R08521609_handson9

May 13, 2021

1 Feedforward Neural Network

1.1 Single-layer Perceptron

A single layer perceptron predicts a binary label $\hat{y}$ for a given input vector $\boldsymbol{x} \in \mathbb{R}^d$ (d presents the number of dimensions of inputs) by using the following formula,

$$\hat{y} = \begin{cases} 1 & \text{if } \boldsymbol{w} \cdot \boldsymbol{x} + b > 0 \\ 0 & \text{else} \end{cases}$$

[ ]:
```
!wget -nc https://raw.githubusercontent.com/brpy/colab-pdf/master/colab_pdf.py
from colab_pdf import colab_pdf
colab_pdf('0513/R08521609_handson9.ipynb')
```

```
--2021-05-13 08:13:34--  https://raw.githubusercontent.com/brpy/colab-
pdf/master/colab_pdf.py
Resolving raw.githubusercontent.com (raw.githubusercontent.com)...
185.199.108.133, 185.199.109.133, 185.199.110.133, ...
Connecting to raw.githubusercontent.com
(raw.githubusercontent.com)|185.199.108.133|:443... connected.
HTTP request sent, awaiting response... 200 OK
Length: 1865 (1.8K) [text/plain]
Saving to: colab_pdf.py

colab_pdf.py        100%[===================>]   1.82K  --.-KB/s    in 0s

2021-05-13 08:13:34 (26.4 MB/s) - colab_pdf.py saved [1865/1865]

Mounted at /content/drive/

WARNING: apt does not have a stable CLI interface. Use with caution in scripts.


WARNING: apt does not have a stable CLI interface. Use with caution in scripts.

Extracting templates from packages: 100%
```

[48]:
```
!gdown --id '1TpMVIPzhCRVkl3HD_QgXbIAf3fL698Ul' --output data.xlsx
```

```
Downloading...
From: https://drive.google.com/uc?id=1TpMVIPzhCRVkl3HD_QgXbIAf3fL698Ul
To: /content/data.xlsx
100% 10.7k/10.7k [00:00<00:00, 10.1MB/s]
```

[49]:
```
# import some useful package
%matplotlib inline
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
from tqdm.notebook import tqdm
```

[50]:
```
# read the Excel file
df = pd.read_excel("data.xlsx")

# select the list where we are interested.
# <hint>: you need to change the format into float for the later calculation
df = df.astype("float64")

# change the list to NumPy array
df_arr = df.to_numpy()

# print the NumPy array
print(df_arr)
```

```
[[ 1.  4. 35. 35.]
 [ 1.  2. 27. 19.]
 [ 1.  4. 36. 37.]
 [ 1.  4. 35. 38.]
 [ 1.  4. 14. 25.]
 [ 1.  3. 35. 32.]
 [ 1.  3. 31. 18.]
 [ 1.  3. 36. 31.]
 [ 1.  3. 35. 27.]
 [ 1.  3. 36. 29.]
 [ 1.  3. 35. 33.]
 [ 1.  3. 30. 28.]
 [ 1.  3. 32. 23.]
 [ 1.  3. 32. 36.]
 [ 1.  3. 36. 27.]
 [ 1.  3. 35. 26.]
 [ 1.  3. 28. 28.]
 [ 1.  3. 37. 25.]
 [ 1.  3. 31. 23.]
 [ 1.  3. 34. 26.]
```

```
[ 1.  3. 38. 28.]
[ 1.  3. 36. 31.]
[ 1.  3. 38. 33.]
[ 1.  3. 36. 32.]
[ 1.  3. 19. 18.]
[ 0.  1.  7.  7.]
[ 0.  1.  3.  3.]
[ 0.  1.  2.  2.]
[ 0.  1.  2.  2.]
[ 0.  1.  2.  2.]
[ 0.  1.  2.  2.]
[ 0.  1.  2.  2.]
[ 0.  1.  2.  2.]
[ 0.  1.  1.  1.]
[ 0.  1.  1.  1.]
[ 0.  1.  4.  5.]
[ 0.  1.  7.  7.]
[ 0.  1.  9.  9.]
[ 0.  1. 10. 10.]
[ 0.  1. 10. 10.]
[ 0.  1. 10. 10.]
[ 0.  1. 11. 12.]
[ 0.  1. 12. 12.]
[ 0.  1. 12. 12.]
[ 0.  1. 12. 12.]
[ 0.  1. 12. 12.]
[ 0.  1. 13. 13.]
[ 0.  1. 13. 13.]
[ 0.  1. 13. 13.]
[ 0.  1. 13. 13.]]
```

Taka a look at the equation, $\boldsymbol{w}$ is a weight vector; b is a bias weight; and $g(.)$ denotes a Heaviside step function (we assume $g(0) = 0$).

$$\hat{y} = g(\boldsymbol{w} \cdot \boldsymbol{x} + b) = g(w_1 x_1 + w_2 x_2 + ... + w_d x_d + b)$$

In order to train a weight vector and bias weight in a unified code, we include a bias term as an additional dimension to inputs. More concretely, we append 1 to each input, Then, the formula of the single-layer perceptron becomes,

$$\hat{y} = g((w_1, w_2, \ldots, w_n) \cdot \boldsymbol{x}') = g(w_1 x_1 + w_2 x_2 + \ldots + w_n)$$

In other words, w_1 and w_2 present weights for x_1 and x_2, respectively, and w_n does a bias weight.

