Theoretical Astrophysics I: Physics of Sun and Stars
Lecture 6: Radiative Energy Transport

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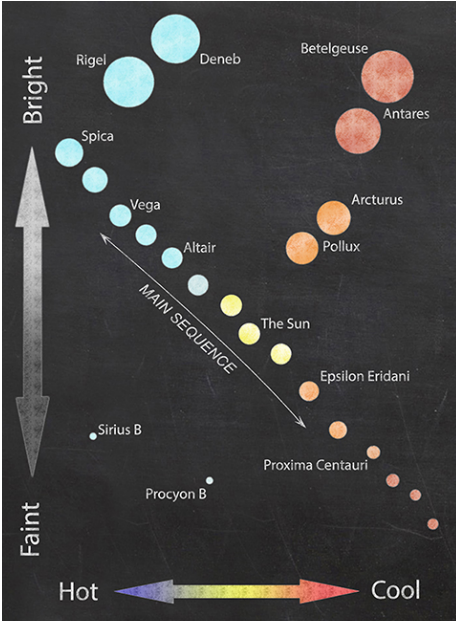
Brief recap

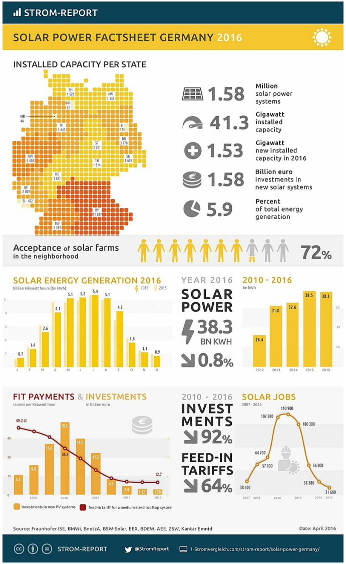
- We started with describing observed properties of the stars. One quantity that we measure (and want to reproduce) is the **luminosity**, and conversely **flux** (or specific flux).
- We wrote down equations that govern stellar structure and evolution. Solving them for proper boundary conditions will yield the structure of a star: $p(r)$, $T(r)$, $\rho(r)$, $F(r)$, etc...
- Analyzing their variation in time allows us to model stellar evolution.
- We spent last two lectures talking about a difficult problem of convection. But there is another way to transport energy: via **radiation**.



HR diagram

Photons vs particles

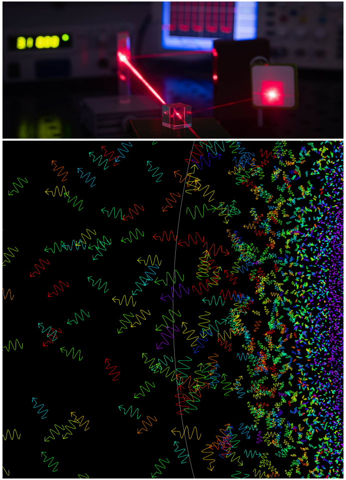
- It is obvious that radiation can carry energy.
- We treat radiation using photons, but they are clearly different from atoms, molecules, ions and electrons.
- Photons do not have mass and move with speed of light.
- The number of photons is not conserved.
- Photons can also be treated like a gas (e.g. see the derivation of Stefan-Boltzmann law by Boltzmann)
- They still observe conservation of energy, momentum, angular momentum, etc.



Credits: Strom Report

Photons vs particles

- It will be essential to understand photon-matter interaction. As Ivan Hubeny said:
- *...In other words radiation in fact determines the structure of the medium yet the medium is probed only by this radiation.*
- Radiation: constituent in energy transport (and equation of state).
- Also: diagnostics that allows us to understand physical properties of the medium.
- Contrary to the lab: we need to treat wavelength and angular dependence of the radiation field.



Credits: LabRoots.com (up), Prof. Rob Rutten (bottom)

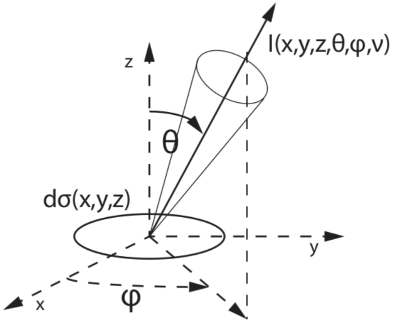
Specific monochromatic intensity

- We need to treat wavelength and angular dependence of the radiation field
- Intensity: energy transported through given area in given time per given solid angle and frequency/wavelength bin (note the deprojection factor $\cos\theta$).

$$I_\nu = \frac{dE}{dS\,dt\,d\Omega\,d\nu\,\cos\theta} \tag{1}$$

- Going to number of photons:

$$n(\theta, \phi\,\nu) = \frac{I_\nu}{h\nu} \tag{2}$$



Credits: IM thesis (2014, University of Belgrade)

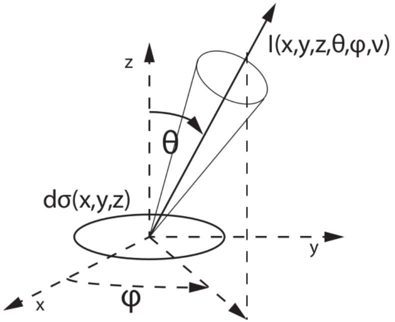
Other "moments" of the radiation field

- Intensity fully describes the radiation field (without polarization). But often we need some derived quantities:
- Mean intensity:

$$J_\nu = \frac{1}{4\pi} \oint I_\nu(\theta, \phi) \sin\theta d\theta d\phi \quad (3)$$

