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Space Physics
Practical 2.5
Kinetic Theory

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SpaceMaster

1. Write a function that gets mass (m), density (n), temperature(T), bulk velocity (u) and particle's velocity (v) and calculates the Maxwellian distribution function (f_{sM}). [0.5p]

```
function f = maxwellian(m, n, T, u, v)
kb = 1.38066e-23; % Boltzmann constant

%%%%%% WRITE YOUR CODE HERE %%%%%%%%
f = n*((m/(2*pi*kb*T))^(3/2))* exp(-1 * (m *(v-u)*(v-u)')/ (2*kb*T) );
end
```

2. Assume that the solar wind protons number density (n) is $3*10^6$ m^{-3}, the temperature (T) is 10^6 K and the bulk velocity (u) is $3*10^5$ m/s. Bin the velocity space in one-dimension (e.g. along x) equally from 0 to $6*10^5$ m/s with velocity step $\Delta v=5*10^3$ m/s. Put a proton at each binned velocity and find the proton distribution function. Plot the distribution function and interpret your observations. [0.5p]

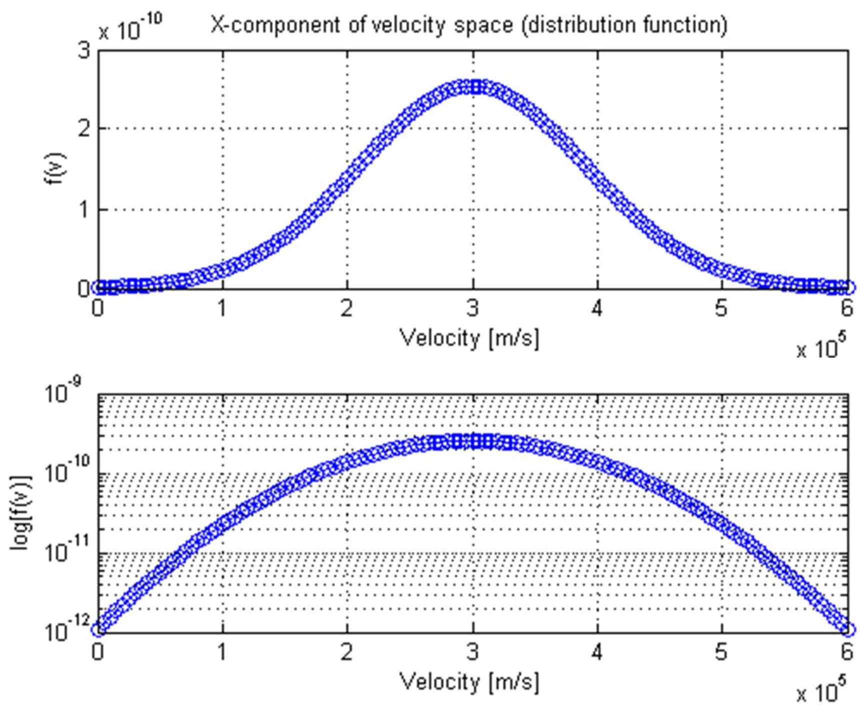

Figure 1: Distribution function with velocity step = $5*10^3$ m/s.

The plots show the Maxwellian distribution values as a function of velocity. As it can be seen the velocity step value used on the distribution provides a good undisturbed curve that allowed estimating the distribution value for a rather large number of points and therefore being able to appreciate the progression of the distribution as the value of velocity increases. It can be seen that largest value of the distribution is obtained at a velocity of $3*10^5$ m/s which is the middle value of the entire velocity range. This means that the distribution is in fact a Maxwellian distribution with the largest value at the middle point and the decreasing equally towards the extreme range values.

