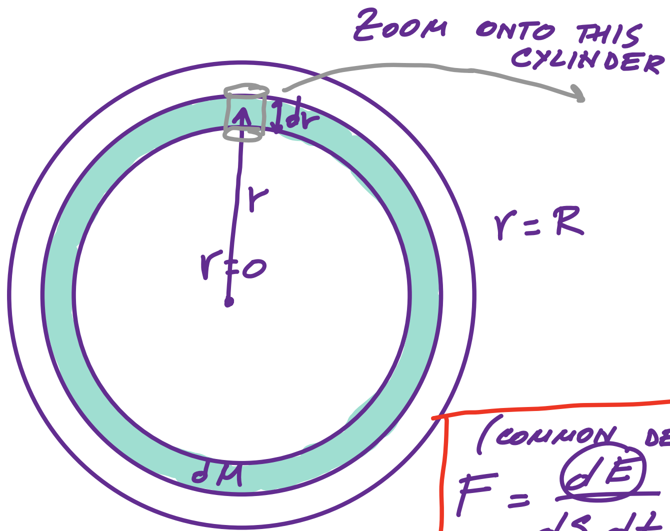
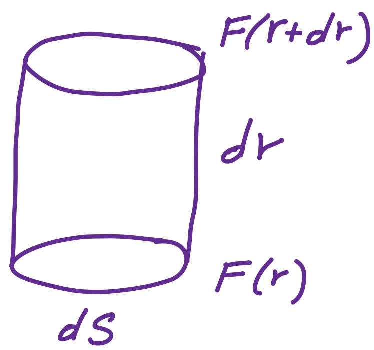
Theoretical Astrophysics I

Basic quantities "Cheat Sheet"





(common def.)

$$\boxed{F = \frac{dE}{dS\,dt}}$$

"Flux" or "Flux density" or "Specific flux"

Here dE is the <u>net</u> energy coming into the surface.

$dE > 0 \Rightarrow$ Energy goes in the r direction

$dE < 0 \Rightarrow$ Energy goes toward center

So, at the surface:

$$\boxed{\begin{aligned} F(R) &= \sigma T_{eff}^4 \\ L_* &= 4\pi R^2 F(R) \end{aligned}}$$

Now, 1^{st} law of TD: $\quad u = \dfrac{dE}{dV}$ Internal energy density

$$\delta(u\,dV) = \delta Q + \delta W$$

Traditionally, we would cast quantities in terms of dV. But, to follow book, we go and cast it in terms of dm (page below).

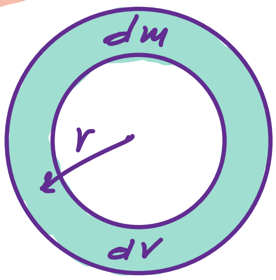
Main dif: F now becomes $\boxed{\dfrac{dE}{dt}\,[W]}$

1ST LAW OF TD IN dm INSTEAD

$$\boxed{F = \frac{dE}{dt}}$$

← THIS IS ENERGY FLOW THROUGH THE WHOLE SHELL dm.

$$\boxed{F(R) = F(M) = L_*}$$ ← SURFACE



dm STAYS THE SAME, dV CAN CHANGE

$$dm = \rho\, dV = \rho\, 4 r^2 \pi\, dr$$

$$u = \frac{dE}{dm}$$ THIS IS INTERNAL ENERGY

$$q = \frac{dE}{dm\, dt}$$

Now, I ST LAW OF TD:

$$\delta u\, dm = \delta Q + \delta W$$

$$\boxed{\delta u\, dm = q\, dm\, \delta t - \frac{\partial F}{\partial m} dm\, \delta t - p\, \delta dV}$$

CHANGE OF INTERNAL E

ENERGY GENERATION

FLUX GRADIENT

WORK

Now, DIVIDE WITH dm AND δt, TAKING $\delta t \to 0$

$$\boxed{\frac{\partial u}{\partial t} = q - \frac{\partial F}{\partial m} - P \frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)}$$ ← THIS IS FROM THE BOOK

$$\dot{u} + P\left(\dot{\frac{1}{\rho}}\right) = q - \frac{\partial F}{\partial m}$$ (REMEMBER $F = \frac{dE}{dt}$)

CHANGE OF INTERNAL ENERGY (HEATING / COOLING)

WORK

ENERGY PRODUCTION (FUSION)

GRADIENT OF FLUX $\left(\frac{dE}{dt\, dm}\right)$

FOR A STABLE STAR: $\dot{u} = 0$, $\dot{\rho} = 0$, SO

$$q = \frac{\partial F}{\partial m}$$

→ $q > 0$, $\frac{\partial F}{\partial m} > 0$

→ $q = 0$, $F = \text{const}$!

NOW, CAN WE CAST THIS IN TERMS OF r?

$q = \frac{dE}{dm\,dt}$ → STAYS THE SAME AS PRODUCTION OF E PER UNIT MAS MAKES SENSE.

$$q = \frac{dE}{4\pi r^2 dr\,\rho\,dt}$$

BUT FOR F WE PREFER $F = \frac{dE}{dS\,dt}$

SO WE HAVE TO REPLACE THE OLD FLUX WITH: $\boxed{F 4\pi r^2}$

$$q = \frac{\partial F}{\partial m} 4\pi r^2 = \frac{\partial F}{\partial r\,\cancel{4 r^2 \pi}\,\rho}\,\cancel{4\pi r^2}$$

$$\boxed{q = \frac{1}{\rho}\frac{\partial F}{\partial r}}$$

← THE MEANING OF FLUX IS DIFFERENT NOW!

$$\frac{dE^{\text{NUCLEAR}}}{4\pi r^2 \cancel{\rho}\,dr\,dt} = \frac{1}{\cancel{\rho}}\frac{\partial F}{\partial r}$$

$$\frac{dE}{dr\,dt} = q = 4\pi r^2 \frac{\partial F}{\partial r}$$

THIS WOULD BE A NEW DEFINITION OF q

IF $q = 0$, THEN $\boxed{4\pi r^2 F = \text{const}}$