

# Theoretical Astrophysics I: Physics of Sun and Stars

## Lecture 5: Convection in stars versus convection simulations

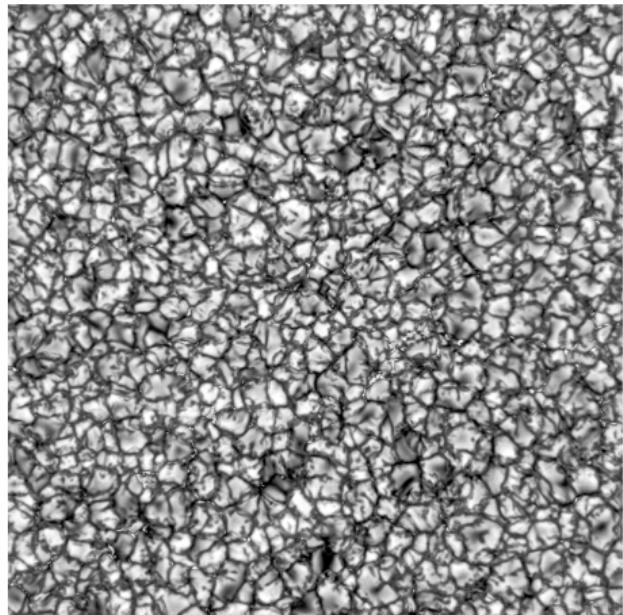
Petri Käpylä Ivan Milić  
[pkapyla,milic@leibniz-kis.de](mailto:pkapyla,milic@leibniz-kis.de)

Institut für Sonnenphysik - KIS, Freiburg

May 14, 2024

# Convection in the Sun

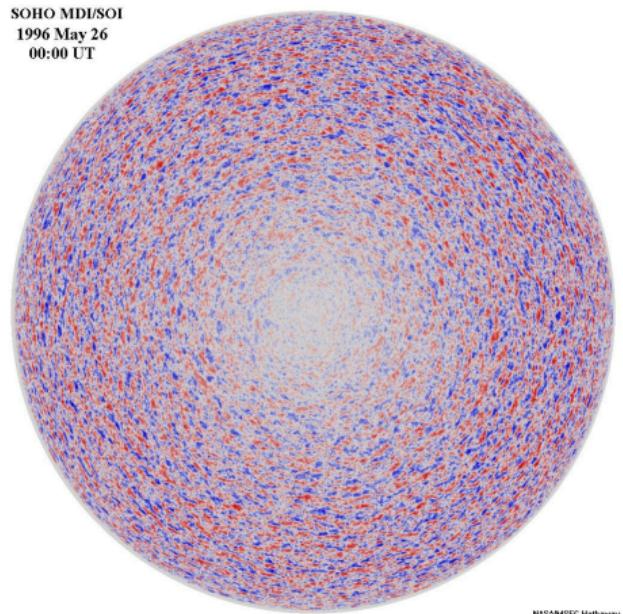
- ▶ Several types of convective flows are conjectured to exist in the Sun.
- ▶ Granulation is directly observed. Velocity  $u \sim 1 \text{ km/s}$ ,  $\ell \sim 10^3 \text{ km}$ , turnover time  $\tau_c \sim$  a few minutes.



Solar granulation from the Daniel K. Inoue Solar Telescope (DKIST).

# Convection in the Sun

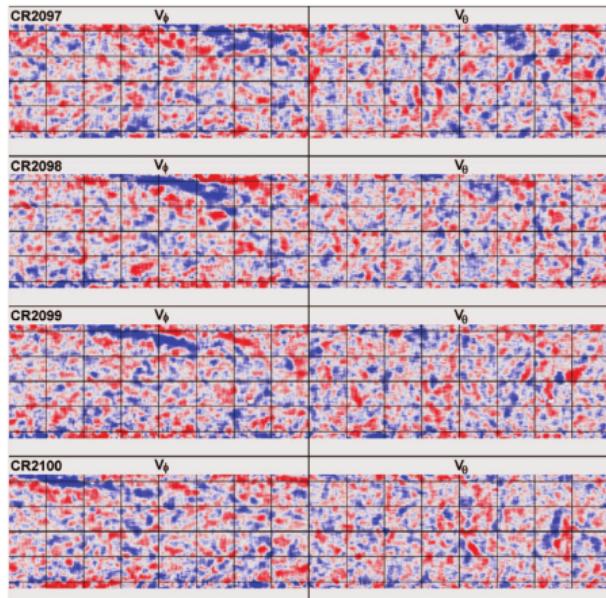
- ▶ Several types of convective flows are conjectured to exist in the Sun.
- ▶ Granulation is directly observed. Velocity  $u \sim 1 \text{ km/s}$ ,  $\ell \sim 10^3 \text{ km}$ , turnover time  $\tau_c \sim$  a few minutes.
- ▶ Supergranulation is observed using the Doppler effect. Velocity  $u_h \sim 300 \text{ m/s}$ ,  $u_v \sim 40 \text{ m/s}$ ,  $\ell \sim 20 \dots 35 \text{ Mm}$ , turnover time  $\tau_c \sim$  about a day.
- ▶ Why is the signal so weak near the centre of the disk?