*3. Run your code for different velocity steps (e.g. Δv=2*10^4 m/s, Δv=6*10^4 m/s and Δv=9*10^4 m/s) and discuss the results. [0.5p]*

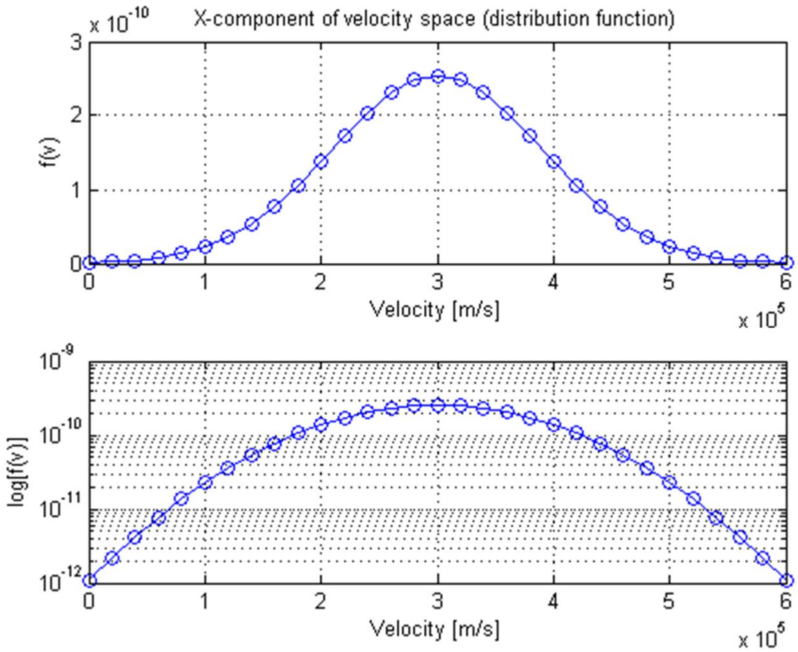

Figure 2: Distribution function with velocity step = 2*10^4 m/s.

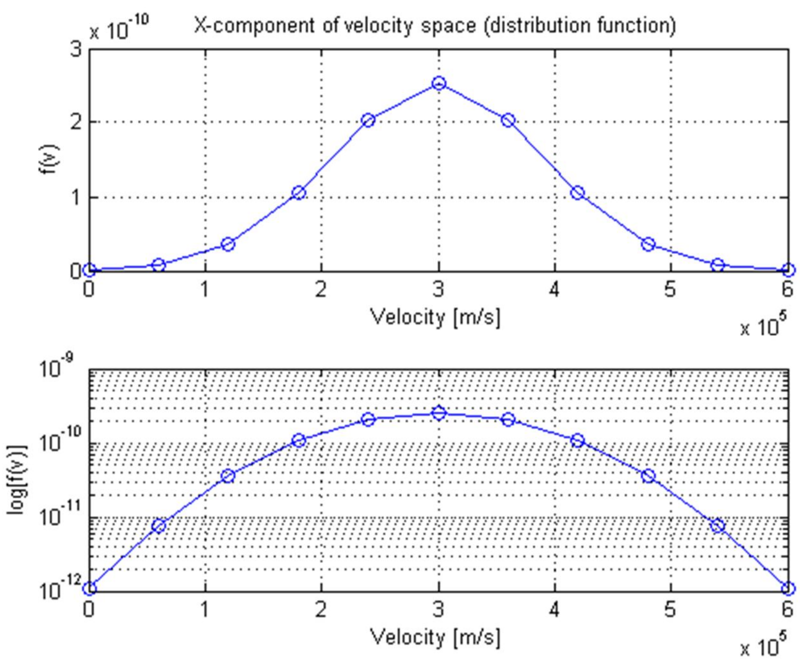

Figure 3: Distribution function with velocity step = 6*10^4 m/s.

