1. Non-dimensionalize:

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\cdot\nabla\bar{u} = -\bar{\nabla}\varpi + \nu\bar{\nabla}^2\bar{u} + \alpha g T\hat{\bar{z}}$$

Choose:

$$x = l_c\tilde{x},\quad t = \tau_c\tilde{t},\quad u = u_c\tilde{u},\quad \varpi = \varpi_c\tilde{\varpi},\quad T = T_c\tilde{T}$$

$$\Rightarrow \frac{u_c}{\tau_c}\frac{\partial\tilde{\bar{u}}}{\partial\tilde{t}} + \frac{u_c^2}{l_c}\tilde{\bar{u}}\cdot\tilde{\bar{\nabla}}\tilde{\bar{u}} = -\frac{\varpi_c}{l_c}\tilde{\bar{\nabla}}\tilde{\varpi} + \frac{u_c}{l_c^2}\nu\tilde{\bar{\nabla}}^2\tilde{\bar{u}} + \alpha g T_c\tilde{T}\hat{\bar{z}}.$$

Choose:

$$l_c = L,\quad \tau_c = \frac{L^2}{\chi},\quad u_c = \frac{L}{\tau_c}$$

Now we can drop the tildes and simplify by multiplying with $\frac{\tau_c}{u_c}$:

$$\frac{\partial\bar{u}}{\partial t} + \underbrace{\frac{\tau_c}{\cancel{u_c}}\frac{u_c^{\cancel{2}}}{l_c}}_{=\frac{\tau_c}{L}u_c = \frac{\tau_c}{L}\frac{L}{\tau_c} = 1}\bar{u}\cdot\bar{\nabla}\bar{u} = -\underbrace{\frac{\tau_c}{u_c}\frac{\varpi_c}{l_c}}_{=\frac{\varpi_c}{u_c}\frac{\tau_c}{L} = \frac{\varpi_c}{u_c^2}}\bar{\nabla}\varpi + \underbrace{\frac{\tau_c}{\cancel{u_c}}\frac{\cancel{u_c}}{l_c^2}}_{=\frac{\tau_c\nu}{L^2} = \frac{\nu}{\chi} = Pr}\nu\bar{\nabla}^2\bar{u} + \underbrace{\frac{\tau_c}{u_c}}\alpha g T_c T\hat{\bar{z}}.$$

$$\frac{\tau_c}{u_c}\alpha g T_c T\hat{\bar{z}} = \frac{\nu}{\nu}\frac{\tau_c^2}{L}\alpha g T_c T\hat{\bar{z}} = \frac{\nu}{\nu}\frac{L^{\cancel{4}\,3}}{\chi^2\cancel{L}}\alpha g T_c T\hat{\bar{z}} = Pr\frac{\alpha g T_c}{\chi\nu}T\hat{\bar{z}} = Pr\,Ra\,T\hat{\bar{z}}.$$

$$\varpi_c = \frac{p_c}{\rho_c} = c_s^2$$

$$\Rightarrow \frac{\varpi_c}{u_c^2} = \frac{c_s^2}{u_c^2} = \frac{1}{Ma}$$

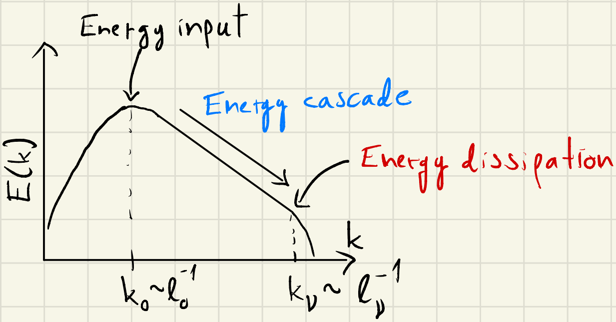
Ma: Mach number

Therefore, we have the non-dimensional equation:

$$\frac{\partial\bar{u}}{\partial t} + \bar{u}\cdot\nabla\bar{u} = -\frac{1}{Ma^2}\bar{\nabla}\varpi + Pr\,\bar{\nabla}^2\bar{u} + Ra\,Pr\,T\hat{\bar{z}},$$

where Ma, Pr, and Ra define the system uniquely.

