

# Problem 1: Support Vector Machines

## Instructions:

1. Please use this q1.ipynb file to complete SVM using cvxopt
2. You may create new cells for discussions or visualizations

```
In [1]: # Import modules
import numpy as np
import matplotlib.pyplot as plt
from cvxopt import matrix, solvers
```

### a): Linearly Separable Dataset

```
In [64]: data = np.loadtxt('clean_lin.txt', delimiter='\t')
x = data[:, 0:2]
y = data[:, 2]

""" Assign color per label, then scatter plot dataset """
pred_color = {1: 'g', -1: 'm'}
colors = [pred_color[i] for i in y]
plt.figure(figsize = (7.5, 7.5))
plt.scatter(x[:, 0], x[:, 1], c=colors)

""" Modifying y to work with NumPy broadcasting,
concatenating a stack of ones onto x for bias term """
y = y[:, np.newaxis]
x = np.hstack((x, np.ones_like(y)))

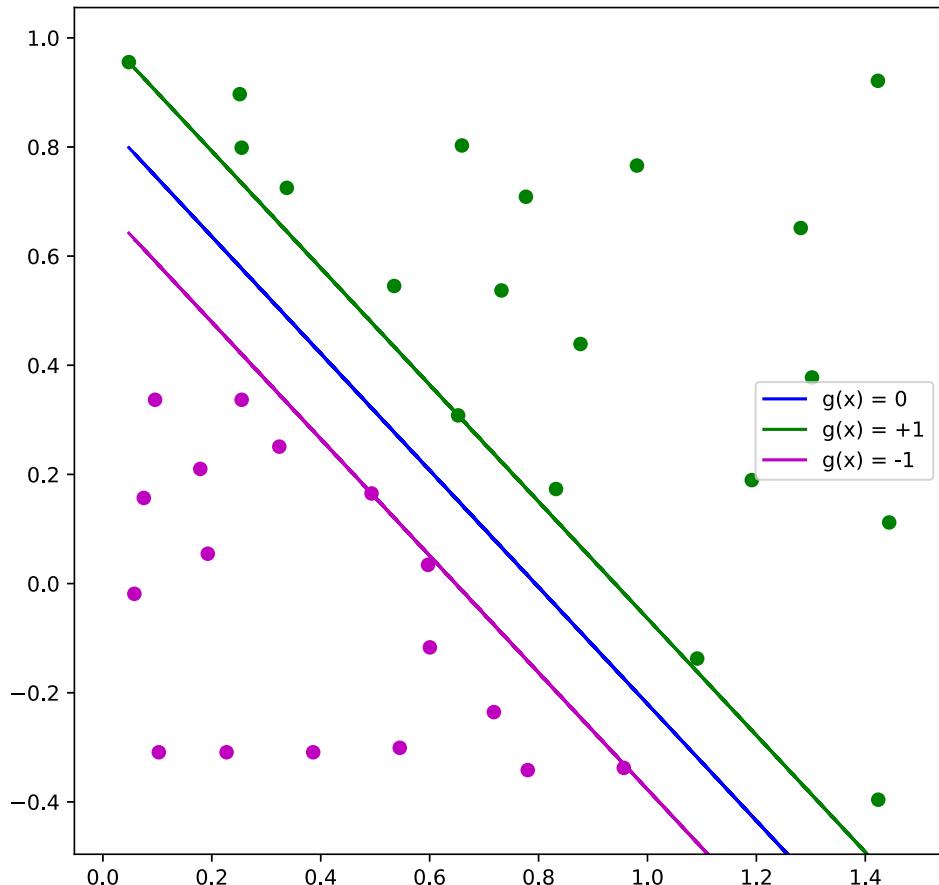
""" Quadratic programming """
Q = matrix([[0.5, 0, 0], [0, 0.5, 0], [0, 0, 0]], tc='d')
p = matrix([0, 0, 0], tc='d')
G = matrix(-y * x, tc='d')
h = matrix(-1*np.ones_like(y), tc='d')
sol = solvers.qp(Q, p, G, h)
w1, w2, b = sol['x'][0], sol['x'][1], sol['x'][2]

""" Plot boundary and margin boundaries """
plt.plot(x[:, 0], (-w1*x[:, 0]-b)/w2, c='b', label='g(x) = 0')
plt.plot(x[:, 0], (-w1*x[:, 0]-b+1)/w2, c='g', label='g(x) = +1')
plt.plot(x[:, 0], (-w1*x[:, 0]-b-1)/w2, c='m', label='g(x) = -1')
plt.xlim((x[:, 0].min()-0.1, x[:, 0].max()+0.1))
plt.ylim((x[:, 1].min()-0.1, x[:, 1].max()+0.1))
plt.legend()
plt.show()
```

	pcost	dcost	gap	pres	dres
0:	8.7483e-01	3.5428e+01	1e+02	2e+00	3e+01
1:	8.4879e+00	-3.2104e+00	4e+01	6e-01	9e+00
2:	1.6650e+01	1.1151e+01	1e+01	1e-01	2e+00
3:	2.1484e+01	1.6526e+01	6e+00	4e-02	5e-01
4:	2.1785e+01	2.0581e+01	2e+00	9e-03	1e-01
5:	2.1870e+01	2.1694e+01	2e-01	2e-04	3e-03

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```
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6: 2.1867e+01 2.1801e+01 7e-02 5e-05 7e-04
7: 2.1861e+01 2.1860e+01 1e-03 8e-07 1e-05
8: 2.1861e+01 2.1861e+01 1e-05 8e-09 1e-07
9: 2.1861e+01 2.1861e+01 1e-07 8e-11 1e-09
Optimal solution found.
```



## b): Linearly Non-separable Dataset

In [66]:

```
# Load the data set that is not linearly separable
data = np.loadtxt('dirty_nonlin.txt', delimiter='\t')
x = data[:, 0:2]
y = data[:, 2]

""" Assign color per label, then scatter plot dataset """
pred_color = {1:'g', -1:'m'}
colors = [pred_color[i] for i in y]
plt.figure(figsize = (7.5, 7.5))
plt.scatter(x[:, 0], x[:, 1], c=colors)

""" Modifying y to work with NumPy broadcasting,
concatenating a stack of ones onto x for bias term """
y = y[:, np.newaxis]
x = np.hstack((x, np.ones_like(y)))
```

file:///C:/Users/ivanw/Documents/College & Grad School/Carnegie Mellon/24-787 Machine Learning/HW6/q1.html

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```

""" Quadratic programming """
Q = np.zeros((len(y)+3, len(y)+3))
Q[0:2, 0:2] = 0.5*np.eye(2)
Q = matrix(Q, tc='d')

C = 0.05
p = C*np.ones(len(y))[np.newaxis, :]
p = matrix(np.hstack((np.zeros((1, 3)), p)).T, tc='d')

G = np.block([[[-y*x, -1*np.eye(len(y))],
               [np.zeros((len(y), 3)), -1*np.eye(len(y))]]])
G = matrix(G, tc='d')

h = matrix(np.vstack((-1*np.ones_like(y), np.zeros_like(y))), tc='d')

sol = solvers.qp(Q, p, G, h)
w1, w2, b = sol['x'][0], sol['x'][1], sol['x'][2]

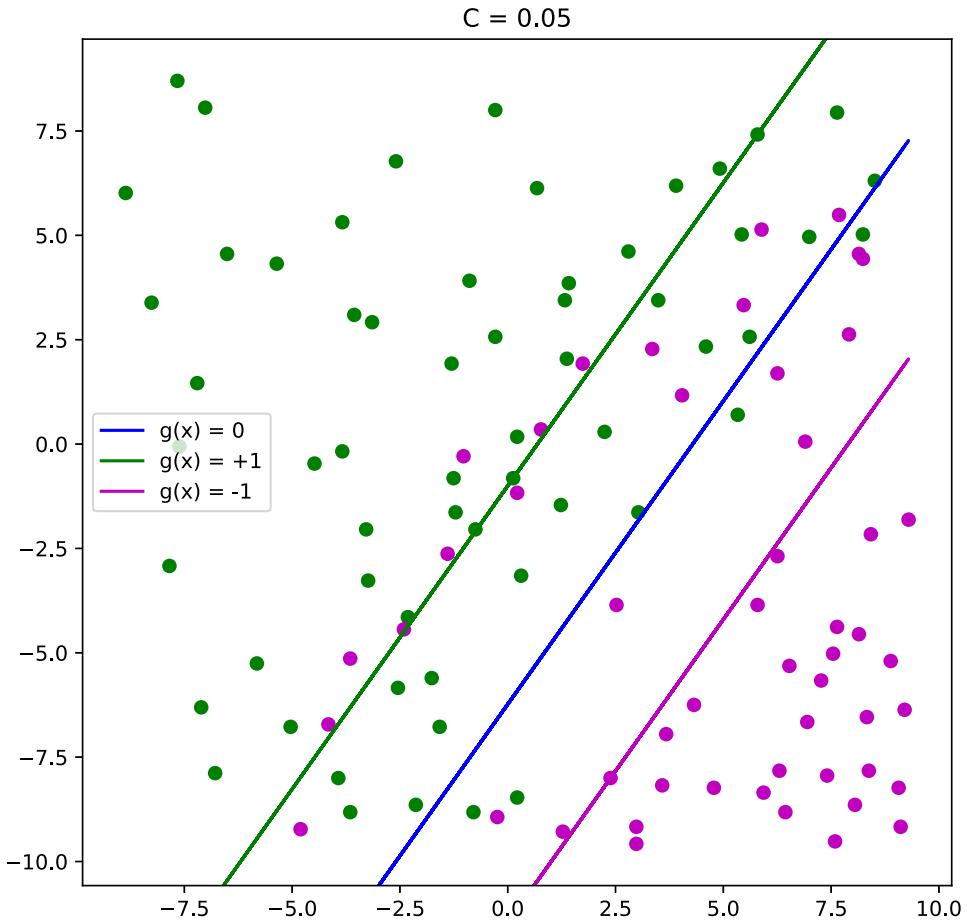
""" Plot boundary and margin boundaries """
plt.plot(x[:, 0], (-w1*x[:, 0]-b)/w2, c='b', label='g(x) = 0')
plt.plot(x[:, 0], (-w1*x[:, 0]-b+1)/w2, c='g', label='g(x) = +1')
plt.plot(x[:, 0], (-w1*x[:, 0]-b-1)/w2, c='m', label='g(x) = -1')
plt.xlim((x[:, 0].min()-1, x[:, 0].max()+1))
plt.ylim((x[:, 1].min()-1, x[:, 1].max()+1))
plt.title("C = {}".format(C))
plt.legend()
plt.show()

```

```

pcost      dcost      gap      pres      dres
0: 1.2369e+00 3.9714e+01 6e+02 2e+00 7e+02
1: 1.0368e+01 -1.5829e+01 3e+01 8e-02 2e+01
2: 6.1832e+00 7.5736e-02 6e+00 1e-02 3e+00
3: 2.6957e+00 1.9060e+00 8e-01 1e-03 3e-01
4: 2.3622e+00 2.1461e+00 2e-01 3e-04 8e-02
5: 2.3003e+00 2.2049e+00 1e-01 7e-05 2e-02
6: 2.2686e+00 2.2290e+00 4e-02 2e-05 6e-03
7: 2.2515e+00 2.2425e+00 9e-03 3e-06 7e-04
8: 2.2470e+00 2.2462e+00 8e-04 1e-07 3e-05
9: 2.2466e+00 2.2466e+00 9e-06 1e-09 3e-07
10: 2.2466e+00 2.2466e+00 9e-08 1e-11 3e-09
Optimal solution found.

```



c): Output 4 plots & Explain your observations here:

```
In [71]: C = [0.1, 1, 100, 1000000]
for i in range(len(C)):
    plt.figure(i, figsize = (7.5, 7.5))
    plt.title("C = {}".format(C[i]))
    p = C[i]*np.ones(len(y))[np.newaxis, :]
    p = matrix(np.hstack((np.zeros((1, 3)), p)).T, tc='d')

    sol = solvers.qp(Q, p, G, h)
    w1, w2, b = sol['x'][0], sol['x'][1], sol['x'][2]

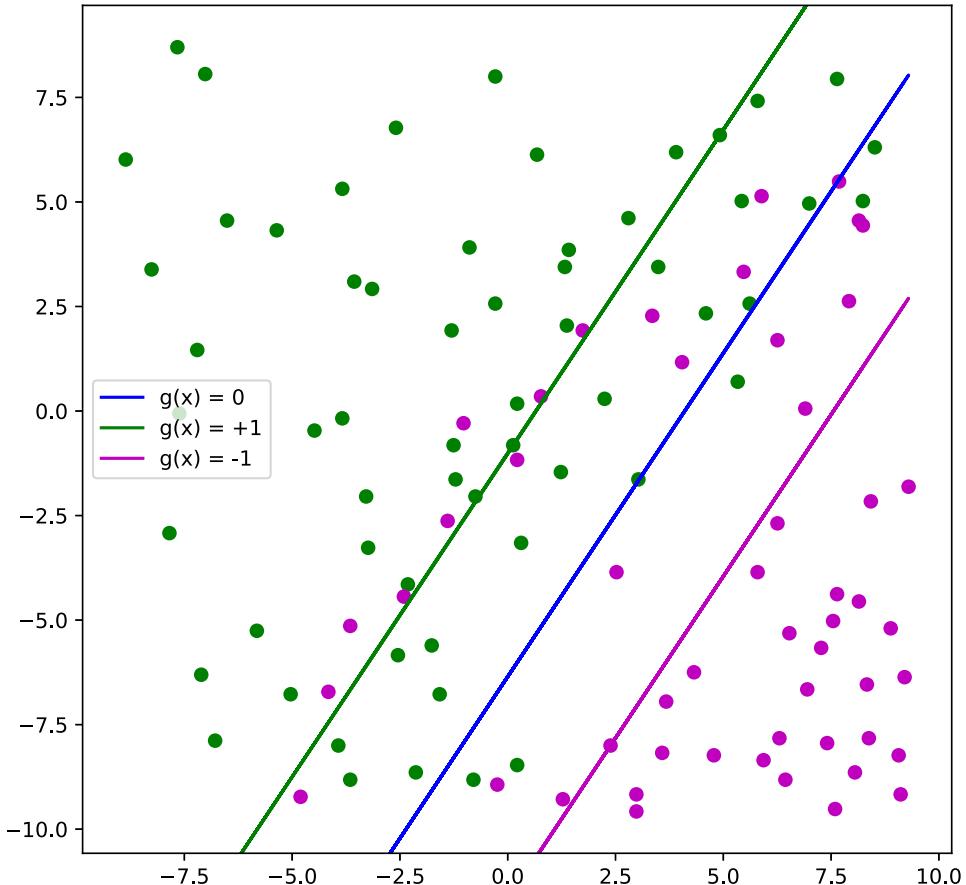
    plt.scatter(x[:, 0], x[:, 1], c=colors)
    plt.plot(x[:, 0], (-w1*x[:, 0]-b)/w2, c='b', label='g(x) = 0')
    plt.plot(x[:, 0], (-w1*x[:, 0]-b+1)/w2, c='g', label='g(x) = +1')
    plt.plot(x[:, 0], (-w1*x[:, 0]-b-1)/w2, c='m', label='g(x) = -1')
    plt.xlim((x[:, 0].min()-1, x[:, 0].max()+1))
    plt.ylim((x[:, 1].min()-1, x[:, 1].max()+1))
    plt.legend()
    plt.show()
```

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q1

	pconst	dconst	gap	pres	dres
0:	2.0586e+00	5.0520e+01	7e+02	3e+00	6e+02
1:	1.9677e+01	-2.3176e+01	5e+01	1e-01	3e+01
2:	1.1129e+01	1.0321e+00	1e+01	2e-02	4e+00
3:	5.1180e+00	3.8916e+00	1e+00	2e-03	5e-01
4:	4.6561e+00	4.2742e+00	4e-01	5e-04	1e-01
5:	4.5701e+00	4.3758e+00	2e-01	2e-04	5e-02
6:	4.5183e+00	4.4169e+00	1e-01	6e-05	2e-02
7:	4.4859e+00	4.4444e+00	4e-02	2e-05	6e-03
8:	4.4679e+00	4.4615e+00	6e-03	8e-07	2e-04
9:	4.4647e+00	4.4644e+00	3e-04	3e-08	8e-06
10:	4.4645e+00	4.4645e+00	3e-06	3e-10	9e-08

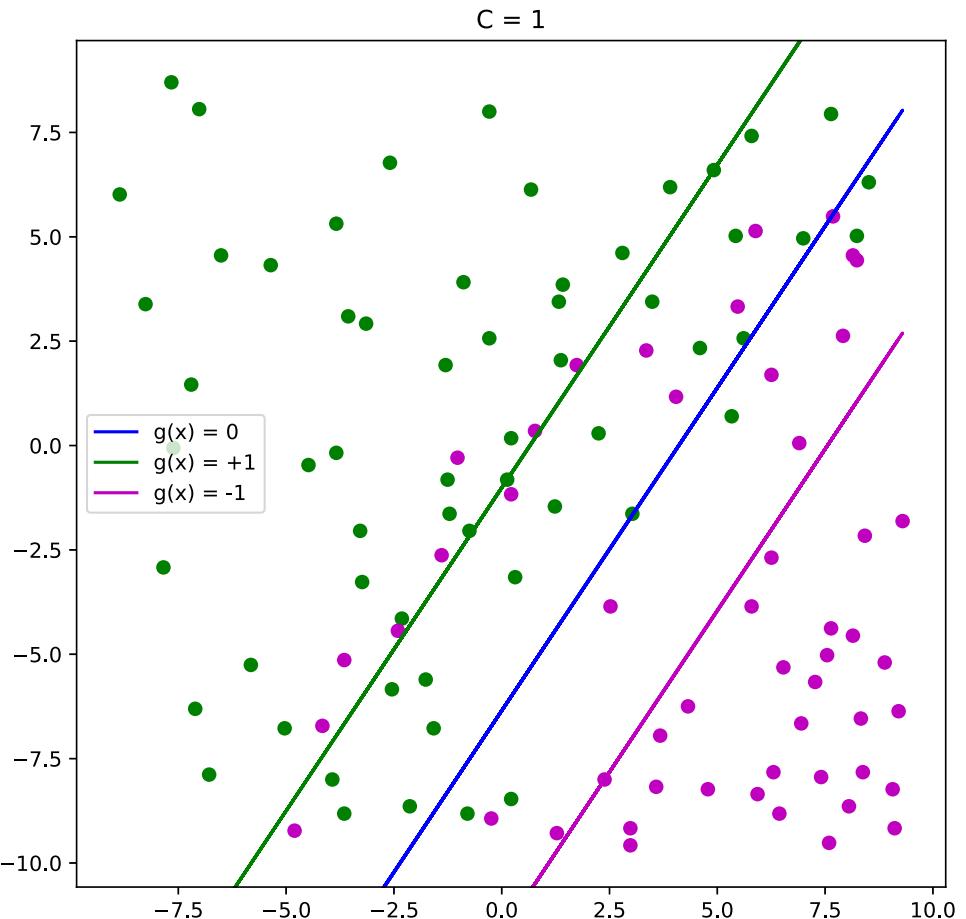
Optimal solution found.