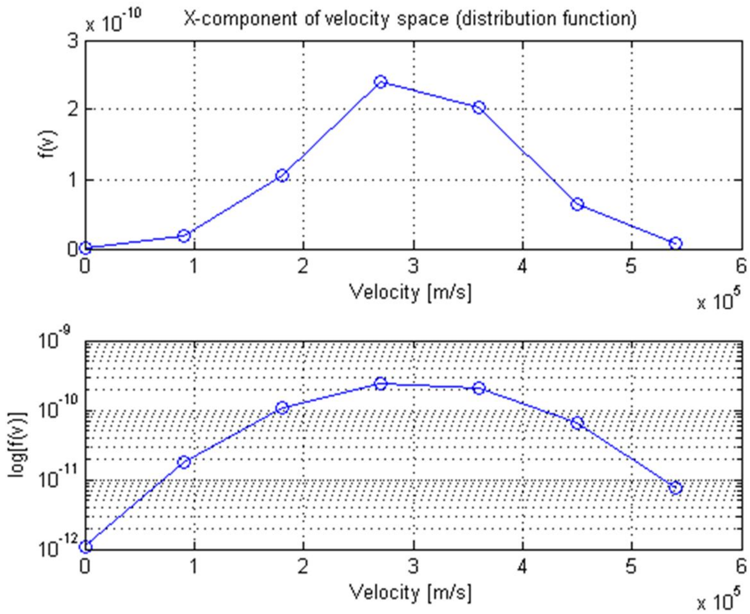

Figure 4: Distribution function with velocity step = $9*10^4$ m/s.

The three plots above show the same Maxwellian distribution with different step velocities. It can be seen how as the step velocity increases the smoothness of the curve decreases. The plots continue showing a bell shape however, this shape deteriorates as the step value becomes too large and the middle range value is not the highest value anymore (figure 4). A small step value might be more recommendable since it provides more information about the distribution and gives a better approximation of the distribution behavior.

4. Now we want to test 3D velocity space. Download the code from: http://sspt.irf.se/Members/shahab/practical3/vsd.m Try to understand the code and run it. Assume non-drifting solar wind protons have the density of $8*10^6$ m^{-3} and the temperature of 10^5 K. Bin the velocity space from -200 to 200 km/s with $\Delta v = 10$ km/s and find the 3D Maxwellian velocity space distribution.

After running the code, the Maxwellian velocity space distribution is stored in the variable f which is a matrix of the size 41X41X41.

5. We already know that:

$$n = \int_{-\infty}^{+\infty} f_{xM}(\boldsymbol{r}, \boldsymbol{v}, t) d^3\boldsymbol{v} \equiv \sum_{v_i, v_j, v_k} f(v_i, v_j, v_k) \Delta v_i \Delta v_j \Delta v_k$$

How much is the calculated number density (use above equation)? Compare it with the solar wind number density. [0.5p]

When using the above equation, the calculated number density is $8*10^6$ which is the same value as the defined solar wind proton density. The fact that both values match means that the selected step velocity provides a good resolution for the Maxwellian distribution.

6. Change the solar wind number density to $n=5*10^6$ m^{-3}. Run the code and discuss the results. [0.5p]

After changing the solar wind number density and running the code we still get that the calculated number density is the same as the defined solar wind proton velocity. This means that the selected step velocity still provides reliable data. However, if we were to continue increasing the step velocity instead of reducing we would have noticed discrepancies between the estimated density and the defined one, these discrepancies would have increased with the increased step velocity.