- Flux (in z direction):

$$\mathcal{F}_\nu = \oint I_\nu(\theta, \phi) \cos\theta \sin\theta d\theta d\phi \quad (4)$$



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Some de-confusion of the term flux

- Typically we say that (spectral, monochromatic) flux is:

$$F_\nu = \frac{dE}{dtdSd\nu} \tag{5}$$

- But in the book typically:

$$F_\nu = \frac{dE}{dtd\nu} \rightarrow F = \frac{dE}{dt} \tag{6}$$

But then, to make situation works, in stellar atmospheres theory flux is:

$$\mathcal{F}_\nu = \frac{dE}{dtdSd\nu} \tag{7}$$

then astrophysical flux:

$$F_\nu = \frac{1}{\pi}\frac{dE}{dtdSd\nu} \tag{8}$$

and Eddington flux (which the book uses and calls the radiation flux) is:

$$H_\nu = \frac{1}{4\pi}\frac{dE}{dtdSd\nu} = \frac{1}{4\pi}\mathcal{F}_\nu \tag{9}$$

More moments of the radiation field

- Mean intensity:

$$J_\nu = \frac{1}{4\pi} \oint I_\nu(\theta, \phi) \sin\theta d\theta d\phi \tag{10}$$

- Radiation flux:

$$\mathcal{H}_\nu = \frac{1}{4\pi} \oint I_\nu(\theta, \phi) \cos\theta \sin\theta d\theta d\phi \tag{11}$$

- and the so called K-integral, which is proportional to the pressure of the radiation field:

$$\mathcal{K}_\nu = \frac{1}{4\pi} \oint I_\nu(\theta, \phi) \cos^2\theta \sin\theta d\theta d\phi = \frac{p_\nu c}{4\pi} \tag{12}$$

- **Note that we can define all these in the frequency/wavelength-integrated form**

More moments of the radiation field

- Mean intensity:

$$J = \frac{1}{4\pi} \int_0^\infty \oint I_\nu(\theta, \phi) \sin\theta d\theta d\phi d\nu \tag{13}$$

- Radiation flux:

$$\mathcal{H} = \frac{1}{4\pi} \int_0^\infty \oint I_\nu(\theta, \phi) \cos\theta \sin\theta d\theta d\phi d\nu \tag{14}$$

- and the so called K-integral, which is proportional to the pressure of the radiation field:

$$\mathcal{K} = \frac{1}{4\pi} \int_0^\infty \oint I_\nu(\theta, \phi) \cos^2\theta \sin\theta d\theta d\phi d\nu = \frac{p\,c}{4\pi} \tag{15}$$

- **Here we integrated over all frequencies**

A quick question:

- What would be the radiation flux if the intensity was isotropic?

$$\mathcal{H}_\nu = \frac{1}{4\pi} \oint I_\nu(\theta, \phi) \cos\theta \sin\theta d\theta d\phi \tag{16}$$

A quick question:

- What would be the radiation flux if the intensity was isotropic?

$$\mathcal{H}_\nu = \frac{1}{4\pi} \oint I_\nu(\theta, \phi) \cos\theta \sin\theta d\theta d\phi \tag{17}$$

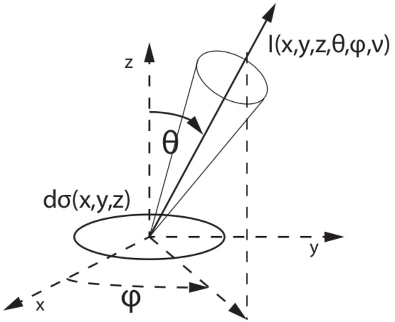
- A common substitute in this case is to integrate ϕ to 2π and then set $\cos\theta = \mu$ (this is again an another μ).

$$\mathcal{H}_\nu = \frac{1}{2} \int_{-1}^{1} I_\nu(\mu) \mu d\mu = 0 \tag{18}$$

- If the radiation is completely isotropic, there is no energy transport. In order to transport the energy outward toward the surface the radiation has to be slightly anisotropic.

Modeling the radiation field

- Our task is not to model and understand intensity and its relationship with other physical quantities (density, temperature, pressure, chemical composition). For that we need to:
- Understand the interaction between the radiation and matter (absorption, emission, scattering coefficients).
- Mathematically express relationship between these coefficients and the intensity.



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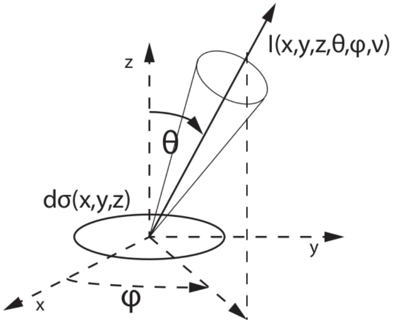
Radiative Transfer Equation (RTE)

- This formulation is (more or less) due to Kirchhoff. The change of intensity "along-the-ray" over a distance *ds* is:

$$dI_\nu = \eta_\nu ds - \chi_\nu I_\nu ds \tag{19}$$

- The terms of the right represent emission and **total** absorption (both true absorption and scattering) per unit volume.



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