Theoretical Astrophysics I: Physics of Sun and Stars
Lecture 2: Stellar colors and luminosities. Basic equations of stellar structure and evolution

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Stefan-Boltzmann law

- The simplest way for us to relate the emitted energy to the temperature is through the Stefan-Boltzmann law.
- The emissivity (emitted energy per unit surface in the outgoing direction) is equal to:

$$\epsilon = \sigma T^4 \tag{1}$$

- Then, the luminosity (emitted energy per second) of such object is: $L = 4\pi R^2 \sigma T^4$.
- If you substitute the numbers, you will get $T = 5777\,\mathrm{K}$.
- What temperature would this be?
- What does this question even mean?

Departures from the ideal case

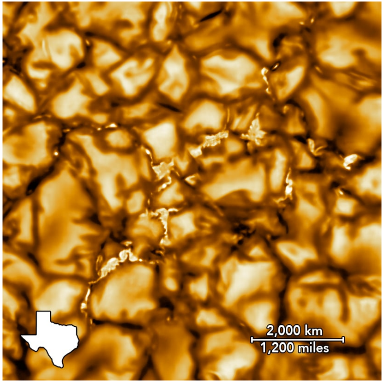
We made a series of approximations in the process of inferring this temperature.

- First we assumed that the solar radiance is constant in time (*excellent approximation*)
- Then we assumed that the emission is isotropic (*this is not a good approximation but see below*)
- Then we assumed that the solar surface is homogeneous (*on small scales it is not, but let's discuss this for a while*)
- Finally, we assumed that the Sun emits like a blackbody to infer the called effective temperature - 5777K.
- Let's discuss a bit what this concept means - 5 mins

Real situations

The Sun (and other stars) depart severely from these assumptions:

- Stars are inhomogeneous in all 3 dimensions (especially radially).
- They emit non-Plankian spectra, in a generally anisotropic manner
- And they also change on scales from seconds to billions of years.
- **This course is about all these aspects**



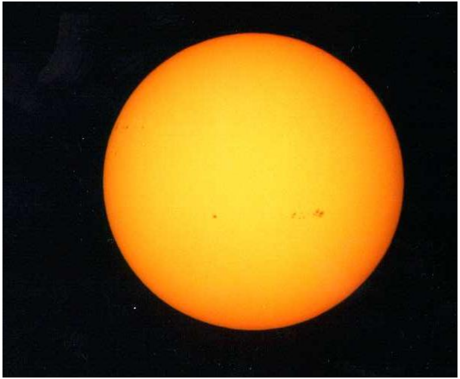
Credits: NSO/AURA/NSF/DKIST

The concept of stellar atmosphere

- Energy is generated in the cores of stars and transported outwards (in multiple ways)
- As we move away from the stellar center - the medium becomes less opaque, at some point the photons can escape the star. Then they contribute to the total stellar luminosity that we ultimately detect.
- The layer of the last emission is referred to as $\tau = 1$ layer, where τ is the so called **optical depth**.
- As the stars are made of plasma - we don't have an obvious surface. $\tau = 1$ layer serves as a loose definition of a solar surface (observationally).
- Next week, we will describe a star using equations of stellar structure - we will again have to make a definition of solar surface.

The concept of stellar atmosphere

- The physical location of $\tau = 1$ depends on the density of the medium and how the photon interacts with the medium - **described by the opacity**.
- Solar plasma has different opacity at different wavelengths - photosphere has different physical location.
- Also, when we look at the Sun under different angles, we see different depths.
- This is the so called limb-darkening.



Why study the stellar atmospheres?

- The stellar light gives us direct information about the stellar surface (the photosphere).
- Are there ways to probe the stellar interior? Can the stellar atmosphere help?

Why study the stellar atmospheres?

- Are there ways to probe the stellar interior? Can the stellar atmosphere help?
- Stellar structure determines the structure of the atmosphere, and $T_{\rm eff}$.
- Stellar atmosphere oscillates because of the processes in stellar interior.
- At the surface, we measure the magnetic fields generated in the interior.
- Neutrinos produced in the core leave the star directly.
- **The structure of the whole star leaves an observable imprint on its surface.**

Understanding the stars

- **The structure of the whole star leaves an observable imprint on its surface.**
- We measure these observable imprints.
- We can observe *a lot* of stars - so we can have accurate statistics.
- **We devise theoretical models that try to describe the stellar structure and the evolution and compare the outputs of these models both to the measurable properties of individual stars and to the statistical distributions.**

Luminosity vs the color

- Effective temperature describes the total luminosity of the star.
- We used a blackbody to approximate solar *surface*.
- Now, different temperatures imply different colors (i.e. different spectral distributions).

