

R08521609_handson5

April 14, 2021

#Single View Geometry ##Introduction The purpose of this lab is to practice single view metrology. Answer the following questions and explain your solution in detail. #Single view metrology Part1: For the below image from the NTU Liberal Arts building construction site, shown below, estimate the positions (in the image plane) of the three major vanishing points, corresponding to the building orientations. Use the pre-selected lines to solve for each vanishing point.

- Specify their vanishing points (u, v) .
- Plot the vanishing points and the vanishing lines used to estimate them on the image plane.
- Plot the ground horizon line and specify its parameters in the form $au + bv + c = 0$.

```
[1]: import numpy as np
import cv2
from google.colab.patches import cv2_imshow

# download the image
!gdown --id '1rXoS2vQBJcwaWy67vkftzrmNYvaqXbBW' --output fengyu.jpg
filename = 'fengyu.jpg'

# use openCV to read the original image
img = cv2.imread(filename)
cv2_imshow(img)
```

Downloading...

From: <https://drive.google.com/uc?id=1rXoS2vQBJcwaWy67vkftzrmNYvaqXbBW>

To: /content/fengyu.jpg

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this block shows the result of line segments plotted on the image, all you need to do here is to download the image and see the result

```
[2]: # download the image with line segments on it
!gdown --id '18p66e4l-JL_ZWlcyLEvwk6KdgrQzt0eE' --output fengyu_lines.jpg

# use openCV to read the image with line segments
lnSegImg = cv2.imread('fengyu_lines.jpg')
cv2_imshow(lnSegImg)
```

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step 1: download the segment points of nine line segments shown above and print them out

```
[3]: # download the segment points of nine line segments
import json

!gdown --id '1CvZMg3zhm3sdX92ydJhAHOVNPqpFtff' --output lineSegPts.json
jsonName = 'lineSegPts.json'
# create a list to store the segment points of nine line segments
cls_pts = []

# read the segment points of line segments from lineSegPts.json
with open(jsonName, 'r') as json_file:
    data = json.load(json_file)

# append three clusters of line segment points to the cls_pts
for i, cls in enumerate(data):
    cls_pts.append(data[cls])

# print out the content in lineSegPts.json
print("\nThree sets of segment points in cluster 1: \n", cls_pts[0])
print("\nThree sets of segment points in cluster 2: \n", cls_pts[1])
print("\nThe start point(a1,b1) and end point(a2,b2) of first line segment in_
→cluster 1: \n",cls_pts[0][0])
```

```
print("\nThe start point(a1,b1) and end point(a2,b2) of second line segment_
↪in cluster 3: \n",cls_pts[2][1])
```

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From: <https://drive.google.com/uc?id=1CvZMg3zhm3sdX92ydJhAHOVNPqpfFtff>

To: /content/lineSegPts.json

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Three sets of segment points in cluster 1:

```
[[[294.3813509900795, 107.95841141703897], [508.53601569973927,
194.49437797318717]], [[287.3885456118049, 174.39006251064768],
[505.9137136828862, 247.814518982531]], [[280.39574023353026,
238.19941158740346], [504.16551233831757, 300.2605593195905]]]
```

Three sets of segment points in cluster 2:

```
[[[949.0827545310392, 3.066330742919945], [518.1511230948668,
190.99797528404986]], [[964.816566632157, 91.3504986436368], [519.8993244394355,
248.68861965481528]], [[935.09714377449, 179.63466654435365],
[519.0252237671511, 301.13465999187486]]]
```

The start point(a1,b1) and end point(a2,b2) of first line segment in cluster 1:

```
[[294.3813509900795, 107.95841141703897], [508.53601569973927,
194.49437797318717]]
```

The start point(a1,b1) and end point(a2,b2) of second line segment in cluster 3:

```
[[426.37055250501265, 490.8145058775734], [433.3633578832873,
322.98717679898294]]
```

