



5th NCSP DKIST Data-Training Workshop: He I Diagnostics in the Solar Atmosphere



Modeling He I 10830/D3 spectra using **slab model**

Ivan Milic (CU Boulder / LASP) ; ivan.milic@colorado.edu

Brief recap of today's lectures (and past schools)

- Spectrum is the distribution of the intensity (i.e. number of photons) over wavelengths
- To synthesize (model, calculate) a spectrum, we need to calculate the intensity for the desired set of wavelengths
- To do that, we need to solve radiative transfer equation, throughout the whole atmosphere
- And for that, we need to find opacity and emissivity on a given geometric grid
- That is, as we have heard, often very complicated!

Radiative Transfer Equation (RTE):

$$\frac{I_\lambda}{ds} = -\chi_\lambda I_\lambda + j_\lambda$$

Or, introducing the so called optical depth scale, we can cast it as:

$$\text{Intensity} \quad \frac{I_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda \quad \text{Source function}$$

Optical Depth (our new coordinate)

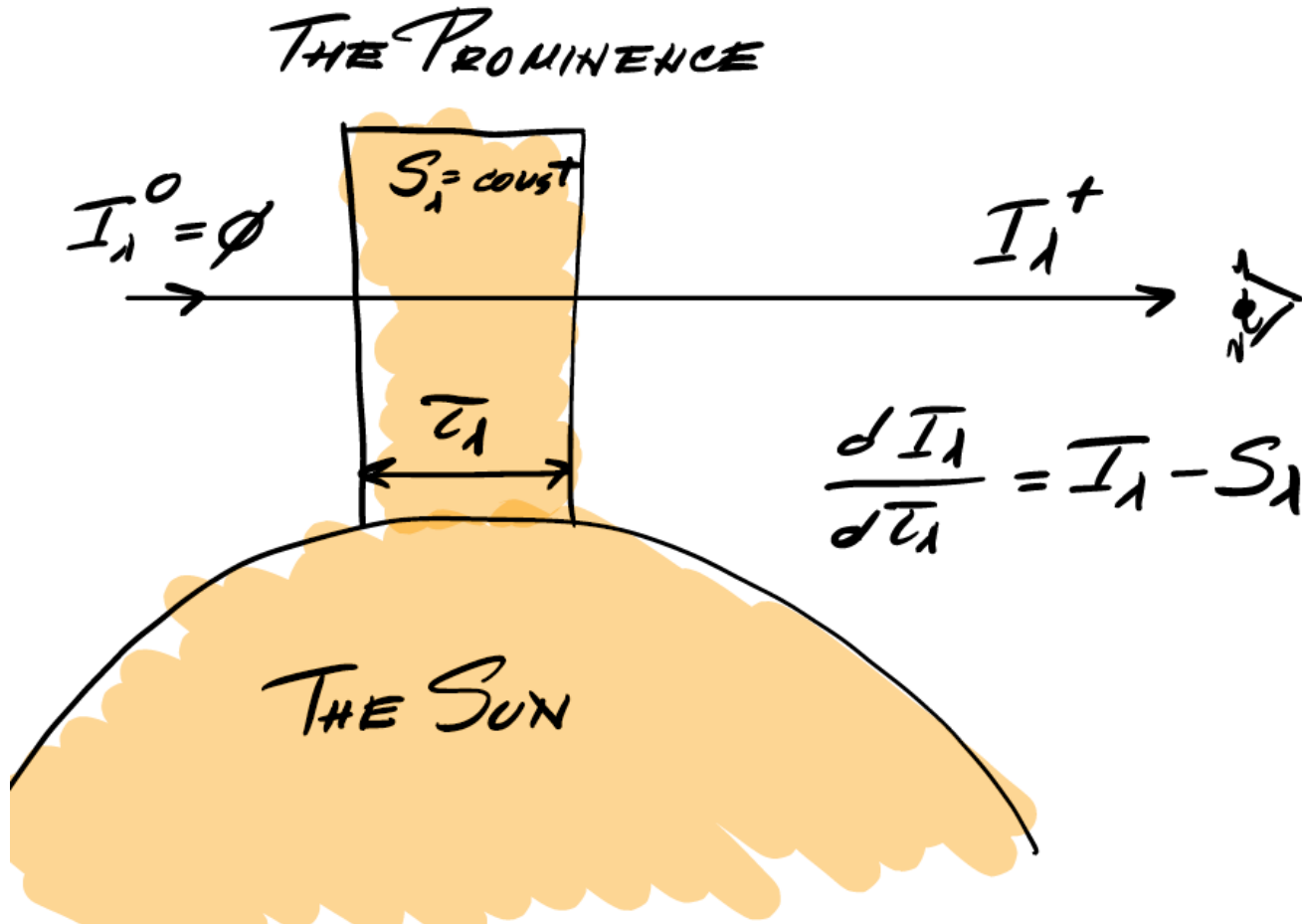
Formal solution

- If we know all the opacities and emissivities, then we can calculate source function and the optical depth, and solve the integral (numerically, or, with certain assumptions, analytically), to get:

$$I_{\lambda}^{+} = I_{\lambda}^0 e^{-\tau_{\lambda}} + \int_0^{\tau_{\lambda}} S(t) e^{-t} dt$$