Solar supergranulation from SOHO MDI.

# Convection in the Sun

- ▶ Several types of convective flows are conjectured to exist in the Sun.
- ▶ Granulation is directly observed. Velocity  $u \sim 1 \text{ km/s}$ ,  $\ell \sim 10^3 \text{ km}$ , turnover time  $\tau_c \sim \text{a few minutes}$ .
- ▶ Supergranulation is observed using the Doppler effect. Velocity  $u_h \sim 300 \text{ m/s}$ ,  $u_v \sim 40 \text{ m/s}$ ,  $\ell \sim 20 \dots 35 \text{ Mm}$ , turnover time  $\tau_c \sim \text{about a day}$ .
- ▶ Giant cells?  $u \sim 10 \text{ m/s}$ ,  $\ell \sim 200 \text{ Mm}$ ,  $\tau_c \sim \text{a month}$ .
- ▶ Why would we even expect giant cells?



Hathaway et al. (2013), Science, 342, 1217.

# Simulations of convection

- ▶ Simulations solve the equations of (magneto)hydrodynamics in a coordinate system most suited to the physical problem:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \mu_0 \mathbf{J}),$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}),$$

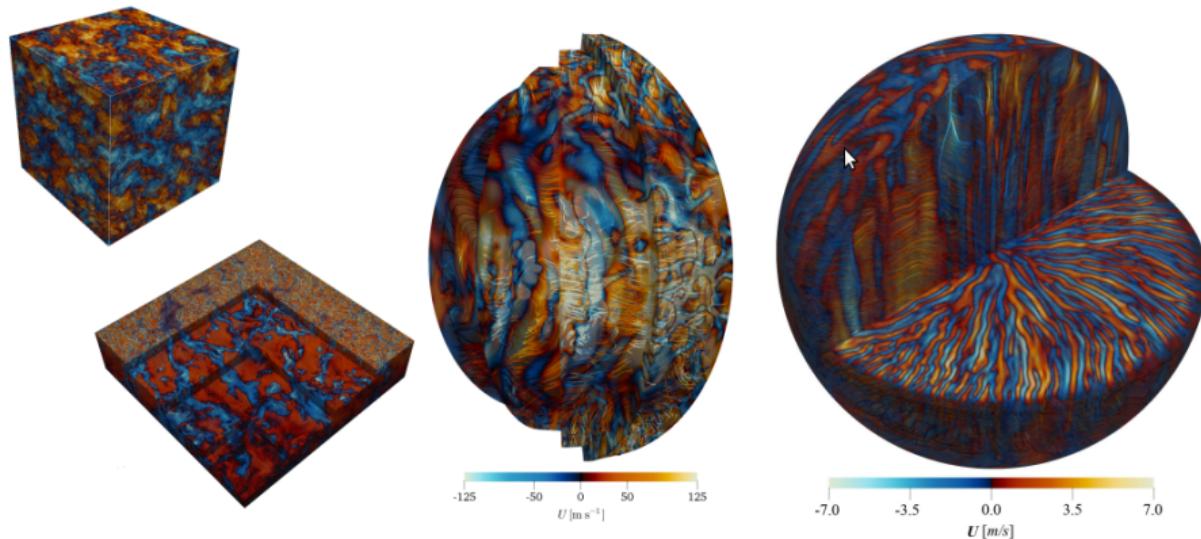
$$\rho \frac{\partial \mathbf{u}}{\partial t} = -\nabla \cdot (\rho \mathbf{u} \mathbf{u}) + \rho \mathbf{g} - \nabla p - 2\rho \boldsymbol{\Omega}_0 \times \mathbf{U} + \mathbf{J} \times \mathbf{B} + \nabla \cdot \mathbf{F}^{\text{visc}},$$

$$\rho T \frac{\partial s}{\partial t} = -\nabla \cdot (\rho s \mathbf{u}) + \nabla \cdot \mathcal{F} + \mathcal{H} + 2\nu \rho \mathbf{S}^2 + \eta \mu_0 \mathbf{J}^2,$$

- ▶ Here,  $\mathbf{F}_{\text{visc}} = 2\nu \rho \mathbf{S}$ , where  $S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) - \frac{1}{3}\delta_{ij}\nabla \cdot \mathbf{u}$  is the traceless rate-of-strain tensor, and  $\mathcal{F}$  contains the radiative flux  $\mathcal{F}_{\text{rad}} = -K \nabla T$ .