7. Plot the distribution function in the v_x- v_y plane, using MATLAB surf command. [0.5p]

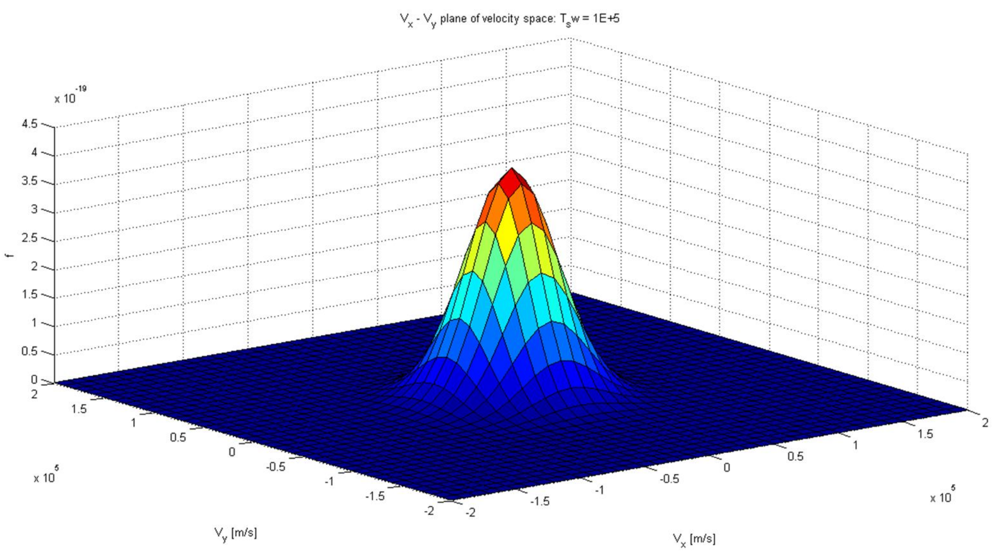

Figure 5: Distribution function in the v_x- v_y plane with temperature of 10^5 K.

The calculated number density for this case, as previously stated, is $5*10^6$ m^{-3}.

8. Plot the distribution function in the v_x-v_y plane and compare it with distribution function in the v_y – v_z plane, using MATLAB contour command. [0.5p]

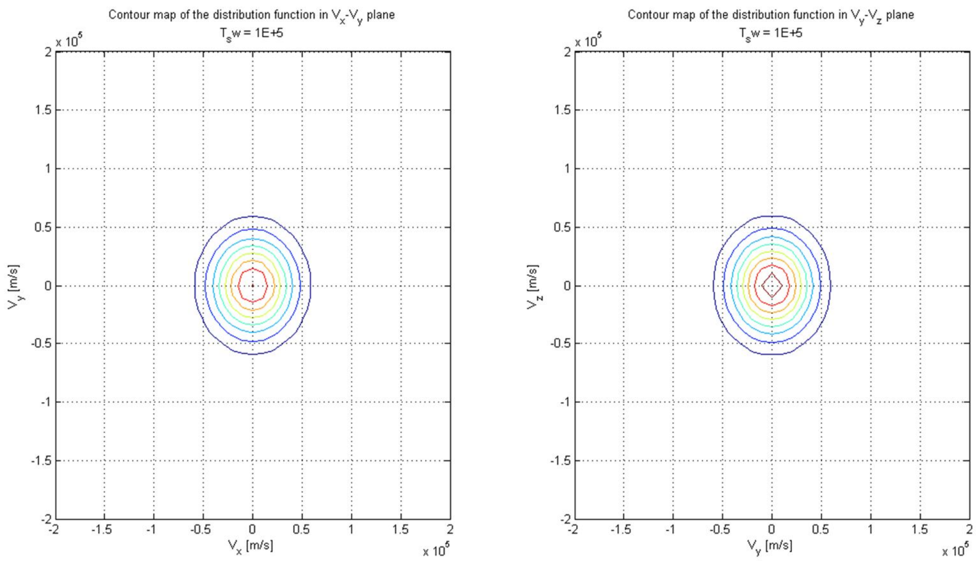

Figure 6: Distribution functions in the v_x- v_y and v_y- v_z planes with temperature of 10^5 K

As it can be seen from the plots, the distribution on both planes is the same without variations between them.

9. Calculate the solar wind protons number density for $T=5*10^5$ K and $T=20*10^5$ K. What are the densities you get and why are they different? How can we solve this problem? [0.5p]