2. Kolmogorov scale.



- Energy transfer rate:
$$\frac{u^2}{\tau} \sim \frac{u^2}{\ell/u} \sim \frac{u^3}{\ell} = \epsilon$$
- ϵ = const. between ℓ_0 and ℓ_ν.
- ϵ equals dissipation at ℓ_ν.
- Present ℓ_ν in terms of ϵ and ν.

- Dimensional analysis:
$$[\epsilon] = \frac{m^3}{s^3\, m} = \frac{m^2}{s^3}, \quad [\nu] = \frac{m^2}{s}, \quad [\ell_\nu] = m.$$
- Assume that:
$$\ell_\nu = \epsilon^a \nu^b$$
And therefore:
$$m = m^{2a+2b}\, s^{-3a-b}$$
$$\Rightarrow \begin{array}{l} 1 = 2a + 2b \\ 0 = -3a - b \Rightarrow b = -3a \end{array} \quad 1 = 2a - 6a \Rightarrow a = -\frac{1}{4} \quad \Rightarrow b = \frac{3}{4}$$

- Thus:
$$\underline{\underline{\ell_\nu = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}.}}$$

- Ratio ℓ_ν / L:
$$\frac{\ell_\nu}{L} = \left(\frac{\nu^3}{\epsilon}\right)^{1/4} \frac{1}{L} = \left(\frac{\nu^3 \cancel{L}}{u^3}\right)^{1/4} \frac{1}{L^{3/4}} = \underline{\underline{Re_L^{-3/4}}}\ ; \text{ where } Re_L = \frac{uL}{\nu}.$$

- The ratio L/ℓ_ν gives the order of magnitude of grid points that are needed <u>per direction</u>. Therefore $(Re_L^{3/4})^3$ points are needed.
- For $Re = 10^{12}$ this is $\underline{\underline{10^{27}.}}$

- 10^3 cores for $1000^3 = 10^9$ grid points.
- Assume ideal weak scaling, i.e., linear scaling between problem size and number of cores:

$$n_{cores} = \frac{10^{27}}{10^{\cancel{9}6}} \cdot \cancel{10^3} = \underline{\underline{10^{21}}}.$$

Power requirement: fastests current CPUs have $\sim 10^2$ cores and TDP ~ 400W.
$\Rightarrow 4 \cdot 10^{21}$ W which is the luminosity of a small M type dwarf.

Time constraints:

$$\delta t_{sound} \sim \frac{\Delta x}{c_s} \sim \frac{0{,}2\,m}{2 \cdot 10^5 \frac{m}{s}} \sim 10^{-6}\,s.$$

$$\delta t_{conv} \sim \frac{\Delta x}{u_{conv}} \sim \frac{0{,}2\,m}{2 \cdot 10^3 \frac{m}{s}} \sim 10^{-4}\,s.$$

For a solar cycle $\tau_{cyc} \sim 11$ yr:

$$\underbrace{\frac{\tau_{cyc}}{\delta t}}_{\text{number of time steps}} \cdot \underbrace{t_{wc}}_{\text{wallclock time per timestep}} = \text{time-to-solution} = \tau_{tts}.$$

$$\tau_{tts}^{sound} \sim 3{,}5 \cdot 10^{14} \cdot \tau_{wc} \sim 3{,}5 \cdot 10^{13}\,s \sim \underline{\underline{1{,}1 \cdot 10^6\,yr}}$$

$$\tau_{tts}^{conv} \sim 3{,}5 \cdot 10^{12} \cdot \tau_{wc} \sim 3{,}5 \cdot 10^{11}\,s \sim \underline{\underline{1{,}1 \cdot 10^4\,yr}}$$

This means that to do the simulation in an acceptable time (say 1 year), the single core performance would need to improve by a factor 10^6 (10^4). This is very unlikely for semiconductor-based CPUs (or GPUs).