\mathbf{C}^{-1}$ that transforms $\mathbf{S}'$ back to $\mathbf{S}$.

(4 marks)

B3. **Show your working clearly for this question.**

(a) Convert 11010011001.1001_2 to its decimal representation.

(3 marks)

(b) Convert 216.375_{10} to its binary and hexadecimal representations.

(7 marks)

B4. Let the universal set $U = \{x \in \mathbb{Z} | -3 \le x \le 6\}$ and define the following sets within U :

$$A = \{x \in \mathbb{N} | x \le 3\}$$

$$B = \{x \in \mathbb{Z} | 2x - 5 < 0\}$$

(a) Rewrite sets *U*, *A* and *B* by listing.

(3 marks)

(b) Find $\overline{A} \cap B$ and $A - \overline{B}$.

(4 marks)

(c) Draw a Venn diagram showing sets *U*, *A* and *B*, indicating all the elements clearly.

(3 marks)

B5. (a) Is the following statement a proposition? Justify your answer.

It rains once a week.

(2 marks)

(b) Let p and q be simple propositions.

(i) If the truth values of p and q are both false, determine the truth value of $p \vee (p \Leftrightarrow q)$.

(ii) By constructing a truth table, determine whether the compound proposition $((p \Rightarrow q) \wedge \neg q) \Rightarrow \neg p$ is a tautology or a contradiction.

(8 marks)

B6. In a beauty contest, 4 Asians, 4 Americans and 4 Europeans are to line up in a single file for a procession. How many ways can these people line up if

(a) there are no restrictions?

(2 marks)

(b) the first four people in line must be Asians?

(2 marks)

(c) the last six people in line must be Americans or Europeans?

(3 marks)

(d) all the Asians, all the Americans and all the Europeans must each line up together?

(3 marks)

B7. In a programming class, 60% of the students are proficient in Python, 80% of the students are proficient in JavaScript and 50% of the students are proficient in both Python and JavaScript.

What is the probability that a randomly selected student

(a) is proficient in Python or JavaScript?

(2 marks)

(b) is proficient in neither Python nor JavaScript?

(2 marks)

(c) is proficient in Python, given that the student is proficient in JavaScript?

(2 marks)

(d) is proficient in JavaScript, given that the student is **not** proficient in Python?

(4 marks)

<u>SECTION C</u> (40 marks)

Answer ALL **THREE** questions.

C1. Let $\mathbf{A} = \begin{bmatrix} 1 & -1 & 1 \\ -1 & -1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$.

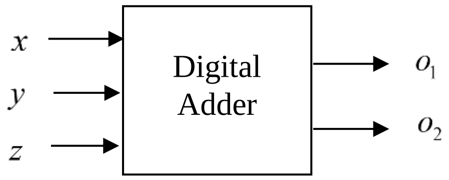
(a) If $\mathbf{A}^3 - \mathbf{A}^2 - \mathbf{A} = n\mathbf{I}$, where $\mathbf{I}$ is the 3×3 identity matrix, find the value of n.

(7 marks)

(b) Hence, find $\mathbf{A}^{-1}$.

(5 marks)

C2. Refer to the diagram below.



A digital adder takes in three binary inputs x, y and z and returns their binary sum as two binary outputs o_1 and o_2, where o_1 is the most significant bit of the sum and o_2 is the least significant bit of the sum.

For example, when $x=0$, $y=1$ and $z=1$, their binary sum will be $0+1+1=10_2$, thus $o_1=1$ and $o_2=0$.

(a) Construct a truth table for the digital adder, depicting all possible combinations of the inputs x, y and z with the outputs o_1 and o_2.

(5 marks)

(b) Derive the **product-of-sums** expression for o_1 and the **sum-of-products** expression for o_2. Do not simplify the expressions obtained.