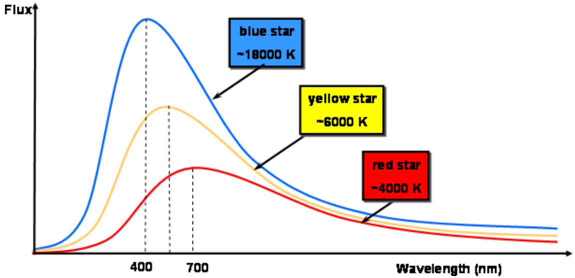


Figure: Planck curves for blackbodies of different temperatures. Credits: University of Swinburne

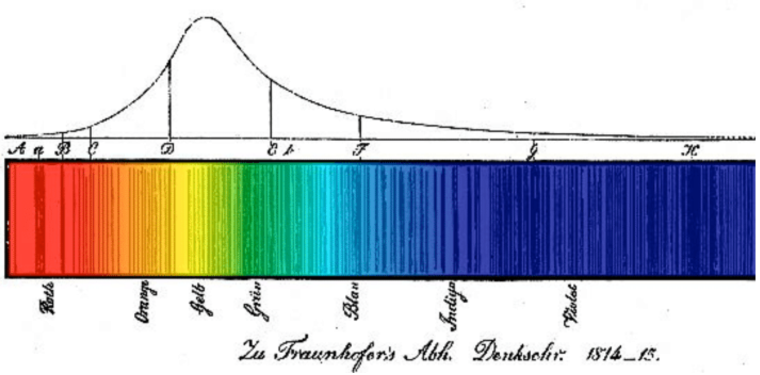
A side quest

- Can someone convince me that a star **cannot** be a blackbody? - 2-3 mins

Stars are not black bodies!

- Star needs to transport energy outwards. For that a gradient of temperature is needed - not a blackbody.
- Harder question is can stellar photosphere be a blackbody?
- In theory - yes, no fundamental law would be broken if it was.
- In practice - it can't. Different wavelengths *always* have different opacities and thus sample different depths.

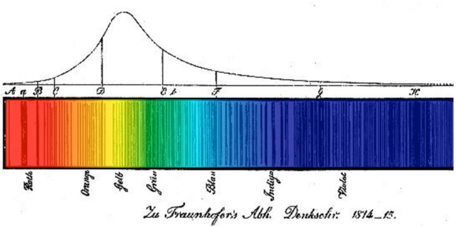
Solar spectrum is NOT a blackbody spectrum



Credits: Fraunhofer (1814)

- Wollastone (1802) and Fraunhofer (1814) discovered dark lanes in the solar spectrum,
- These lines represent loss of light at specific wavelengths - solar atmosphere is much more opaque at these wavelengths. Why?

Solar spectrum is NOT a blackbody spectrum



Credits: Fraunhofer (1814)

- These lines represent loss of light at specific wavelengths - solar atmosphere is much more opaque at these wavelengths. Why?
- Kirchoff and Bunsen (1860's) related them to chemical elements.
- Birth of quantum mechanics: Bohr's model can explain hydrogen lines. **Energy jumps between discrete energy levels**.
- Discovery of Helium by Janssen (1868) in the solar spectrum.

Planck Law

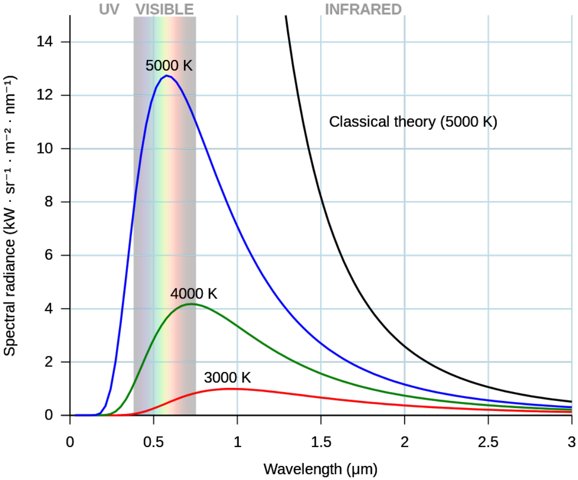
- Kirchoff (again) reckognized the importance of an universal law that connects emission and absorption properties of a medium in an equilibrium state. - *"... It is of utmost importance to derive this law theoretically*".
- We had some pieces - Stefan-Boltzmann law that we just talked about...
- ... Then we had Wien's law: Emission of bodies of different temperature peaks at different wavelengths (colors).
- People also experimentally captured the dependence $B(T)$, where B is some "brightness" or essentially luminosity.
- *Interesting:* Both Boltzmann and Wien (as well as Rayleigh and later Jeans) derived the corresponding laws theoretically. Do you know how?
- Recommendation: "Theoretical Concepts in Physics" (Malcom Longair, 2020, Cambridge)

