

Theoretical Astrophysics: Physics of Sun and Stars

Homework 2

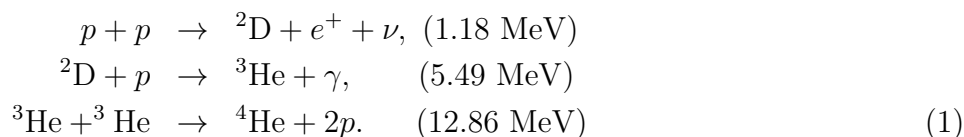
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Deadline for this homework is 11/06 23:59

Problem 1: The solar luminosity is $L_{\odot} = 3.83 \cdot 10^{26}$ W. Assume that all of the energy for this luminosity is provided by the p – p 1 chain, and that neutrinos carry off 3% of the energy. How many neutrinos are produced per second? What is the neutrino flux (i.e. the number of neutrinos per second per cm^2) at Earth?

The p – p 1 chain is comprised of the reactions:



where the first two reactions need to happen twice for the last reaction to occur, and where ν signifies an emitted neutrino. Energy released (in $\text{MeV} = 10^6 \text{ eV}$) is given in brackets after each reaction.

Problem 2:

- a) Why does convection transport heat radially outward although there is no net mass flux?
- b) Estimate the superadiabatic temperature gradient (in order of magnitude fashion) in the Sun, by making use of the mixing length expression for F_{conv} :

$$F_{\text{conv}} = c_p \rho T g^{1/2} \frac{\ell^2}{4\sqrt{2}H_p^{3/2}} (\nabla - \nabla_{\text{ad}})^{3/2}. \tag{2}$$

Use the average values of density and temperature of the Sun, and $c_p = 2.07 \cdot 10^4 \text{ J K}^{-1} \text{ kg}^{-1}$.

Hint: Recall that $\ell = \alpha_{\text{MLT}} H_p$ and bear in mind the definition of the pressure scale height from previous homework. When is this result a good approximation and when not? Why?

Problem 3: Use the equations of hydrostatic equilibrium and mass conservation:

$$\frac{dp}{dr} = -\rho \frac{Gm}{r^2}, \quad \frac{dm}{dr} = 4\pi r^2 \rho, \tag{3}$$

assuming a polytropic equation of state

$$p = K\rho^\gamma, \quad (4)$$

where $\gamma = 1 + \frac{1}{n}$ is the polytropic exponent and n is the polytropic index, to derive the Lane-Emden equation:

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left(\xi^2 \frac{d\theta}{d\xi} \right) = -\theta^n. \quad (5)$$

Here θ and ξ are the non-dimensional density and radius defined as

$$\rho = \rho_c \theta^n, \quad \text{and} \quad r = \alpha \xi, \quad (6)$$

and where α^2 equals a constant that arises in the derivation of Eq.(5).

The boundary conditions at $\xi = 0$ for the Lane-Emden equation are:

$$\theta = 1, \quad \text{and} \quad \frac{d\theta}{d\xi} = 0. \quad (7)$$

What do these boundary conditions correspond to?

Solve the Lane-Emden equation for $n = 0, 1$, and 5 . For these values the equation can be solved *analytically*.

Solve the Lane-Emden equation for $n = 1.5$ and $n = 3$, and compare the results with the ones found in the textbook by D. Prialnik. For this you will have to solve the equation *numerically*.

Useful physical constants

- $R_\odot = 696 \times 10^6 \text{ m}$
 - $M_\odot = 1.989 \times 10^{30} \text{ kg}$
 - $L_\odot = 3.83 \times 10^{26} \text{ W}$
 - $T_\odot^{\text{eff}} = 5777 \text{ K}$
 - $1 \text{ AU} = 1.496 \times 10^8 \text{ km}$
 - $c = 2.997 \times 10^8 \text{ m/s}$
 - $G = 6.674 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$
 - $k = 1.38 \cdot 10^{-23} \text{ J/K}$
 - $m_{\text{H}} = 1.67 \cdot 10^{-27} \text{ kg}$
 - $h = 6.626 \times 10^{-34} \text{ J s}$.
 - $k = 1.38 \times 10^{-23} \text{ J/K}$.
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