1.2 Steps

1. Initialize the weights and the threshold. Weights may be initialized to 0 or to a small random value. In the example below, we use 0

2. For each example i, perform the following steps over the input x_i and desired output o :

- Calculate the actual output:

$$\hat{y} = g(w_1 x_1 + w_2 x_2 + \ldots + w_n)$$

- Update the weights:

$$w_i(t+1) = w_i(t) + \eta \cdot (o - \hat{y}(t)) x_i$$

η means learning rate. In the example below, we use 0.5 And the steps of the training is set as a fixed number of iterations (10000 times)

[51]:
```
# Data setting
b = np.ones(df_arr.shape[0])
x = np.column_stack((df_arr[:,1:],b.T))
y = df_arr[:,0]
w = np.zeros(df_arr.shape[1])

# Hyperparameter setting
eta = 0.5
step = 10000

# Training loop
for t in tqdm(range(step)):
    for i in range(len(y)):
        y_pred = np.heaviside(np.dot(x[i], w), 0)
        w += eta * (y[i] - y_pred) * x[i]
```

```
HBox(children=(FloatProgress(value=0.0, max=10000.0), HTML(value='')))
```

[52]:
```
# To see the value of weight vector
w
```

[52]:
```
array([ -30. ,    4.5,    7.5, -128. ])
```

[53]:
```
# To see the prediction
np.heaviside(np.dot(x, w), 0)
```

[53]:
```
array([1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1.,
       1., 1., 1., 1., 1., 1., 1., 1., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.])
```

1.3 Single-layer Perceptron with batch

In order to reduce the execusion run by the Python interpreter, which is relatively slow. The common technique to speed up a machine-learning code written in Python is to to execute computations within the matrix library (e.g., numpy). The single-layer perceptron makes predictions for four inputs,

$$\hat{y}_1 = g(\boldsymbol{x}_1 \cdot \boldsymbol{w}) \hat{y}_2 = g(\boldsymbol{x}_2 \cdot \boldsymbol{w}) \vdots \hat{y}_n = g(\boldsymbol{x}_n \cdot \boldsymbol{w})$$

Here, we define $\hat{Y} \in \mathbb{R}^{n \times 1}$ and $X \in \mathbb{R}^{n \times d}$ as,

$$\hat{Y} = \begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{pmatrix}, X = \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_n \end{pmatrix}$$

Then, we can write the all predictions in one dot-product computation,

$$\hat{Y} = X \cdot \boldsymbol{w}$$

[54]:
```
# Data setting
b = np.ones(df_arr.shape[0])
x = np.column_stack((df_arr[:,1:],b.T))
y = df_arr[:,0]
w = np.zeros(df_arr.shape[1])

# Training loop
# <hint>steps are very similar to above one. The only difference is that you␣
↪don't need to calculate one by one.
for t in tqdm(range(step)):
    y_pred = np.heaviside(np.dot(x, w), 0)
    w += eta * np.dot((y - y_pred), x)
```

```
HBox(children=(FloatProgress(value=0.0, max=10000.0), HTML(value='')))
```

[55]:
```
# To see the value of weight vector
w
```

[55]:
```
array([  256. ,   148. ,   -55.5, -1679.5])
```

[56]:
```
# To see the prediction
np.heaviside(np.dot(x, w), 0)
```

[56]:
```
array([1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1.,
       1., 1., 1., 1., 1., 1., 1., 1., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.])
```

1.4 Single-layer Perceptron with different activation function

1.4.1 ReLU Function

The most popular choice, due to both simplicity of implementation and its good performance on a variety of predictive tasks, is the *rectified linear unit* (*ReLU*). [**ReLU provides a very simple**

nonlinear transformation]. Given an element x, the function is defined as the maximum of that element and 0:

$$\mathrm{ReLU}(x) = \max(x, 0).$$

sigmoid function [**The *sigmoid function* transforms its inputs**], for which values lie in the domain $\mathbb{R}$, (**to outputs that lie on the interval (0, 1).**) For that reason, the sigmoid is often called a *squashing function*: it squashes any input in the range (-inf, inf) to some value in the range (0, 1):

$$\mathrm{sigmoid}(x) = \frac{1}{1 + \exp(-x)}.$$

[57]:
```
# define the activation function
def ReLU(x):
    return np.maximum(0, x)
def sigmoid(x):
    return 1.0 / (1 + np.exp(-x))
```