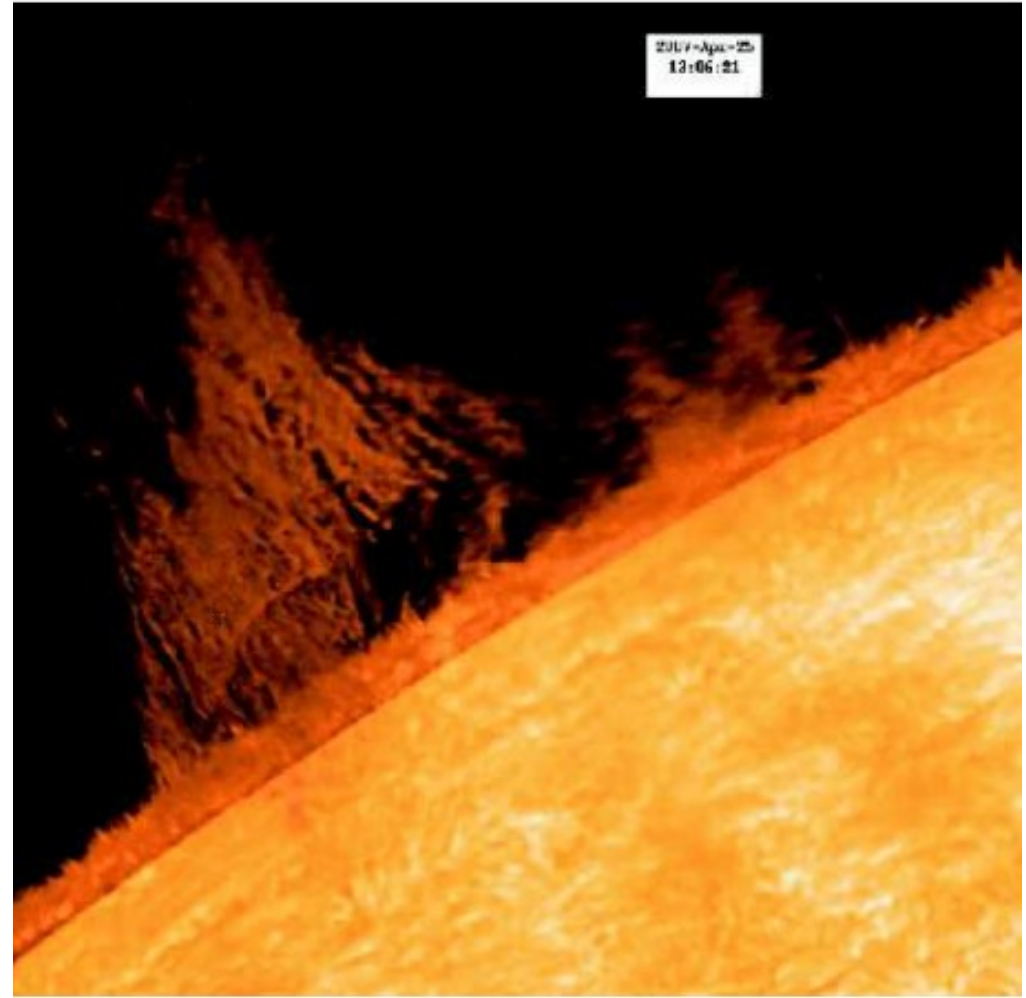
- But, as we have heard, calculating these can be extremely complicated. So, for simple diagnostics, or when speed is important, we often make a lot of simplifying assumptions.

We will model the object (a prominence in this case), as a **slab**



Are they actually slabs?

- Well, this one looks like a “sheet” (recall the filament from the intro slide). It also has complex spatial structure
- But, we can treat each pixel in the prominence as a separate slab!
- Why do we see this prominence?

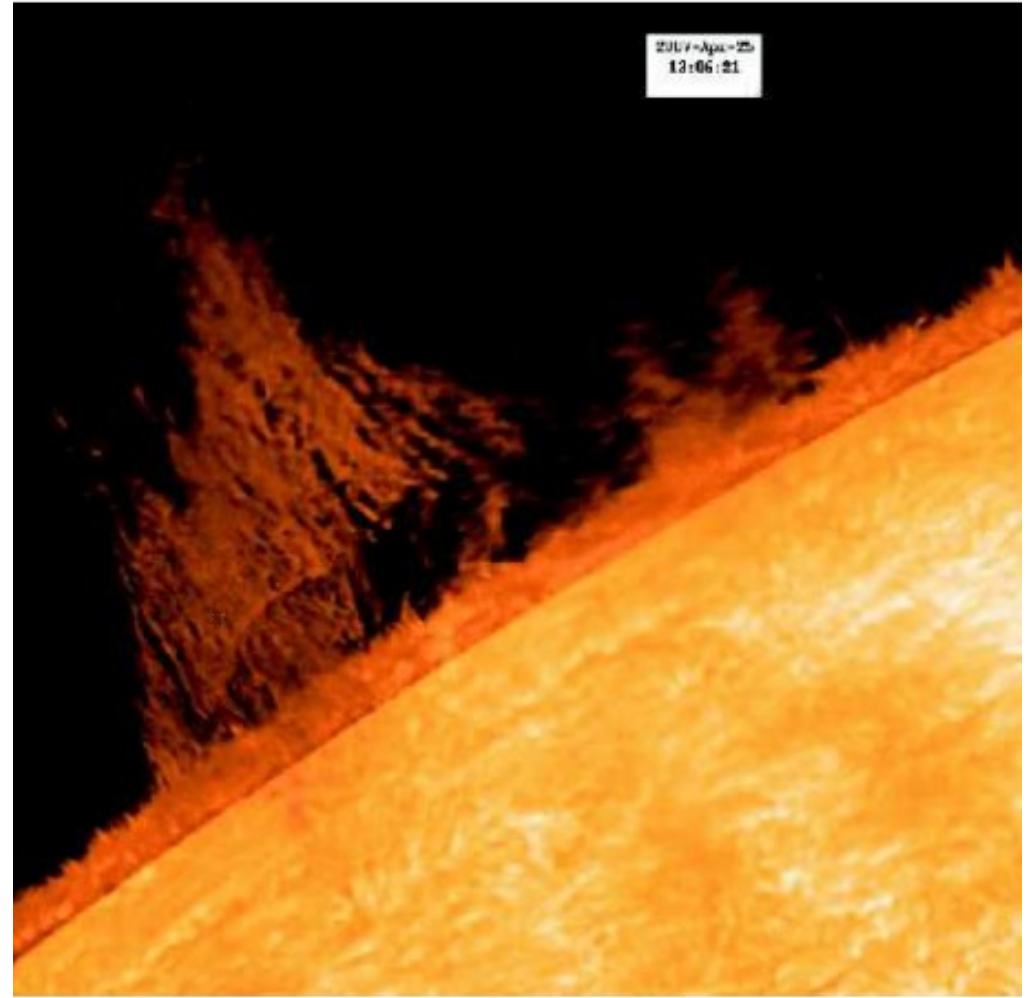


Hinode NFI observations of a prominence in H alpha, Henzel et al. 2008.

Are they actually slabs?

Why do we see this prominence?

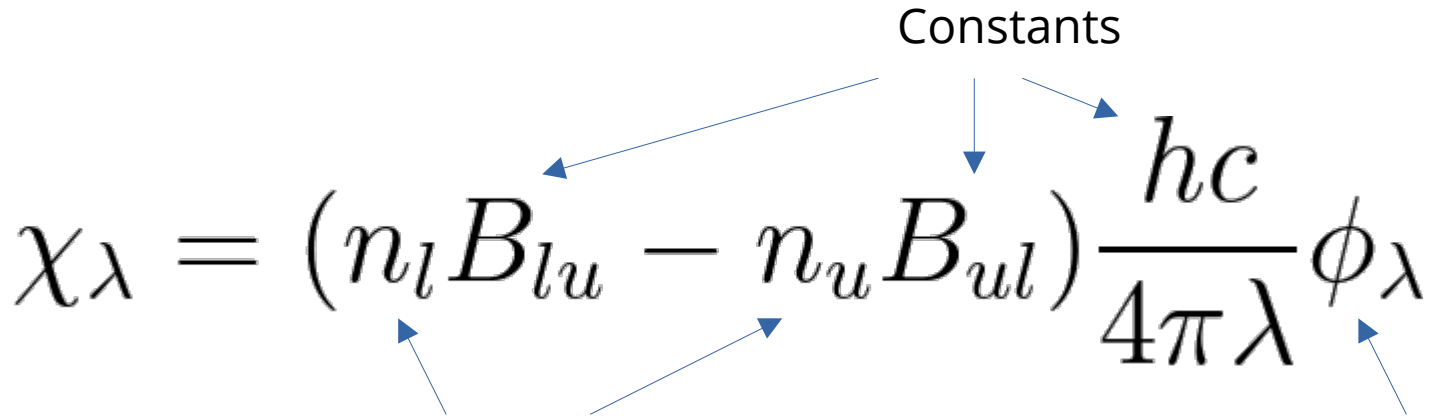
- There are some atoms in the second level of H alpha, and these atoms “scatter” (absorb, and then re-emit in a different direction), incoming radiation
- For 10830, the mechanism is completely the same



