Theoretical Astrophysics I: Physics of Sun and Stars
Lecture 5: Convection, magnetic fields, and dynamo

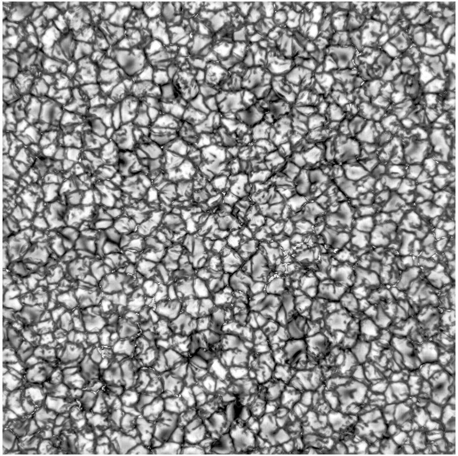
Petri Käpylä Ivan Milić
pkapyla,milic@leibniz-kis.de

Institut für Sonnenphysik - KIS, Freiburg

May 13, 2024

Convection in the Sun

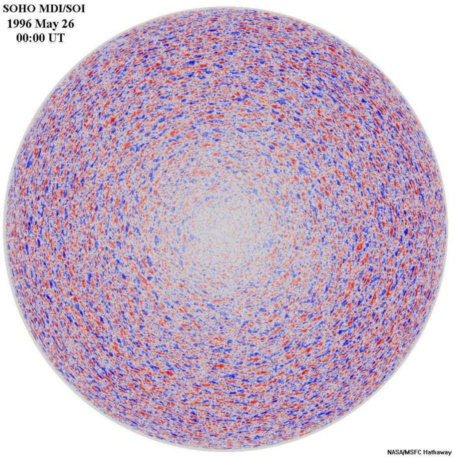
- Several types of convective flows are conjectured to exist in the Sun.
- Granulation is directly observed. Velocity $u \sim 1$ km/s, $\ell \sim 10^3$ km, turnover time $\tau_{\rm c} \sim$ a few minutes.



Solar granulation from the Daniel K. Inoue Solar Telescope (DKIST).

Convection in the Sun

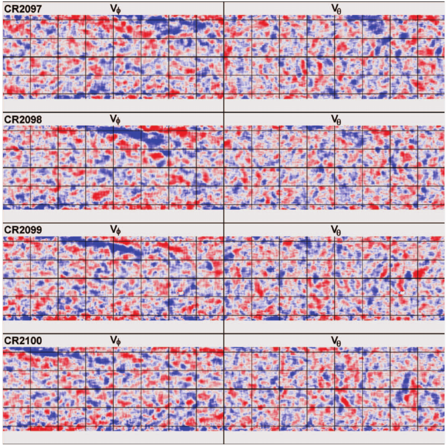
- Several types of convective flows are conjectured to exist in the Sun.
- Granulation is directly observed. Velocity $u \sim 1$ km/s, $\ell \sim 10^3$ km, turnover time $\tau_c \sim$ a few minutes.
- Supergranulation is observed using the Doppler effect. Velocity $u_h \sim 300$ m/s, $u_v \sim 40$ m/s, $\ell \sim 20 \ldots 35$ Mm, turnover time $\tau_c \sim$ about a day.
- Why is the signal so weak near the centre of the disk?



Solar supergranulation from SOHO MDI.

Convection in the Sun

- Several types of convective flows are conjectured to exist in the Sun.
- Granulation is directly observed. Velocity $u \sim 1$ km/s, $\ell \sim 10^3$ km, turnover time $\tau_c \sim$ a few minutes.
- Supergranulation is observed using the Doppler effect. Velocity $u_h \sim 300$ m/s, $u_v \sim 40$ m/s, $\ell \sim 20 \dots 35$ Mm, turnover time $\tau_c \sim$ about a day.
- Giant cells? $u \sim 10$m/s, $\ell \sim 200$ Mm, $\tau_c \sim$ a month.
- Why would we even expect giant cells?



