# Theoretical Astrophysics: Physics of Sun and Stars
# Homework 2

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**Deadline for this homework is 11/06 23:59**

**Problem 1:** The solar luminosity is $L_\odot = 3.83 \cdot 10^{26}$ W. Assume that all of the energy for this luminosity is provided by the $\mathrm{p-p\,1}$ chain, and that neutrinos carry off 3% of the energy. How many neutrinos are produced per second? What is the neutrino flux (i.e. the number of neutrinos per second per cm$^2$) at Earth?

The $\mathrm{p-p\,1}$ chain is comprised of the reactions:

$$\begin{aligned}
p + p &\rightarrow {}^2\mathrm{D} + e^+ + \nu, && (1.18\ \mathrm{MeV}) \\
{}^2\mathrm{D} + p &\rightarrow {}^3\mathrm{He} + \gamma, && (5.49\ \mathrm{MeV}) \\
{}^3\mathrm{He} + {}^3\,\mathrm{He} &\rightarrow {}^4\mathrm{He} + 2p. && (12.86\ \mathrm{MeV})
\end{aligned} \tag{1}$$

where the first two reactions need to happen twice for the last reaction to occur, and where $\nu$ signifies an emitted neutrino. Energy released (in MeV $= 10^6$ eV) is given in brackets after each reaction.

**Problem 2:**

a) Why does convection transport heat radially outward although there is no net mass flux?

b) Estimate the superadiabatic temperature gradient (in order of magnitude fashion) in the Sun, by making use of the mixing length expression for $F_{\mathrm{conv}}$:

$$F_{\mathrm{conv}} = c_p \rho T g^{1/2} \frac{\ell^2}{4\sqrt{2} H_p^{3/2}} (\nabla - \nabla_{\mathrm{ad}})^{3/2}. \tag{2}$$

Use the average values of density and temperature of the Sun, and $c_p = 2.07 \cdot 10^4$ J K$^{-1}$ kg$^{-1}$.

Hint: Recall that $\ell = \alpha_{\mathrm{MLT}} H_p$ and bear in mind the definition of the pressure scale height from previous homework. When is this result a good approximation and when not? Why?

**Problem 3:** Use the equations of hydrostatic equilibrium and mass conservation:

$$\frac{dp}{dr} = -\rho \frac{Gm}{r^2}, \quad \frac{dm}{dr} = 4\pi r^2 \rho, \tag{3}$$

assuming a polytropic equation of state

$$p = K\rho^{\gamma}, \tag{4}$$

where $\gamma = 1 + \frac{1}{n}$ is the polytropic exponent and $n$ is the polytropic index, to derive the Lane-Emden equation:

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2 \frac{d\theta}{d\xi}\right) = -\theta^n. \tag{5}$$

Here $\theta$ and $\xi$ are the non-dimensional density and radius defined as

$$\rho = \rho_c \theta^n, \quad \text{and} \quad r = \alpha\xi, \tag{6}$$

and where $\alpha^2$ equals a constant that arises in the derivation of Eq.(5).

The boundary conditions at $\xi = 0$ for the Lane-Emden equation are:

$$\theta = 1, \text{ and } \frac{d\theta}{d\xi} = 0. \tag{7}$$

What do these boundary conditions correspond to?

Solve the Lane-Emden equation for $n = 0, 1,$ and $5$. For these values the equation can be solved *analytically*.

Solve the Lane-Emden equation for $n = 1.5$ and $n = 3$, and compare the resuls with the ones found in the textbook by D. Prialnik. For this you will have to solve the equation *numerically*.

**Useful physical constants**

- $R_\odot = 696 \times 10^6\,\mathrm{m}$
- $M_\odot = 1.989 \times 10^{30}\,\mathrm{kg}$
- $L_\odot = 3.83 \times 10^{26}$ W
- $T_\odot^{\mathrm{eff}} = 5777\,\mathrm{K}$
- $1\,\mathrm{AU} = 1.496 \times 10^8\,\mathrm{km}$
- $c = 2.997 \times 10^8\,\mathrm{m/s}$
- $G = 6.674 \times 10^{-11}$ $\mathrm{Nm^2/kg^2}$
- $k = 1.38 \cdot 10^{-23}$ J/K
- $m_{\mathrm{H}} = 1.67 \cdot 10^{-27}$ kg
- $h = 6.626 \times 10^{-34}$ J s.
- $k = 1.38 \times 10^{-23}$ J/K.