Hands-on exercises 8: Mixed problems from stellar structure

P. Käpylä, I. Milić

June 14, 2024

Problem 1: Mass of the Sun and its luminosity can be measured with relative ease (discuss how). If Sun turned *all* the hydrogen into Helium, what would be the lifetime of the Sun?

The energy produced by one p-p chain is 24.5 eV.

Problem 2: One of the reasons why the Sun will not spend all the hydrogen is that the fusion happens only in the core. In the red giant phase, the hydrogen is "spent" in the core and core produces no energy (the envelope around it does, though). Derive the so called Schönberg-Chandrasekhar limit, which gives us an approximation for the size of the stellar core in the red giant phase. They assumed an isothermal core composed of an ideal gas.

Problem 3: (The one we did not get to do two weeks ago):

For the gas of pure hydrogen, with given p and T, calculate the ionization fraction and mass density.

Note! Assume that the hydrogen is not necessarily completely ionized and that it can come as neutral or ionized hydrogen (no $H-$, no H_2).

To solve the problem you will need the Saha ionization equation:

$$\frac{n_{i+1}n_e}{n_i} = \frac{2}{\lambda^3}\frac{g_{i+1}}{g_i}\exp(-E_i/k_BT) \tag{1}$$

Plot the solution for various combinations of T and p in python. If the time permits - introduce the concept of negative ion of Hydrogen and compare the number densities of the negative ion of hydrogen and neutral hydoren in the *second excited state*.

Useful physical constants

- $R_\odot = 696 \times 10^6\,\mathrm{m}$
- $M_\odot = 1.989 \times 10^{30}\,\mathrm{kg}$
- $L_\odot = 3.83 \times 10^{26}\ \mathrm{W}$
- $T_\odot^{\mathrm{eff}} = 5777\,\mathrm{K}$

- $1\,\mathrm{AU} = 1.496 \times 10^{8}\,\mathrm{km}$
- $c = 2.997 \times 10^{8}\,\mathrm{m/s}$
- $G = 6.674 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$
- $k = 1.38 \cdot 10^{-23}\ \mathrm{J/K}$
- $m_\mathrm{e} = 9.11 \cdot 10^{-31}\ \mathrm{kg}$
- $m_\mathrm{H} = 1.67 \cdot 10^{-27}\ \mathrm{kg}$