Hinode NFI observations of a prominence in H alpha, Henzel et al. 2008.

So we need opacity and emissivity

- Or, rather **optical depth and the source function**
- Exact expression for the opacity looks like this:

$$\chi_{\lambda} = (n_l B_{lu} - n_u B_{ul}) \frac{hc}{4\pi\lambda} \phi_{\lambda}$$


The diagram shows the equation $\chi_{\lambda} = (n_l B_{lu} - n_u B_{ul}) \frac{hc}{4\pi\lambda} \phi_{\lambda}$ with three labels and arrows pointing to specific parts: 'Constants' points to h , c , and 4π ; 'Atomic level "populations"' points to n_l and n_u ; and 'Line absorption profile, not identical to the shape of the line in the spectrum, as we will see' points to ϕ_{λ} .

Atomic level "populations"

Line absorption profile,
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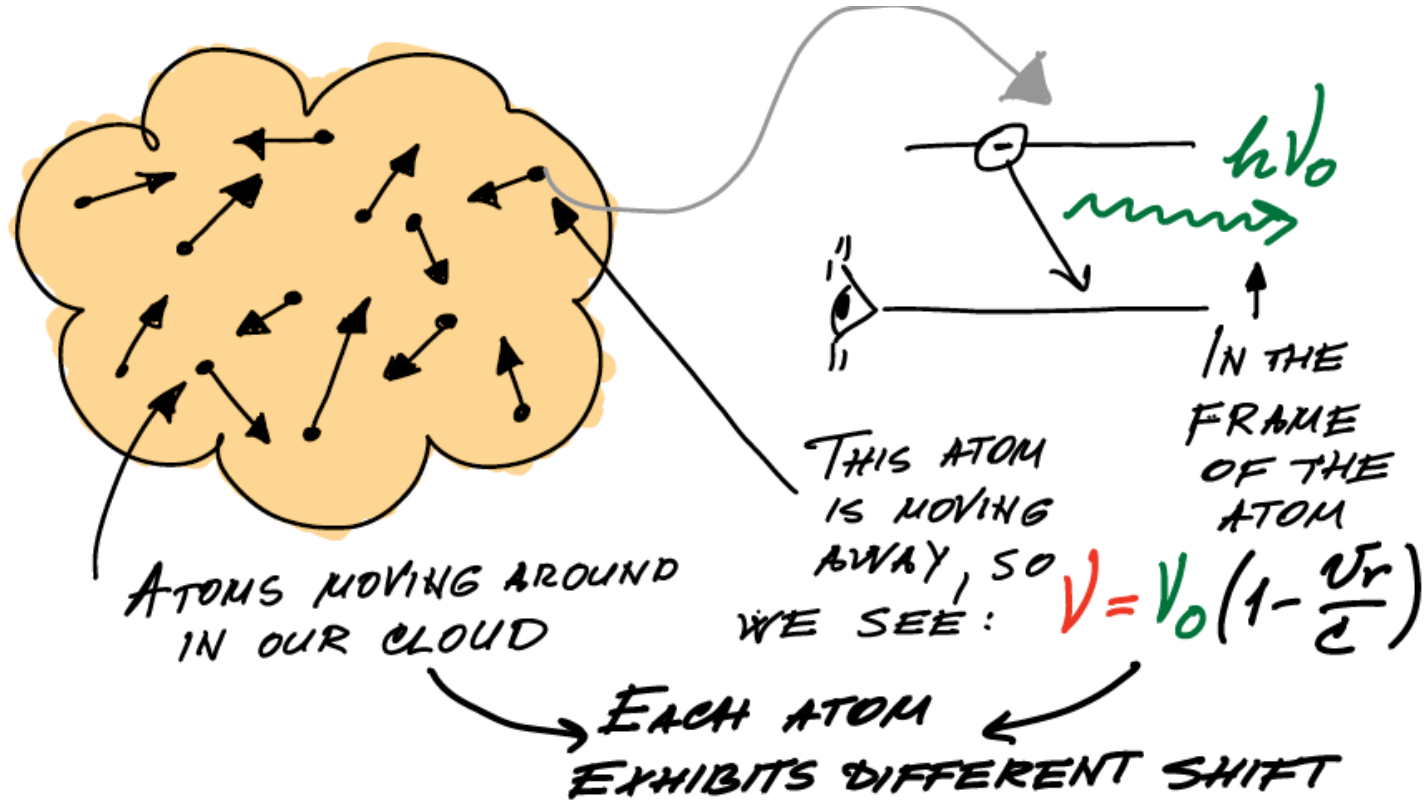
Finding the level populations is **very hard**

- It would require us to solve a complicated NLTE problem, and to know a lot of input parameters (boundary conditions), which is in principle doable but not very practical
- To understand (and to diagnose), we then simply treat the wavelength dependent optical depth of the whole slab as a **free** parameter:

$$\tau_{\lambda} = \tau \phi_{\lambda}$$

- Where τ is the total optical depth of the prominence, or optical depth at some referent wavelength (like in Hazel code), and ϕ is the line profile that describes line shift (los velocity) and broadening (Doppler velocity, damping)