 $C = 0.1$ 

	pconst	dconst	gap	pres	dres
0:	-5.3462e+01	2.5872e+02	1e+03	4e+00	7e+01
1:	1.3078e+02	-3.0840e+01	2e+02	5e-01	8e+00
2:	5.6284e+01	3.4499e+01	2e+01	3e-02	5e-01
3:	4.9731e+01	4.0109e+01	1e+01	1e-02	2e-01
4:	4.6211e+01	4.2662e+01	4e+00	4e-03	7e-02
5:	4.5260e+01	4.3579e+01	2e+00	1e-03	2e-02
6:	4.4780e+01	4.4030e+01	8e-01	5e-04	8e-03
7:	4.4567e+01	4.4205e+01	4e-01	2e-04	3e-03
8:	4.4427e+01	4.4336e+01	9e-02	4e-05	6e-04
9:	4.4408e+01	4.4349e+01	6e-02	2e-05	3e-04

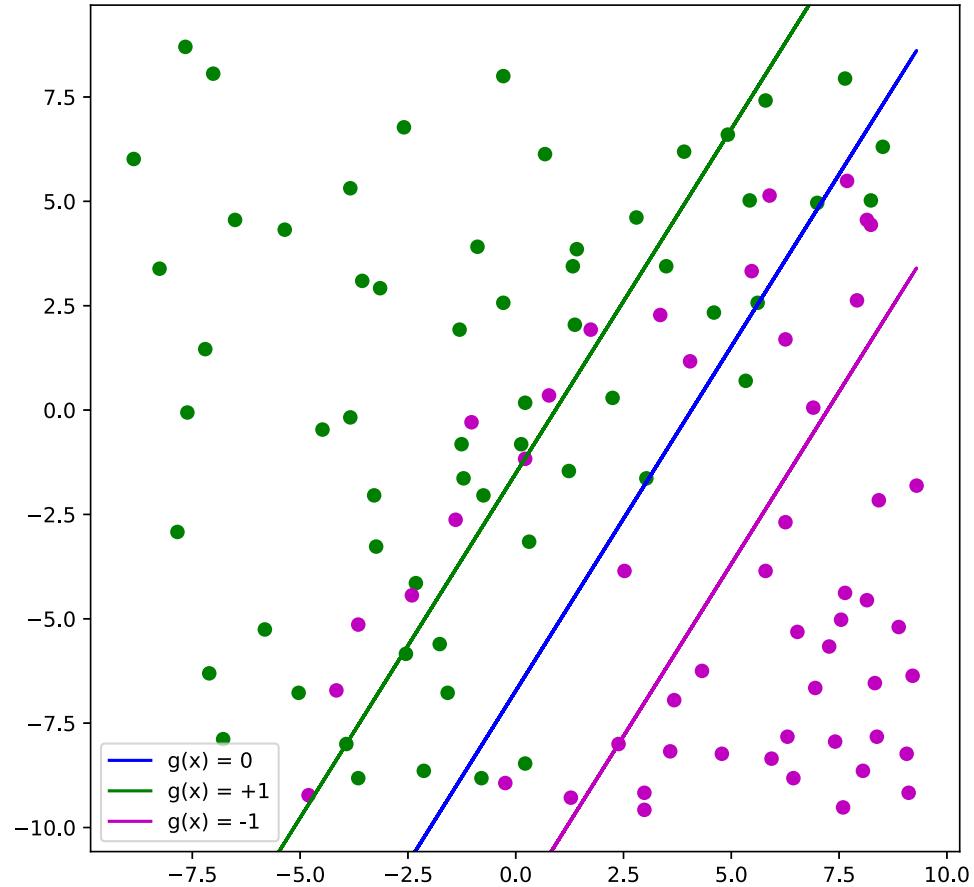
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q1  
10: 4.4385e+01 4.4371e+01 1e-02 4e-16 8e-14  
11: 4.4378e+01 4.4377e+01 2e-04 4e-16 1e-12  
12: 4.4377e+01 4.4377e+01 2e-06 4e-16 3e-13  
Optimal solution found.



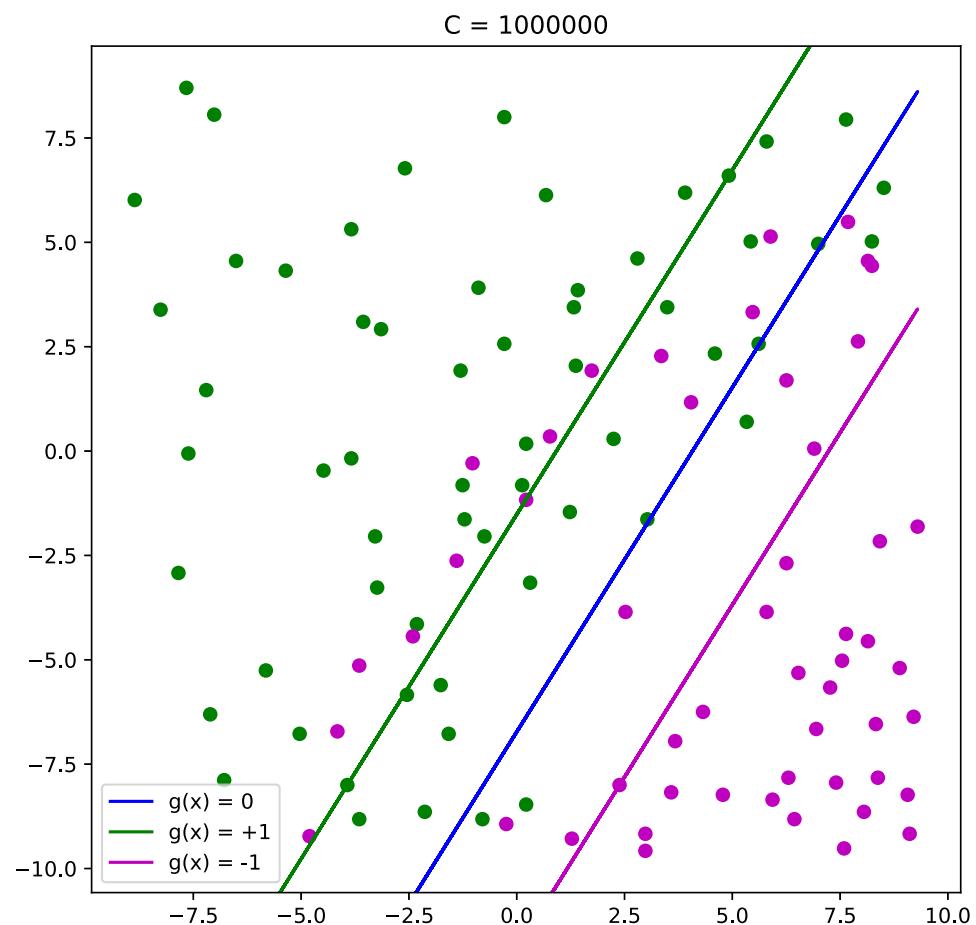
	pcost	dcost	gap	pres	dres
0:	-8.1948e+05	4.6956e+05	2e+06	2e+02	2e+01
1:	1.4936e+05	-6.1257e+03	2e+05	4e+00	3e-01
2:	7.9786e+03	2.6759e+03	6e+03	1e-01	8e-03
3:	5.3173e+03	3.5802e+03	2e+03	3e-02	2e-03
4:	5.1659e+03	3.8326e+03	1e+03	2e-02	1e-03
5:	4.7745e+03	4.1018e+03	7e+02	8e-03	6e-04
6:	4.6479e+03	4.2535e+03	4e+02	4e-03	3e-04
7:	4.4923e+03	4.3953e+03	1e+02	2e-04	2e-05
8:	4.4686e+03	4.4071e+03	6e+01	7e-05	5e-06
9:	4.4355e+03	4.4342e+03	1e+00	9e-07	7e-08
10:	4.4348e+03	4.4347e+03	5e-02	3e-08	3e-09
11:	4.4348e+03	4.4348e+03	8e-04	4e-10	4e-11

Optimal solution found.

$C = 100$ 

	$p_{cost}$	$d_{cost}$	$gap$	$pres$	$dres$
0:	-8.2235e+13	4.5105e+13	2e+14	2e+06	2e+01
1:	1.4250e+13	-6.9400e+11	2e+13	3e+04	2e-01
2:	2.4964e+11	-1.7815e+08	3e+11	4e+02	3e-03
3:	2.5672e+09	2.6019e+07	3e+09	4e+00	3e-05
4:	9.4700e+07	2.6383e+07	8e+07	1e-01	8e-07
5:	5.5257e+07	3.4069e+07	2e+07	3e-02	2e-07
6:	5.2212e+07	3.7734e+07	2e+07	2e-02	1e-07
7:	4.8927e+07	4.0549e+07	9e+06	7e-03	5e-08
8:	4.6421e+07	4.2681e+07	4e+06	2e-03	2e-08
9:	4.4985e+07	4.3923e+07	1e+06	7e-05	5e-10
10:	4.4770e+07	4.4021e+07	8e+05	3e-05	2e-10
11:	4.4415e+07	4.4293e+07	1e+05	5e-06	4e-11
12:	4.4349e+07	4.4346e+07	3e+03	1e-07	8e-13
13:	4.4347e+07	4.4347e+07	2e+02	6e-09	2e-12
14:	4.4347e+07	4.4347e+07	2e+00	6e-11	2e-12

Optimal solution found.

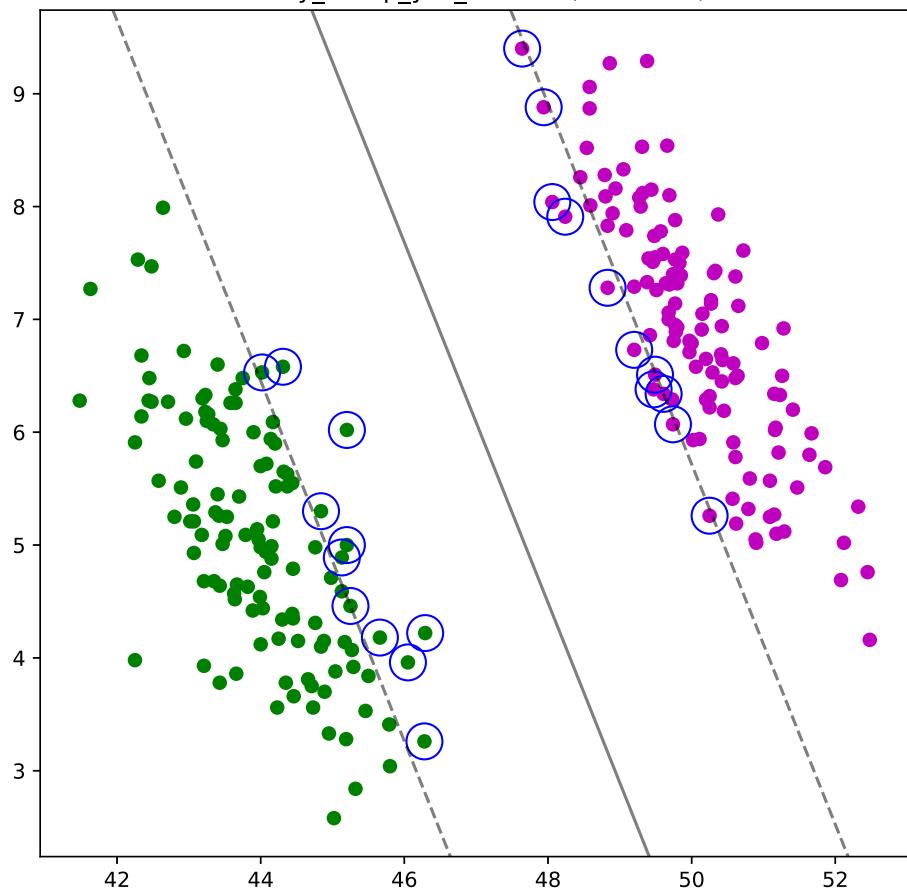


```
In [1]:  
import numpy as np  
import matplotlib.pyplot as plt  
from sklearn.svm import SVC  
from plot_svc_decision_function import plot_svc_decision_function
```

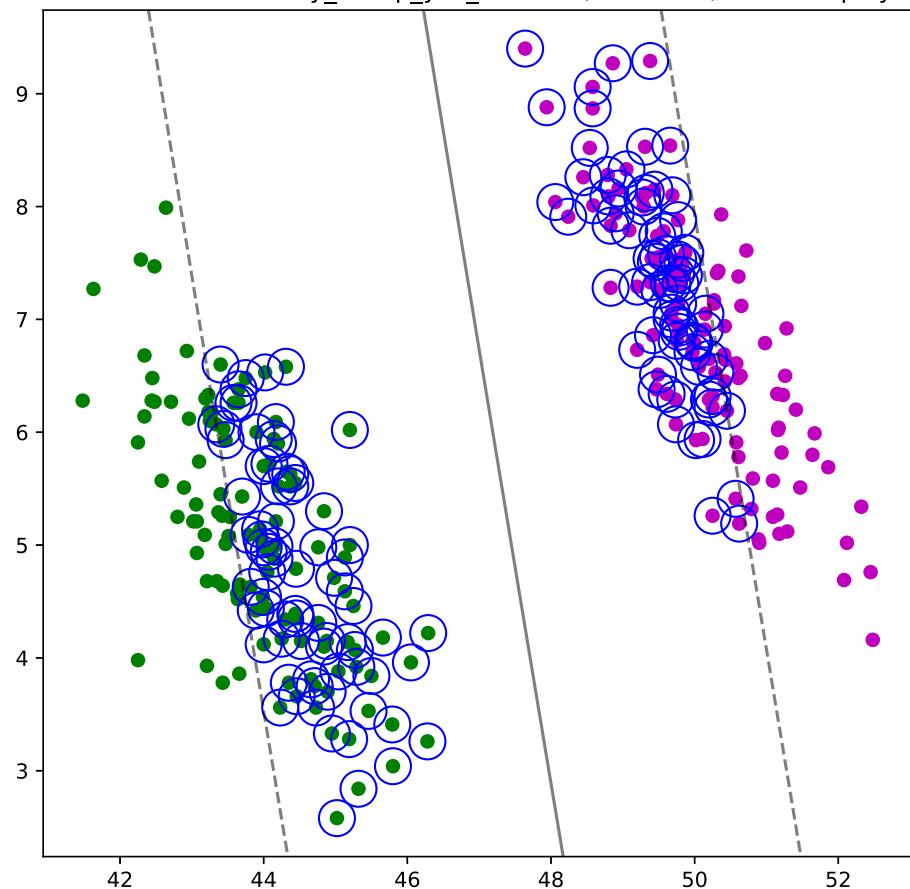
## Problem 2a: Support Vector Machine with sklearn

```
In [2]:  
file_list = ["binary_linsep_yes_n240.txt", "binary_linsep_no_n240.txt",  
            "binary_moons_linsep_no_n100.txt", "concentriccircles_binary.txt"]  
kernels = ["linear", "poly", "rbf"]  
Cs = [0.01, 10, 1000]  
count = 0  
  
for i in range(len(file_list)):  
    data = np.genfromtxt(file_list[i], delimiter=' ')  
    X = data[:, 0:2]  
    y = data[:, 2]  
    pred_color = {0:'m', 1:'g'}  
    colors = [pred_color[i] for i in y]  
    for j in range(len(Cs)):  
        for k in range(len(kernels)):  
            model = SVC(kernel=kernels[k], C=Cs[j]).fit(X, y)  
            plt.figure(count, figsize=(7.5, 7.5))  
            plt.scatter(X[:, 0], X[:, 1], c=colors)  
            plt.title("Plot {}: Data = {}, C = {}, Kernel = {}".format(count+1, file_li  
            plot_svc_decision_function(model)  
            if i == 2 and j == 2 and k == 2:  
                SV = model.support_vectors_  
            count += 1  
print("Support vector values (Shape = {}) for case of interest:\n".format(SV.shape))  
print(SV)
```

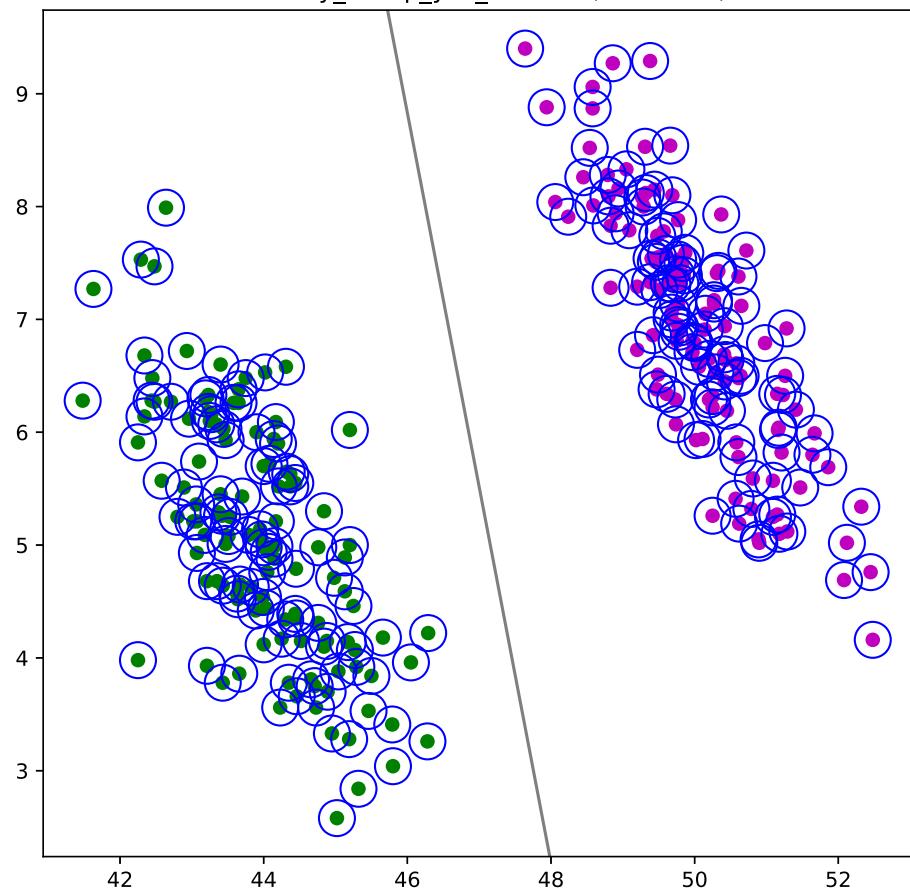
Plot 1: Data = binary\_linsep\_yes\_n240.txt, C = 0.01, Kernel = linear



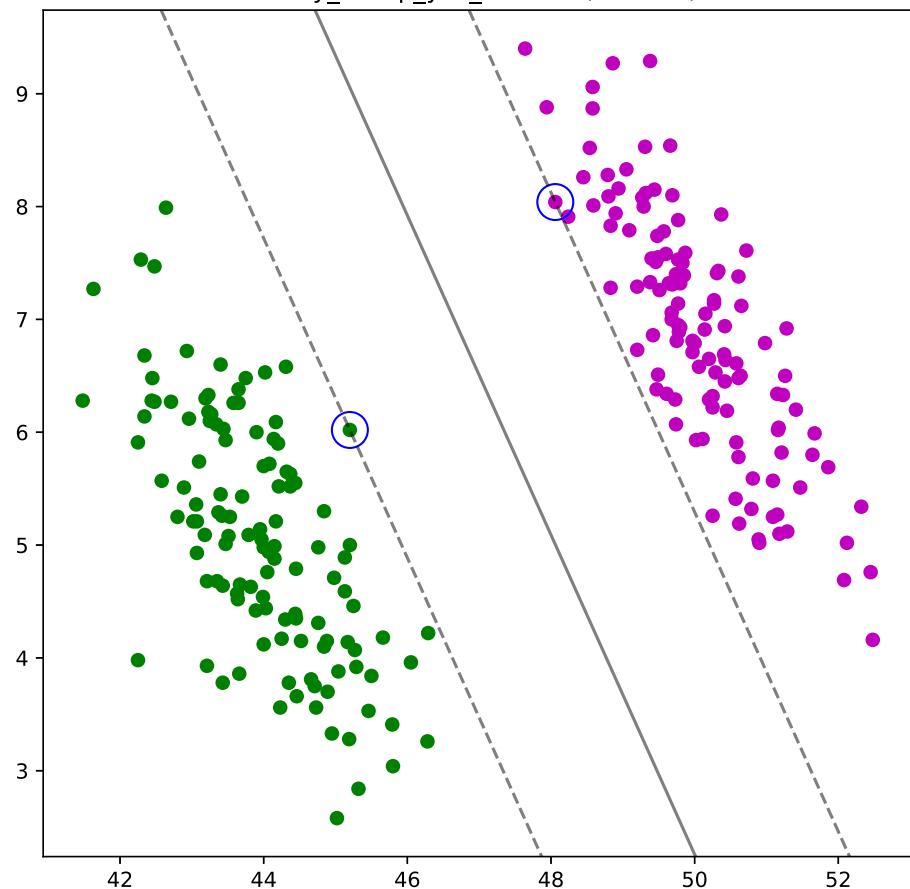
Plot 2: Data = binary\_linsep\_yes\_n240.txt, C = 0.01, Kernel = poly



