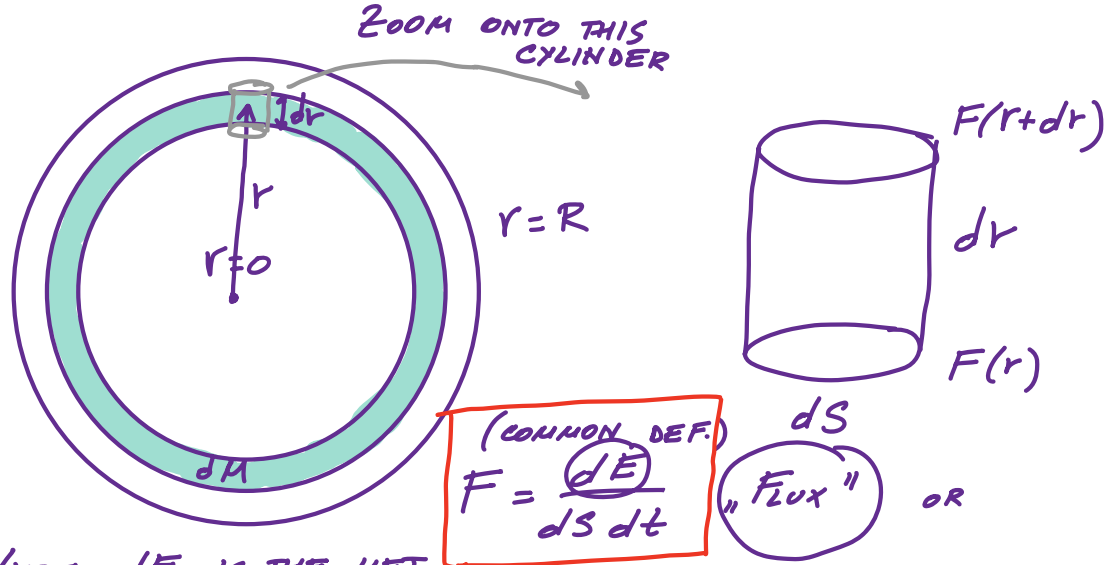
Theoretical Astrophysics I

Basic quantities "Cheat Sheet"



(common def.)

$$F = \frac{dE}{dS\,dt}$$

"Flux" or "Flux density" or "Specific flux"

Here dE is the **net** energy coming into the surface.

$dE > 0 \Rightarrow$ Energy goes in the r direction

$dE < 0 \Rightarrow$ Energy goes toward center

So, at the surface:

$$F(R) = \sigma T_{eff}^4$$

$$L_* = 4\pi R^2 F(R)$$

Now, 1st law of TD: $u = \frac{dE}{dV}$ Internal energy density

$$\delta(u\,dV) = \delta Q + \delta W$$

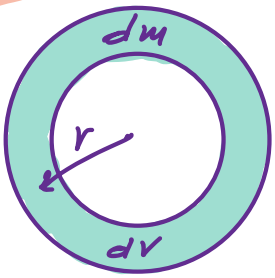
Traditionally, we would cast quantities in terms of dV. But, to follow book, we go and cast it in terms of dm (page below).

Main dif: F now becomes $\frac{dE}{dt}\,[W]$

1ST LAW OF TD IN dm INSTEAD

$$F = \frac{dE}{dt}$$ ← THIS IS ENERGY FLOW THROUGH THE WHOLE SHELL dm.

$$F(R) = F(M) = L_*$$ ← SURFACE



dm STAYS THE SAME, dV CAN CHANGE

$$dm = \rho dV = \rho 4\pi r^2 dr$$

$$u = \frac{dE}{dm}$$ THIS IS INTERNAL ENERGY

NOW, 1ST LAW OF TD:

$$\delta u\, dm = \delta Q + \delta W$$

$$q = \frac{dE}{dm\, dt}$$

$$\delta u\, dm = q\, dm\, \delta t - \frac{\partial F}{\partial m} dm\, \delta t - p\, \delta dV$$

- $\delta u\, dm$: CHANGE OF INTERNAL E
- $q\, dm\, \delta t$: ENERGY GENERATION
- $\frac{\partial F}{\partial m} dm\, \delta t$: FLUX GRADIENT
- $p\, \delta dV$: WORK

NOW, DIVIDE WITH dm AND δt, TAKING $\delta t \to 0$

$$\frac{\partial u}{\partial t} = q - \frac{\partial F}{\partial m} - p \frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)$$ ← THIS IS FROM THE BOOK

$$\dot{u} + p\left(\frac{\dot{1}}{\rho}\right) = q - \frac{\partial F}{\partial m}$$ (REMEMBER $F = \frac{dE}{dt}$)

- $\dot{u}$: CHANGE OF INTERNAL ENERGY (HEATING/COOLING)
- $p\left(\frac{\dot{1}}{\rho}\right)$: WORK
- q: ENERGY PRODUCTION (FUSION)
- $\frac{\partial F}{\partial m}$: GRADIENT OF FLUX ($\frac{dE}{dt\, dm}$)

FOR A STABLE STAR: $\dot{u} = 0$, $\dot{\rho} = 0$, SO

$$q = \frac{\partial F}{\partial m}$$

- $q > 0$, $\frac{\partial F}{\partial m} > 0$
- $q = 0$, $F = \text{const}$!

NOW, CAN WE CAST THIS IN TERMS OF r?

$q = \frac{dE}{dm\,dt}$ → STAYS THE SAME AS PRODUCTION OF E PER UNIT MAS MAKES SENSE.

$$q = \frac{dE}{4\pi r^2 dr \rho dt}$$

BUT FOR F WE PREFER $F = \frac{dE}{dS\,dt}$

SO WE HAVE TO REPLACE THE OLD FLUX WITH: $F 4\pi r^2$

$$q = \frac{\partial F}{\partial u} 4\pi r^2 = \frac{\partial F}{\partial r} \cancel{4\pi r^2} \cancel{4\pi r^2 \rho}$$

$$q = \frac{1}{\rho} \frac{\partial F}{\partial r}$$ ← THE MEANING OF FLUX IS DIFFERENT NOW!