step 2: calculate vanishing points based on the pre-calculated line segment points

```
[4]: import math

# create an empty list for appending vanishing point
list_vp = []
# iterate through each cluster of line segments in order to estimate each
↪vanishing point
for n in range(len(cls_pts)):
    # create an empty list for adding slope and intercept of three line segments
    lines = []
    for i in range(3):

        # get the start point (a1,b1) and end point (a2,b2) of the line segment
        # start point (a1,b1)
        a1,b1 = cls_pts[n][i][0][0], cls_pts[n][i][0][1]
        # end point (a2,b2)
        a2,b2 = cls_pts[n][i][1][0], cls_pts[n][i][1][1]

        # determine the x and y interval of line segment
```

```

intv_x = [a1, a2]
intv_y = [b1, b2]

# calculate slope and intercept for each line segment
slope = (intv_y[1] - intv_y[0])/(intv_x[1] - intv_x[0])
intercept = intv_y[1] - slope*(intv_x[1])
lines.append([slope,intercept])

# find ang1 between the first and second line
ang1 = math.degrees(math.atan((lines[0][0] - lines[1][0])/(1+
↪lines[1][0]*lines[0][0])))
# find ang2 between the second and third line
ang2 = math.degrees(math.atan((lines[1][0] - lines[2][0])/(1+
↪lines[2][0]*lines[1][0])))

# calibrate ang1 and ang2,
# if angle is greater than 90 degree, it should be subtracted from 180
if ang1 > 90:
    ang1 = 180 - ang1
if ang2 > 90:
    ang2 = 180 - ang2

# compare ang1 and ang2
# the vanish point is determined based on two lines with smaller angle
if ang1 < ang2:
    x = (lines[0][1] - lines[1][1])/(lines[1][0] - lines[0][0])
    y = lines[1][0]*x + lines[1][1]
    vp = (x, y)
    list_vp.append(vp)

else:
    x = (lines[2][1] - lines[1][1])/(lines[1][0] - lines[2][0])
    y = lines[1][0]*x + lines[1][1]
    vp = (x, y)
    list_vp.append(vp)

# print out three vanishing points corresponds to three clusters
print('The VP coordinate of cluster 1 is: ', list_vp[0])
print('The VP coordinate of cluster 2 is: ', list_vp[1])
print('The VP coordinate of cluster 3 is: ', list_vp[2])

```

The VP coordinate of cluster 1 is: (1408.3050710184996, 551.0180150472971)
The VP coordinate of cluster 2 is: (-188.8724388178565, 499.33482277134294)
The VP coordinate of cluster 3 is: (516.4503127506019, -1671.0997400165634)

step 3: plot vanishing points, vanishing lines and ground horizon line on the original image


```

[5]: import matplotlib.pyplot as plt
import pylab # this allows you to control figure size
pylab.rcParams['figure.figsize'] = (25, 20) # this controls figure size in the
↳notebook

# convert bgr to rgb, and show the image via plt.imshow()
im_rgb = cv2.cvtColor(img, cv2.COLOR_BGR2RGB)
plt.imshow(im_rgb)

# plot vanishing points and vanishing lines on the original image
for n in range(len(cls_pts)):
    for i in range(3):

        # determine the x and y interval of ray from start point to vanishing point
        intv_x = [cls_pts[n][i][0][0], list_vp[n][0]]
        intv_y = [cls_pts[n][i][0][1], list_vp[n][1]]

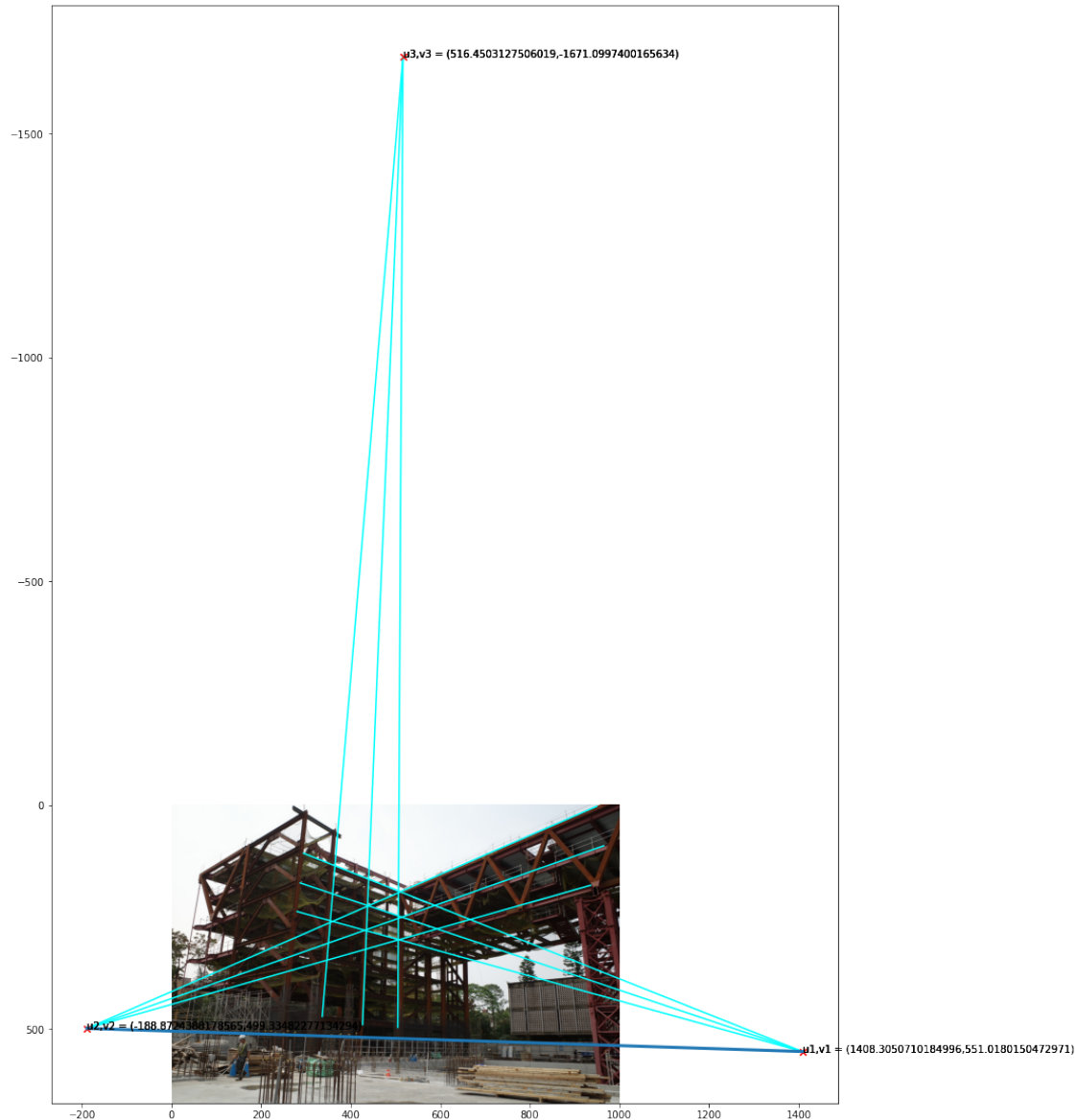
        # plot vanishing lines
        plt.plot(intv_x, intv_y, color='cyan')

        # plot vanishing points
        plt.plot(list_vp[n][0], list_vp[n][1], marker = 'x', color = "red")

        # plot each coordinates (u,v) on the original image
        plt.text(list_vp[n][0], list_vp[n][1], 'u{0},v{0} = ({1},{2})'.
↳format(n+1,list_vp[n][0],list_vp[n][1]))

# plot horizontal line on the original image
plt.plot([list_vp[0][0], list_vp[1][0]], [list_vp[0][1], list_vp[1][1]],
↳linewidth = 3)
plt.show()