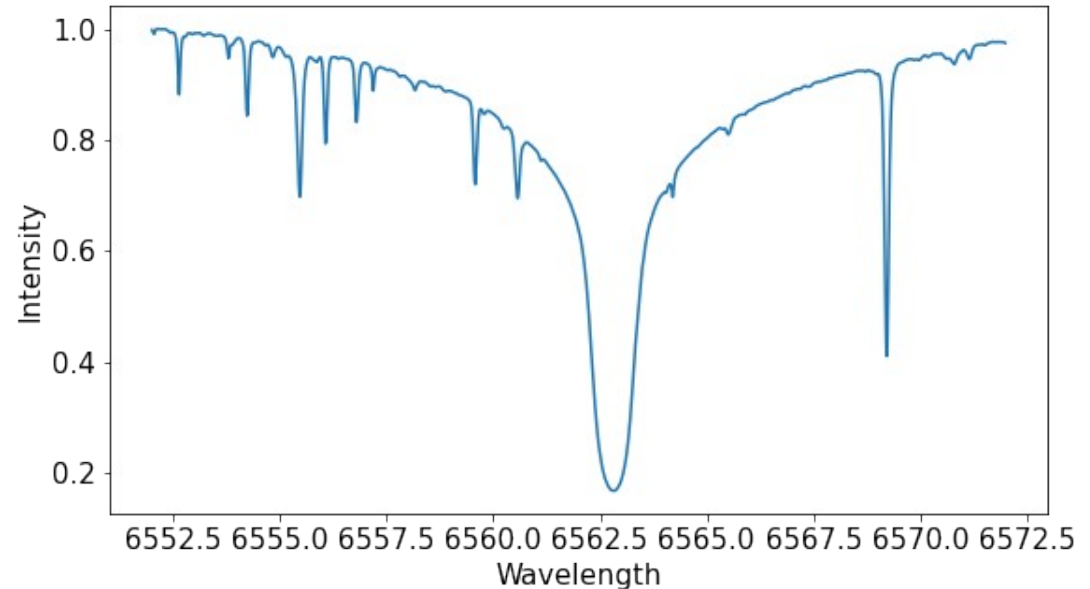
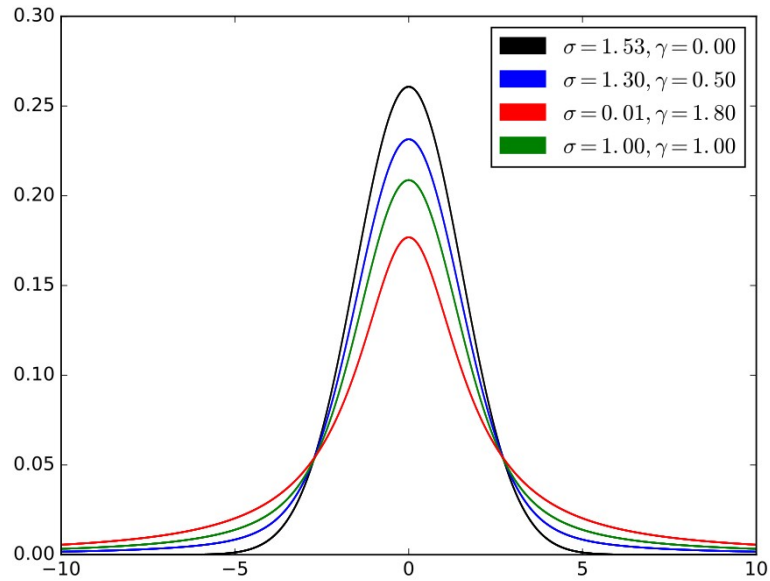
Simple spectral line broadening



The line-of-sight velocity distribution follows Gaussian distribution. Then, when we sum up emissions (absorptions) by all the particles, Doppler-shifted frequency (and wavelength) will follow Gaussian Distribution too!

The actual shape is more complicated – **Voigt function**

- Voigt profile enhances the line “wings”, and it is very visible in some observed spectral lines. We will get to experiment with it soon!

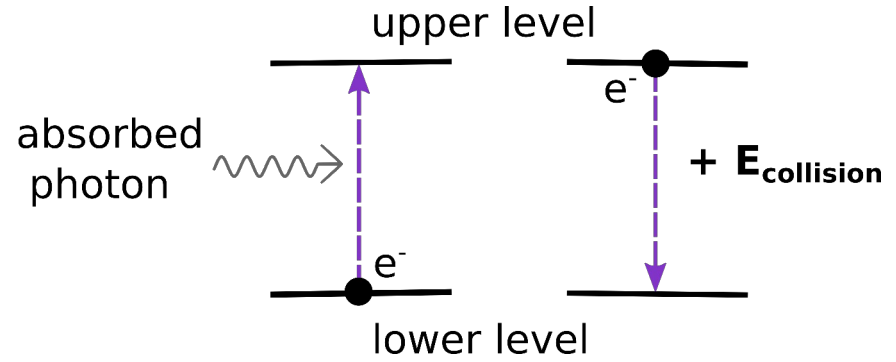
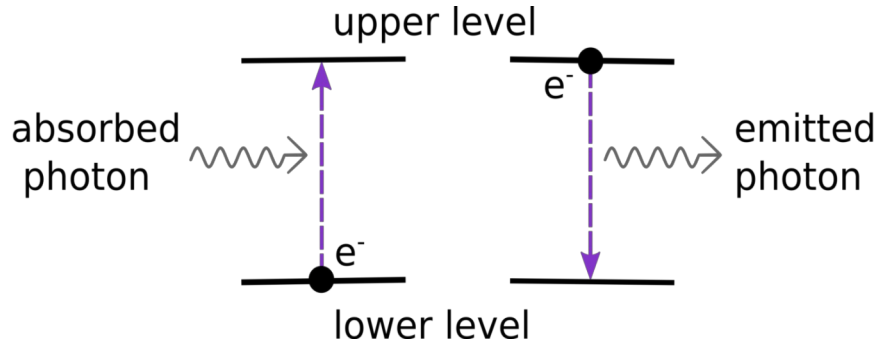


Ok, what about the source function?

- How do these objects emit? Why do we see, e.g. prominences in H alpha but not in broadband?



Spectral line scattering



- Absorption **immediately** followed by emission.
- I will argue that the probability of getting “true” absorption in which the photon is destroyed is:

for prominences this is practically zero!