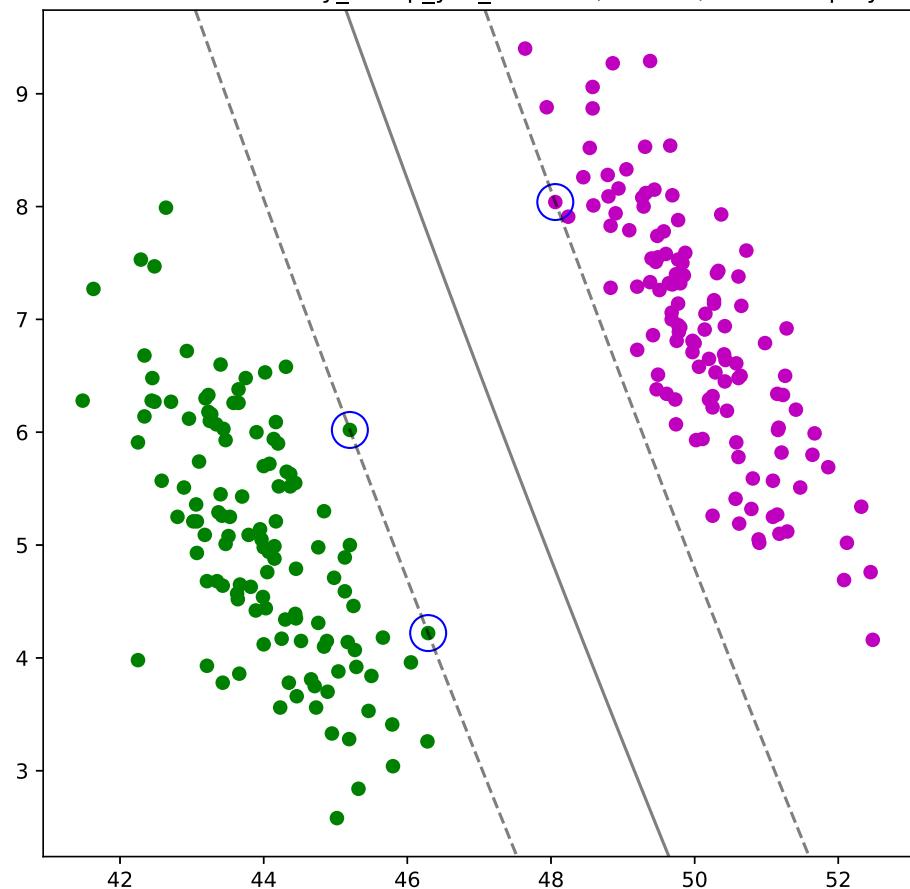
Plot 3: Data = binary\_linsep\_yes\_n240.txt, C = 0.01, Kernel = rbf



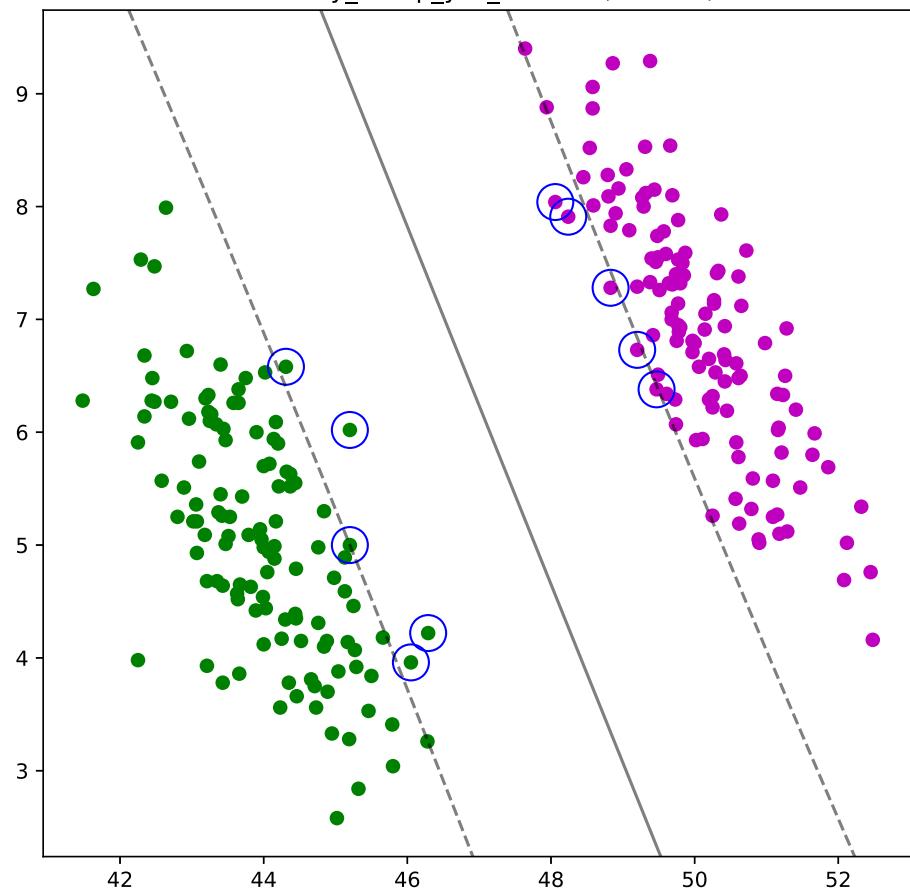
Plot 4: Data = binary\_linsep\_yes\_n240.txt, C = 10, Kernel = linear



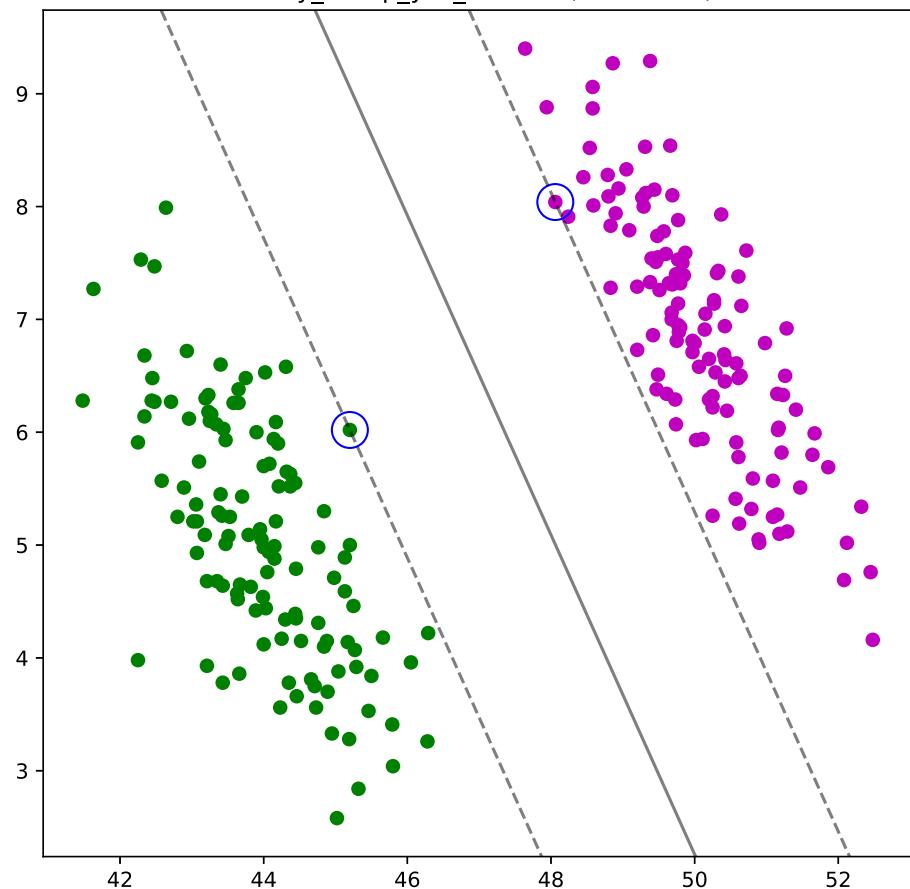
Plot 5: Data = binary\_linsep\_yes\_n240.txt, C = 10, Kernel = poly



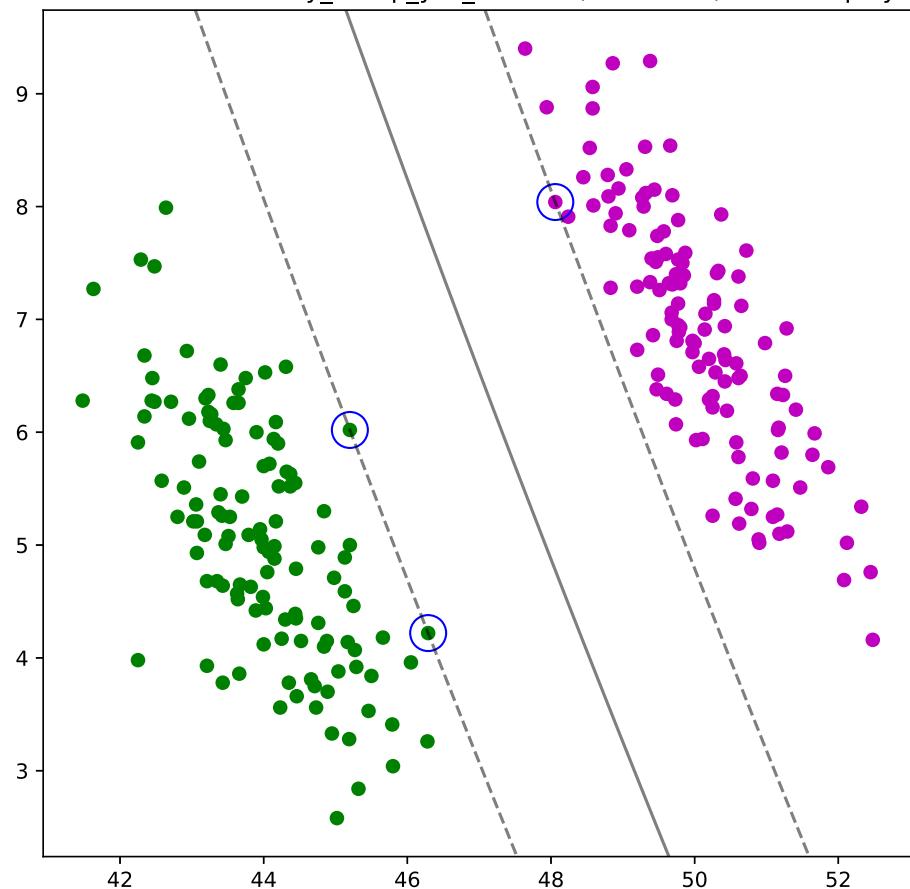
Plot 6: Data = binary\_linsep\_yes\_n240.txt, C = 10, Kernel = rbf



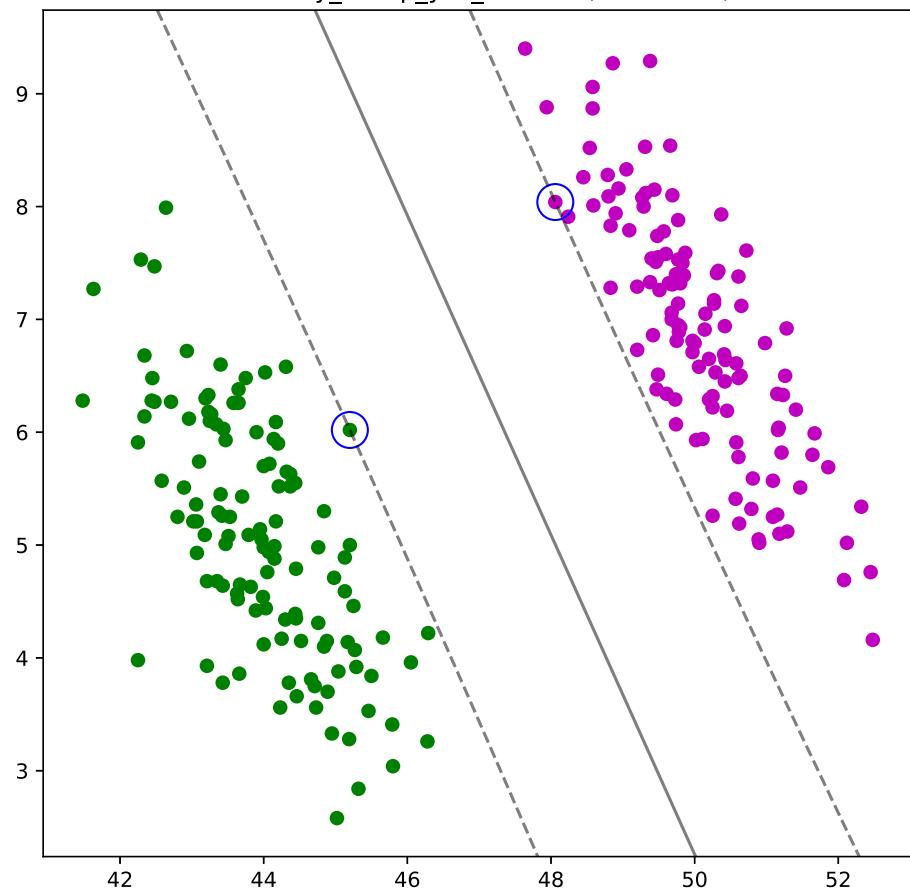
Plot 7: Data = binary\_linsep\_yes\_n240.txt, C = 1000, Kernel = linear



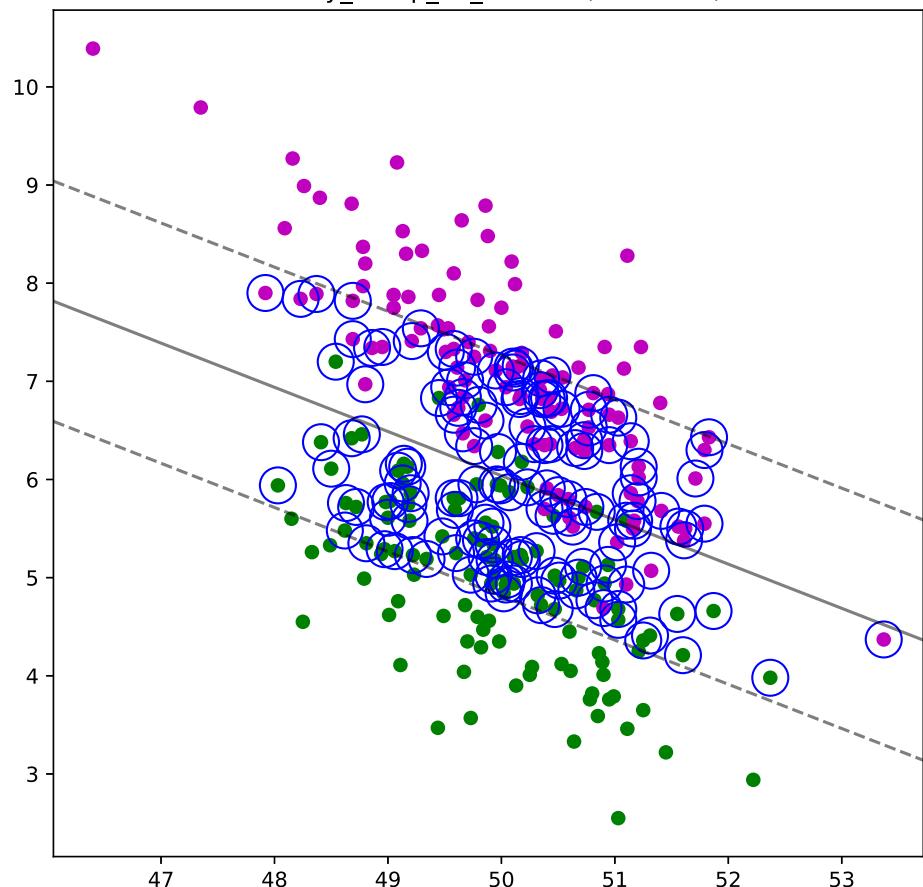
Plot 8: Data = binary\_linsep\_yes\_n240.txt, C = 1000, Kernel = poly



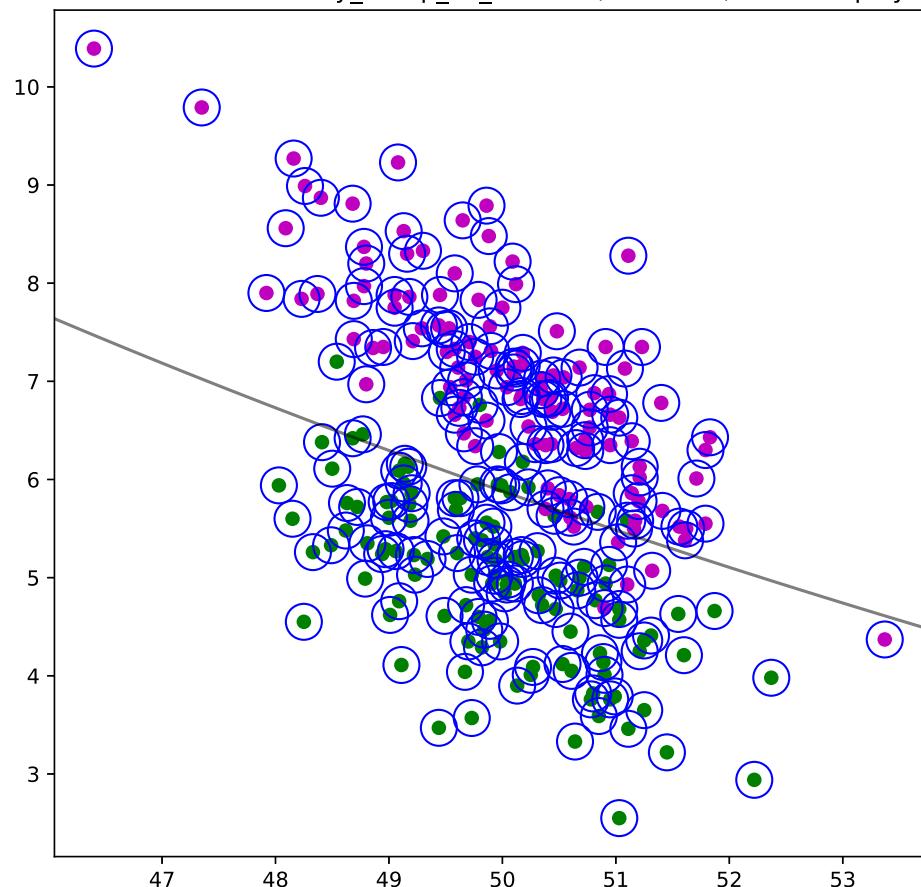
Plot 9: Data = binary\_linsep\_yes\_n240.txt, C = 1000, Kernel = rbf



