Theoretical Astrophysics I: Physics of Sun and Stars
Lecture 9: Stellar Atmospheres

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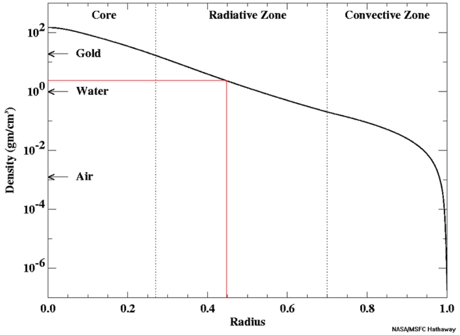
Recap

- So far we have dealt with interiors of the stars - these contain an absolute majority of stellar material.
- We studied the equations that govern stellar structure and the evolution and solved them in more or less detail to understand how the stars look into the inside and how they evolve on the HR diagram.
- None of these processes were **directly** observable. The only observable quantity we talked about was the stellar luminosity (L).
- Now we are going to talk about the structures that we can observe - stellar atmospheres, and their observational signatures - stellar spectra.

Why study stellar atmospheres?

- Stellar atmosphere is the miniscule part on the very right of this diagram.
- How can it be important? (Salpeter once asked the same question)
- First: Stellar atmosphere is the only part we can observe: insight into what is going on inside.
- Second: It can influence the evolution of the star, at certain evolution stages atmospheres can be huge and influence the rest.
- Third: There is a lot of interesting physics going on (especially the case for the Sun, talk next week!)



Credits: NASA/MSFC Hathaway

Differences between stellar atmospheres and stellar interiors

Interior

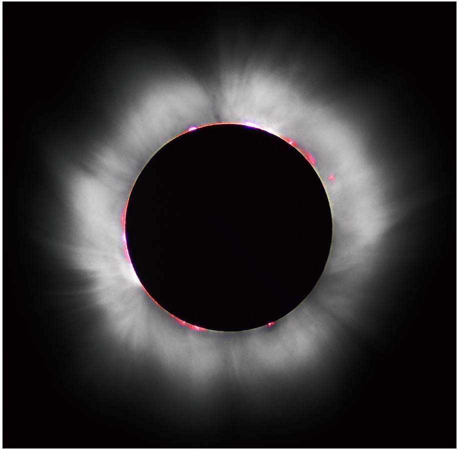
- $T = 10^5 - 10^7$ K
- $\rho = 10^1 - 10^5 \mathrm{kg/m^3}$
- Completely ionized
- Ideal gas
- LTE

Atmospheres

- $T = 10^4 - 10^5$ K
- $\rho = 10^{-1} - 10^{-8} \mathrm{kg/m^3}$
- Neutral, partially ionized, totally ionized - depending on the layer and species
- Ideal gas
- Photosphere in LTE, chromosphere and up in NLTE, but some things are tricky (e.g. photoionization)

Sketch of the atmosphere

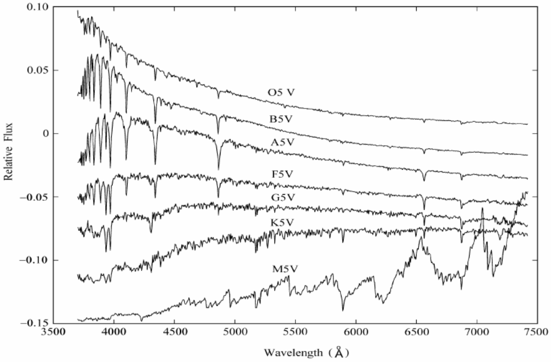
- Atmospheres of main sequence stars are typically very thin: plane-parallel approximation is valid.
- The transport of energy goes from convective to radiative, and then the energy leaves the star.
- Radiative equilibrium is often used: $H = const$.
- To understand the observational signatures of the atmosphere we need to:
- i) mathematically describe propagation of radiation;
- ii) take into account appropriate physical processes that contribute to opacity and emissivity;



Credits: Wikipedia

Spectral classses

- One of the main features we want to understand are the differences between the different spectral classes.
- For a while - people thought that these differences are due to different chemical composition.
- But it turns out stellar spectra are **extremely** sensitive to the temperature and the pressure.
- Today, we will try to explain that.