Hathaway et al. (2013), Science, 342, 1217.

Simulations of convection

- Simulations solve the equations of (magneto)hydrodynamics in a coordinate system most suited to the physical problem:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B} - \eta\mu_0 \boldsymbol{J}),$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{u}),$$

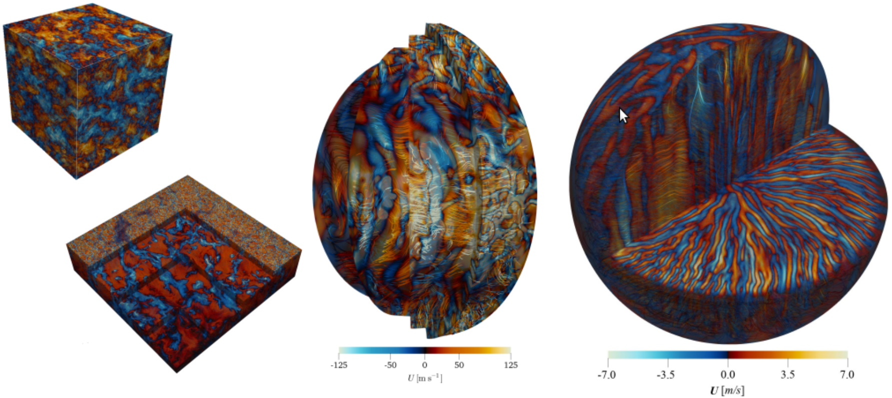
$$\rho \frac{\partial \boldsymbol{u}}{\partial t} = -\nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u}) + \rho \boldsymbol{g} - \nabla p - 2\rho \boldsymbol{\Omega}_0 \times \boldsymbol{U} + \boldsymbol{J} \times \boldsymbol{B} + \nabla \cdot \boldsymbol{F}^{\mathrm{visc}},$$

$$\rho T \frac{\partial s}{\partial t} = -\nabla \cdot (\rho s \boldsymbol{u}) + \nabla \cdot \mathcal{F} + \mathcal{H} + 2\nu\rho \mathbf{S}^2 + \eta\mu_0 \boldsymbol{J}^2,$$

- Here, $\boldsymbol{F}_{\mathrm{visc}} = 2\nu\rho\mathbf{S}$, where $\mathrm{S}_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) - \frac{1}{3}\delta_{ij}\boldsymbol{\nabla} \cdot \boldsymbol{u}$ is the traceless rate-of-strain tensor, and $\mathcal{F}$ contains the radiative flux $\mathcal{F}_{\mathrm{rad}} = -K\boldsymbol{\nabla} T$.

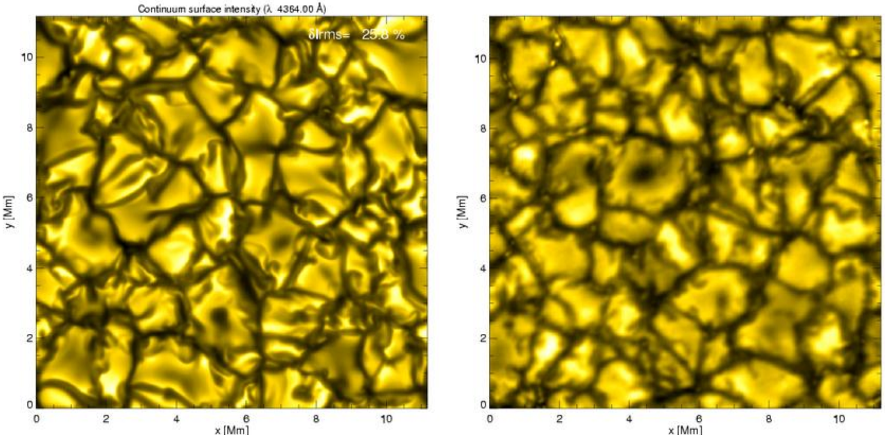
Simulations of convection

- Simulations solve the equations of (magneto)hydrodynamics in a coordinate system most suited to the physical problem:



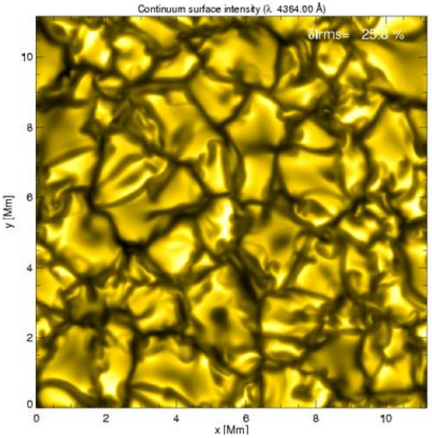
PENCIL CODE, https://pencil-code.org/

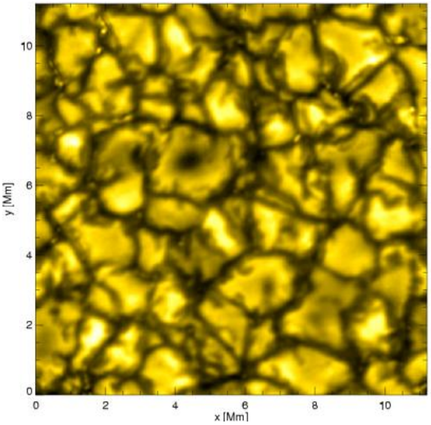
Simulations of solar surface convection



Which one is simulation and which on observation?

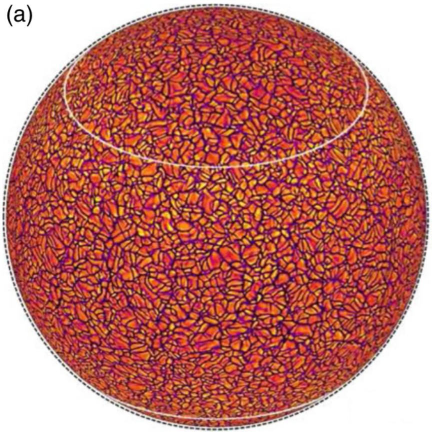
Simulations of solar surface convection

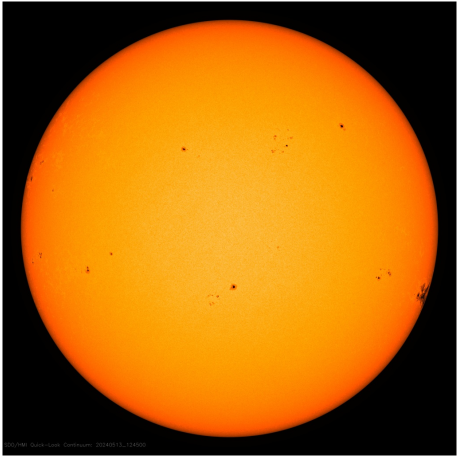




Left: Simulated solar surface convection (credit: Matthias Steffen). Right: Observed granulation from Swedish Solar Telescop (SST) (credit: Mats Carlsson).

Simulations of global solar convection





Left: Radial velocity near the surface of a high-resolution global simulation of the Sun (Hotta et al. 2015, Astrophys. J., 798, 51). Right: HMI Intensitygram from Solar Dynamics Observatory (SDO) 13.05.2024.

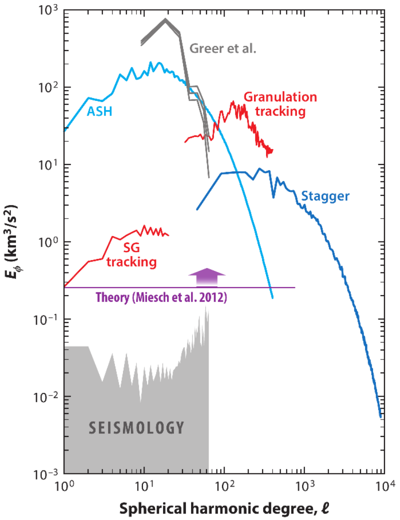
Convective conundrum

- Convective velocities observed in the Sun can be used to compute the power spectrum of the velocity:

$$E_{\mathrm{K}} = \frac{1}{2} \int E(k) dk, \qquad (1)$$

with $E_{\mathrm{K}} = \frac{1}{2}\boldsymbol{u}^2$ and $E(k) = \sum_k |\hat{\boldsymbol{u}}(k)|^2$.