Planck Law

- Finally, Planck derived this law theoretically.
- After that, multiple other approaces to derivation were made (for example one by Bose and Einstein).

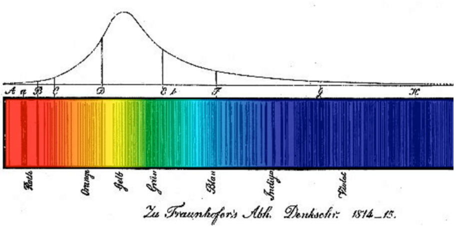
$$B(\lambda, T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1} \qquad (2)$$

- This equation describes the *intensity* of the light in a state where photons are in equilibrium.



Planck curves for few different temperatures.
Credits: Wikipedia

Solar spectrum is NOT a blackbody spectrum



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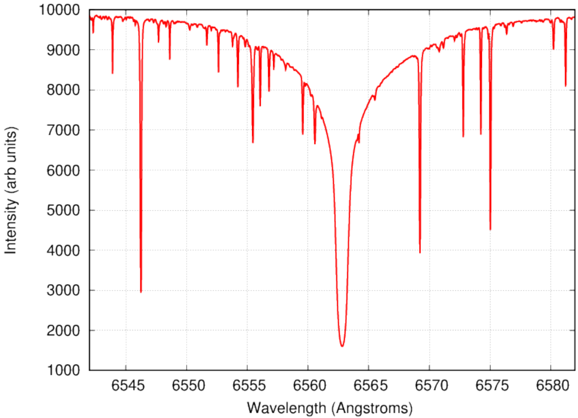
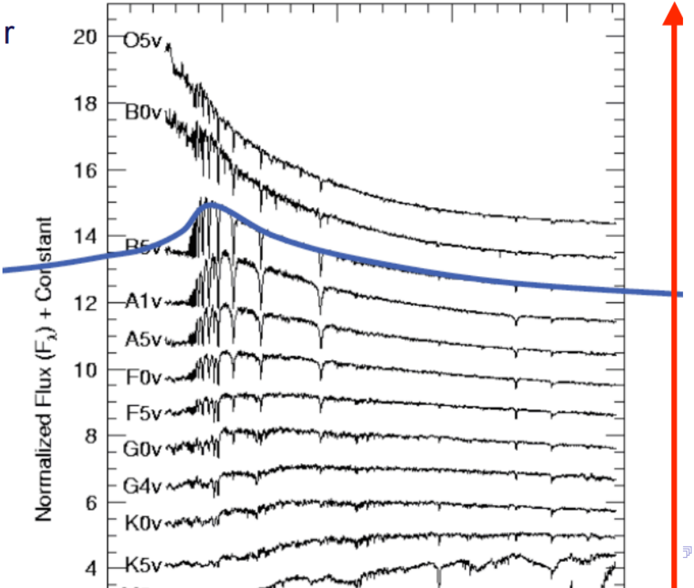
Today we can do better than 200 years ago ...



Hα spectral line in solar spectrum: notice all the weak spectral lines around!

And we can do the same for other stars.