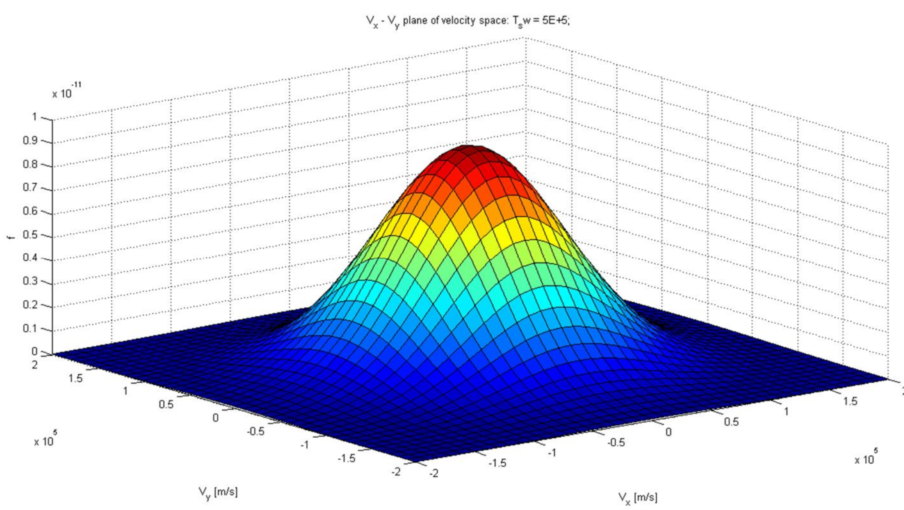

Figure 7: Distribution function in the v_x- v_y plane with temperature of $5*10^5$ K.

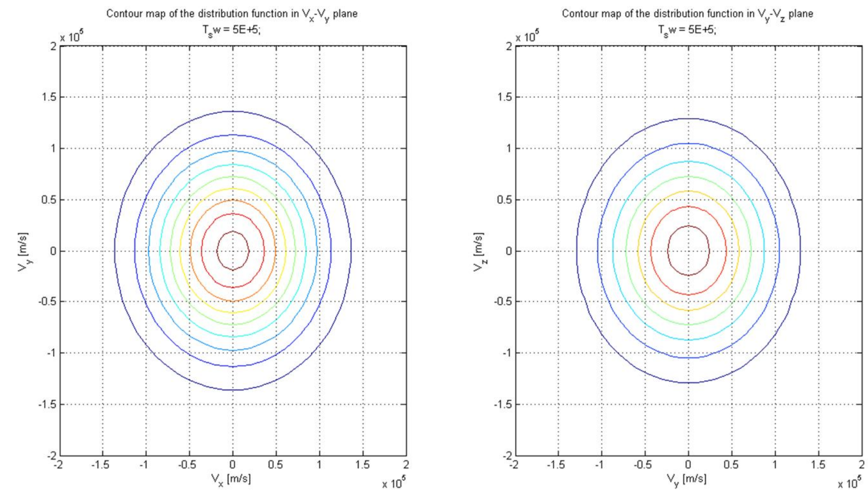

Figure 8: Distribution functions in the v_x- v_y and v_y- v_z planes with temperature of $5*10^5$ K

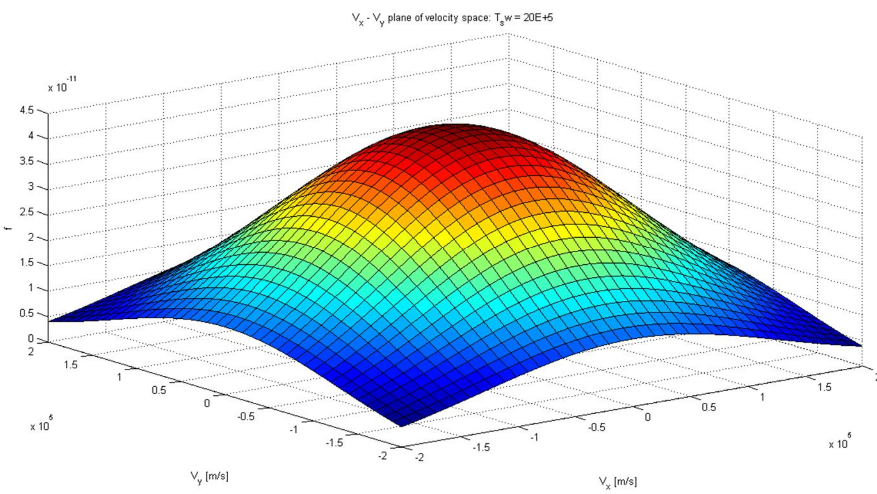

Figure 9: Distribution function in the v_x- v_y plane with temperature of $20*10^5$ K.