Credits: Wikipedia

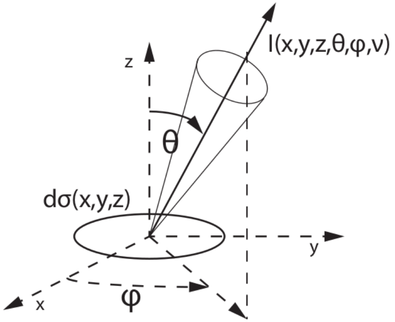
Specific monochromatic intensity

- We need to treat wavelength and angular dependence of the radiation field
- Intensity: energy transported through given area in given time per given solid angle and frequency/wavelength bin (note the deprojection factor $\cos\theta$).

$$I_\nu = \frac{dE}{dS\, dt\, d\Omega\, d\nu\, \cos\theta} \tag{1}$$

- Going to number of photons:

$$n(\theta, \phi\, \nu) = \frac{I_\nu}{c\, h\nu} \tag{2}$$



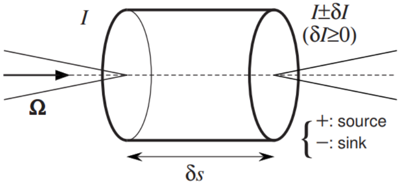
Credits: IM thesis (2014, University of Belgrade)

Radiative Transfer Equation (RTE)

- This formulation is (more or less) due to Kirchhoff. The change of intensity "along-the-ray" over a distance ds is:

$$dI_\nu = \eta_\nu ds - \chi_\nu I_\nu ds \tag{3}$$

- The terms of the right represent emission and **total** absorption (both true absorption and scattering) per unit volume.



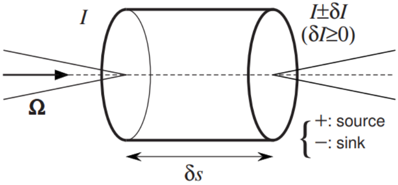
Credit: Anthony B. Davis and Yuri Knyazikhin

Radiative Transfer Equation

- Now, it makes sense that the absorption and emission properties of the medium depend on:
- Amount of matter capable of absorbing/emitting
- The inherent properties of the matter at the given temperature (T is very important!)
- So we define:

$$\kappa_\nu = \chi_\nu/\rho \quad (4)$$

$$j_\nu = \eta_\nu/\rho \quad (5)$$



Credit: Anthony B. Davis and Yuri Knyazikhin

- So our equation becomes:

$$\frac{1}{\rho}\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu \quad (6)$$

Optical depth and Source function

- We often do the following:

$$\frac{dI_\nu}{-\rho\kappa_\nu ds} = I_\nu - j_\nu/\kappa_\nu \tag{7}$$

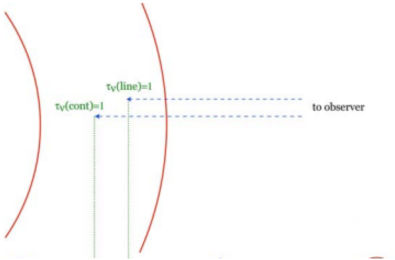
- And we get:

$$\frac{dI_\nu}{-\rho\kappa_\nu ds} = I_\nu - j_\nu/(\rho\kappa_\nu) \tag{8}$$

$$\kappa_\nu = \chi_\nu/\rho;\ j_\nu = \eta_\nu/\rho \tag{9}$$

- So our equation becomes:

$$\frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu \tag{10}$$



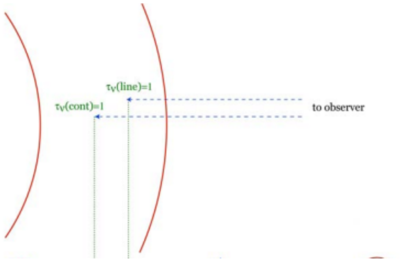
Credit: Rolf Kudritzki

$$\frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu \tag{11}$$

- Solving this equation if everything is known is relatively straightforward:

$$dI_\nu = I_\nu^0 e^{-\tau_\nu} + \int_0^{\tau_\nu} S(t) e^{-t} dt \tag{12}$$

- Here t is the dummy variable for frequency dependent optical depth.
- **Please note that the optical depth depends on the frequency!** Discuss!

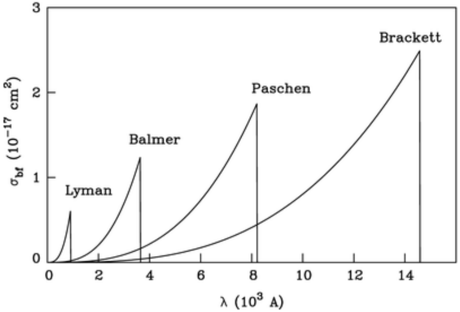


Credit: Rolf Kudritzki

Optical depth

$$\tau_\nu = \int -\rho\kappa_\nu ds \tag{13}$$

- As the absorption and scattering properties (opacity) of the medium depend on the wavelength, so does the optical depth.
- Now, why does the opacity depend on the wavelength? Discuss!
- Right: opacity due to bound-free absorption of neutral hydrogen atom.



