Hands-on exercises 4: Equations of radiative transfer in the stellar interiors

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These are, again, mostly analytical exercises.

Problem 1: Starting from the the radiative transfer equation:

$$\cos\theta \frac{dI_\lambda}{dz} = -\kappa_\lambda \rho I_\lambda + j_\lambda \rho \tag{1}$$

Derive the equations for the transfer of the mean intensity, flux density, and the K-integral, these are all called moments of the radiation field. Convince yourself that when total absorbed radiation by an element is equal to the total emitted radiation (so called radiative equilibrium) the following statement holds:

$$\oint \kappa_\lambda I_\lambda \sin\theta d\theta d\phi = \oint j_\lambda \sin\theta d\theta d\phi. \tag{2}$$

Show that radiative equilibrium leads to the conservation of the flux.

Problem 2: Assuming that the K integral is proportional to the radiation pressure (this proof will be a homework), show that the radiative flux in stellar interiors is proportional to $-T^3 \frac{dT}{dz}$ (so called diffusion approximation).

Problem 3: Convince yourself that using assumption that the radiation field is isotropic is fine for the calculation of the K-integral, but not for the calculation of the Flux density.

Useful physical constants

- $R_\odot = 696 \times 10^6\,\mathrm{m}$
- $M_\odot = 1.989 \times 10^{30}\,\mathrm{kg}$
- $L_\odot = 3.83 \times 10^{26}$ W
- $T_\odot^{\mathrm{eff}} = 5777\,\mathrm{K}$
- $1\,\mathrm{AU} = 1.496 \times 10^8\,\mathrm{km}$
- $c = 2.997 \times 10^8\,\mathrm{m/s}$
- $G = 6.674 \times 10^{-11}$ $\mathrm{Nm^2/kg^2}$

- $k = 1.38 \cdot 10^{-23}$ J/K
- $m_{\mathrm{H}} = 1.67 \cdot 10^{-27}$ kg