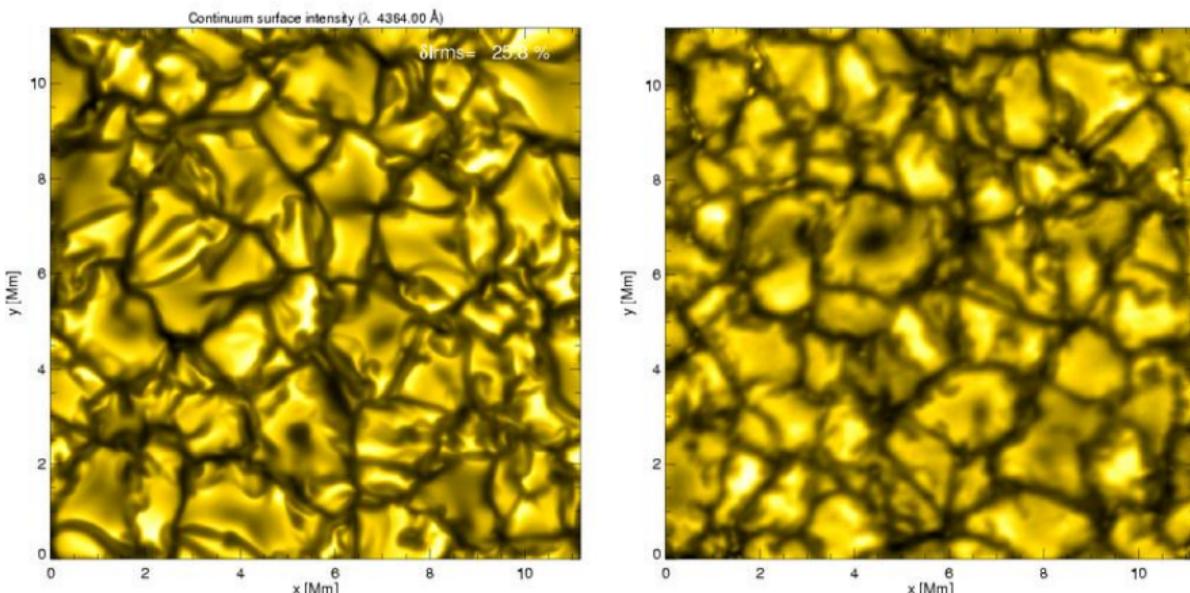
# Simulations of convection

- ▶ Simulations solve the equations of (magneto)hydrodynamics in a coordinate system most suited to the physical problem:



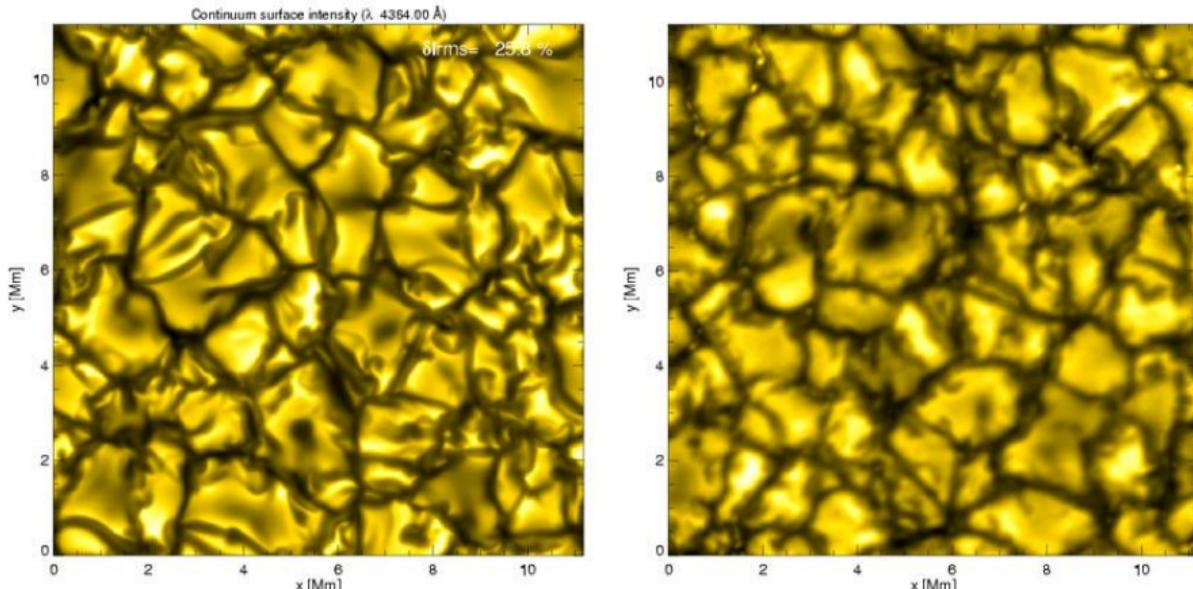
PENCIL CODE, <https://pencil-code.org/>

# Simulations of solar surface convection



Which one is simulation and which on observation?