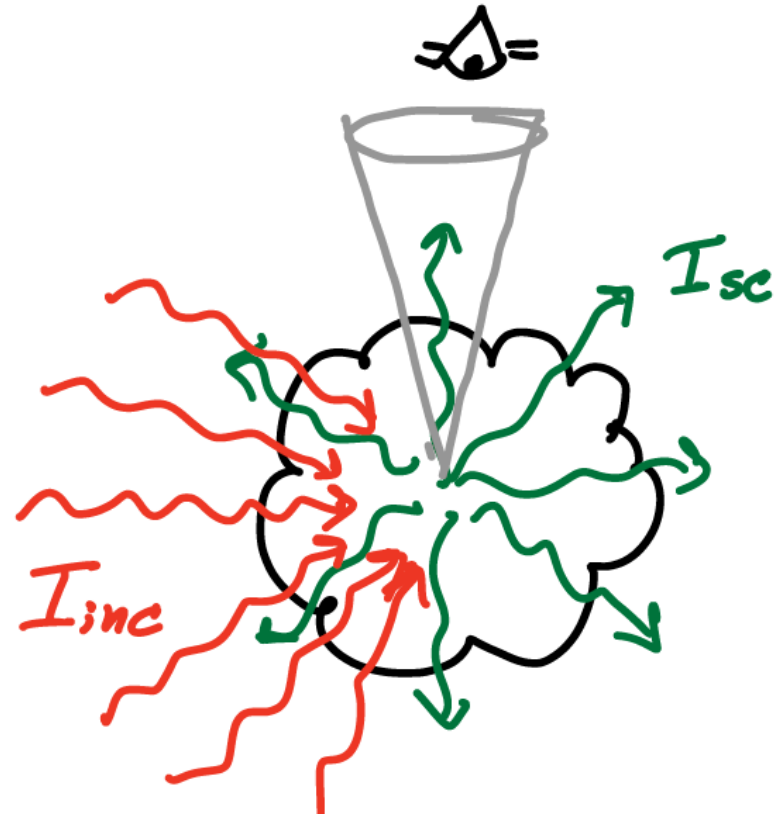
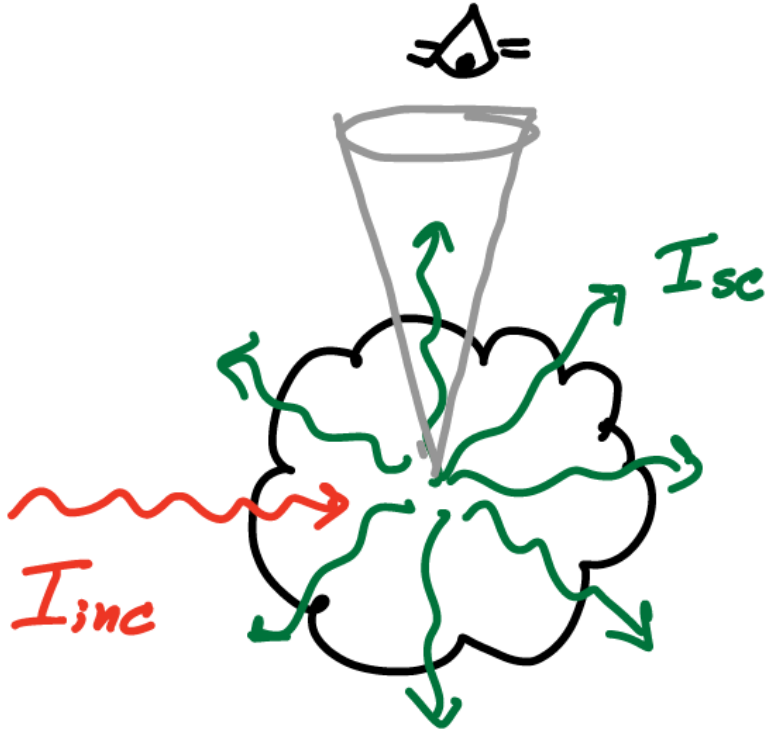
$$\epsilon = \frac{C_{ul}}{A_{ul} + C_{ul}} = \frac{\chi_{\text{abs}}}{\chi_{\text{abs}} + \chi_{\text{scatt}}}$$

How to calculate emissivity (source function) in scattering?

- We can solve SE equations, but that is hard. Let's go simpler
- We are considering one wavelength only (for now)
- The scattering is **coherent**, meaning no change in wavelength (good for continuum problems)
- The scattering is **isotropic**, meaning: no matter where they come from, photons scatter equally in all the directions
- We will go even simpler say there is no "truly" absorbed photons in our slab. i.e. only scattering exists (i.e. "epsilon" is zero)

In which case will there be more scattered radiation?

- Or: "In which case will **scattering emissivity** be higher?"



Right, more “total” incoming radiation, more emission

- We need to integrate the intensity over directions:

This is “scattered”
flux. It describes
the absorption.

$$\frac{dF_{\text{sc}}}{dl} = \chi_{\text{sc}} \oint I(\lambda, \hat{\Omega}) d\Omega$$

$$j_{\text{scattered}} = \frac{\chi_{\text{sc}}}{4\pi} \oint I(\lambda, \hat{\Omega}) d\Omega$$

But that same flux is then
scattered in all directions
equally, so we divide with 4 pi

So, we treat scattered photons as
being removed from our intensity, and
then some of them are added back
through the emission

Scattering source function is equal to **mean intensity**!

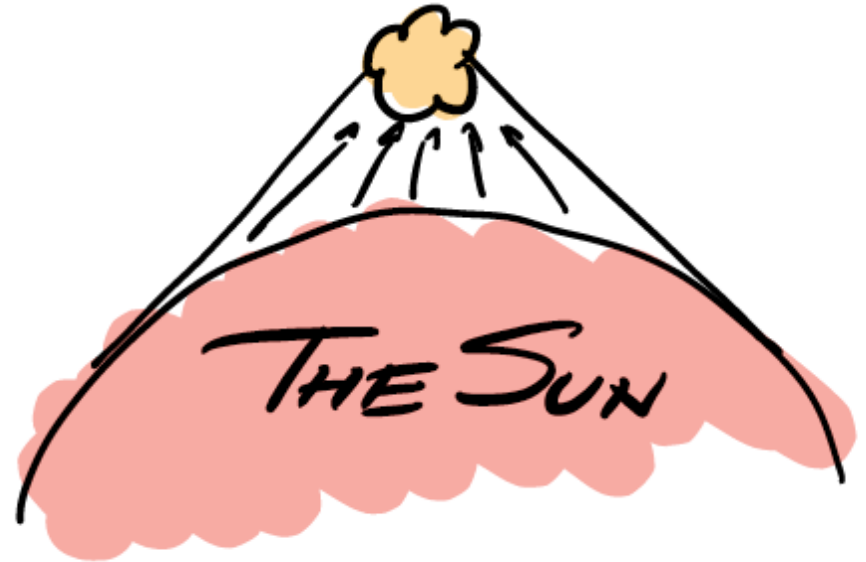
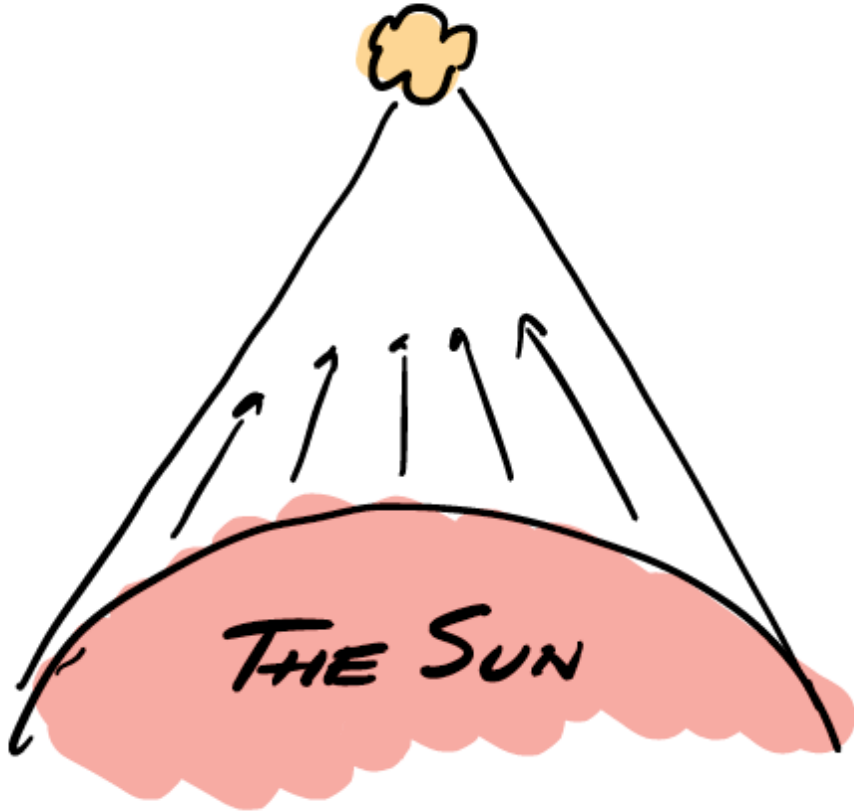
- Remember we only have the scattering so:

$$j = \chi \frac{1}{4\pi} \oint I(\lambda, \theta, \phi) \sin \theta d\theta d\phi = \chi J$$

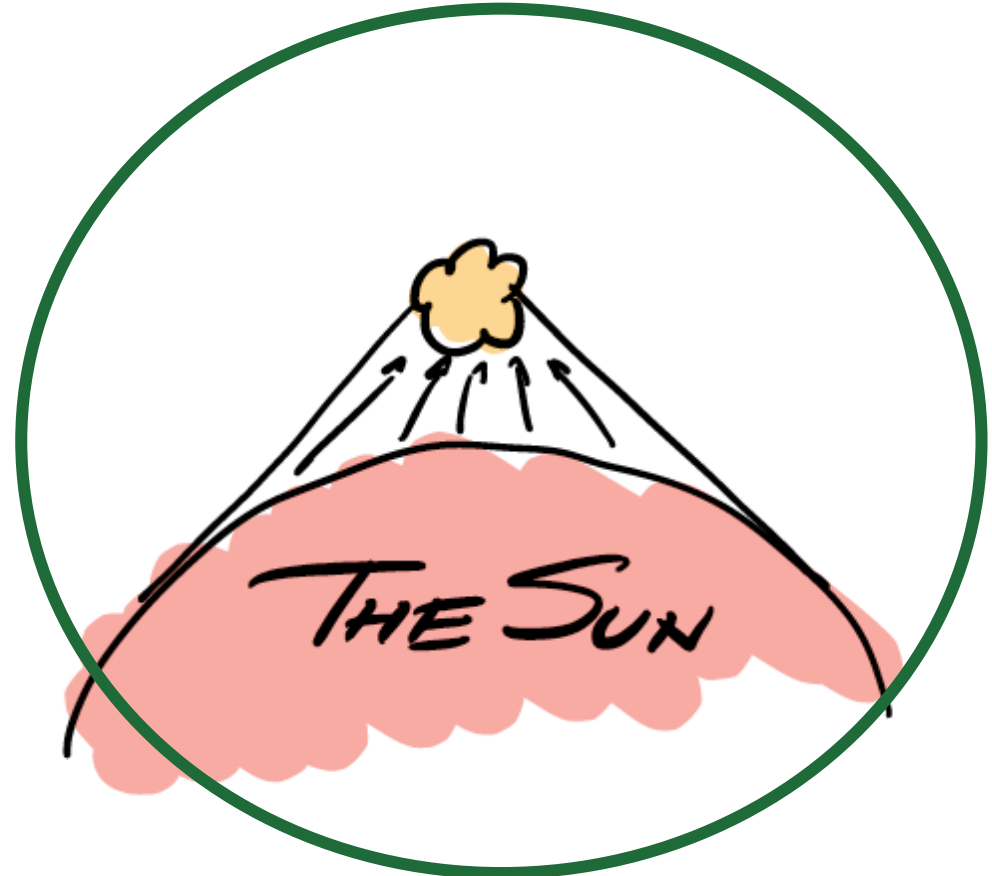
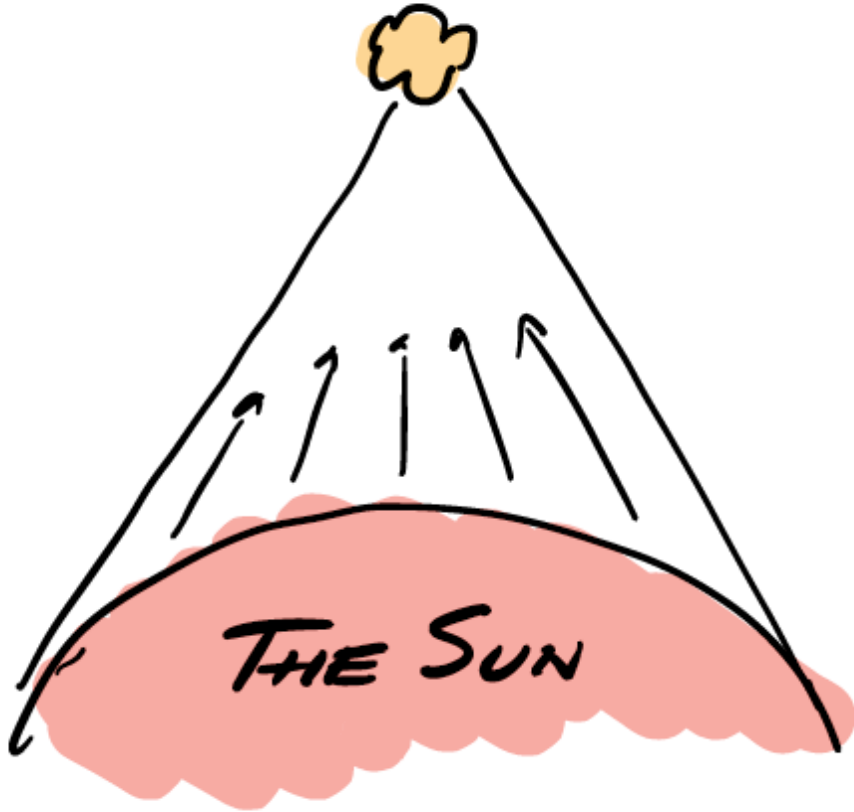
- Or (small caps j and big caps J are not the same thing!)

$$S = \frac{j}{\chi} = J$$

For which of these two cases will we have higher source function in the “blob”?