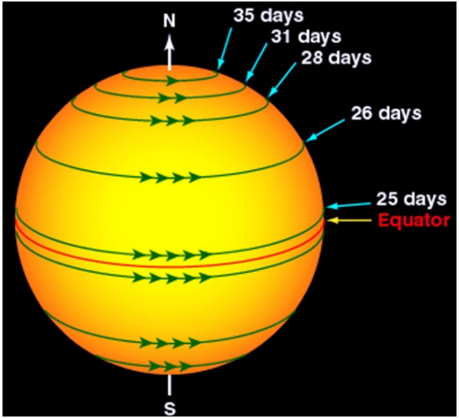
- While the surface simulations (Stagger) seem to be compatible with the granulation tracking observations, global simulations (ASH) suggest velocities that are orders of magnitude larger than the (helio)seismology or supergranulation (SG) tracking.
- This is one of the manifestations of the "Convective conundrum," or the discrepancy between models and reality.



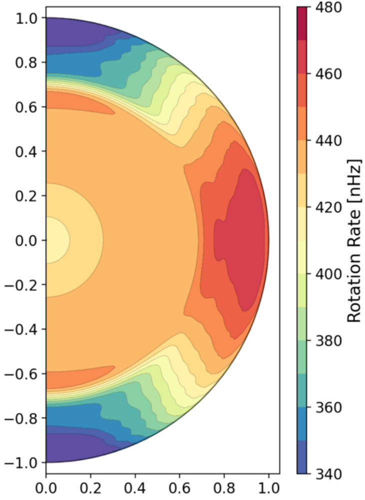
Hanasoge et al. (2016)

Differential rotation

The Sun rotates *differentially*, i.e., not like a solid body.



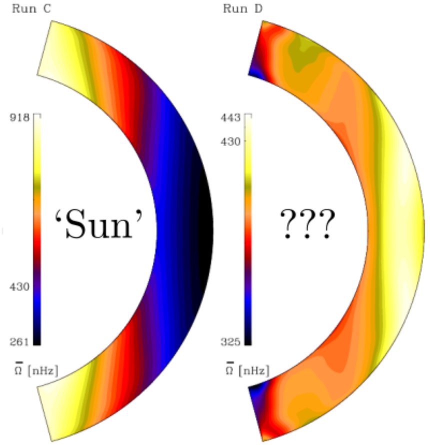
Source: NASA.



Source: Larson and Schou (2018).

Differential rotation in simulations

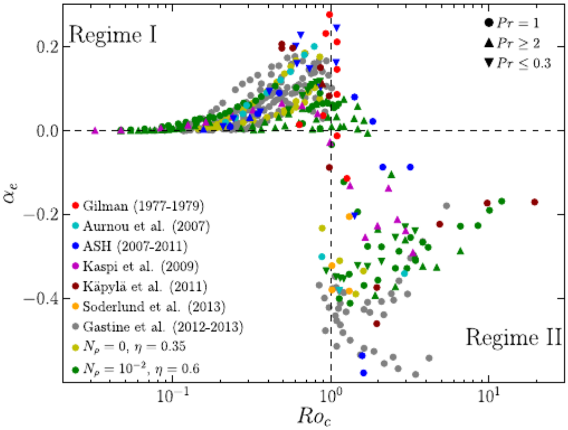
- The interaction of the convective flows and the global solar rotation lead to differential rotation.
- Simulations with solar parameters (only luminosity and rotation rate as we will see below!) very often yield "anti-solar" differential rotation with fast poles and slow equator.
- What simple argument can explain the anti-solar states?