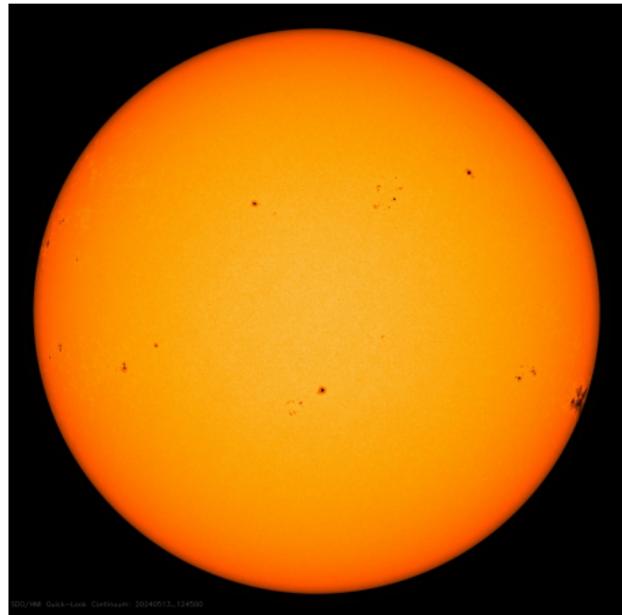
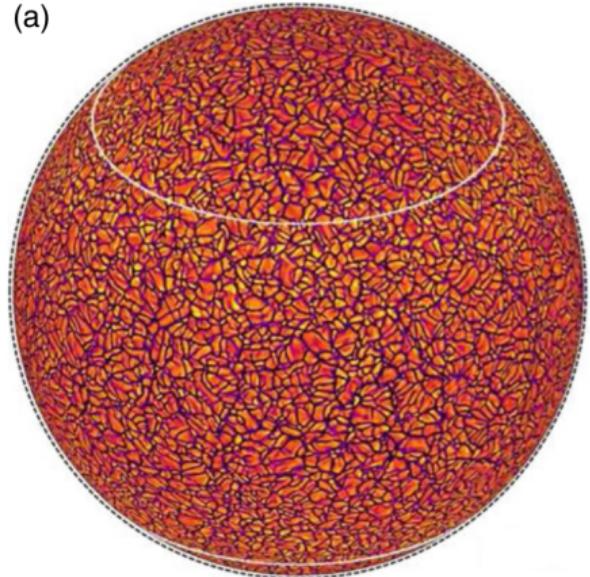
# Simulations of solar surface convection



Left: [Simulated solar surface convection](#) (credit: Matthias Steffen). Right: [Observed granulation](#) from Swedish Solar Telescope (SST) (credit: Mats Carlsson).

# Simulations of global solar convection

(a)



Left: Radial velocity near the surface of a high-resolution global simulation of the Sun (Hotta et al. 2015, *Astrophys. J.*, 798, 51). Right: HMI Intensitygram from Solar Dynamics Observatory (SDO) 13.05.2024.

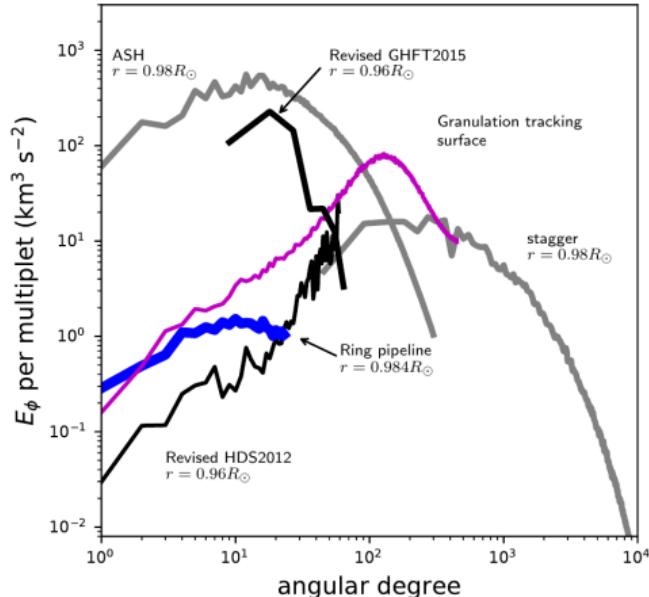
# Convective conundrum

- ▶ Convective velocities observed in the Sun can be used to compute the power spectrum of the velocity:

$$E_K = \frac{1}{2} \int E(k) dk, \quad (1)$$

with  $E_K = \frac{1}{2} \mathbf{u}^2$  and  $E(k) = \sum_k |\hat{\mathbf{u}}(k)|^2$ .

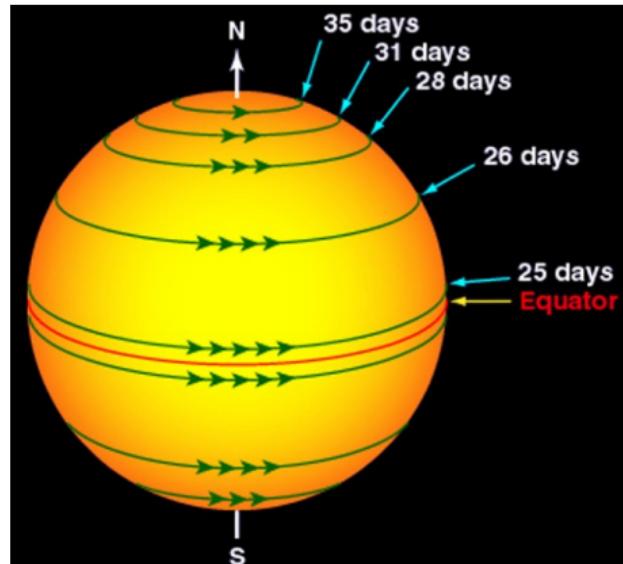
- ▶ While the surface simulations (Stagger) seem to be largely compatible with the granulation tracking observations, global simulations (ASH) suggest velocities that are much larger.
- ▶ This is one of the manifestations of the “Convective conundrum,” or the discrepancy between models and reality.