Credit: Rolf Kudritzki

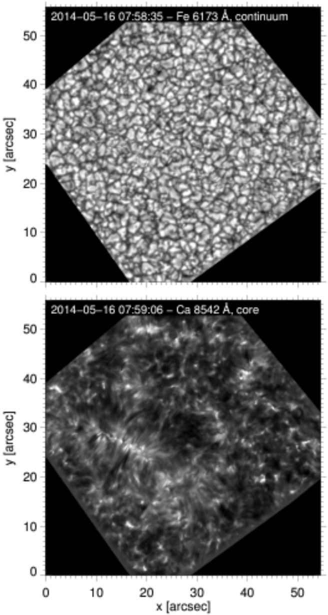
Constant source function

$$dI_\nu = I_\nu^0 e^{-\tau_\nu} + \int_0^{\tau_\nu} S(t) e^{-t} dt \qquad (14)$$

- For a slab of constant source function, we get:

$$dI_\nu = I_\nu^0 e^{-\tau_\nu} + S_\nu (1 - e^{-\tau_\nu}) \qquad (15)$$

- Depending on the τ_ν we can get either dominant contribution of the background radiation, or the slab.
- Discuss the image on the right (solar atmosphere imaged at two different wavelengths)

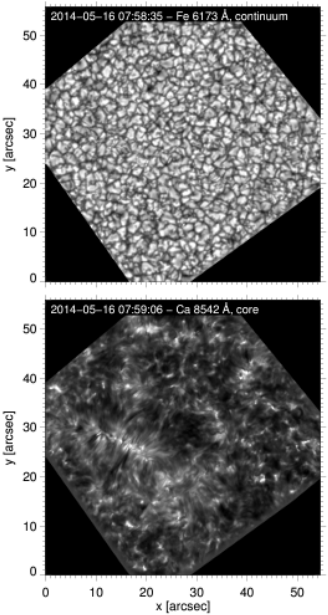


Credit: Gosic et al., 2018

Constant source function

$$dI_\nu = I_\nu^0 e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu}) \qquad (16)$$

- We can now frame this differently:
- If we somehow **know** that, at a wavelength the medium is **optical thick**, we know we are seeing that medium.
- If it is **transparent**, we know we are seeing the light below.
- So we can use different wavelengths to probe different regions.
- But, sometimes differences between S and



Credit: Gosic et al., 2018

Linear source function with depth

- A very often used assumption in the atmosphere modeling is the so called, Milne-Eddington model, it assumes that, at some *referent* wavelength, we have:

$$S_\nu = a + b\tau_r \tag{17}$$

- where τ_r is the optical depth at that referent wavelength.
- We define r_ν, which is the ratio of opacities at other frequencies (wavelengths) to the referent one.
- Additionally, we will assume that $S_\nu = B_\nu$ (LTE), and that $\tau_\nu >> 1$, we get:

$$dI_\nu = \int_0^\infty (a + b\tau_r)e^{-\tau_r r_\nu} d\tau_r r_\nu \tag{18}$$

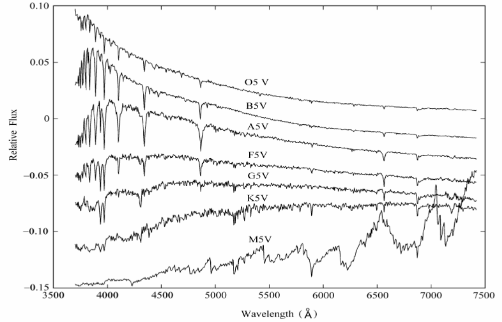
- That yields (blackboard!):

$$dI_\nu = a + b/r_\nu \tag{19}$$

Linear source function with depth

$$dI_\nu = a + b/r_\nu \tag{20}$$

- It is reasonable to assume that S increases inward in the solar atmosphere (because T increases), so a and b are positive.
- For more opaque regions we will get less intensity.
- Wow - not really insightful after all!
- Extremely important: we also understood that a **gradient** of temperature in the atmosphere is necessary!



Credits: Wikipedia

Why the Milne-Eddington approximation

- Why choose linear, why not quadratic? Why not some other functional dependence?
- Beginning of XX century, Eddington, Milne, Schwarzschild and company were trying to solve the structure of the stellar atmosphere in radiative equilibrium.
- Given the outgoing specific flux (H), find the structure of the atmosphere.
- Radiative equilibrium (your hw):

$$\int_0^\infty \kappa_\nu J_\nu d_\nu = \int_0^\infty \kappa_\nu S_\nu d_\nu \tag{21}$$

- Was simplified, by assuming that there is some mean $\overline{\kappa}$ (gray atmosphere), so that:

$$J = S \tag{22}$$

- Furthermore, it was reasonable to assume LTE, so that $S = B = \sigma T^4 = J$.
- This is a direct connection between the radiation and the temperature.
- But how to find $J(z)$, or better to say $J(\tau)$?