```



step 4: specify ground horizon line parameters in the form $au + bv + c = 0$ using two different methods:

method I:

```
[6]: # calculate the difference x and y between vanishing point 1 and 2
x1, y1 = list_vp[0]
x2, y2 = list_vp[1]
a, b = (y1 - y2), (x1 - x2)

# calculate the slope of horizon line
slope = a/b
```

```

# calculate the intercept of horizon line
intercept = y1 - slope*(x1)

# the parameters of GHL equation
param = [a, b, intercept*b]

# normalize the parameter of equation
param_norm = [float(i)/sum(param) for i in param]

# print out the result of GHL
print("The equation is:", param_norm[0], "x - ", param_norm[1], "y + \u2192", param_norm[2], " = 0")

```

The equation is: 6.389027684752692e-05 x - 0.0019744158358725233 y + 0.99796169388728 = 0

method II:

The ground horizon line that passes through two vanishing points $vp1$ and $vp2$ can be also obtained via the cross product:

$$GHL = cross(vp1, vp2)$$

where the 2D vanishing point (u, v) need to be represented in homogeneous coordinates as a 3-vector:

$$vp = [u, v, 1]^T$$

```

[7]: #represent 2D vanishing point in homogeneous coordinates as a 3-vector:
x1, y1 = list_vp[0]
x2, y2 = list_vp[1]
vp1 = [x1, y1, 1]
vp2 = [x2, y2, 1]

#cross product between two 3-vectors: vp1 and vp2
param = np.cross(vp1, vp2)

# normalize the parameter of equation
param_norm = [float(i)/sum(param) for i in param]

# print out the result of GHL
print("The equation is:", param_norm[0], "x - ", param_norm[1], "y + \u2192", param_norm[2], " = 0")

```

The equation is: 6.414356900435008e-05 x - -0.0019822433813177784 y + 1.0019180998123134 = 0

step 5: Based on the vanishing points, calculate Camera's Focal Length and Optical Center:

Hint: The estimation of focal length and optical center can be realized based on the equation:

$$P_i = KRX_i$$

where P_i is the Vanishing Point i , K is the camera's intrinsic parameters matrix, and R is the Rotation Matrix.