Mean angular velocity $\overline{\Omega} = \overline{u}_\phi / r \sin\theta + \Omega_0$. From Käpylä et al. (2014), Astron. Astrophys., 570, 43

Differential rotation in simulations

- Simulations show that the flip fron anti-solar (Regime II) to solar-like (Regime I) occurs around $\mathrm{Ro}_c \sim \mathrm{Co}^{-1} \sim 1$.
- The Sun appears to be near this transition and simulations tend to underestimate Co.



Source: Gastine et al. (2014), MNRAS, 438, L76.

Convection and rotation

- As we discussed in the last tutorial, the order of magnitude of the convective velocity and length scale are

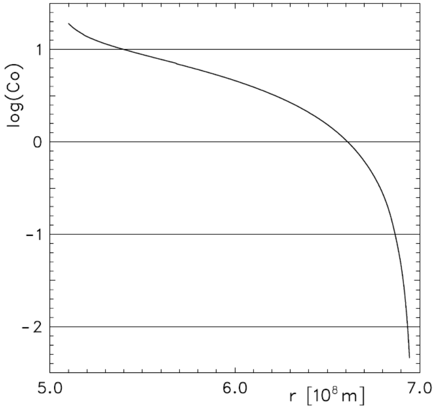
$$u \sim \left(\frac{F_{\rm tot}}{\rho}\right)^{1/3} \equiv u_\star, \text{ and } \ell \sim H_p, \qquad (2)$$

where $F_{\rm tot} = \frac{L}{4\pi r^2}$ and ρ is a reference density.

- We also saw on the previous lecture that according to the mixing length theory the Coriolis number

$$\mathrm{Co} = \frac{2\Omega_\odot \ell}{u}, \qquad (3)$$

varies between 10^{-3} near the surface and around 10 at the base of the convection zone.



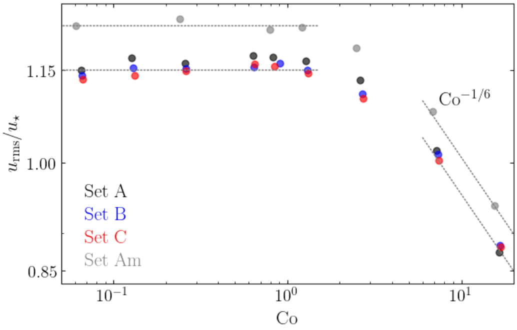
Käpylä et al. (2005), Astron. Astrophys., 438, 403.

Convection and rotation

- When rotation is sufficiently rapid, the rotating mixing length theory (e.g. Stevenson, 1979, Geophys. Astrophys. Fluid Dyn., 12, 139) predicts that the convective velocity changes and is

$$u \sim \left(\frac{F_{\rm tot}}{\rho}\right)^{1/3} \mathrm{Co}^{-1/6} = u_\star \mathrm{Co}^{-1/6}, \quad (4)$$

- Where is the Sun in this diagram?

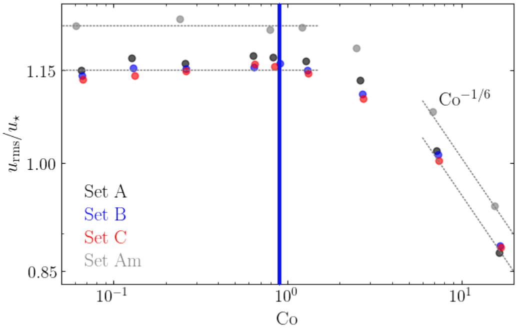


Convective velocity divided by $u_\star$ from several sets of simulations. From Käpylä (2024), Astron. Astrophys., 683, 221.