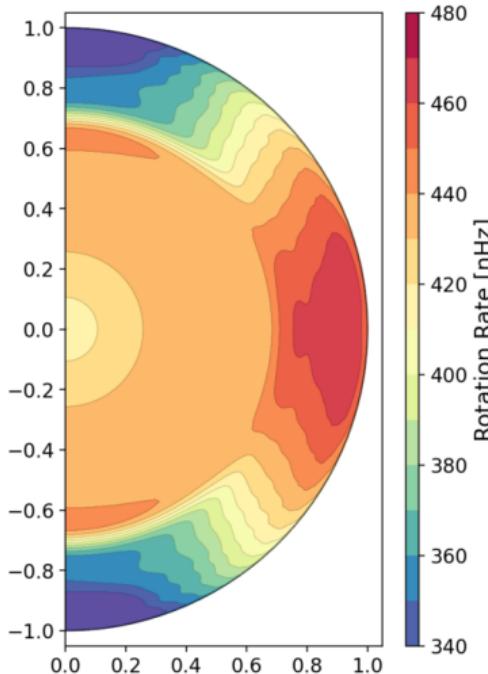
Proxauf (2021), PhD Thesis (Göttingen Univ.)

# Differential rotation

The Sun rotates *differentially*, i.e., not like a solid body. Here  $\Omega = u_\phi / r \sin \theta$ .



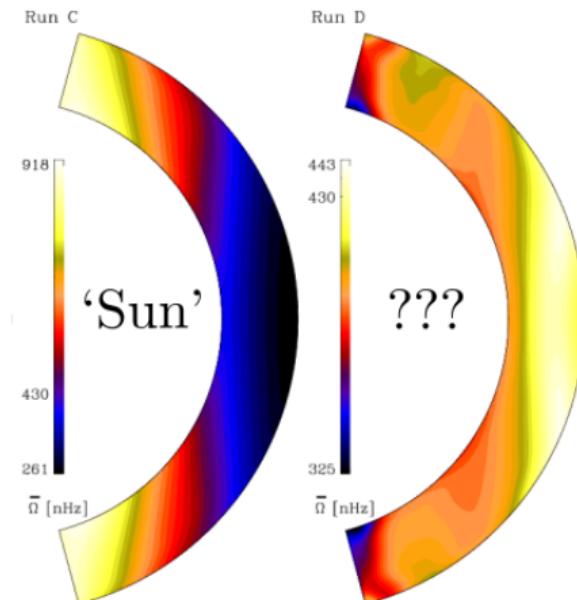
Source: NASA.



Source: Larson and Schou (2018).

# Differential rotation in simulations

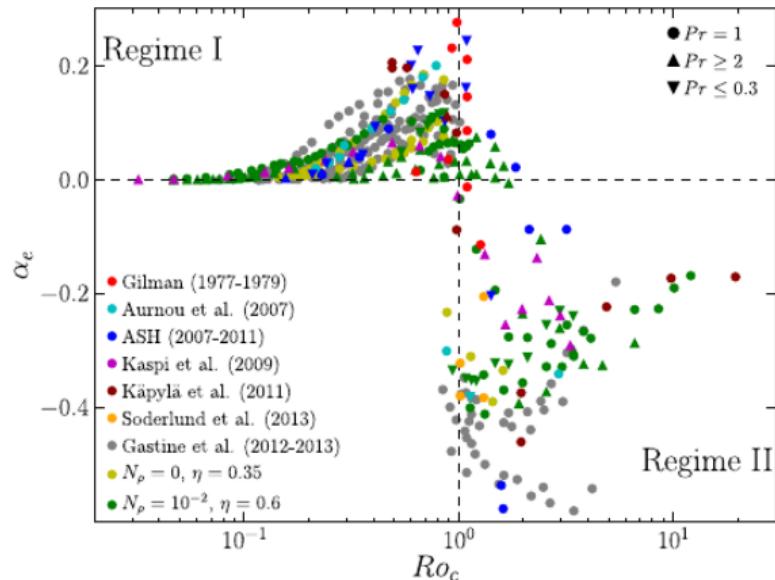
- ▶ The interaction of convection and the global solar rotation lead to differential rotation.
- ▶ Simulations with solar parameters (only luminosity and rotation rate as we will see below!) very often yield “anti-solar” differential rotation with fast poles and slow equator.
- ▶ What simple argument can explain the anti-solar states?



Mean angular velocity  $\bar{\Omega} = \bar{u}_\phi / r \sin \theta + \Omega_0$ . From Käpylä et al. (2014), Astron. Astrophys., 570, 43

# Differential rotation in simulations

- ▶ Simulations show that the flip from anti-solar (Regime II) to solar-like (Regime I) occurs around  $\text{Ro}_c \sim \text{Co}^{-1} \sim 1$ .
- ▶ The Sun appears to be near this transition and simulations tend to underestimate Co.



Source: Gastine et al. (2014), MNRAS, 438, L76.