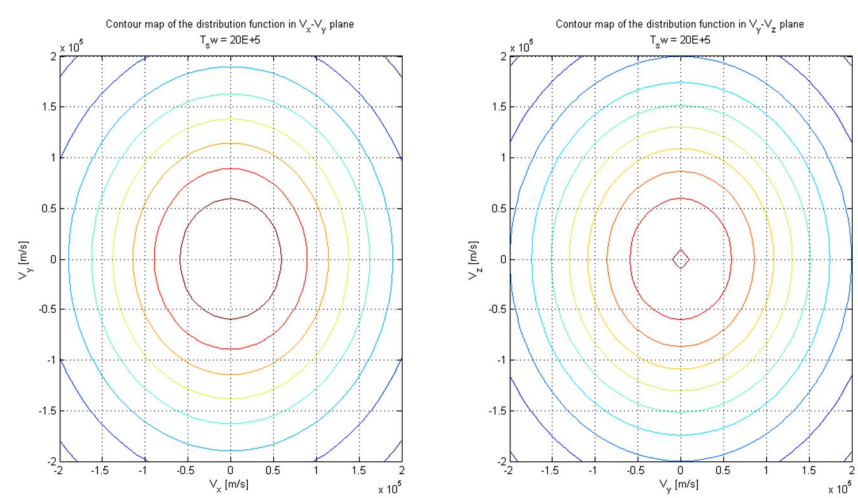

Figure 10: Distribution functions in the v_x- v_y and v_y- v_z planes with temperature of $20*10^5$ K

The calculated solar wind proton number density for $T=5*10^5$ K is $4.9789*10^6$ m^{-3} and for $T=20*10^5$ K is $3.5187*10^6$ m^{-3}. As it can be seen from the above plots and the calculated density values, these values are smaller than the calculated density for $T=10^5$ K. This shows that as we increase the temperature value the density values start to decrease; this is due to the fact that the size of the distribution function increase and the defined velocity range is not enough in order to provide accurate data. The contour plots (figure 8 and 10) show a bigger diameter of the distribution as compared to the contour plot for $T=10^5$ K (figure 6).

10. Do it by your own! Download your group '.mat' file (e.g. group N should download the file 'groupN.mat') from: ftp://ftp.irf.se/pub/tmp/outgoing/shahab/
Each file corresponds the solar wind protons velocity space distributions obtained from a particle simulation code for an observer located at ~1.0 AU distance from the Sun. The simulated observer collects only the solar wind protons with directional velocity range from 0 to $8*10^5$ m/s and velocity resolution of $5*10^3$ m/s in all directions. The magnetic field is constant (+5 $\hat{y}$ nT), and the solar wind flows along -x axis.

(a) Load the file into MATLAB. (the velocity space distribution values are stored in 'f').

load('groupN.mat');

A variable called F is created and the result is a matrix of 321x321x321.

(b) Calculate the solar wind protons number density. [0.5p]

The calculated solar wind proton number density for this data is $7.099*10^6$ m^{-3}.

(c) Calculate the solar wind protons fluxes in all directions and find the solar wind bulk flow velocity. (Attach your MATLAB code to the report) [1p]

The calculated solar wind proton flux is the following:

- x-direction: $-2.485*10^{12}$ [$1/m^2s$]
- y-direction: 0.0 [$1/m^2s$]
- z-direction: 0.0 [$1/m^2s$]

The calculated solar wind bulk flow velocity is the following:

- x-direction: $-3.5*10^5$ m/s
- y-direction: 0.0 m/s
- z-direction: 0.0 m/s

The MatLab code for the entire task 10 is included at the end of the report as an attachment. The section entitled “Calculation of the flux and bulk velocity” describes how the above values were calculated.

(d) Plot the distribution function in the v_x-v_y plane and compare it with distribution function in the v_y-v_z plane. The center of the distribution contours should clearly show the bulk velocity in each plane. [1p]