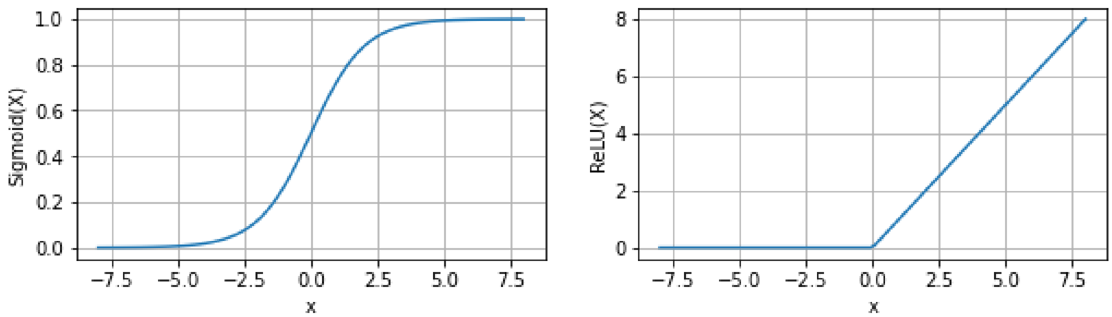
[58]:
```
# Plot the two activation functions with x from -8 to 8
# <hint>some useful funtions in plt: plot, subplot, xlabel, ylebel, grid,␣
↪figsize

# Import matplotlib, numpy and math
import matplotlib.pyplot as plt
import numpy as np
import math

x = np.linspace(-8, 8, 100)
z = sigmoid(x)
r = ReLU(x)

plt.figure(figsize = (10, 2.5))
plt.subplot(1, 2, 1)
plt.plot(x, z)
plt.xlabel("x")
plt.ylabel("Sigmoid(X)")
plt.grid()

plt.subplot(1, 2, 2)
plt.plot(x, r)
plt.xlabel("x")
plt.ylabel("ReLU(X)")
plt.grid()
plt.show()
```



1.5 Single-layer Perceptron with ReLU

[59]:
```
# Training data

# Data setting
b = np.ones(df_arr.shape[0])
x = np.column_stack((df_arr[:,1:],b.T))
y = df_arr[:,0]
w = np.zeros(df_arr.shape[1])

# Training loop
for t in tqdm(range(step)):
    y_pred = ReLU(np.dot(x, w))
    w += eta * np.dot((y - y_pred), x)
```

```
HBox(children=(FloatProgress(value=0.0, max=10000.0), HTML(value='')))
```

[60]:
```
# To see the value of weight vector
w
```

[60]:
```
array([ -602944.5 , -6480401.5 , -5735531.75,  -240394.5 ])
```

[61]:
```
# To see the prediction
ReLU(np.dot(x, w))
```

[61]:
```
array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
       0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.])
```

1.6 Single-layer Perceptron with sigmoid

maybe you will meet the warning --RuntimeWarning: overflow encountered in exp--

It is because of the calculation with the exponential function. It is OK if you don't solve this problem and the code still works.

[70]:
```
# Training data
b = np.ones(df_arr.shape[0])
x = np.column_stack((df_arr[:,1:],b.T))
y = df_arr[:,0]
w = np.zeros(x.shape[1])


for t in tqdm(range(step)):
    y_pred = sigmoid(np.dot(x, w))
    w += np.dot((y - y_pred), x)
```

```
HBox(children=(FloatProgress(value=0.0, max=10000.0), HTML(value='')))
```

```
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:5: RuntimeWarning:
overflow encountered in exp
  """
```

[71]:
```
# To see the value of weight vector
w
```

[71]:
```
array([  450.63847531,   295.043629  ,  -106.97020591, -3164.3296378 ])
```

[72]:
```
# To see the prediction
sigmoid(np.dot(x, w))
```

```
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:5: RuntimeWarning:
overflow encountered in exp
  """
```

[72]:
```
array([1.00000000e+000, 1.00000000e+000, 1.00000000e+000, 1.00000000e+000,
       1.00000000e+000, 1.00000000e+000, 1.00000000e+000, 1.00000000e+000,
       1.00000000e+000, 1.00000000e+000, 1.00000000e+000, 1.00000000e+000,
       1.00000000e+000, 1.00000000e+000, 1.00000000e+000, 1.00000000e+000,
       1.00000000e+000, 1.00000000e+000, 1.00000000e+000, 1.00000000e+000,
       1.00000000e+000, 1.00000000e+000, 1.00000000e+000, 1.00000000e+000,
       1.00000000e+000, 0.00000000e+000, 0.00000000e+000, 0.00000000e+000,
       0.00000000e+000, 0.00000000e+000, 0.00000000e+000, 0.00000000e+000,
       0.00000000e+000, 0.00000000e+000, 0.00000000e+000, 0.00000000e+000,
       0.00000000e+000, 0.00000000e+000, 0.00000000e+000, 0.00000000e+000,
       0.00000000e+000, 0.00000000e+000, 4.07287064e-199, 4.07287064e-199,
       4.07287064e-199, 4.07287064e-199, 1.94603415e-117, 1.94603415e-117,
       1.94603415e-117, 1.94603415e-117])
```

1.7