Theoretical Astrophysics: Physics of Sun and Stars
Homework 1

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Deadline for this homework is 31/05/2025 23:59

Problem 1: Calculate the mean temperature of the Sun. Compare it with the effective temperature of the Sun. Hint: Use the virial theorem and bear in mind the ideal gas equation.

Problem 2: For a gas in hydrostatic equilibrium with a constant mean molecular mass and mean temperature, in a constant gravitational field, show that the pressure falls off exponentially with height (z):

$$p = p_0 e^{-z/H} \tag{1}$$

where H is the pressure scale height. Find the expression for the scale height and calculate its value for the Earth's atmosphere and the solar atmosphere (for the moment, assume the Sun is composed of pure neutral hydrogen atoms).

Problem 3: Calculate the values of the dynamic, thermal, and nuclear timescales for the Sun and for a red giant (you can look up physical properties of your favorite red giant from the internet). This is an order of magnitude (OOM) estimation - do not get too hung-up on the constants. Think about the physical meaning of the timescales.

Problem 4: Stefan-Boltzmann law describes the emergent flux from a blackbody. That is, total energy per unit surface in unit time, emitted into the 2π solid angle (full sphere is 4π but we are only looking at the outgoing half).

Planck law describes the intensity of radiation in a black body, intensity is defined as (see the Appendix one of the book):

$$I_\lambda = \frac{dE}{dtdAd\Omega d\lambda \cos\theta} \tag{2}$$

where Ω is the solid angle $d\Omega = \sin\theta d\theta d\phi$, and θ and ϕ describe the direction of propagation of radiation. For Planck's law:

$$I_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}. \tag{3}$$

Using Planck's law, derive the Stefan-Boltzmann law:

$$F = \frac{dE}{dtdA} = \sigma T^4 \tag{4}$$

Problem 5: The solar luminosity is $L_\odot = 3.83 \cdot 10^{26}$ W. Assume that all of the energy for this luminosity is provided by the pp1 chain, and that neutrinos carry off 3% of the energy. How many neutrinos are produced per second? What is the neutrino flux (i.e., the number of neutrinos per second per cm^2) at Earth?

The pp1 chain consists of the reactions:

$$\begin{aligned} p + p &\rightarrow {}^2\mathrm{D} + e^+ + \nu, && (1.18 \text{ MeV}) \\ {}^2\mathrm{D} + p &\rightarrow {}^3\mathrm{He} + \gamma, && (5.49 \text{ MeV}) \\ {}^3\mathrm{He} + {}^3\mathrm{He} &\rightarrow {}^4\mathrm{He} + 2p. && (12.86 \text{ MeV}) \end{aligned} \tag{5}$$

where the first two reactions need to happen twice for the last reaction to occur, and where ν signifies an emitted neutrino. Energy released (in MeV $= 10^6$ eV) is given in brackets after each reaction.

Useful physical constants

- $R_\odot = 696 \times 10^6$ m
- $M_\odot = 1.989 \times 10^{30}$ kg
- $L_\odot = 3.83 \times 10^{26}$ W
- $T_\odot^{\mathrm{eff}} = 5777$ K
- $1\,\mathrm{AU} = 1.496 \times 10^8$ km
- $c = 2.997 \times 10^8$ m/s
- $G = 6.674 \times 10^{-11}$ Nm^2/kg^2
- $k = 1.38 \cdot 10^{-23}$ J/K
- $m_\mathrm{H} = 1.67 \cdot 10^{-27}$ kg
- $h = 6.626 \times 10^{-34}$ J s.
- $k = 1.38 \times 10^{-23}$ J/K.