# Convection and rotation

- ▶ As we discussed in the last tutorial, the order of magnitude of the convective velocity and length scale are

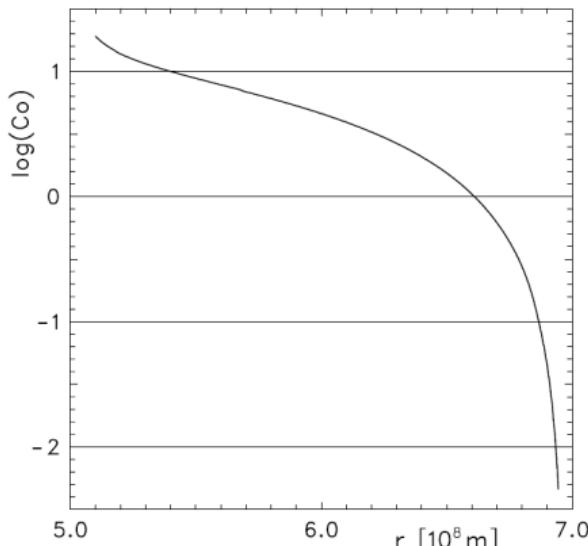
$$u \sim \left( \frac{F_{\text{tot}}}{\rho} \right)^{1/3} \equiv u_*, \text{ and } \ell \sim H_p, \quad (2)$$

where  $F_{\text{tot}} = \frac{L}{4\pi r^2}$  and  $\rho$  is a reference density.

- ▶ We also saw on the previous lecture that according to the mixing length theory the Coriolis number

$$\text{Co} = \frac{2\Omega_{\odot}\ell}{u}, \quad (3)$$

varies between  $10^{-3}$  near the surface and around 10 at the base of the convection zone.



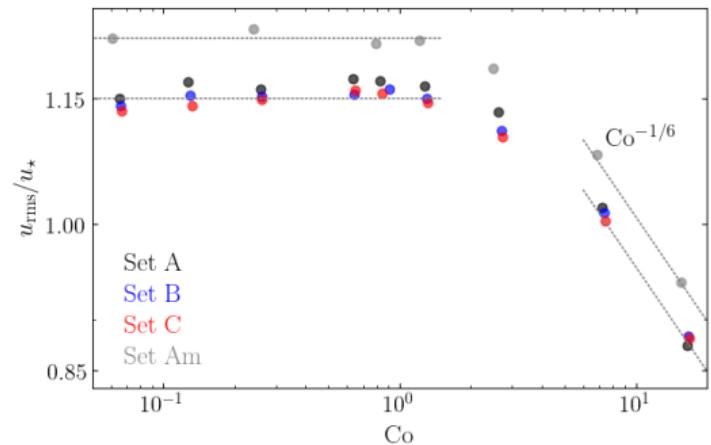
Käpylä et al. (2005), Astron.  
Astrophys., 438, 403.

# Convection and rotation

- When rotation is sufficiently rapid, the rotating mixing length theory (e.g. Stevenson, 1979, Geophys. Astrophys. Fluid Dyn., 12, 139) predicts that the convective velocity changes and is

$$u \sim \left( \frac{F_{\text{tot}}}{\rho} \right)^{1/3} \text{Co}^{-1/6} = u_* \text{Co}^{-1/6},$$

- (4) Convective velocity divided by  $u_*$  from several sets of simulations. From Käpylä (2024), Astron. Astrophys., 683, 221.



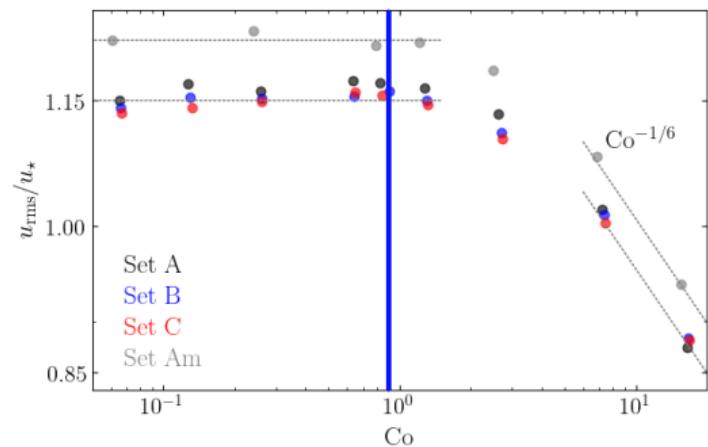
- Where is the Sun in this diagram?

# Convection and rotation

- When rotation is sufficiently rapid, the rotating mixing length theory (e.g. Stevenson, 1979, Geophys. Astrophys. Fluid Dyn., 12, 139) predicts that the convective velocity changes and is

$$u \sim \left( \frac{F_{\text{tot}}}{\rho} \right)^{1/3} \text{Co}^{-1/6} = u_* \text{Co}^{-1/6},$$

- (5) Convective velocity divided by  $u_*$  from several sets of simulations. From Käpylä (2024), Astron. Astrophys., 683, 221.