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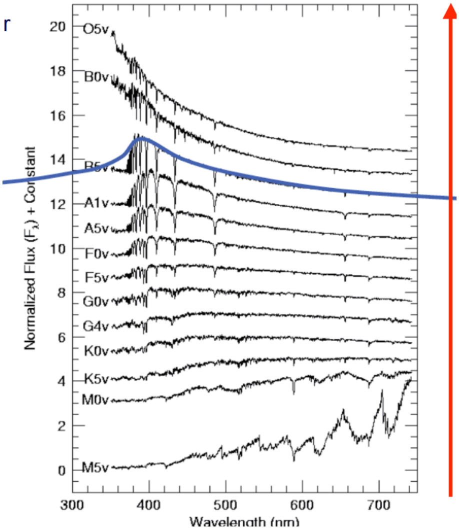


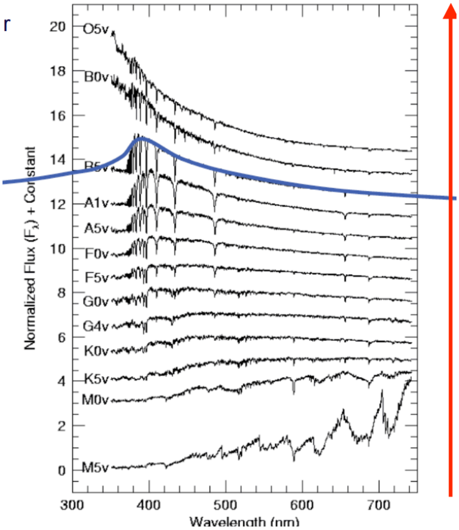
What is the fundamental parameter that determines the shape of the spectrum and the absence/presence of spectral lines?

Credits: Adam Burrows, Princeton

And we can do the same for other stars.

What is the fundamental parameter that determines the shape of the spectrum and the absence/presence of spectral lines?

It is the temperature! It determines the ionization and excitation of the particles and thus their absorption/emission properties.



Credits: Adam Burrows, Princeton

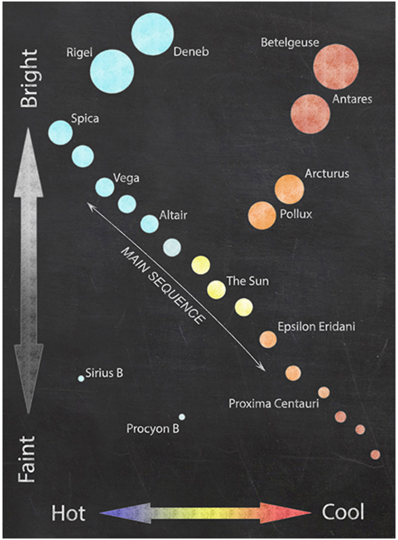
The Harvard Spectral Classification

Class	T_{eff} [K]
O	$\geq$ 30000
B	10000-30000
A	7500-10000
F	6000-7500
G	5000-6000
K	3000-5000
M	2000-3000

Annie Jump Cannon. Credits: Library of Congress, US

HR diagram: basics

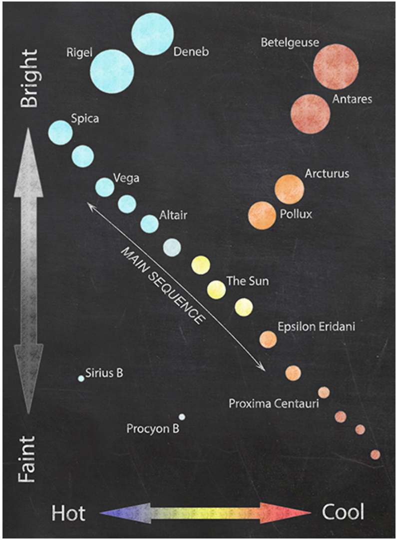
- There are three groups of stars.
- For one (main sequence) the color and brightness are somewhat correlated.
- Other is very hot but faint - white dwarves
- The last is cool but bright - (red) giants (there are also other giants).
- It will turn out these are *phases* in stellar evolution.



Credits: the Open University

HR diagram: basics

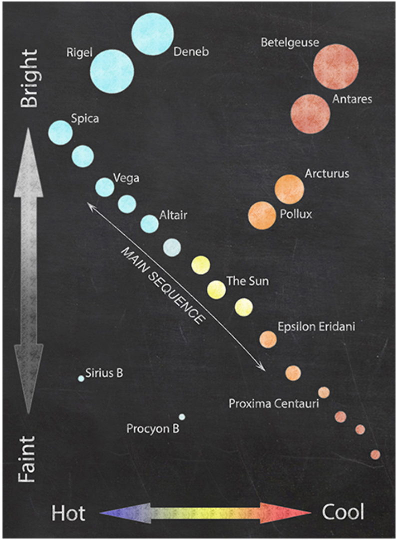
- It will turn out these are *phases* in stellar evolution.
- We **can't** observe stars as they evolve and age.
- But we can observe a lot of stars and try to infer things.
- It is like aliens observed us very briefly: they would need some time that small, weak humans are a first stage in a lifetime of a human being.