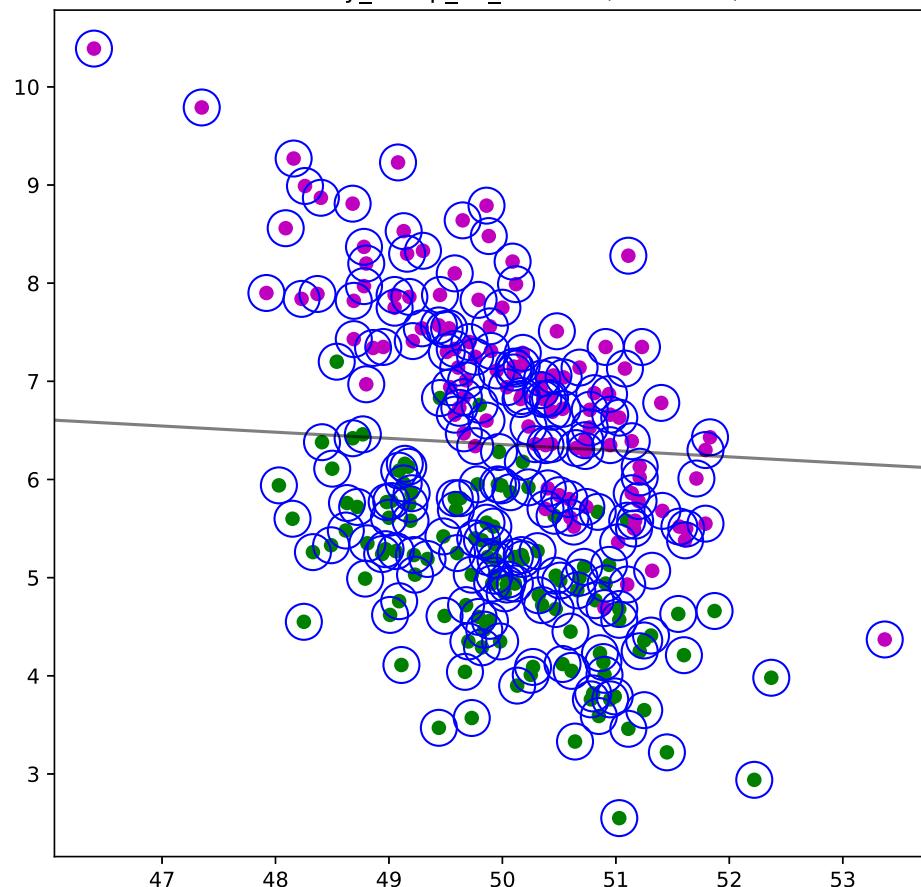
Plot 10: Data = binary\_linsep\_no\_n240.txt, C = 0.01, Kernel = linear



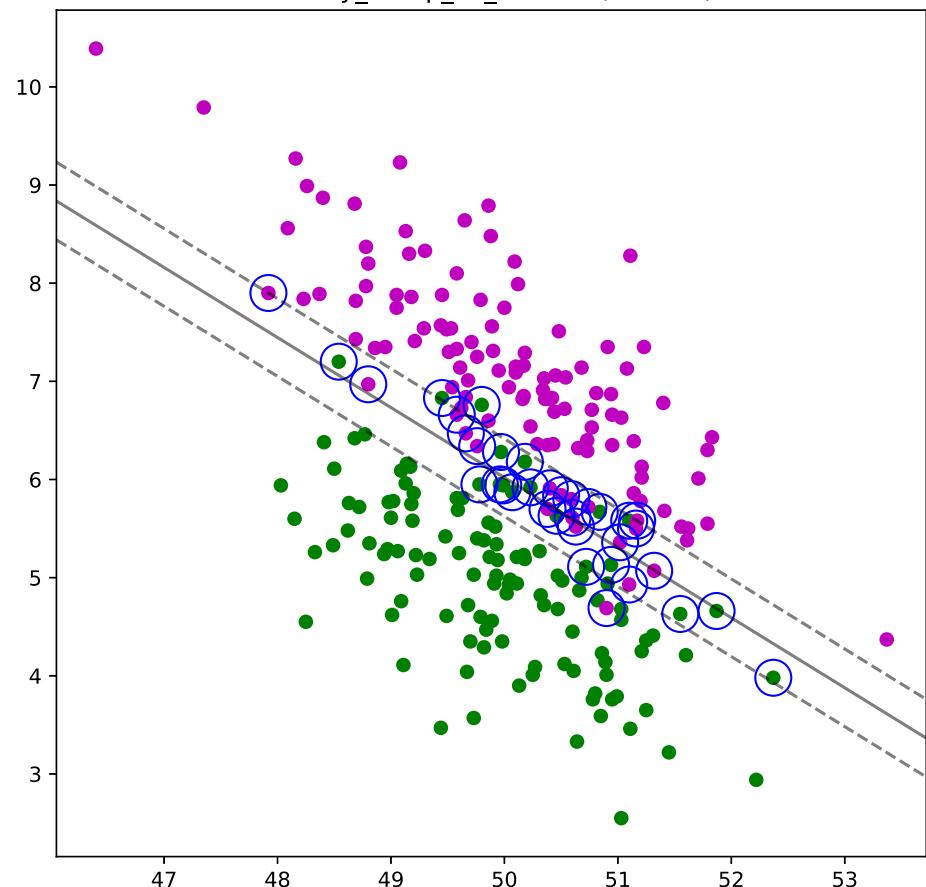
Plot 11: Data = binary\_linsep\_no\_n240.txt, C = 0.01, Kernel = poly



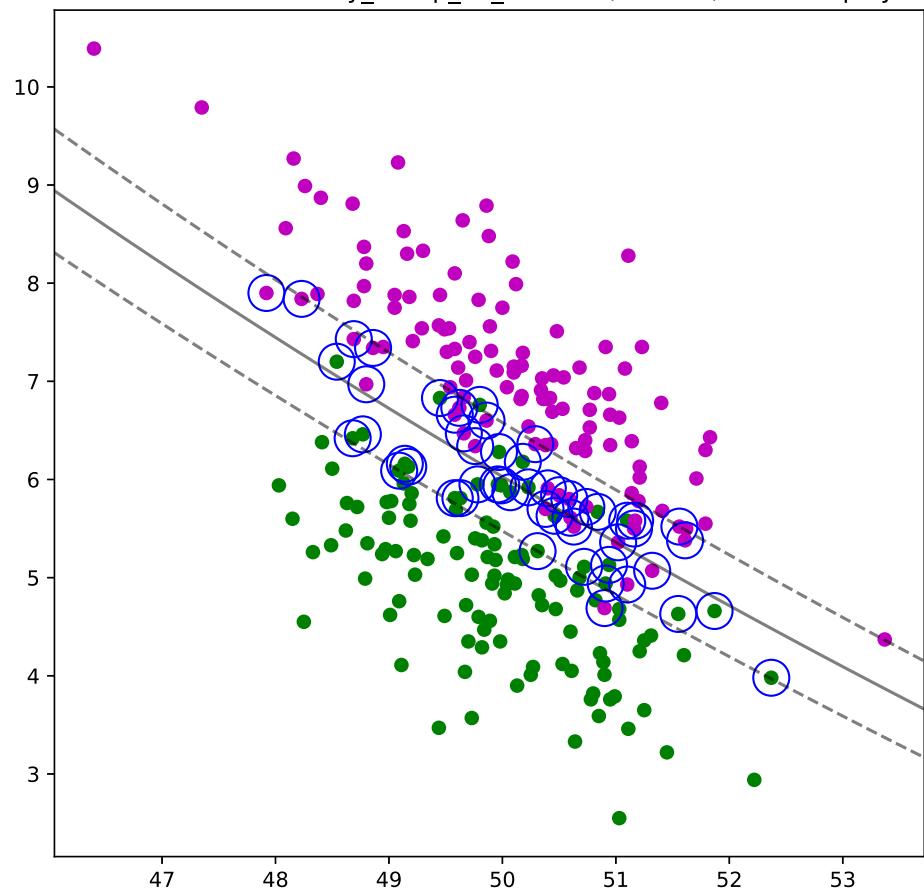
Plot 12: Data = binary\_linsep\_no\_n240.txt, C = 0.01, Kernel = rbf



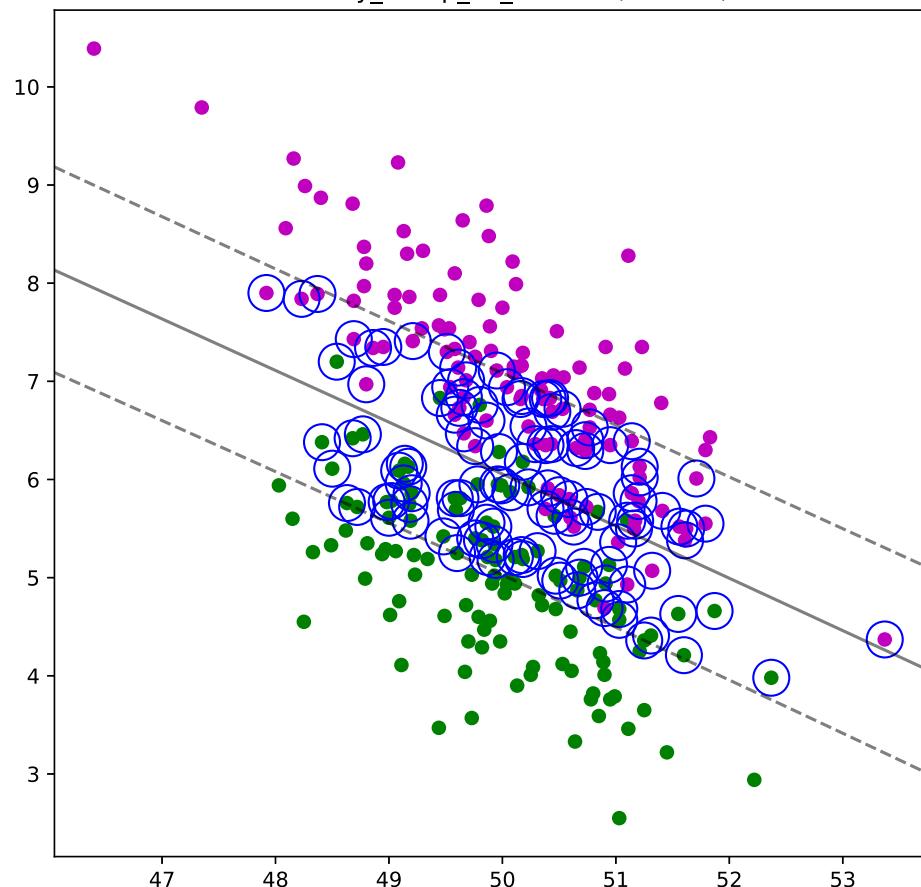
Plot 13: Data = binary\_linsep\_no\_n240.txt, C = 10, Kernel = linear



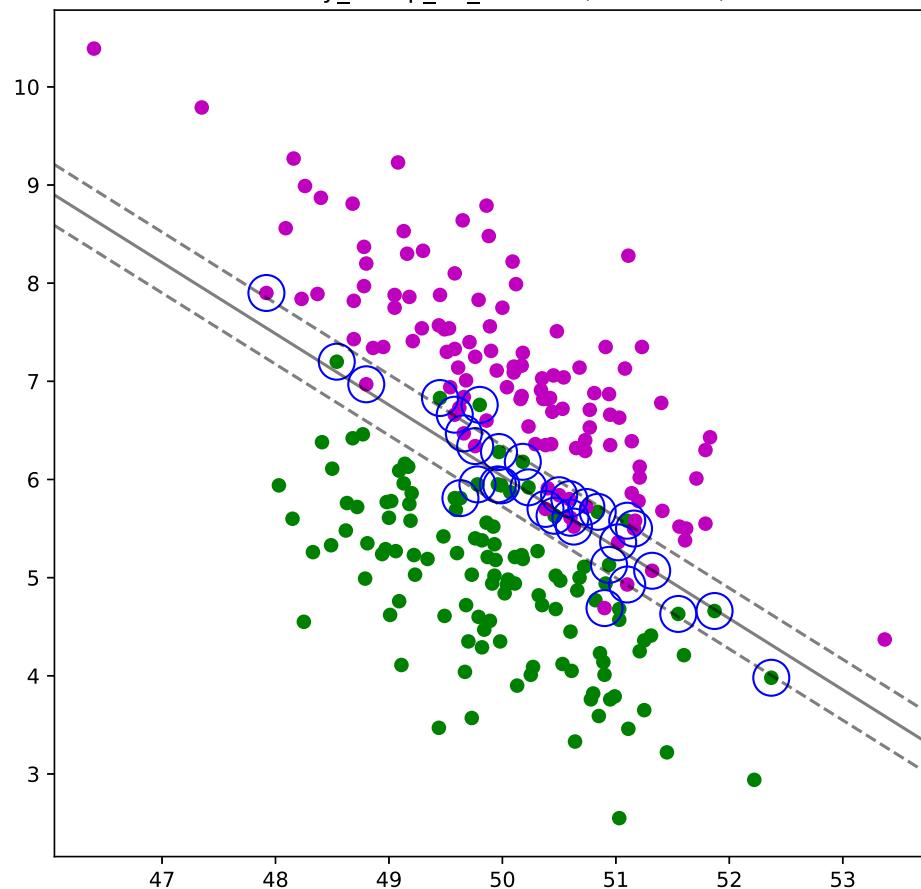
Plot 14: Data = binary\_linsep\_no\_n240.txt, C = 10, Kernel = poly



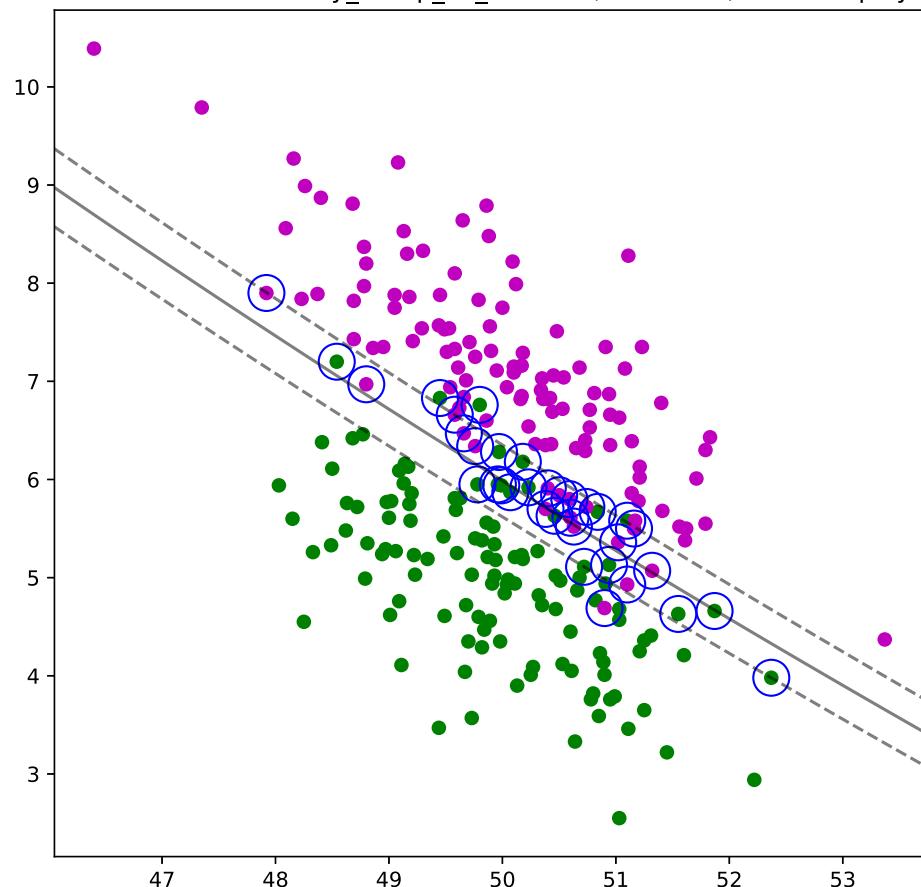
Plot 15: Data = binary\_linsep\_no\_n240.txt, C = 10, Kernel = rbf



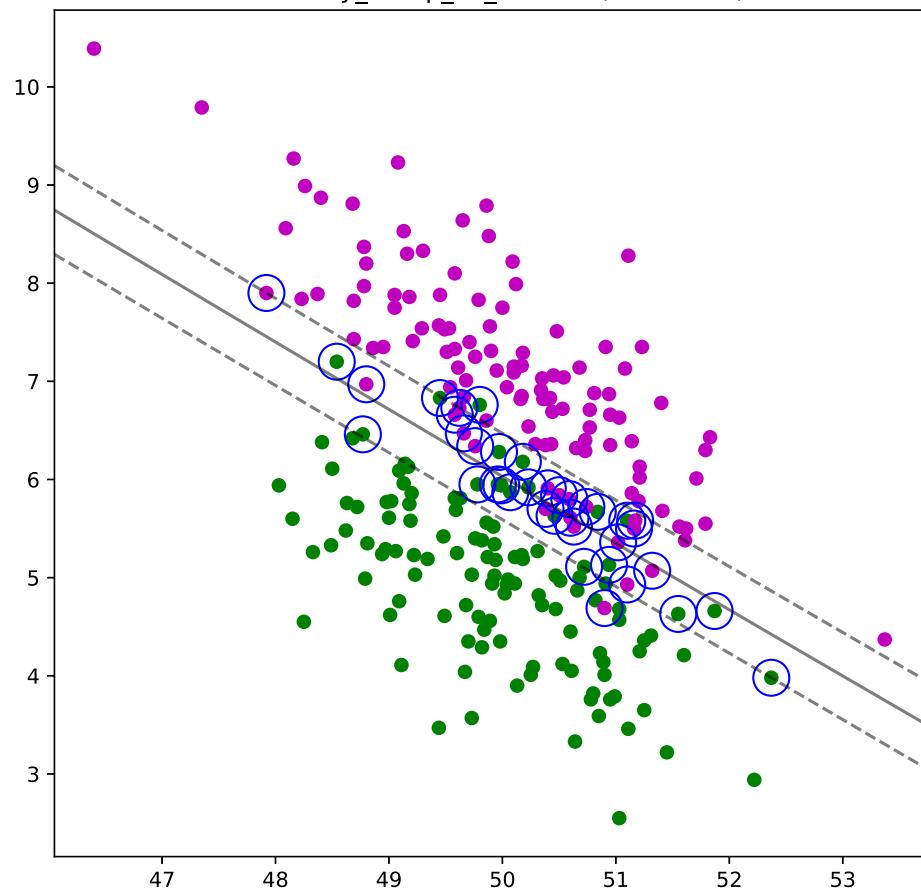
Plot 16: Data = binary\_linsep\_no\_n240.txt, C = 1000, Kernel = linear