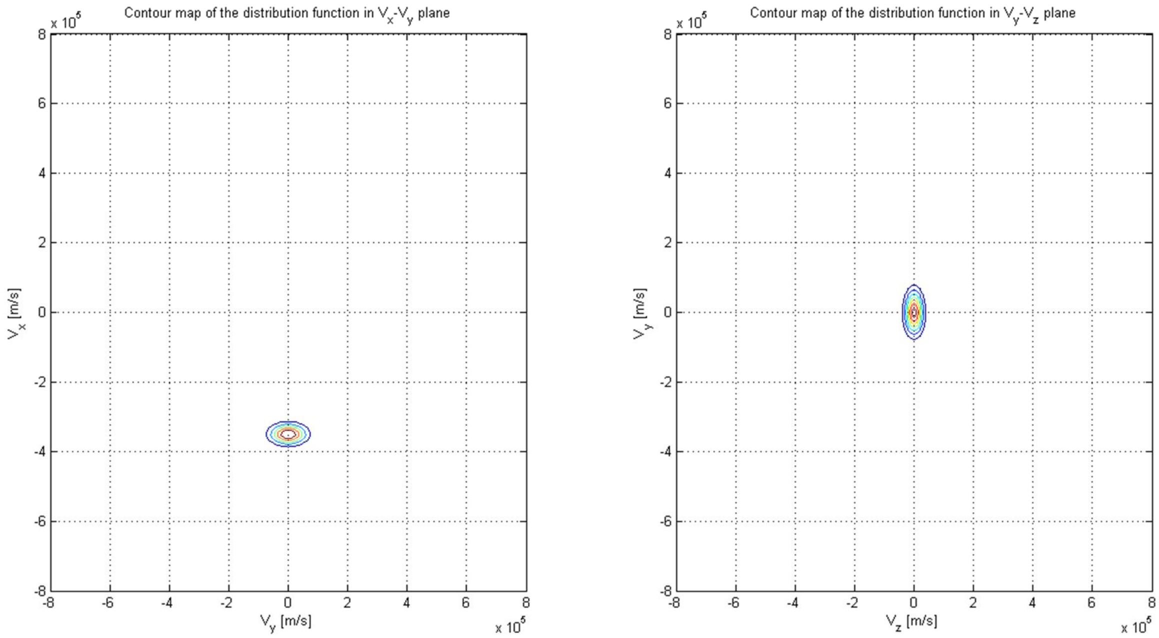

Figure 11: Distribution functions in the v_x- v_y and v_y- v_z planes

The above plots shows the distribution functions for the v_x- v_y and v_y- v_z planes, it can be seen that the calculated velocities for v_y and v_z are zero, as calculated on the previous question and that for v_x the bulk velocity is -3.5*10^5. This is the reason that the contour plot is not at zero, there is a drift velocity on v_x.

(e) What is the type of the distribution function, Maxwellian or Bi-Maxwellian? Why? [0.5p]

The type of the distribution function is Bi-Maxwellian. The reason can be seen on figures 11 and 12 (below); the contour plots do not show a perfect circle for the distribution function of the v_x- v_y and v_y- v_z planes in figure 11 however it is a perfect circle for the v_x- v_z plane in figure 12. It can be seen that the distribution function stretches on the y-direction. The reason been that the distribution is parallel to the magnetic field direction which act on the y-direction too.

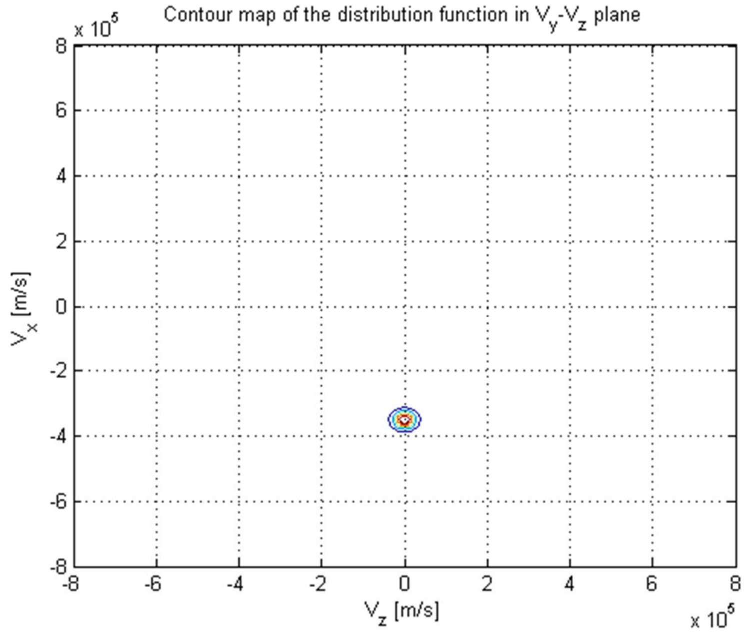

Figure 12: Distribution function in the v_x- v_z plane

(f) Calculate the pressure tensor and scalar pressure. (Attach your MATLAB code to the report)[1.5p]

The calculated pressure tensor is the following:

- x-direction: $4.5581*10^{-12}$ Pa
- y-direction: $1.8233*10^{-11}$ Pa
- z-direction: $4.5581*10^{-12}$ Pa

The calculated scalar pressure is $9.1164*10^{-12}$ Pa.