- When rotation is sufficiently rapid, the rotating mixing length theory (e.g. Stevenson, 1979, Geophys. Astrophys. Fluid Dyn., 12, 139) predicts that the convective velocity changes and is

$$u \sim \left(\frac{F_{\rm tot}}{\rho}\right)^{1/3} {\rm Co}^{-1/6} = u_\star {\rm Co}^{-1/6}, \quad (5)$$

- Where is the Sun in this diagram?

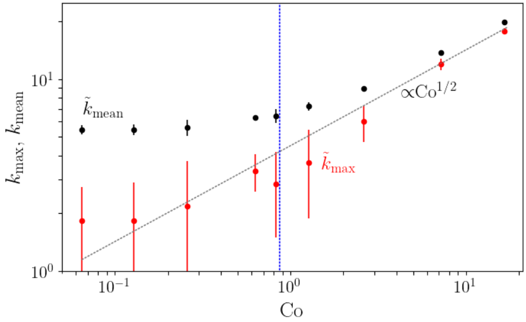


Convective velocity divided by $u_\star$ from several sets of simulations. From Käpylä (2024), Astron. Astrophys., 683, 221.

- The convective length scale also changes

$$\ell \sim H_p \mathrm{Co}^{-1/2}. \quad (6)$$

- It appears that convection in the deep parts of solar convection zone are less rotationally constrained than previously thought.



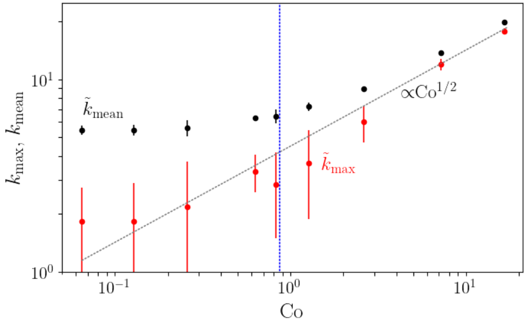
Two measures of the scale ($k = 2\pi/\ell$) of convective structures in simulations as functions of Co. From Käpylä (2023), arXiv:2311.09082.

Convection and rotation

- The convective length scale also changes

$$\ell \sim H_p \mathrm{Co}^{-1/2}. \tag{7}$$

- It appears that convection in the deep parts of solar convection zone are less rotationally constrained than previously thought.
- There is a catch, however, and we will come back to that later.



Two measures of the scale ($k = 2\pi/\ell$) of convective structures in simulations as functions of Co. From Käpylä (2023), arXiv:2311.09082.

Why do simulations struggle to reproduce the Sun?

- Any ideas?

Why do simulations struggle to reproduce the Sun?

- The equations of magnetohydrodynamics can be written in a dimensionless form where a number of dimensionless quantities uniquely defining the system appear (tutorial).

$$
\begin{aligned}
\frac{\partial \boldsymbol{B}}{\partial t} &= \nabla \times (\boldsymbol{u} \times \boldsymbol{B} - \eta\mu_0 \boldsymbol{J}),\\
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \boldsymbol{u}),\\
\rho \frac{\partial \boldsymbol{u}}{\partial t} &= -\nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u}) + \rho \boldsymbol{g} - \nabla p - 2\rho \boldsymbol{\Omega}_0 \times \boldsymbol{U} + \boldsymbol{J} \times \boldsymbol{B} + \nabla \cdot \boldsymbol{F}^{\mathrm{visc}},\\
\rho T \frac{\partial s}{\partial t} &= -\nabla \cdot (\rho s \boldsymbol{u}) + \nabla \cdot \mathcal{F} + \mathcal{H} + 2\nu\rho \mathbf{S}^2 + \eta\mu_0 \boldsymbol{J}^2,
\end{aligned}
$$

Why do simulations struggle to reproduce the Sun?