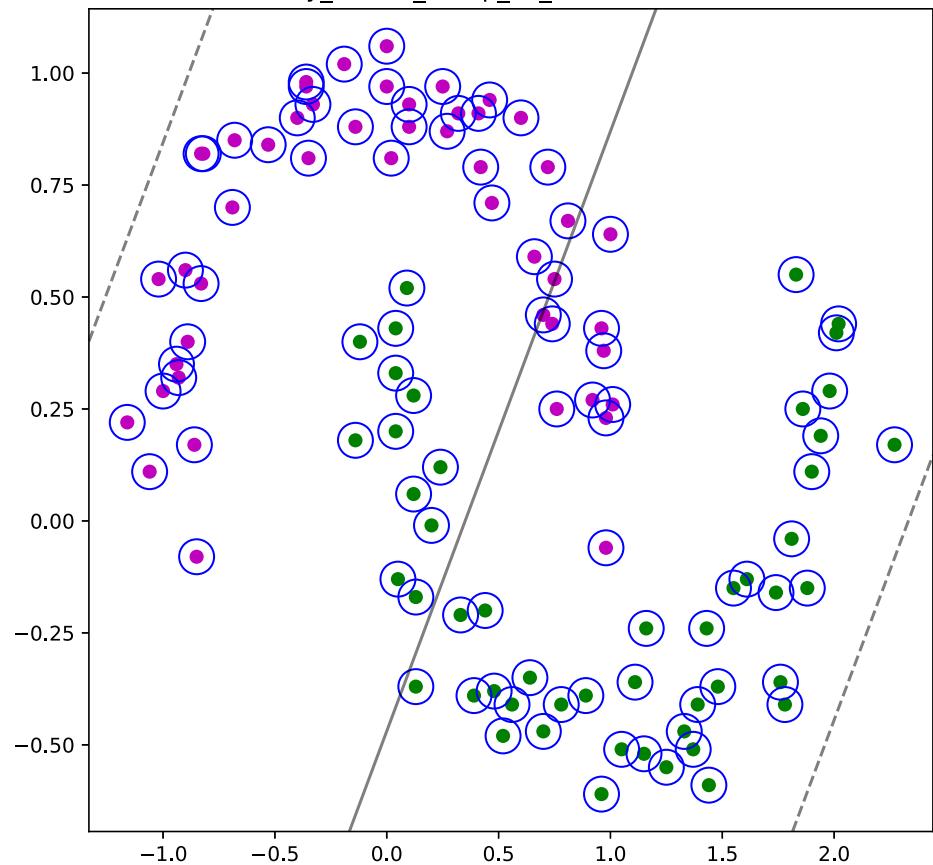
Plot 17: Data = binary\_linsep\_no\_n240.txt, C = 1000, Kernel = poly



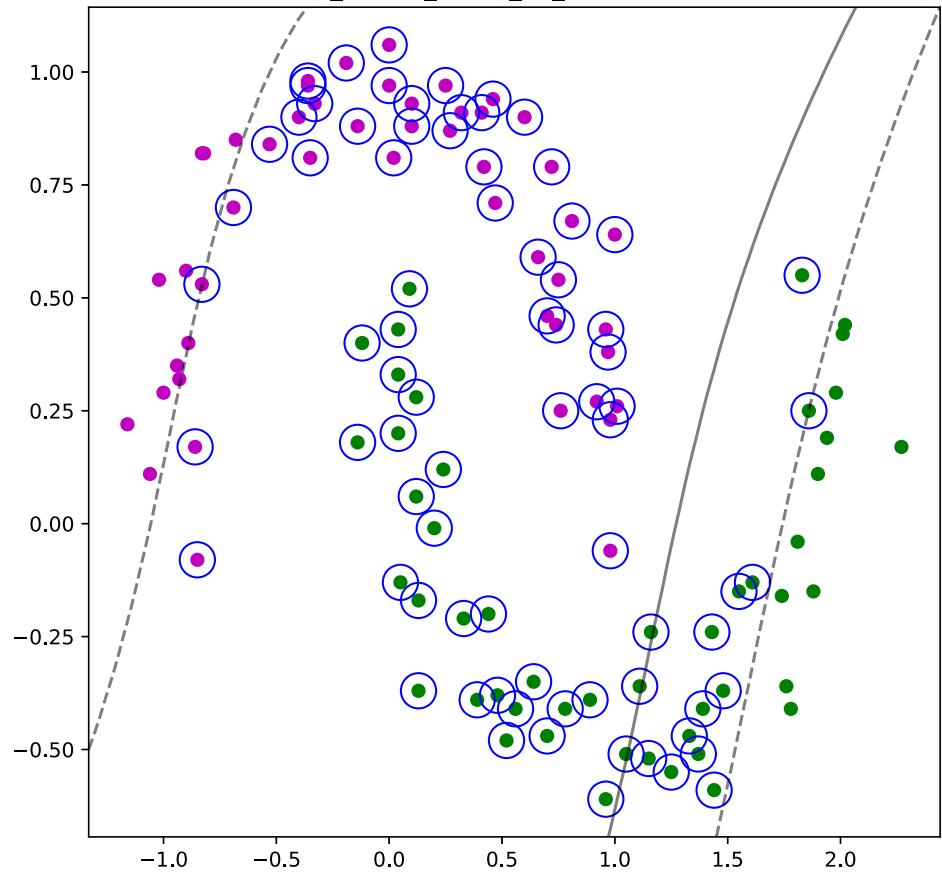
Plot 18: Data = binary\_linsep\_no\_n240.txt, C = 1000, Kernel = rbf



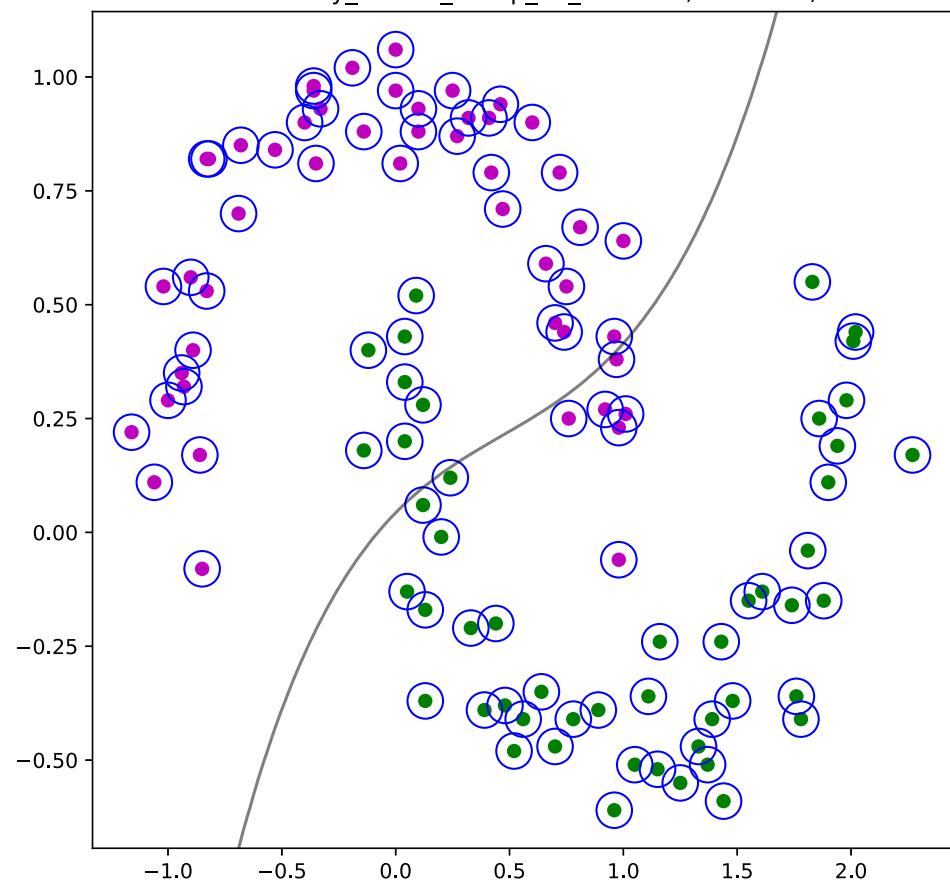
Plot 19: Data = binary\_moons\_linsep\_no\_n100.txt, C = 0.01, Kernel = linear



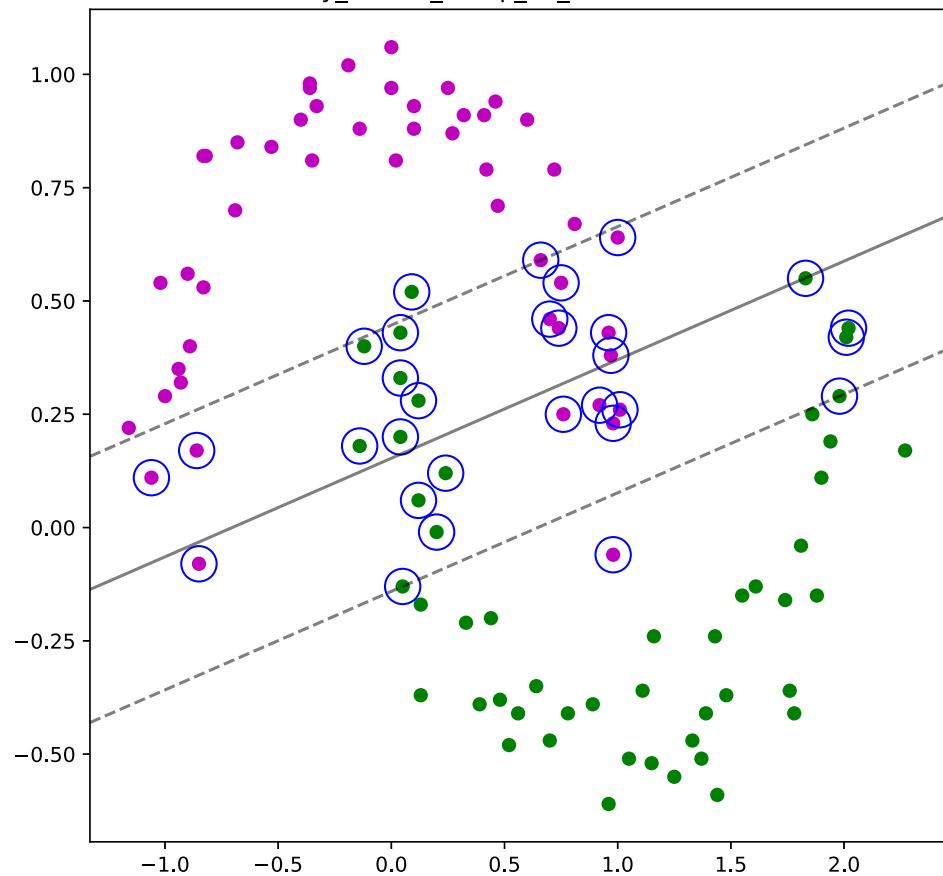
Plot 20: Data = binary\_moons\_linsep\_no\_n100.txt, C = 0.01, Kernel = poly



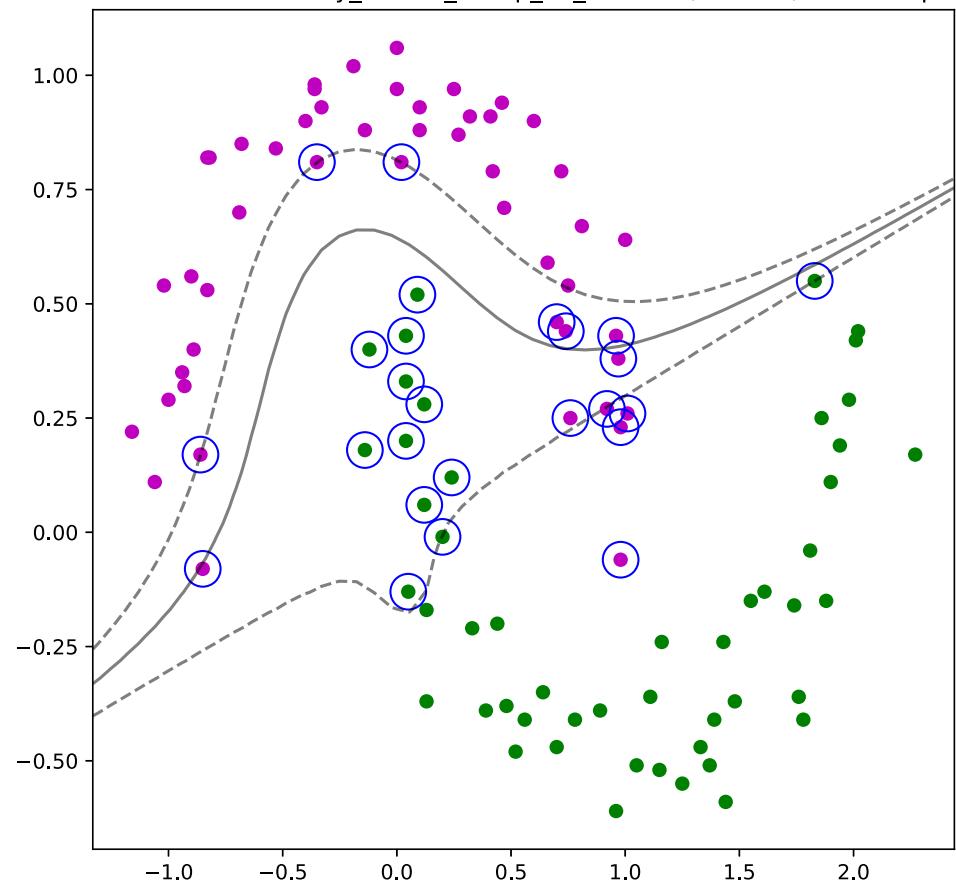
Plot 21: Data = binary\_moons\_linsep\_no\_n100.txt, C = 0.01, Kernel = rbf



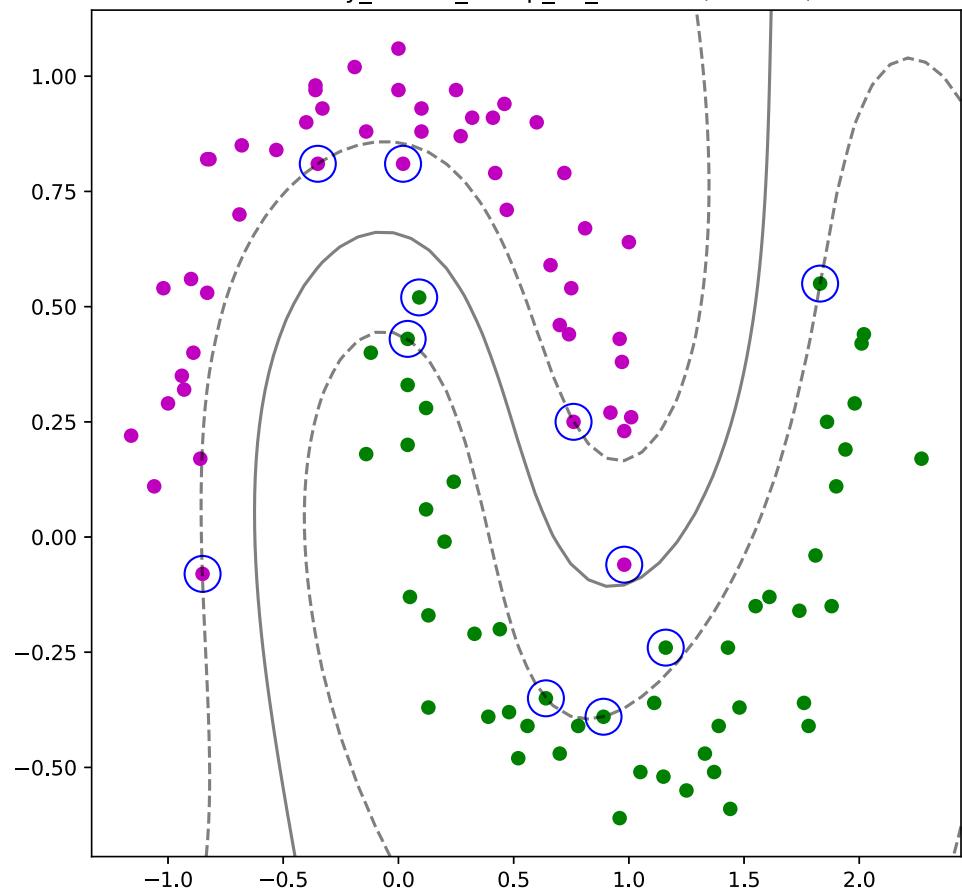
Plot 22: Data = binary\_moons\_linsep\_no\_n100.txt, C = 10, Kernel = linear



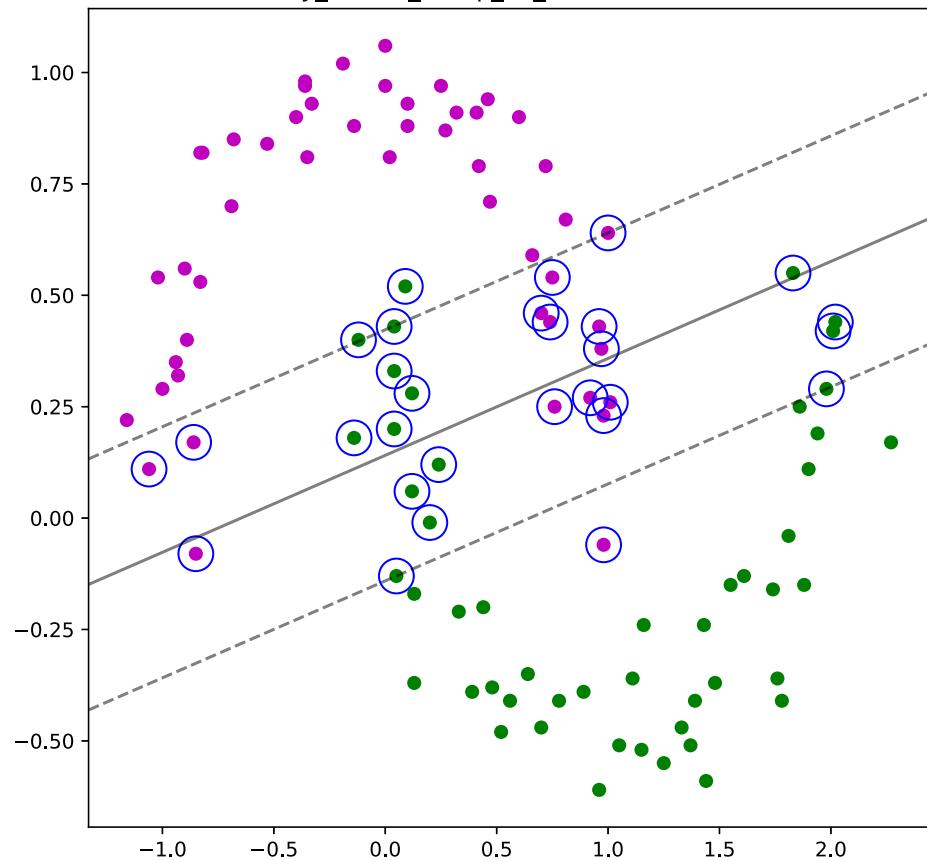
Plot 23: Data = binary\_moons\_linsep\_no\_n100.txt, C = 10, Kernel = poly