Credits: the Open University

HR diagram: questions

- Why are the color and the brightness of the stars on the main sequence correlated?
- In which order these evolutionary stages take place?
- What determines when and how the star changes between these stages?
- What are the fundamental differences between these phases?



Credits: the Open University

How can we infer other stellar parameters?

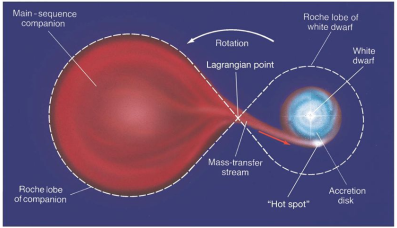
- We saw that we can estimate luminosity from the apparent brightness (irradiance) and distance.
- And that we can infer the effective temperature from the color.
- Combined, these two give us the radius (an estimate of).
- Write down the equations again if necessary.
- What about the mass? Physical composition? Magnetic field? Rotation rate?

Stellar masses

- Are extremely important, yet extremely hard to infer.
- The only accurate opportunity to infer the mass is to observe the star in a dynamical interaction.
- For example in a binary system.
- These binary stars are very important, a big fraction of the stars are in binary (or multiple systems).
- The stars can also interact with each other.

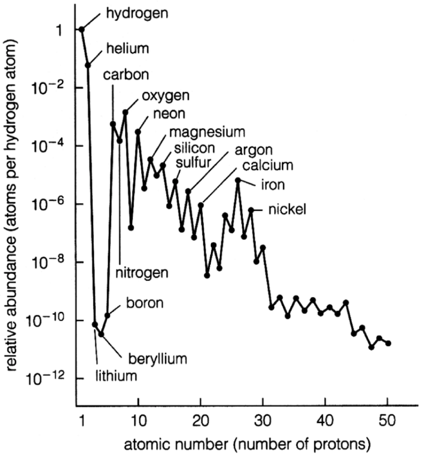
Life after Death for White Dwarfs

A white dwarf that is part of a semidetached binary system can undergo repeated explosions!



Element abundances in the Universe

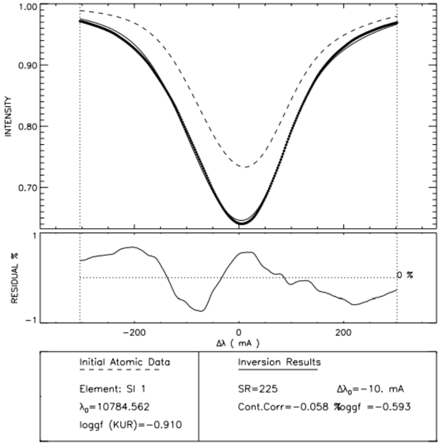
- Most of these elements were generate in stars.
- But they also have implications for generation and evolution of other stars?
- Let's have a relaxed discussion on importance of heavier elements.



Credits: Goldschmidt 1938

These abundances are inferred from the spectra

- Determination of the Silicon Abundance by fitting the solar spectra
- Dots: observations; dashed/solid: low/high Silicon abundance.



Credits: Borrero et al. 2003

Question to think about

- We said that the effective temperature of the Sun is 5777 K.
- We also said that this temperature is not constant, but must be increasing inward, in order to drive energy flux outward.
- What is the temperature in the core?
- What is the source of energy?

Answer

- The temperature at the core is $\approx 1.5 \times 10^7$ K. And the source of energy is nuclear fusion of H in He.
- How do we know? It took us some time.
- First one to even think about this was Eddington. (Maybe check this paper by H. Kragh).
- The previous idea was graviational contraction.
- The temporal scale for such a process is the **Kelvin-Helmholtz timescale**.

Kelvin-Helmholtz timescale

- Let's do an order of magnitude estimation:

$$t_{\mathrm{KH}} = \frac{GM^2}{RL} \tag{3}$$

- For the Sun this amounts to 30 million years.
- We knew back then already that Earth is billions of years old.
- It had to be something else!
- There will be a master course in German on "Applications of thermodynamics in astrophysics" by Prof. Antonio Ferriz Mas, starting in May - keep an eye out!
- See you on Friday! Online / in person?