For vanishing point, understanding that:

$$X_i^\top X_j = 0$$

According to the equations above, we can conclude that a system of 3 equations with 3 unknowns can be formed using all three calculated vanishing points:

$$\begin{aligned} p_1^\top K^{-\top} K^{-1} p_2 &= 0 \\ p_1^\top K^{-\top} K^{-1} p_3 &= 0 \\ p_2^\top K^{-\top} K^{-1} p_3 &= 0 \end{aligned}$$

where

$$K = \begin{pmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

where f is the focal length, and (u_0, v_0) is the optical center.

In order to reduce the computational complexity of calculating the 3 equations system, we can solely estimate focal length f using the equation below:

$$\begin{aligned} x_Q x_R + y_Q y_R + f^2 &= 0 \\ x_Q x_S + y_Q y_S + f^2 &= 0 \\ x_R x_S + y_R y_S + f^2 &= 0 \end{aligned}$$

More detailed about calculation of focal length and optical center is stated in B. Caprile and V. Torre (1990). "Using Vanishing Points for Camera Calibration." International Journal of Computer Vision, 4 (2), 127-140. The paper is provided as supplementary document for this assignment

```
[8]: # Estimation of focal length and optical center
# import sympy packages for solving equations
from sympy import *
import math

# get vanishing points
x1, y1 = list_vp[0]
x2, y2 = list_vp[1]
```

```

x3, y3 = list_vp[2]

# declare 3 sympy variables: f, u0, v0
f, u0, v0 = symbols(['f', 'u0', 'v0'])

# using sympy matrix to represent vanishing point(u,v) in homogeneous
↳ coordinates (u,v,1)
vp1 = Matrix([x1, y1, 1])
vp2 = Matrix([x2, y2, 1])
vp3 = Matrix([x3, y3, 1])

# calculate k, inv_k and trs_inv_k
k = Matrix([[f, 0, u0], [0, f, v0], [0, 0, 1]])
inv_k = k.inv() # inverse k
trs_inv_k = inv_k.T # transpose inverse k
print("K ={0}".format(k[:]))
print("inverse K ={0} ".format(inv_k[:]))
print("transpose inverse K ={0} \n".format(trs_inv_k[:]))

# 3 equations for solving 3 unknowns: f, u0, v0
eq1 = vp1.T * trs_inv_k * inv_k * vp2
eq2 = vp1.T * trs_inv_k * inv_k * vp3
eq3 = vp2.T * trs_inv_k * inv_k * vp3

# specify the system of 3 equations by Eq()
system = [
    Eq(eq1[0], 0),
    Eq(eq2[0], 0),
    Eq(eq3[0], 0)
]

# use solve() to solve algebraic equations
soln = solve(system, [f, u0, v0])
print("Camera's Focal Length = f = {0}".format(abs(soln[0][0])))
print("Camera's Optical Center = ({0}, {1})".format(soln[0][1], soln[0][2]))

```

```

K =[f, 0, u0, 0, f, v0, 0, 0, 1]
inverse K =[1/f, 0, -u0/f, 0, 1/f, -v0/f, 0, 0, 1]
transpose inverse K =[1/f, 0, 0, 0, 1/f, 0, -u0/f, -v0/f, 1]

```

```

Camera's Focal Length = f = 730.503173746431
Camera's Optical Center = (454.573762836756, 241.085431924555)

```

Part2 : Using the height of the worker as a reference, estimate the heights of the steel building, the reinforcing bar and the camera.

- This can be done with PowerPoint, paper and a ruler, or Python.
- Turn in an illustration that shows the ground horizon, and the lines and measurements used to estimate the heights, as well as the estimated heights.

- Given the height of worker is 170 centimeters.

Hint: Using **Cross Ratio method** to calculate the actual heights of the required objects

```
[9]: # download the image
!gdown --id '1z_PpPoKh2HG6IXov_SssH_80BlSAIne5' --output result.jpg
filename = 'result.jpg'

# use openCV to read the original image
img = cv2.imread(filename)
cv2_imshow(img)
```

Downloading...

From: https://drive.google.com/uc?id=1z_PpPoKh2HG6IXov_SssH_80BlSAIne5

To: /content/result.jpg

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0.1 Part II solution:

Known: worker height: 1.7 m

Measured: based on Cross Ratio method

- **Reinforcing bar:**
 - Worker height at Pos2: standard worker height = measured reinforcing bar height : real reinforcing bar height
 - $0.0065 : 1.7 = 0.022 : x_1$ $x_1 = 5.75(m)$
- **Steel building:**
 - Worker height at Pos3: standard worker height = measured steel building height : real steel building height
 - $0.0055 : 1.7 = 0.044 : x_2$ $x_2 = 13.6 (m)$
- **Camera height:**

- Camera height = the intersection of ground horizon line and steel building height
- $13.6 * 0.0025 / 0.044 = 0.773$ (m)

Results: real reinforcing bar height = 5.754 (m) real steel building height = 13.6 (m) camera height = 0.773 (m)