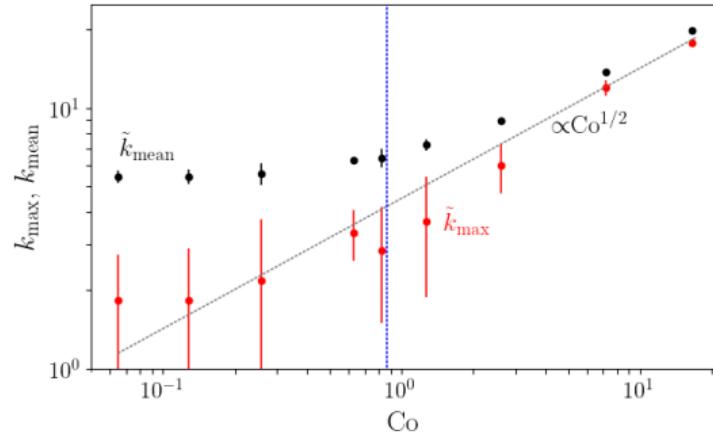
- Where is the Sun in this diagram?

# Convection and rotation

- ▶ The convective length scale also changes

$$\ell \sim H_p \text{Co}^{-1/2}. \quad (6)$$

- ▶ It appears that convection in the deep parts of solar convection zone are less rotationally constrained than previously thought.



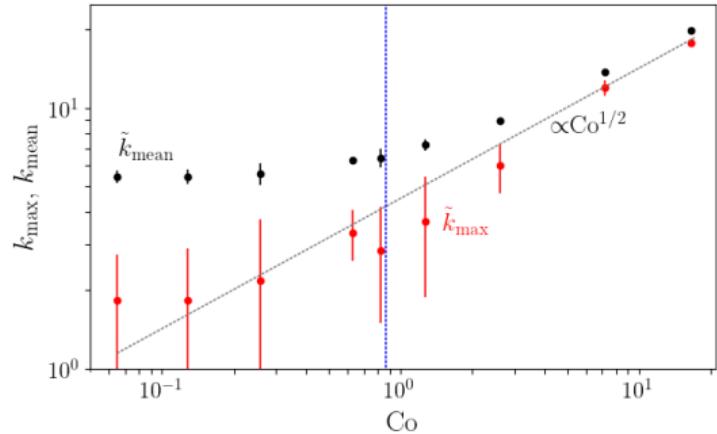
Two measures of the scale ( $k = 2\pi/\ell$ ) of convective structures in simulations as functions of Co. From Käpylä (2023), arXiv:2311.09082.

# Convection and rotation

- ▶ The convective length scale also changes

$$\ell \sim H_p \text{Co}^{-1/2}. \quad (7)$$

- ▶ It appears that convection in the deep parts of solar convection zone are less rotationally constrained than previously thought.
- ▶ There is a catch, however, and we will come back to that later.



Two measures of the scale ( $k = 2\pi/\ell$ ) of convective structures in simulations as functions of Co. From Käpylä (2023), arXiv:2311.09082.

# Why do simulations struggle to reproduce the Sun?

- ▶ Any ideas?

# Why do simulations struggle to reproduce the Sun?

- ▶ The equations of magnetohydrodynamics can be written in a dimensionless form where a number of dimensionless quantities uniquely defining the system appear (tutorial).

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \mu_0 \mathbf{J}),$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}),$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} = -\nabla \cdot (\rho \mathbf{u} \mathbf{u}) + \rho \mathbf{g} - \nabla p - 2\rho \boldsymbol{\Omega}_0 \times \mathbf{U} + \mathbf{J} \times \mathbf{B} + \nabla \cdot \mathbf{F}^{\text{visc}},$$

$$\rho T \frac{\partial s}{\partial t} = -\nabla \cdot (\rho s \mathbf{u}) + \nabla \cdot \mathcal{F} + \mathcal{H} + 2\nu \rho \mathbf{S}^2 + \eta \mu_0 \mathbf{J}^2,$$

# Why do simulations struggle to reproduce the Sun?

- ▶ The equations of magnetohydrodynamics can be written in a dimensionless form where a number of dimensionless quantities uniquely defining the system appear.
- ▶ These include the Prandtl numbers, the Rayleigh number, and the Taylor number:

$$\text{Pr} = \frac{\nu}{\chi}, \quad \text{Pr}_M = \frac{\nu}{\eta}, \quad \text{Ra} = \frac{gd^4}{\nu\chi} \left( -\frac{1}{c_p} \frac{ds}{dr} \right), \quad \text{Ta} = \frac{4\Omega^2 d^4}{\nu^2}, \quad (8)$$

where  $\nu$  is the kinematic viscosity,  $\eta$  is the magnetic diffusivity, and  $\chi = K_{\text{rad}}/c_p\rho$  is the radiative diffusivity.

- ▶ We can also estimate the relative magnitudes of different terms in the equations by OOM (order of magnitude) analysis. These give an additional set of (diagnostic) numbers (Reynolds, Péclet, Coriolis):

$$\text{Re} = \frac{ud}{\nu}, \quad \text{Re}_M = \frac{ud}{\eta}, \quad \text{Pe} = \frac{ud}{\chi}, \quad \text{Co} = \frac{2\Omega d}{u}. \quad (9)$$

- ▶ We need to compare these quantities in the simulations to those in stars.

# Why do simulations struggle to reproduce the Sun?

- ▶ Typically these numbers in stars are either  $\gg 1$  or  $\ll 1$ .
- ▶ Simulations are limited by available computational resources and can for most parameters do only modest values.