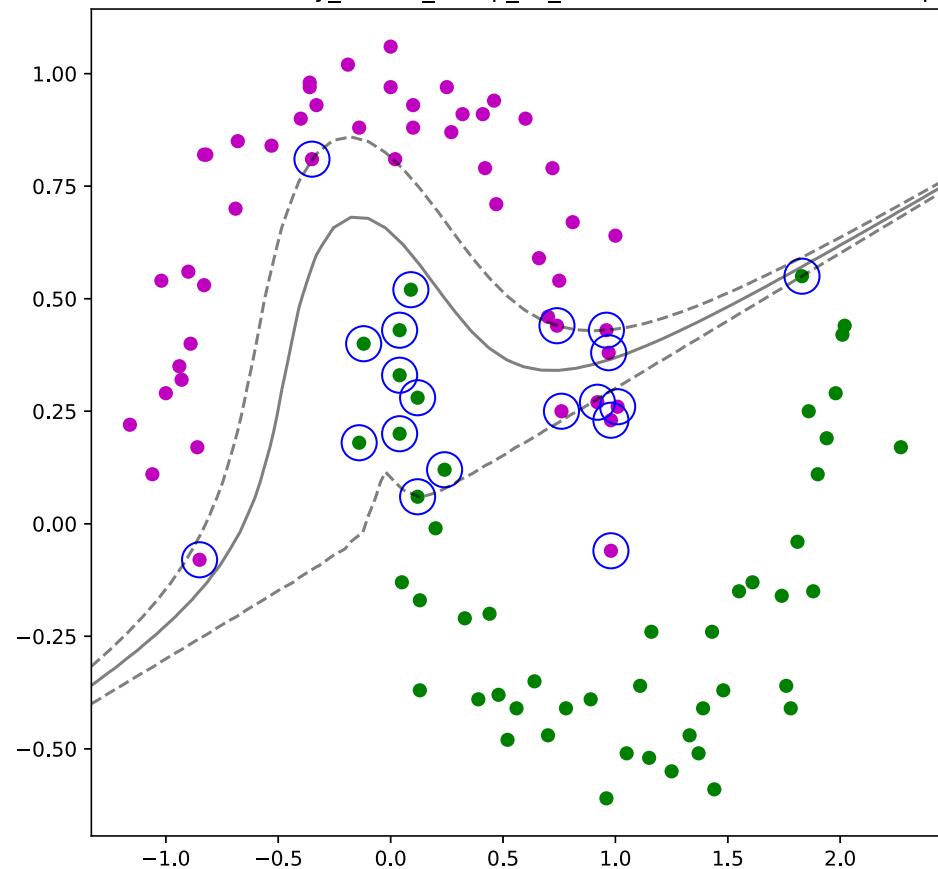
Plot 24: Data = binary\_moons\_linsep\_no\_n100.txt, C = 10, Kernel = rbf



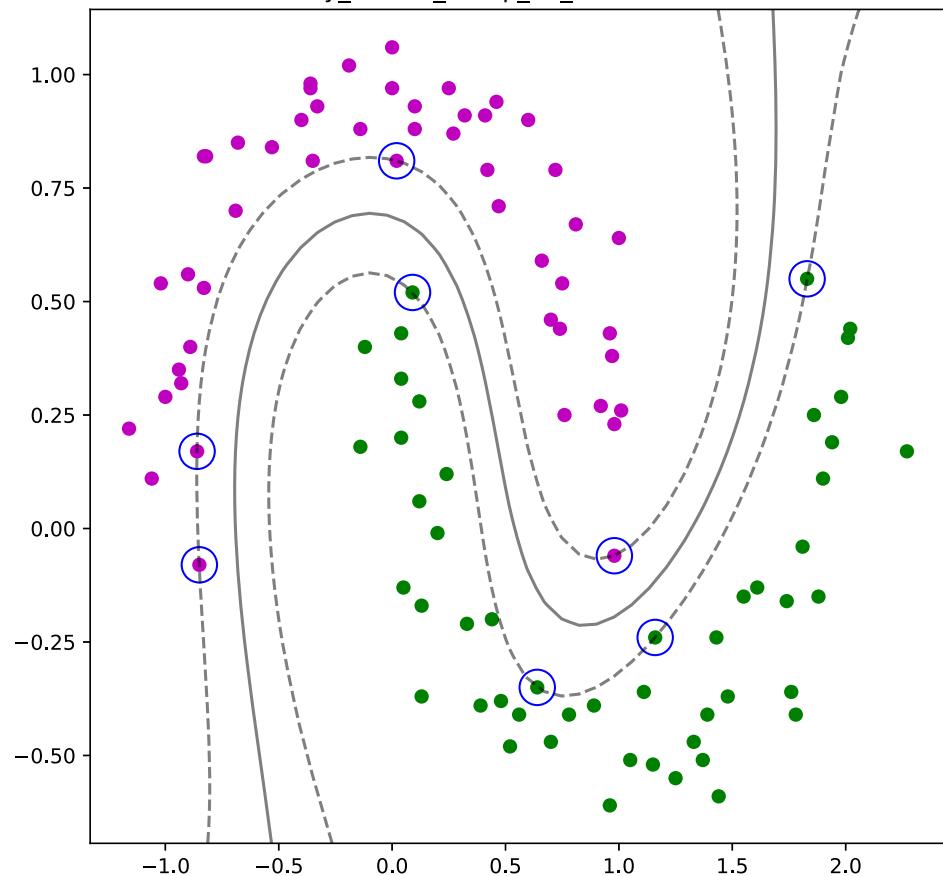
Plot 25: Data = binary\_moons\_linsep\_no\_n100.txt, C = 1000, Kernel = linear



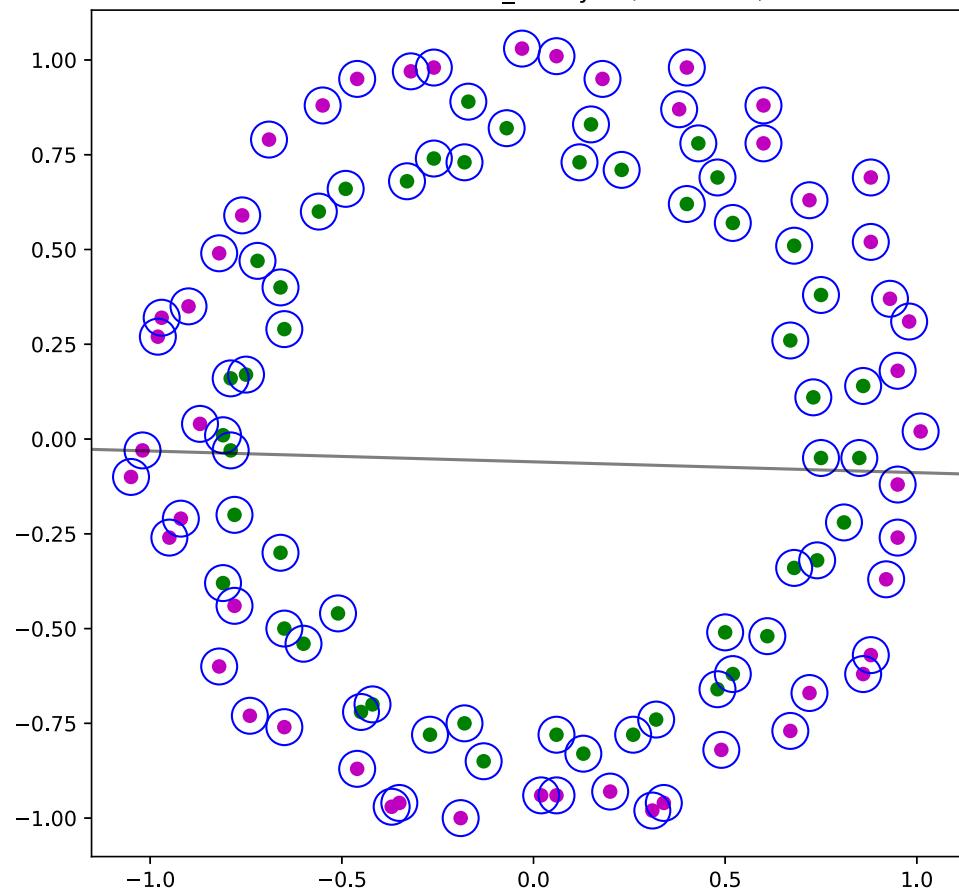
Plot 26: Data = binary\_moons\_linsep\_no\_n100.txt, C = 1000, Kernel = poly



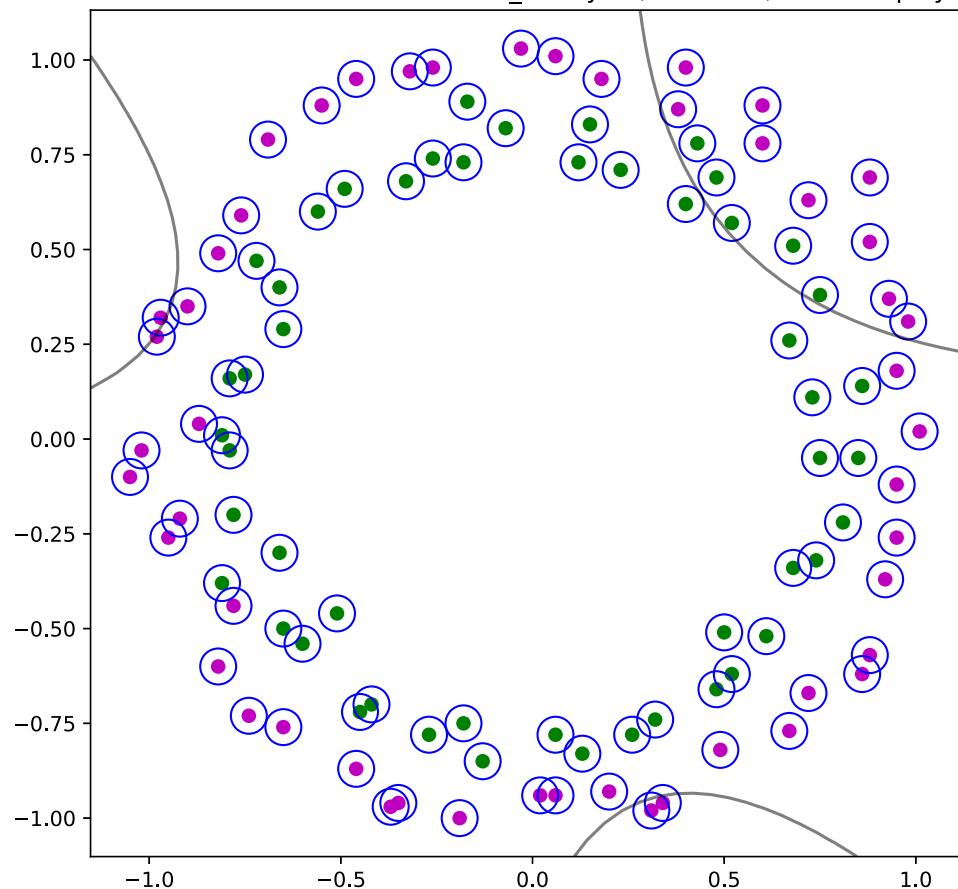
Plot 27: Data = binary\_moons\_linsep\_no\_n100.txt, C = 1000, Kernel = rbf



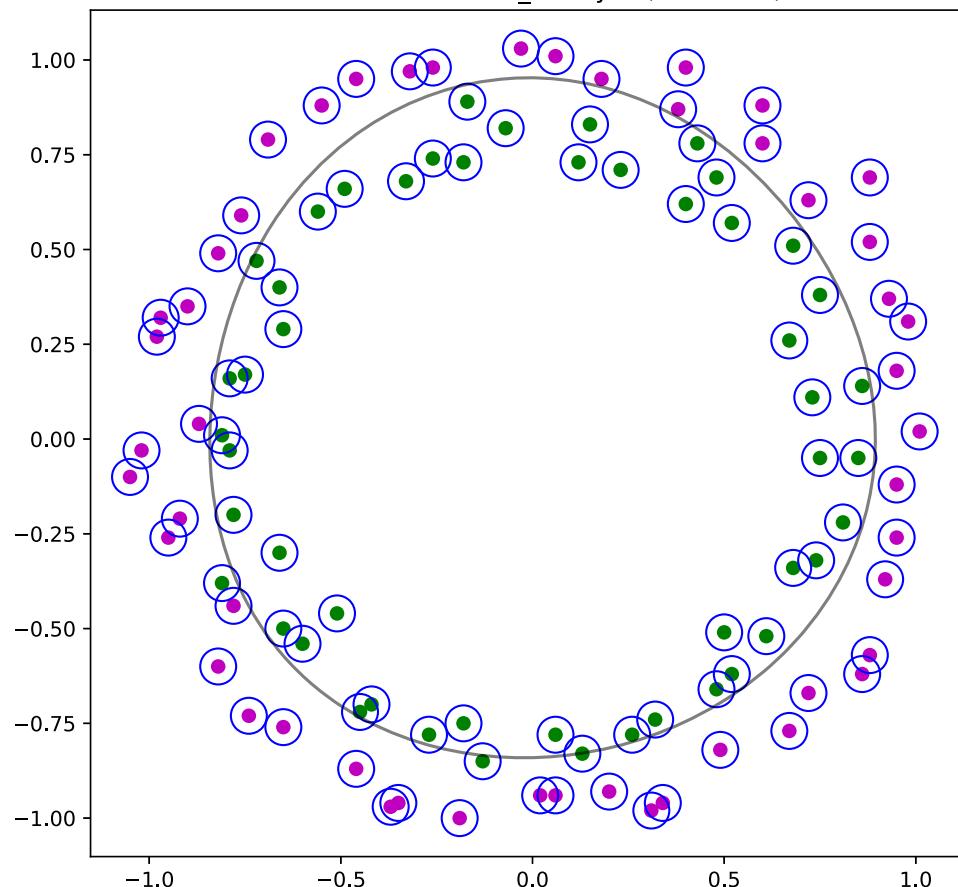
Plot 28: Data = concentriccircles\_binary.txt, C = 0.01, Kernel = linear



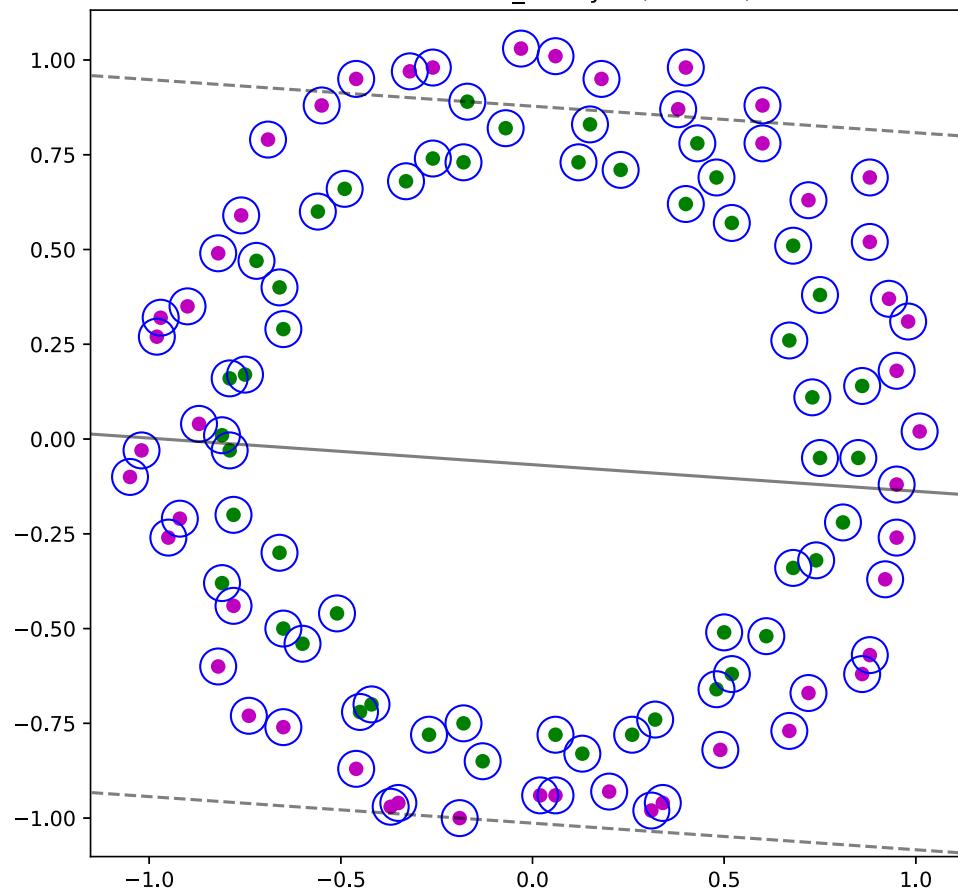
Plot 29: Data = concentriccircles\_binary.txt, C = 0.01, Kernel = poly



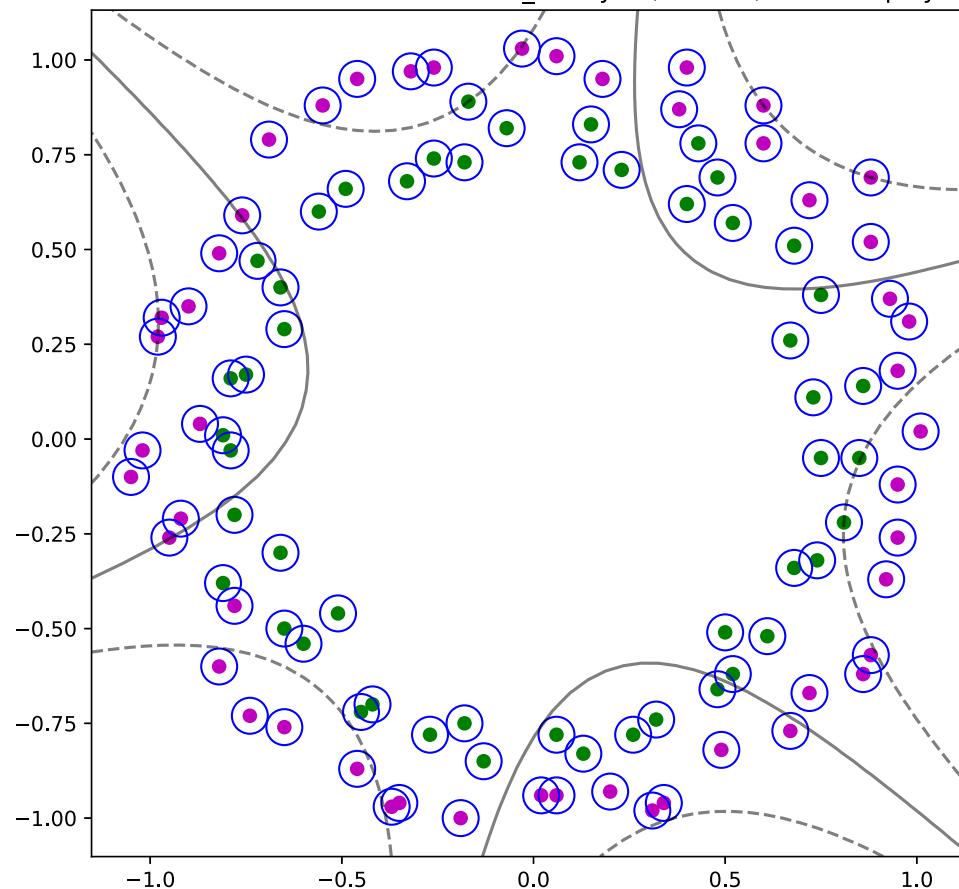
Plot 30: Data = concentriccircles\_binary.txt, C = 0.01, Kernel = rbf



