Hands-on exercises 4: More thermodynamics related to convection

P. Käpylä, I. Milić

May 9, 2024

These are again mostly analytical exercises. But, as we deal with the real objects, sometimes we will want to calculate the physical values of our results. For that, it is very convenient to start a simple python session. We will also encourage you to do so :-)

Problem 1: Convince yourself that

$$\frac{1}{T}\frac{dT}{dr} = -\frac{\nabla}{H_p}, \tag{1}$$

where $\nabla = \frac{p}{T}\frac{dT}{dp}$, and H_p is the pressure scale height.

Problem 2: Show that the adiabatic temperature gradient can be written as

$$\left.\frac{dT}{dr}\right|_{\mathrm{a}} = -\frac{g}{c_p}, \tag{2}$$

where g is the acceleration due to gravity and c_p is the specific heat capacity at constant pressure.

Problem 3: Use the second law of thermodynamics

$$ds = \frac{dQ}{T}, \tag{3}$$

where s is the specific entropy, to show that an adiabatic process is *isentropic*, i.e., that the entropy does not change.

Show that the superadiabatic temperature gradient is related to the specific entropy gradient via:

$$-\frac{1}{c_p}\frac{ds}{dz} = \frac{1}{H_p}(\nabla - \nabla_{\mathrm{a}}). \tag{4}$$

Problem 4: Use the Navier-Stokes equation

$$\rho\frac{\partial \boldsymbol{u}}{\partial t} = -\rho\boldsymbol{u}\cdot\nabla\boldsymbol{u} - \rho\boldsymbol{g} + \boldsymbol{\nabla}p + \boldsymbol{F}_{\mathrm{visc}}, \tag{5}$$

and the definition of the convective enthalpy flux (unit $\mathrm{W/m^2}$)

$$F_{\mathrm{conv}} = c_p\rho u T', \tag{6}$$

to estimate the convective velocity and temperature fluctuations at the middle of the convection zone of the Sun ($r = 0.85R_\odot$). Hint: assume a hydrostatic stationary state, use the tabulated mixing length model from previous lecture and $c_p = 2.07\cdot 10^4$ J $\mathrm{K^{-1}}$ $\mathrm{kg^{-1}}$.

Useful physical constants

- $R_\odot = 696 \times 10^6\,\mathrm{m}$
- $M_\odot = 1.989 \times 10^{30}\,\mathrm{kg}$
- $L_\odot = 3.83 \times 10^{26}$ W
- $T_\odot^{\mathrm{eff}} = 5777\,\mathrm{K}$
- $1\,\mathrm{AU} = 1.496 \times 10^8\,\mathrm{km}$
- $c = 2.997 \times 10^8\,\mathrm{m/s}$
- $G = 6.674 \times 10^{-11}$ $\mathrm{Nm^2/kg^2}$
- $k = 1.38 \cdot 10^{-23}$ J/K
- $m_{\mathrm{H}} = 1.67 \cdot 10^{-27}$ kg