Hands-on exercises 6: Analytical Radiative Transfer

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Problem 1: For isotropic Planckian radiation, calculate the wavelength-integrated mean intensity J, flux H and K-integral K.

Show that $J = 3K$ and convince yourself that when radiation is very slightly anisotropic, $J = 3K$ is a good approximation.

Use it to derive the diffusion approximation that we showed in class and demonstrate that the Rosseland mean opacity is defined via:

$$\frac{1}{\overline{\kappa}} = \frac{\int_0^\infty 1/\kappa_\nu dB_\nu/dT d\nu}{\int_0^\infty dB_\nu/dT d\nu} \tag{1}$$

Problem 2: The *formal* solution of the radiative transfer equation on a line segment with optical depth τ_ν reads:

$$I_\nu = \int_0^{\tau_\nu} S_\nu(t) e^{-t} dt \tag{2}$$

Show that for constant S_ν, small τ leads to intensity being proportional to the amount of emitting/absorbing material, while for large τ it saturates. Discuss!

Problem 3: For the gas of pure hydrogen, with given ρ and T, calculate bound-free opacity at λ = 50 nm and 500 nm (also appears in the slides).

Note! Assume that the hydrogen is not necessarily completely ionized and that it can come as neutral or ionized hydrogen (no $H-$, no H_2).

To solve the problem you will need the Saha ionization equation:

$$\frac{n_{i+1} n_e}{n_i} = \frac{2}{\lambda^3} \frac{g_{i+1}}{g_i} \exp(-E_i/k_B T) \tag{3}$$

Useful physical constants

- $R_\odot = 696 \times 10^6$ m

- $M_{\odot} = 1.989 \times 10^{30}\,\mathrm{kg}$
- $L_{\odot} = 3.83 \times 10^{26}$ W
- $T_{\odot}^{\mathrm{eff}} = 5777\,\mathrm{K}$
- $1\,\mathrm{AU} = 1.496 \times 10^{8}\,\mathrm{km}$
- $c = 2.997 \times 10^{8}\,\mathrm{m/s}$
- $G = 6.674 \times 10^{-11}$ $\mathrm{Nm^2/kg^2}$
- $k = 1.38 \cdot 10^{-23}$ J/K
- $m_{\mathrm{H}} = 1.67 \cdot 10^{-27}$ kg