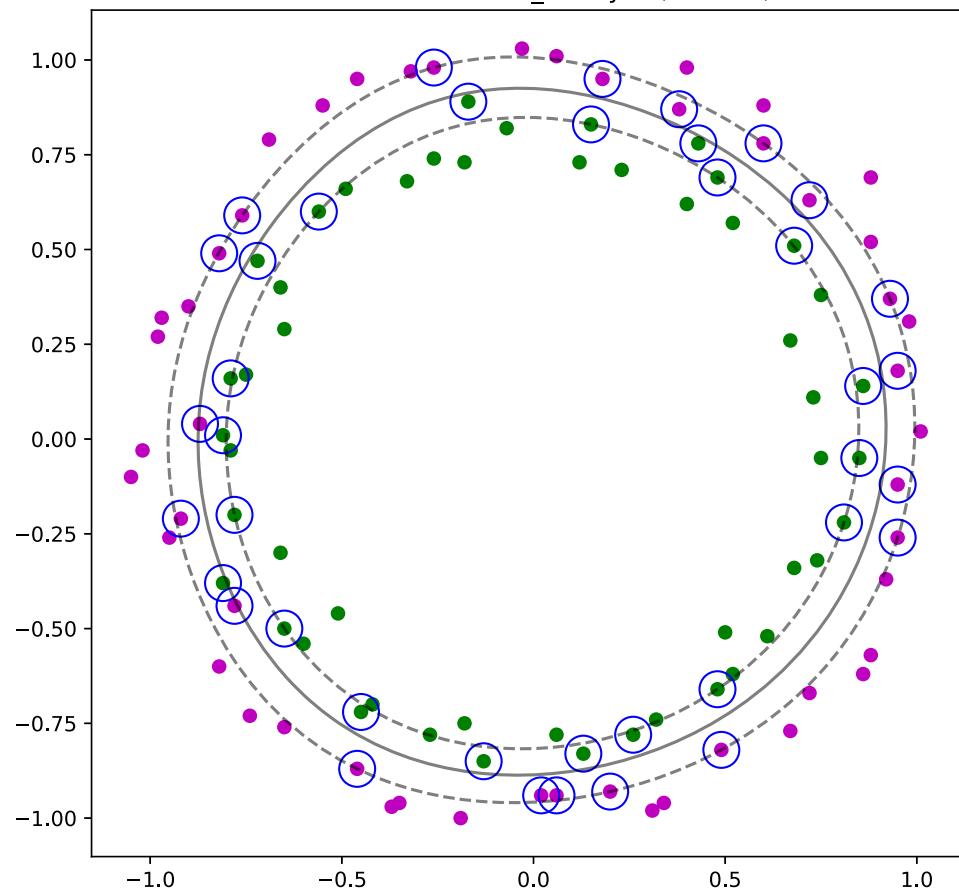
Plot 31: Data = concentriccircles\_binary.txt, C = 10, Kernel = linear



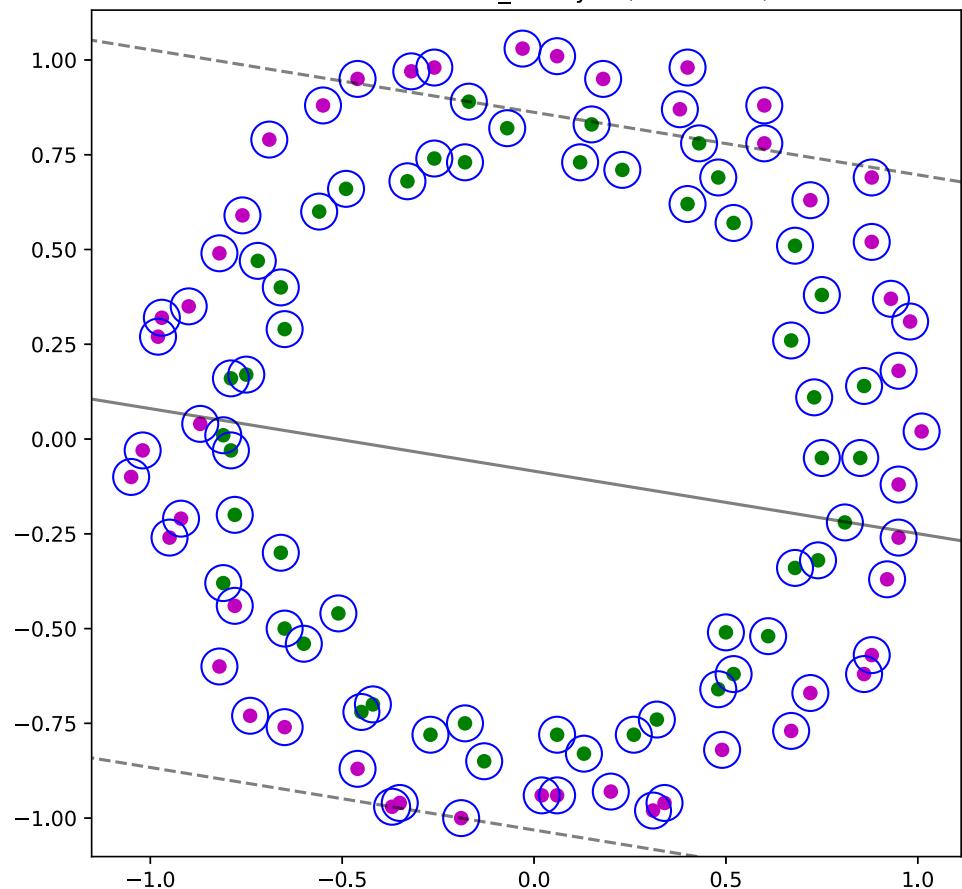
Plot 32: Data = concentriccircles\_binary.txt, C = 10, Kernel = poly



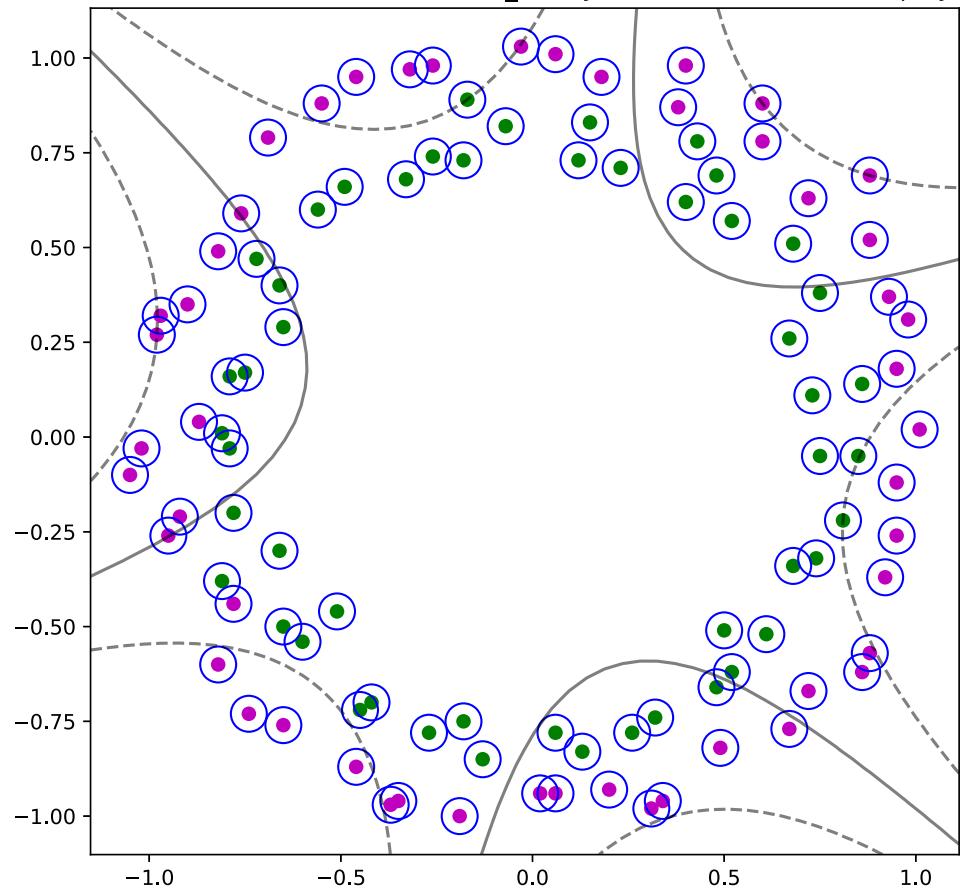
Plot 33: Data = concentriccircles\_binary.txt, C = 10, Kernel = rbf



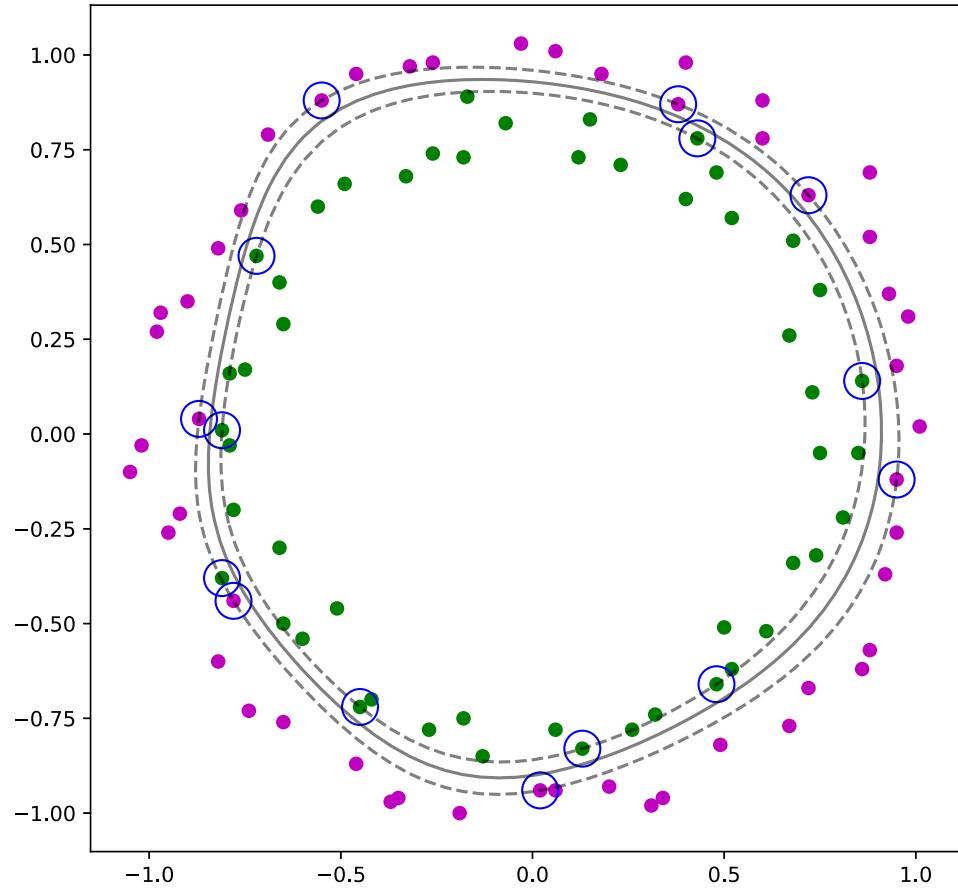
Plot 34: Data = concentriccircles\_binary.txt, C = 1000, Kernel = linear



Plot 35: Data = concentriccircles\_binary.txt, C = 1000, Kernel = poly



Plot 36: Data = concentriccircles\_binary.txt, C = 1000, Kernel = rbf



Support vector values (Shape = (8, 2)) for case of interest:

```
[[ -0.86  0.17]
 [  0.02  0.81]
 [  0.98 -0.06]
 [ -0.85 -0.08]
 [  0.64 -0.35]
 [  0.09  0.52]
 [  1.83  0.55]
 [  1.16 -0.24]]
```

In [ ]:

```
In [1]:  
import numpy as np  
import matplotlib.pyplot as plt  
from mpl_toolkits.mplot3d import Axes3D  
from sklearn.svm import SVR
```

### 3.) Support Vector Regression

```
In [2]:  
""" Helper function to perform data standardization """  
def data_std(X, mean, std):  
    return (X - mean)/std
```

#### Part a.)

```
In [3]:  
""" Load and parse data """  
data = np.genfromtxt("train.txt", delimiter="\t")  
train_X = data[:, 0:2]  
train_y = data[:, 2]  
data = np.genfromtxt("test.txt", delimiter="\t")  
test_X = data[:, 0:2]  
test_y = data[:, 2]  
  
""" Establish standardized data """  
# Mean of training features: (# of features, )  
mean = np.mean(train_X, axis = 0)  
# Standard deviation of training features: (# of features, )  
std = np.std(train_X, axis = 0)  
  
train_X_st = data_std(train_X, mean, std)  
test_X_st = data_std(test_X, mean, std)
```

```
In [4]:  
""" Sklearn SVR model """  
# Default: C = 1, Kernel = rbf  
model = SVR(epsilon=0.0001).fit(train_X_st, train_y)  
pred = model.predict(test_X_st)  
  
""" Compute RMSE """  
RMSE = np.sqrt(np.mean(np.power(test_y - pred, 2)))  
print("RMSE = {}".format(RMSE))
```

RMSE = 24.033860070431174

#### Part b.)

Same as part a.), but loop through permutations of  $C$  and  $\epsilon$

```
In [5]:  
Cs = [0.01, 10, 100]  
eps = [0.0001, 0.01, 1]  
for i in range(len(Cs)):  
    for j in range(len(eps)):  
        model = SVR(C=Cs[i], epsilon=eps[j]).fit(train_X_st, train_y)  
        pred = model.predict(test_X_st)
```

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```
RMSE = np.sqrt(np.mean(np.power(test_y - pred, 2)))
print("C = {}, epsilon = {} -> RMSE = {}".format(Cs[i], eps[j], RMSE))
```

```
C = 0.01, epsilon = 0.0001 -> RMSE = 49.86230821904665
C = 0.01, epsilon = 0.01 -> RMSE = 49.86230821904665
C = 0.01, epsilon = 1 -> RMSE = 49.719198470155014
C = 10, epsilon = 0.0001 -> RMSE = 7.206613901052269
C = 10, epsilon = 0.01 -> RMSE = 7.2030835074726145
C = 10, epsilon = 1 -> RMSE = 7.195908569734601
C = 100, epsilon = 0.0001 -> RMSE = 4.356151814584229
C = 100, epsilon = 0.01 -> RMSE = 4.354976661414509
C = 100, epsilon = 1 -> RMSE = 4.189717810970863
```

The higher penalty strengths  $C$  are much more effective in reducing the classification error, however, only to a certain value before the model is overfitted. While keeping  $C$  constant, it seems that a larger epsilon value yields less RMSE.

## Part c.)

In [6]:

```
""" Create feature mesh and standardize the feature mesh """
xx, yy = np.meshgrid(np.linspace(test_X[:, 0].min(), test_X[:, 0].max(), 100), np.linspace(data_std(np.c_[xx.ravel(), yy.ravel()]), mean, std))

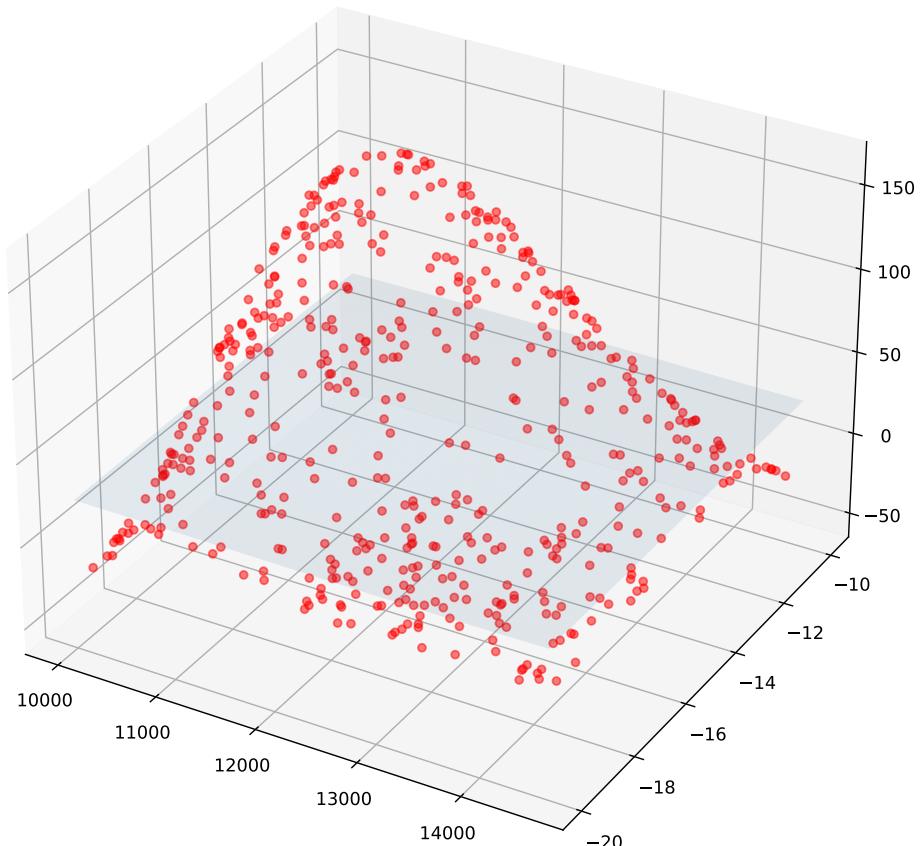
""" Same as part b.), but loop through permutations of C and epsilon """
Cs = [0.01, 10, 100]
eps = [0.0001, 0.01, 1]
count = 0
for i in range(len(Cs)):
    for j in range(len(eps)):
        model = SVR(C=Cs[i], epsilon=eps[j]).fit(train_X_st, train_y)
        pred = model.predict(feat_mesh)