For which of these two cases will we have higher source function in the “blob”?



Two-level atom scattering source function, a bit more complicated

- Bear with me, let's take a moment to agree on this:

All the intensities get mashed up together

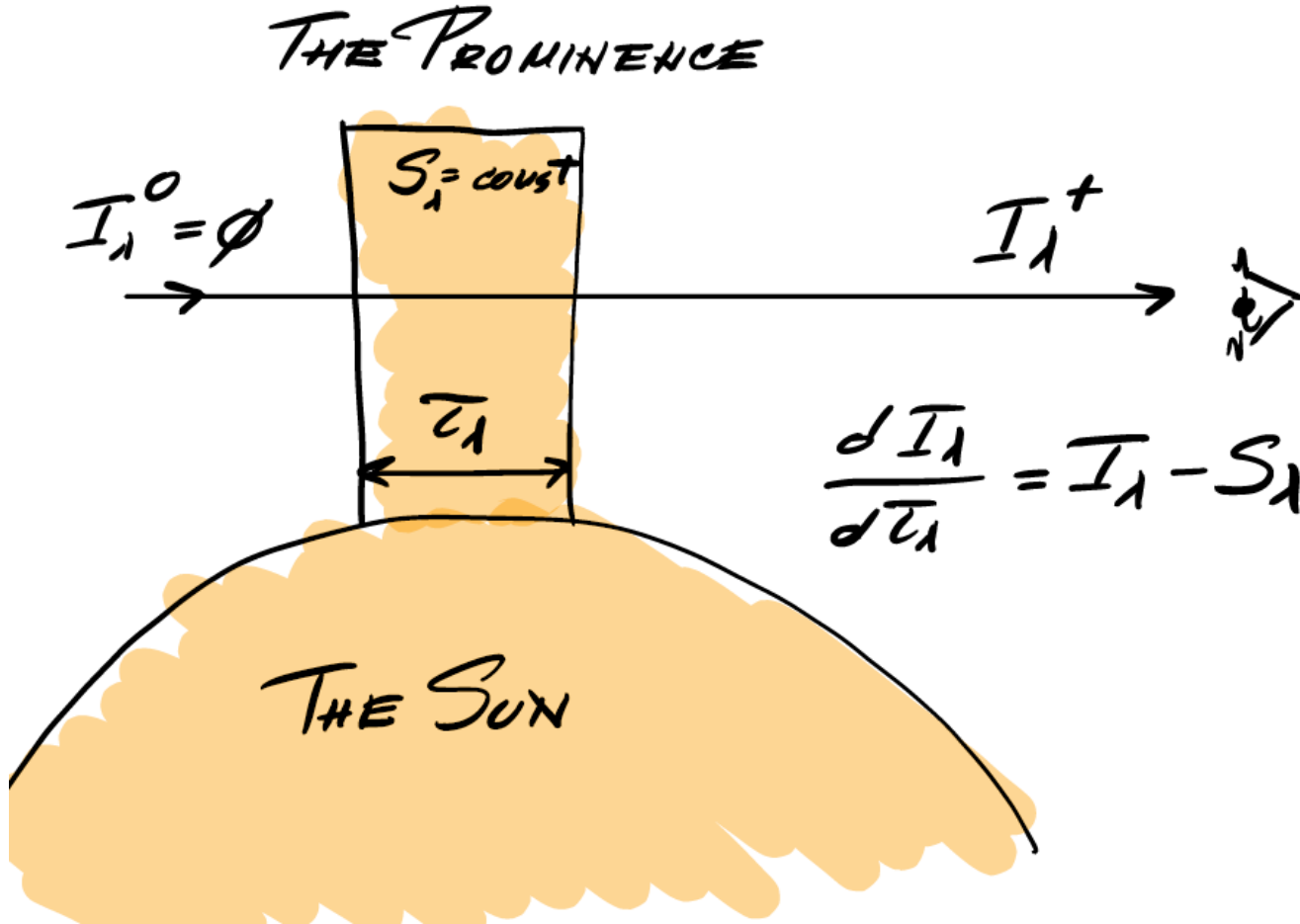
All the direction contribute

$$S = J = \frac{1}{4\pi} \oint \int_0^\infty I(\theta, \varphi, \lambda) \phi_\lambda d\lambda \sin \theta d\theta d\varphi$$

Wavelengths at the line core contribute more (phi is line absorption profile)

Intensity here is “incoming”, photospheric intensity. And for 10830 you can assume it to be “flat”, i.e. constant with wavelength, due to nature of the line absorption profile, it means S can be calculated like in the previous slide!

Now, to solve RTE



- The general solution with no incident radiation:

$$I_\lambda = \int_0^{\tau_\lambda} S_\lambda(t) e^{-t} dt$$

- Assuming constant source function:

$$I_\lambda = S(1 - e^{-\tau_\lambda})$$

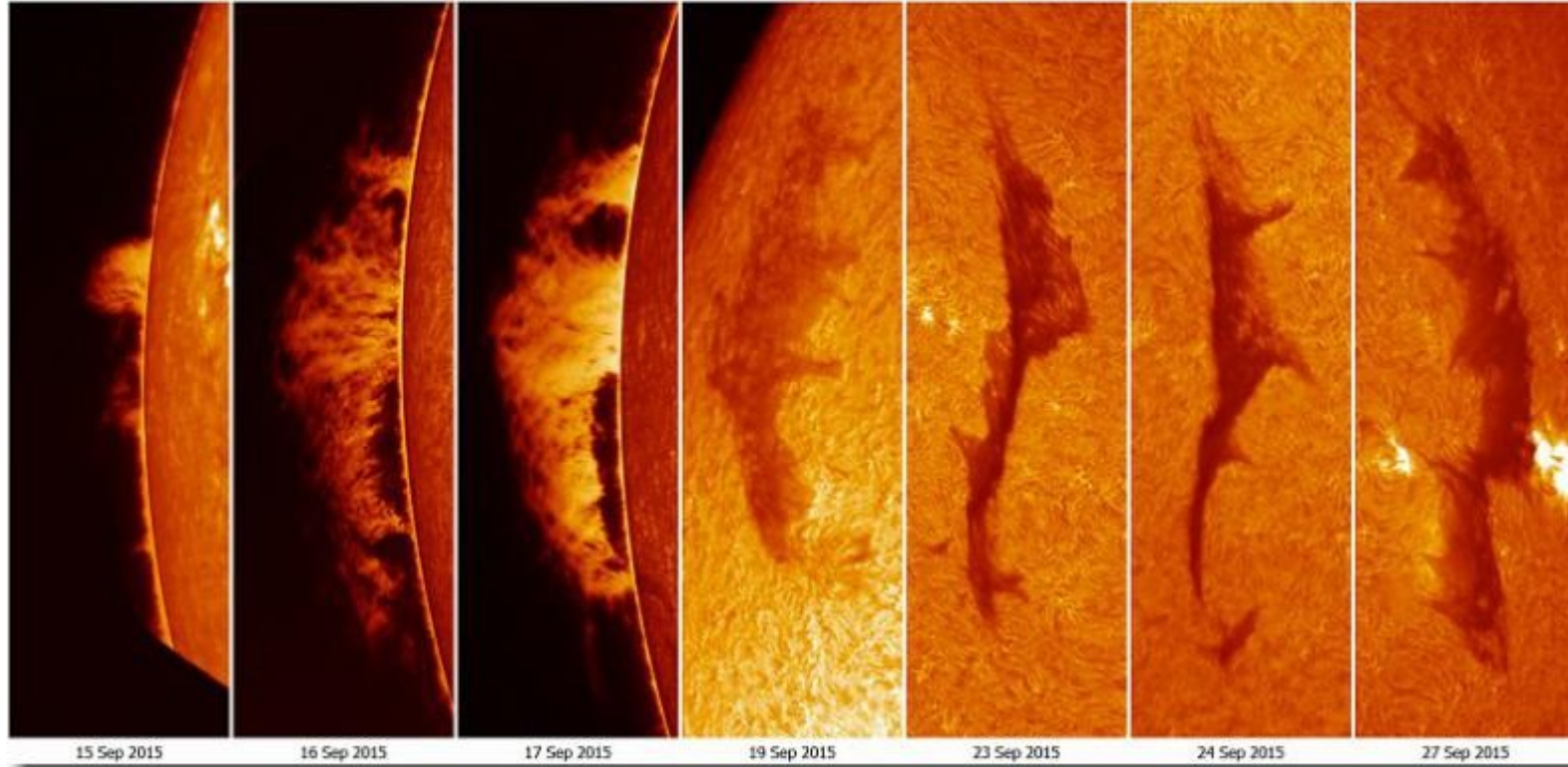
- For small optical depth:

$$I_\lambda = S\tau_\lambda$$

- And for large: $I_\lambda = S$

Prominences vs Filaments

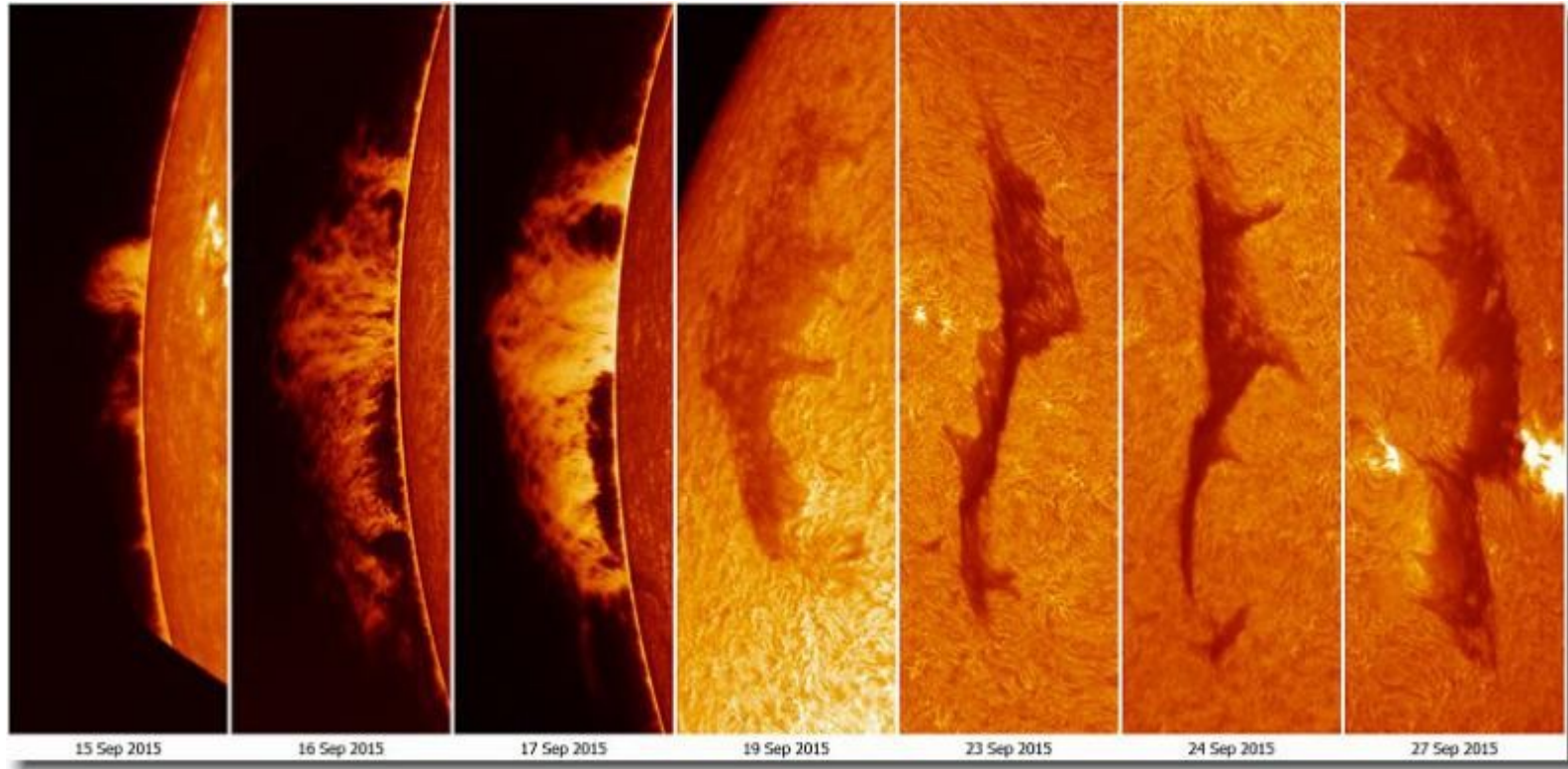
- Filaments are, essentially prominences seen on disk



Credits: Bob Antol (www.cloudynights.com)

Before we carry on, think about the following:

- Why are prominences bright above the limb and dark on the disk?



Credits: Bob Antol (www.cloudynights.com)

Conclusions

- To simply describe absorption / emission in a slab, we **parametrize** optical depth using **four parameters**: total optical depth, line center, broadening and damping
- To calculate the **source function** we assume it is equal to the **mean intensity**, which follows from the height of the slab and the **limb darkening** of the incoming radiation.
- We will now implement this all into a simple python code and explore!