(4 marks)

(c) The simplified Boolean expression for o_1 is given by

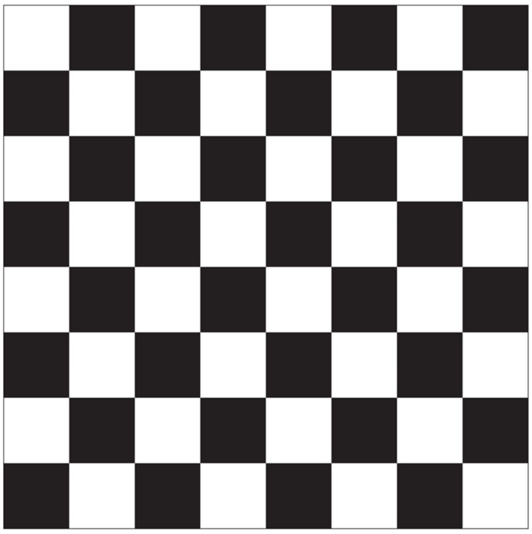
$$f(x,y,z) = x\,y + x\,z + y\,z\,.$$

Rewrite $f(x,y,z)$ using exactly **two** AND $(\bullet)$ operators, **one** OR $(+)$ operator and **one** XOR $(\oplus)$ operator.

(<u>Hint:</u> $x \oplus y = \bar{x}\,y + x\,\bar{y}$)

(5 marks)

C3. A regular chessboard contains 8×8 squares, as shown in the figure below. There are 32 black squares and 32 white squares in total.



(a) The *pawn* is the most basic and fundamental piece in the game of chess. Now, three pawns are randomly placed on three different squares of the chessboard. What is the probability that all three pawns are placed on squares of the same colour?

(3 marks)

(b) The *rook* is a chess piece that is able to attack another rook if they are both on the same row or column of the chessboard. Now, one rook is randomly placed on a white square and another rook is randomly placed on a black square of the chessboard. What is the probability that the two rooks are able to attack each other?

(4 marks)

(c) The *bishop* is a chess piece that is able to attack another bishop if they are both on the same diagonal of the chessboard. Now, two bishops are randomly placed on two different squares of the chessboard. What is the probability that the two bishops are able to attack each other?

(7 marks)

******* END OF PAPER *******

Formula Sheet

Transformation Matrices

Reflection	about the y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	about the x-axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	about the line $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Scaling	relative to the origin	$\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Shearing	in the x-direction	$\begin{bmatrix} 1 & s_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	in the y-direction	$\begin{bmatrix} 1 & 0 & 0 \\ s_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Rotation	about the origin	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Translation		$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}$

Boolean Algebra

Name	Identity
Commutative Laws	$x \bullet y = y \bullet x$ $x + y = y + x$
Associative Laws	$x \bullet (y \bullet z) = (x \bullet y) \bullet z = x \bullet y \bullet z$ $x + (y+z) = (x+y) + z = x + y + z$
Distributive Laws	$x \bullet (y+z) = (x \bullet y) + (x \bullet z)$ $x + (y \bullet z) = (x+y) \bullet (x+z)$
Identity Laws	$x \bullet 1 = x$ $x + 0 = x$
Complement Laws	$x \bullet \overline{x} = 0$ $x + \overline{x} = 1$
Involution Law	$\overline{\overline{x}} = x$
Idempotent Laws	$x \bullet x = x$ $x + x = x$
Bound Laws	$x \bullet 0 = 0$ $x + 1 = 1$
De Morgan's Laws	$\overline{x \bullet y} = \overline{x} + \overline{y}$ $\overline{x + y} = \overline{x} \bullet \overline{y}$
Absorption Laws	$x \bullet (x + y) = x$ $x + (x \bullet y) = x$ $x \bullet (\overline{x} + y) = x \bullet y$ $x + (\overline{x} \bullet y) = x + y$

Probability Rules

Addition	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Subtraction	$P(\overline{A}) = 1 - P(A)$
Multiplication	$P(A \cap B) = P(A)P(B)$ if A and B are independent events
Conditional Probability	$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$