        """ Plot each regression model as a surface """
        fig = plt.figure(count, figsize = (7.5, 7.5))
        ax = Axes3D(fig)
        ax.plot_trisurf(xx.flatten(), yy.flatten(), pred, linewidth=0, antialiased=False)
        ax.scatter3D(test_X[:, 0], test_X[:, 1], test_y, c='r', alpha=0.5)
        plt.title("C = {}, epsilon = {}".format(Cs[i], eps[j]))
        plt.show()
        count += 1
```

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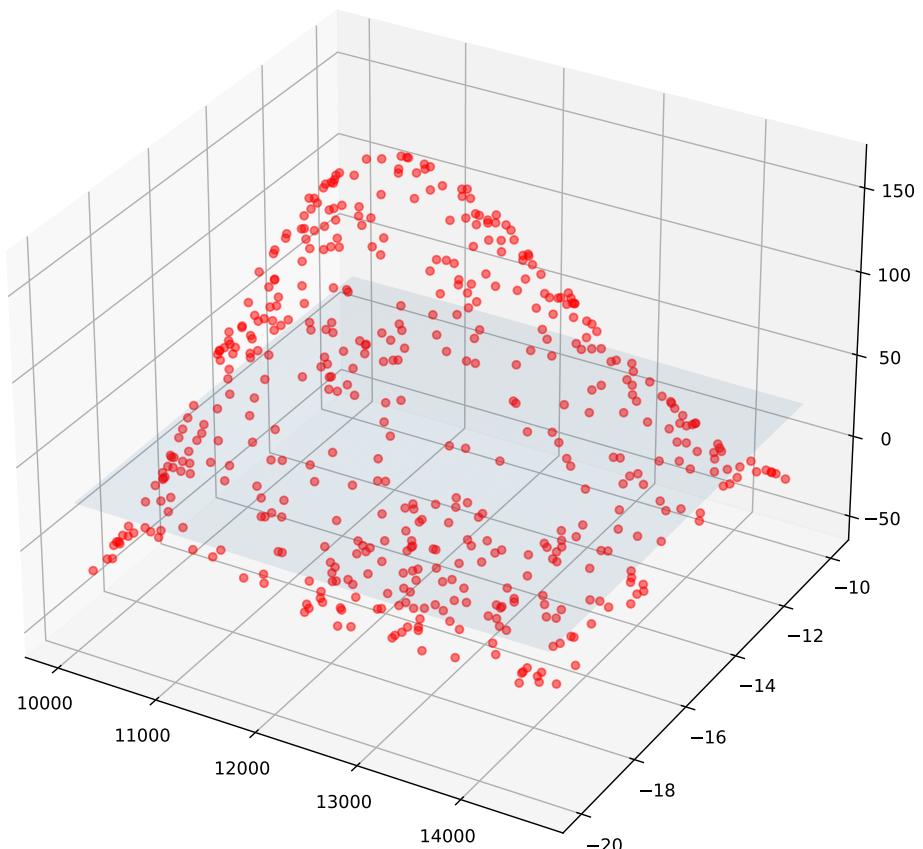
$C = 0.01$ ,  $\epsilon = 0.0001$



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q3

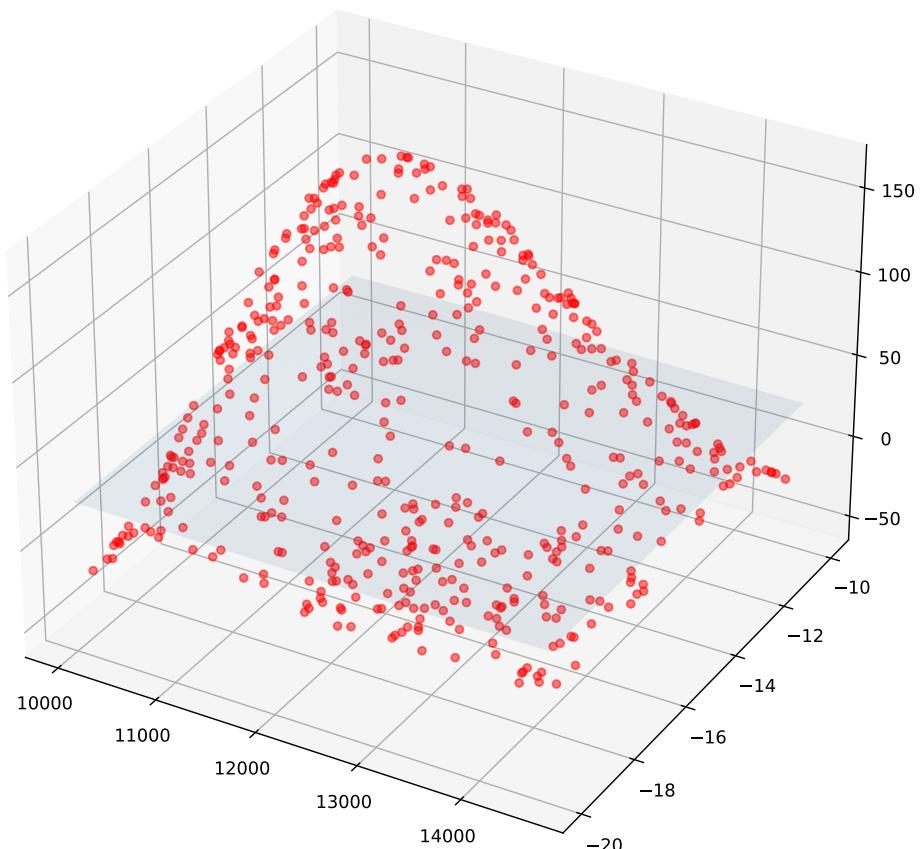
$C = 0.01$ ,  $\epsilon = 0.01$



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q3

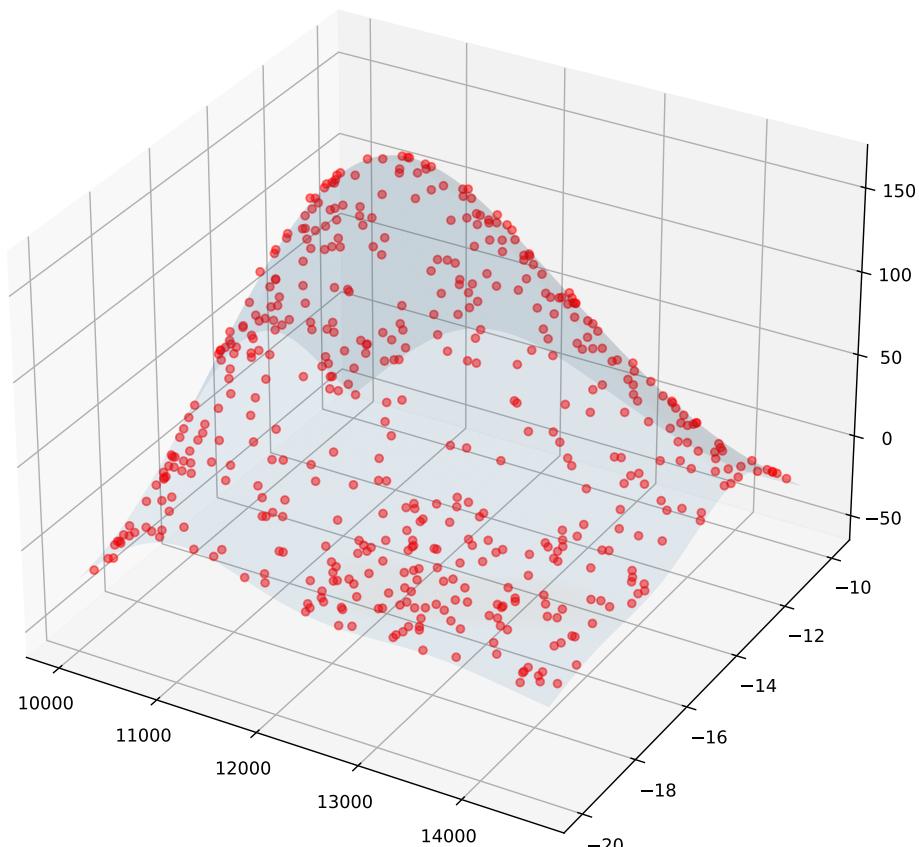
$C = 0.01$ ,  $\epsilon = 1$



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q3

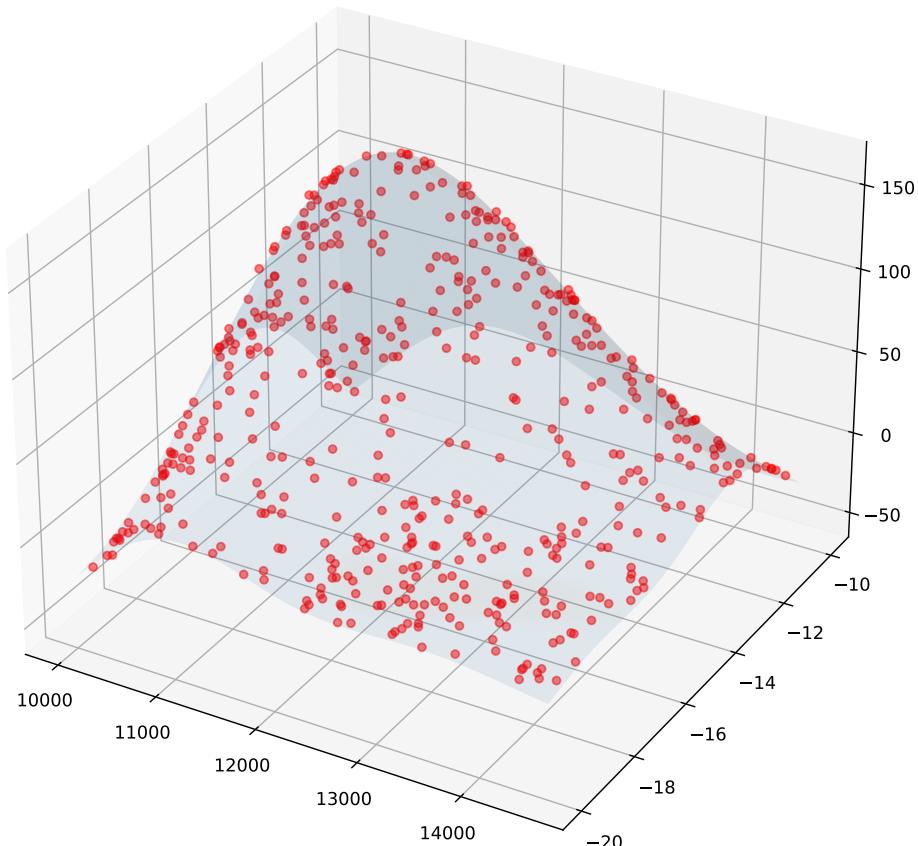
C = 10, epsilon = 0.0001



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q3

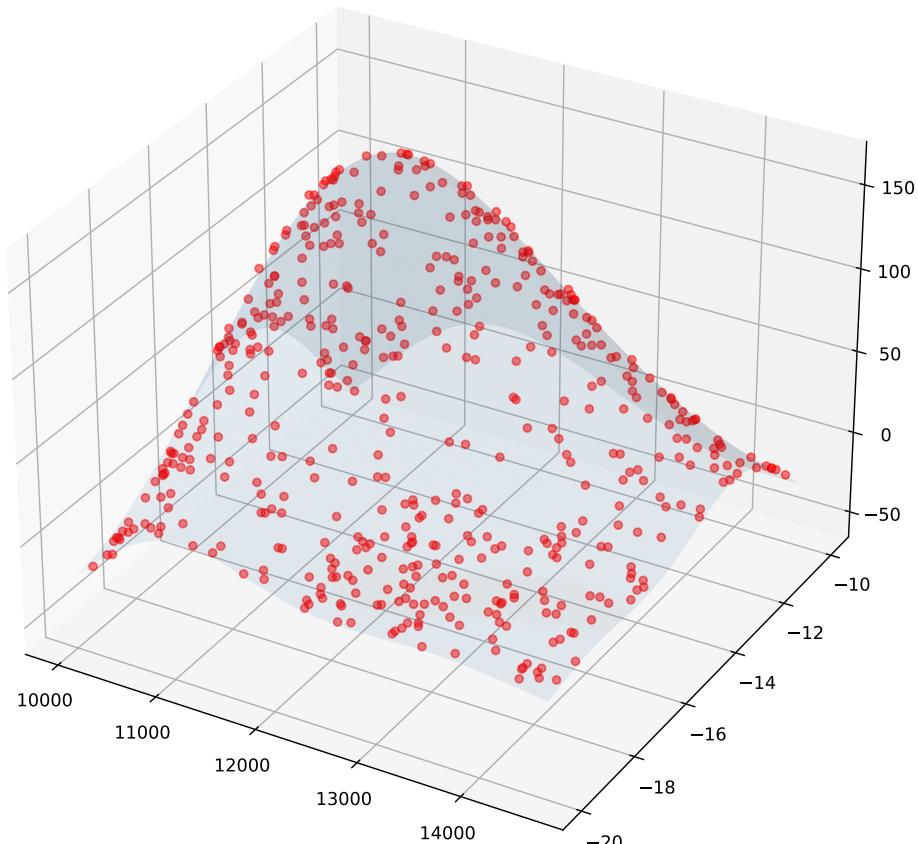
$C = 10$ ,  $\epsilon = 0.01$



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q3

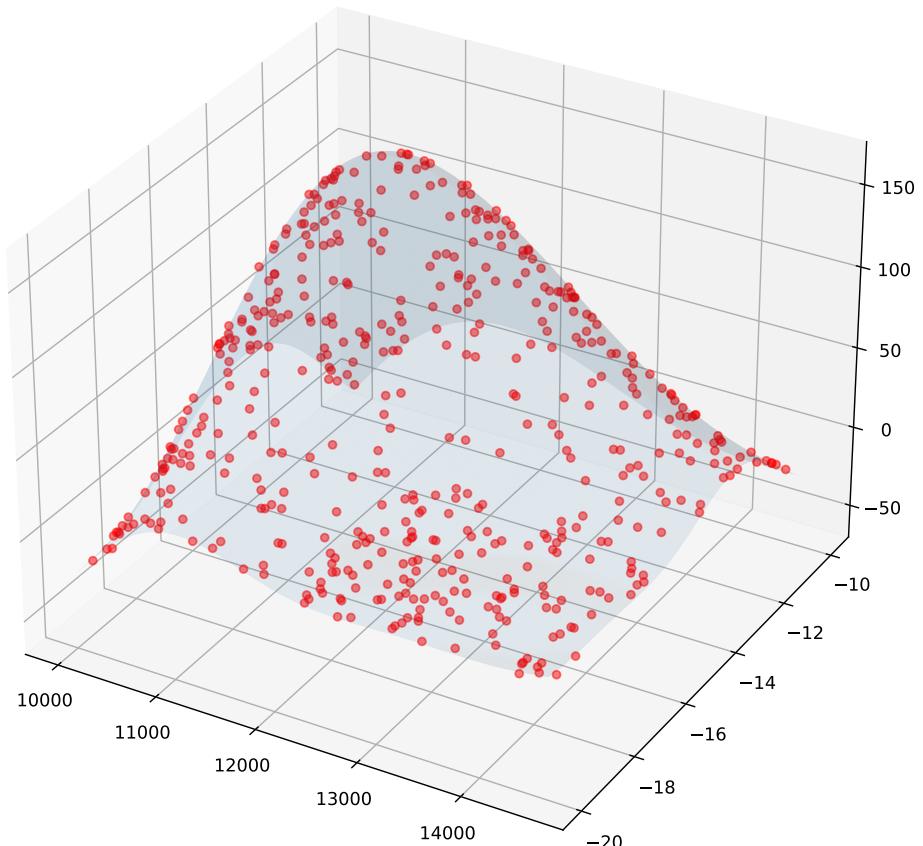
$C = 10, \epsilon = 1$



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q3

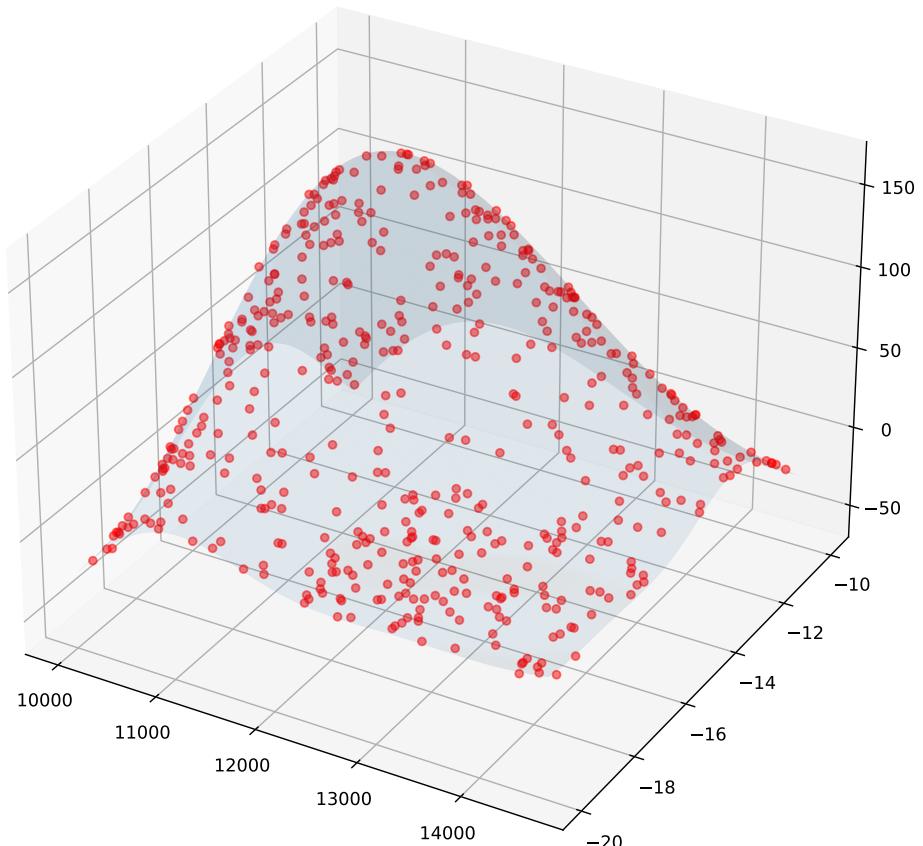
C = 100, epsilon = 0.0001



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q3

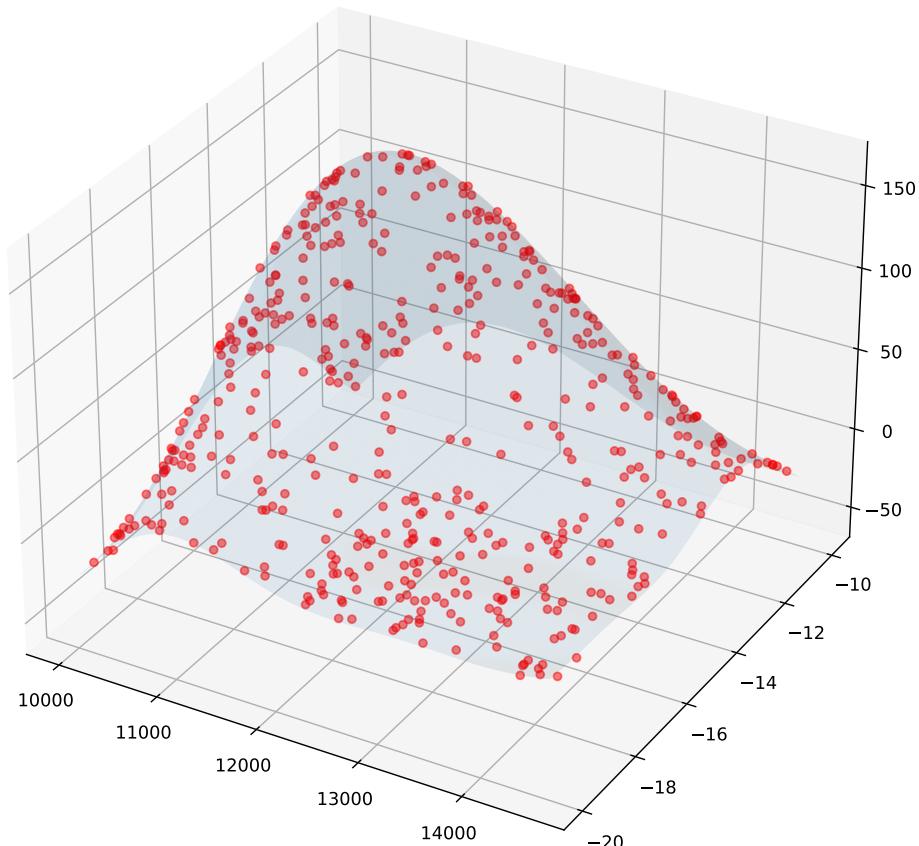
C = 100, epsilon = 0.01



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q3

C = 100, epsilon = 1



In [ ]:

## Homework 6 Written Answers

### Exercise 1c.

From the 4 different penalty strengths  $C$  that are observed above, there seems to be little difference shown across the 4 graphs. The general trend for the classification boundaries of the nonlinear data set is that the main boundary and +1 margin both shift to the right along +x axis, all the while -1 margin seemingly stays put. This behavior shows that the actual margin is shrinking as the value for  $C$  goes up, and alludes to a greater penalty for having a large margin.

### Exercise 2b.

Based on the observations from the graphs above for SVC, it seems like the  $rbf$  kernel provides the best classification boundary contour. In terms of penalty strengths  $C$ , the number of points within the boundary decreases as the value of  $C$  increases. This is true because we are enforcing a harder (narrower) margin in the model. For the Moons dataset, I will pick the one with least support vectors (100 support vectors) as that model is more generalizable onto other data sets being tested.