Milne problem

- But how to find $J(z)$, or better to say $J(\tau)$? We did not use RTE yet. Define that optical depth goes in the negative z direction:

$$\mu \frac{dI}{d\tau} = I - S = I - J \tag{23}$$

- Integrate this over $d\mu$ and $\mu d\mu$. We will need the blackboard here!
- You should get...

Milne problem

- But how to find $J(z)$, or better to say $J(\tau)$? We did not use RTE yet. Define that optical depth goes in the negative z direction:

$$\mu \frac{dI}{d\tau} = I - S = I - J \tag{24}$$

- Integrate this over $d\mu$ and $\mu d\mu$. We will need the blackboard here!
- You should have gotten:

$$\frac{dH}{d\tau} = 0 \tag{25}$$

- and:

$$\frac{dK}{d\tau} = H \tag{26}$$

- This is where Eddington made an approximation to try and "close" the system, he assumed that $K = J/3$, and thus:

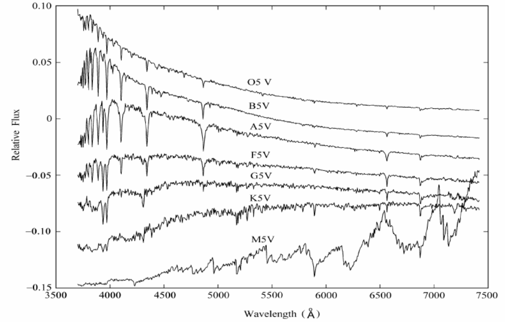
$$J = 3H\tau + \mathrm{const} = a\tau + b \tag{27}$$

- This can later be pursued further to infer the temperature structure of a gray atmosphere, but we will not go there.

Linear source function with depth

$$dI_\nu = a + b/r_\nu \qquad (28)$$

- It is reasonable to assume that S increases inward in the solar atmosphere (because T increases), so a and b are positive.
- For more opaque regions we will get less intensity.
- We "solved" mathematical part. Now is time for physics, because...
- What we still don't understand is why the opacity varies with the wavelength.



Credits: Wikipedia

Opacity sources

- Reminder, we only talk about opacity (κ_ν) because $j_\nu = B_\nu \kappa_\nu$!
- Important sources are:
- Bound-free transitions (photoionization)
- Free-free processes (inverse bremsstrahlung) - not important at low temperatures and optical wavelengths
- Bound-bound processes (spectral lines - very diagnostically important!) Discuss!
- Thomson and Rayleigh scattering.
- **Bound-free and free-free emission of $H-$ (very important for colder stars).**

Bound-free opacity

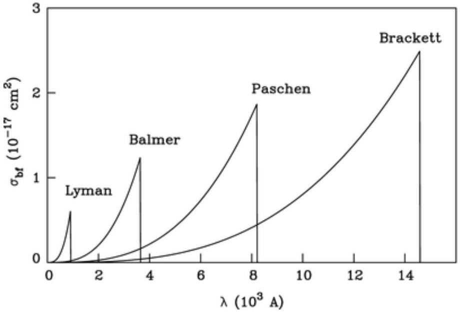
- This is, in fact, photoionization. The energy of the photon needs to be larger than the binding energy of the electron:

$$h\nu > E_{ion} \tag{29}$$

- Higher levels are easier to ionize, so if we have more excited atoms, opacity at lower energies/ higher wavelengths is higher. Talk about this for a while.
- For example, for hydrogen: $E_{ion} = 13.6\mathrm{eV}/i^2$, where i is the principal quantum number of the level.

$$\rho\kappa_\nu^{bf} = \sum_i n_i \sigma_i \tag{30}$$

- where I denoted the cross-section with σ.



Bound-free opacity

- Look carefully at the figure: what levels need to be populated so that hydrogen contributes to the opacity in the visible wavelengths
- The Boltzman equation governs the excitations as:

$$n_i = n_H \frac{g_i e^{-E_i/kT}}{Z_H} \tag{31}$$

- here g_i is called **statistical weight** and Z is the **partition function**. But, we also need the concentration of the neutral hydrogen (Hydrogen can be neutral, ionized, tied into H_2 or even in $H-$). For that we can use Saha equation:

$$\frac{n_{H+} n_e}{n_H} = \frac{}{\Lambda^3} e^{-E_{ion}/kT} \tag{32}$$

How does n_H vary with temperature?