- The equations of magnetohydrodynamics can be written in a dimensionless form where a number of dimensionless quantities uniquely defining the system appear.
- These include the Prandtl numbers, the Rayleigh number, and the Taylor number:

$$\mathrm{Pr} = \frac{\nu}{\chi}, \quad \mathrm{Pr_M} = \frac{\nu}{\eta}, \quad \mathrm{Ra} = \frac{gd^4}{\nu\chi}\left(-\frac{1}{c_p}\frac{ds}{dr}\right), \quad \mathrm{Ta} = \frac{4\Omega^2 d^4}{\nu^2}, \tag{8}$$

  where ν is the kinematic viscosity, η is the magnetic diffusivity, and $\chi = K_{\mathrm{rad}}/c_p\rho$ is the radiative diffusivity.
- We can also estimate the relative magnitudes of different terms in the equations by OOM (order of magnitude) analysis. These give an additional set of (diagnostic) numbers (Reynolds, Péclet, Coriolis):

$$\mathrm{Re} = \frac{ud}{\nu}, \quad \mathrm{Re_M} = \frac{ud}{\eta}, \quad \mathrm{Pe} = \frac{ud}{\chi}, \quad \mathrm{Co} = \frac{2\Omega d}{u}. \tag{9}$$

- We need to compare these quantities in our simulations to those in stars.

Why do simulations struggle to reproduce the Sun?

- Typically these numbers in stars are either $\gg 1$ or $\ll 1$.
- Simulations are limited by available computational resources and can for most parameters do only modest values.

Parameter	Sun ($M_\odot$)	O9 (20 $M_\odot$)	Simulations
Ra	10^{20}	10^{24}	10^{9}
Pr	10^{-6}	10^{-5}	$10^{-1}\ldots 10$
$\mathrm{Pr_M}$	10^{-3}	10^{3}	$10^{-1}\ldots 10$
Re	10^{13}	10^{11}	10^{4}
Pe	10^{7}	10^{6}	10^{4}
$\mathrm{Re_M}$	10^{10}	10^{14}	10^{4}
$\Delta\rho$	10^{6}	3	10^{2}
Ro	$0.1\ldots 1$	1[1]	$10^{-2}\ldots 10^{3}$

Some dimensionless parameters in the Sun and in a $20M_\odot$ O9 star in comparison to typical simulations. From Käpylä et al. (2023), Space Science Rev., 219, 58.

Only the Rossby number (Co^{-1}) can be reproduced!

Why do simulations struggle to capture the Sun?

- More precisely, a Coriolis number can be defined as

$$\mathrm{Co}_\star = \frac{2\Omega H}{u_\star}. \tag{10}$$

It turns out that

$$\mathrm{Co}_\star = (\mathrm{Ra}_\mathrm{F}^\star)^{-1/3}, \quad \text{where} \quad \mathrm{Ra}_\mathrm{F}^\star = \frac{\mathrm{Ra}_\mathrm{F}}{\mathrm{Pr}^2 \mathrm{Ta}^{3/2}}. \tag{11}$$

- The catch mentioned earlier is that it is not enough to reproduce *one* of the system parameters if all the other ones are (too much) off.

Why should we care?

- The evolution of the magnetic field is governed by

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B} - \eta \mu_0 \boldsymbol{J}), \tag{12}$$

  where $\boldsymbol{J} = \mu_0^{-1} \boldsymbol{\nabla} \times \boldsymbol{B}$ is the current density.
- If $\boldsymbol{u} = 0$, the induction equation reduces to a *diffusion equation*:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \eta \nabla^2 \boldsymbol{B}. \tag{13}$$

- Why is this relevant?

Why should we care?

- The evolution of the magnetic field is governed by

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B} - \eta \mu_0 \boldsymbol{J}), \tag{14}$$

  where $\boldsymbol{J} = \mu_0^{-1} \boldsymbol{\nabla} \times \boldsymbol{B}$ is the current density.
- If $\boldsymbol{u} = 0$, the induction equation reduces to a *diffusion equation*:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \eta \nabla^2 \boldsymbol{B}. \tag{15}$$

- Why is this relevant?
- The field cannot be generated without flows. To understand the generation of, e.g., the solar magnetic field, we must now the velocity field. *Study of magnetic phenomena is by far the biggest branch of solar physics!*