The MatLab code for the entire task 10 is included at the end of the report as an attachment. The section entitled “Pressure Tensor” describes how the above values were calculated.

(g) Find the solar wind thermal velocity and the solar wind kinetic temperature in all directions. [1.5p]

The calculated solar wind thermal velocity is the following:

- x-direction: $0.4634*10^{-22}$ m/s
- y-direction: $0.9269*10^{-22}$ m/s
- z-direction: $0.4634*10^{-22}$ m/s

The calculated solar wind kinetic temperature is the following:

- x-direction: $0.4650*10^{5}$ K
- y-direction: $0.4650*10^{5}$ K
- z-direction: $0.4650*10^{5}$ K

Appendix

MatLab code for task 10

```
clear all;                 % freeing up system memory
clc;                       % Clear Command Window

%% CONSTANT VALUES
unit_mass   = 1.6726E-27; % mass of a proton
unit_charge = 1.6022E-19; % unit of charge
k_b = 1.38065E-23;


%% PARTICLES MASS
mass  = 1.0*unit_mass;    % mass of a particle


%% VELOCITY SPACE DEFINITION (Observer c/cs)
dv   = 5E+3;             % velocity step
v_x = -8E+5:dv:8E+5;
v_y = -8E+5:dv:8E+5;
v_z = -8E+5:dv:8E+5;


%% Loading f(v)
load('group11.mat'); %the file group.mat should be in the same directory


%% Compute number density
% integration over x
n = sum(sum(sum(f)))*(dv^3);


%% Calculation of the flux and bulk velocity
% using the given forrmula
x_comp = 0;
y_comp = 0;
z_comp = 0;
for i=1:length(v_x)
for j=1:length(v_y)
for k=1:length(v_z)
x_comp = x_comp + v_x(i)*f(i, j, k);
y_comp = y_comp + v_y(j)*f(i, j, k);
z_comp = z_comp + v_z(k)*f(i, j, k);
end
end
end
flux_vec = [x_comp*(dv^3) y_comp*(dv^3) z_comp*(dv^3)]
sw_v = flux_vec/n
```

```matlab
%% Velocity space
figure
subplot(1,2,1)
%cut_page = (v_x(1,1)- sw_v(1,1))/dv;
%cut_page = abs(cut_page);
contour(v_x,v_y,f(:,:,161))
xlabel('V_y [m/s]')
ylabel('V_x [m/s]')
title('Contour map of the distribution function in V_x-V_y plane')
grid on
subplot(1,2,2)
x=f(161,:,:);
x = reshape(x,321,321);
contour(v_y,v_z,x)
xlabel('V_z [m/s]')
ylabel('V_y [m/s]')
title('Contour map of the distribution function in V_y-V_z plane')
grid on
figure
x=f(:,161,:);
x = reshape(x,321,321);
contour(v_x,v_z,x)
xlabel('V_z [m/s]')
ylabel('V_x [m/s]')
title('Contour map of the distribution function in V_y-V_z plane')
grid on


%% Pressure Tensor
ptensor = zeros(3,3);
for i=1:length(v_x)
for j=1:length(v_y)
for k=1:length(v_z)
c=sw_v-[v_x(i) v_y(j) v_z(k)];
C = mtimes(transpose(c),c);
ptensor= ptensor + C * f( i,j,k);
end
end
end
ptensor = ptensor * (dv^3)* mass;
scalar_p = trace(ptensor)/3.0;


%% Thermal Velocity
T = ptensor /(n*k_b)
v_th = sqrt((2*ptensor/n*unit_mass))
```