Parameter	Sun ( $M_{\odot}$ )	O9 (20 $M_{\odot}$ )	Simulations
Ra	$10^{20}$	$10^{24}$	$10^9$
Pr	$10^{-6}$	$10^{-5}$	$10^{-1} \dots 10$
Pr <sub>M</sub>	$10^{-3}$	$10^3$	$10^{-1} \dots 10$
Re	$10^{13}$	$10^{11}$	$10^4$
Pe	$10^7$	$10^6$	$10^4$
Re <sub>M</sub>	$10^{10}$	$10^{14}$	$10^4$
$\Delta\rho$	$10^6$	3	$10^2$
Ro	$0.1 \dots 1$	1	$10^{-2} \dots 10^3$

Some dimensionless parameters in the Sun and in a  $20M_{\odot}$  O9 star in comparison to typical simulations. From Käpylä et al. (2023), Space Science Rev., 219, 58.

Only the Rossby number ( $\text{Co}^{-1}$ ) can be reproduced!

## Why do simulations struggle to capture the Sun?

- ▶ More precisely, another Coriolis number can be defined as

$$\text{Co}_\star = \frac{2\Omega H_p}{u_\star}, \quad (10)$$

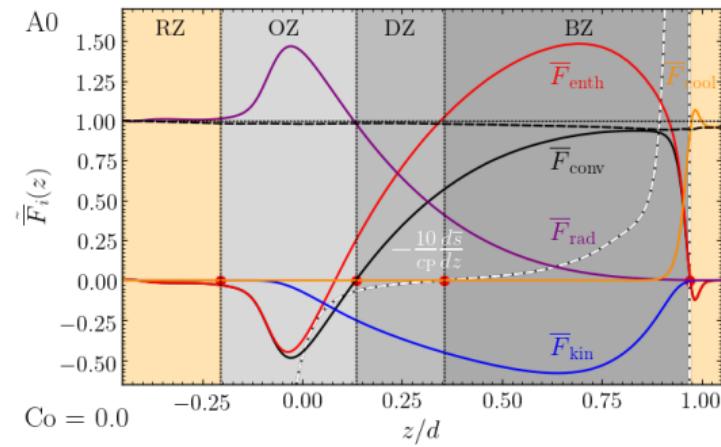
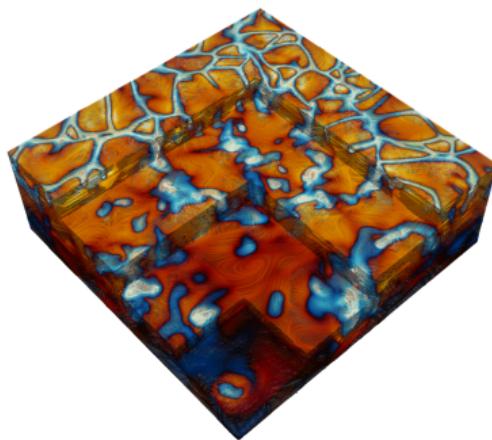
that does not depend on any actual velocity or length scale. It turns out that

$$\text{Co}_\star = (\text{Ra}_F^*)^{-1/3}, \quad \text{where} \quad \text{Ra}_F^* = \frac{\text{Ra}_F}{\text{Pr}^2 \text{Ta}^{3/2}}. \quad (11)$$

- ▶ The catch mentioned earlier is that it is not enough to reproduce *one* of the system parameters if all the other ones are (too much) off.
- ▶ We will come back to this in the tutorial...

# Non-canonical models of convection

- ▶ Heating from the bottom or cooling at the top?
- ▶ Instead of  $\nabla - \nabla_a > 0$  everywhere (as in mixing length theory), perhaps the lower part of the convection zone is *stably stratified*? This happens in the case of surface cooling driven convection.



Left: snapshot of the velocity. Right: Energy fluxes and the negative of the entropy gradient. Grey areas mixed by convection!

# Why should we care?

- ▶ The evolution of the magnetic field is governed by

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \mu_0 \mathbf{J}), \quad (12)$$

where  $\mathbf{J} = \mu_0^{-1} \nabla \times \mathbf{B}$  is the current density.

- ▶ If  $\mathbf{u} = 0$ , the induction equation reduces to a *diffusion equation*:

$$\frac{\partial \mathbf{B}}{\partial t} = \eta \nabla^2 \mathbf{B}. \quad (13)$$

- ▶ Why is this relevant?

# Why should we care?

- ▶ The evolution of the magnetic field is governed by

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \mu_0 \mathbf{J}), \quad (14)$$

where  $\mathbf{J} = \mu_0^{-1} \nabla \times \mathbf{B}$  is the current density.

- ▶ If  $\mathbf{u} = 0$ , the induction equation reduces to a *diffusion equation*:

$$\frac{\partial \mathbf{B}}{\partial t} = \eta \nabla^2 \mathbf{B}. \quad (15)$$

- ▶ Why is this relevant?
- ▶ The field cannot be generated without flows. To understand the generation of, e.g., the solar magnetic field, we must now the velocity field. *Study of magnetic phenomena is by far the most interesting branch of solar physics!*