# Theoretical Astrophysics I: Physics of Sun and Stars
# Lecture 9: Stellar Atmospheres

Petri Käpylä Ivan Milić
pkapyla,milic@leibniz-kis.de

Institut für Sonnenphysik - KIS, Freiburg

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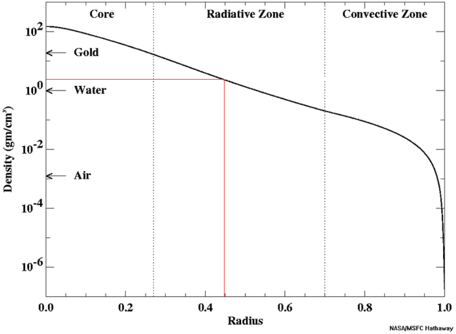
# Recap

- So far we have dealt with interiors of the stars - these contain an absolute majority of stellar material.
- We studied the equations that govern stellar structure and the evolution and solved them in more or less detail to understand how the stars look into the inside and how they evolve on the HR diagram.
- None of these processes were **directly** observable. The only observable quantity we talked about was the stellar luminosity ($L$).
- Now we are going to talk about the structures that we can observe - stellar atmospheres, and their observational signatures - stellar spectra.

# Why study stellar atmospheres?

- Stellar atmosphere is the miniscule part on the very right of this diagram.
- How can it be important? (Salpeter once asked the same question)
- First: Stellar atmosphere is the only part we can observe: insight into what is going on inside.
- Second: It can influence the evolution of the star, at certain evolution stages atmospheres can be huge and influence the rest.
- Third: There is a lot of interesting physics going on (especially the case for the Sun, talk next week!)




Credits: NASA/MSFC Hathaway

# Differences between stellar atmospheres and stellar interiors

Interior

- $T = 10^5 - 10^7$ K
- $\rho = 10^1 - 10^5 \mathrm{kg/m^3}$
- Completely ionized
- Ideal gas
- LTE

Interior

- $T = 10^4 - 10^5$ K
- $\rho = 10^{-1} - 10^{-8} \mathrm{kg/m^3}$
- Neutral, partially ionized, totally ionized - depending on the layer and species
- Ideal gas
- Photosphere in LTE, chromosphere and up in NLTE, but some things are tricky (e.g. photoionization)

# Hayashi zone and pre-main-sequence phase

- The photosphere needs to be able to radiate away all of the incoming energy flux. This is determined by the thermodynamic structure, i.e., the drop of $p$, $\rho$, and $T$ accross it.
- In Hydrostatic equilibrium

$$\frac{dp}{dr} \approx -\rho \frac{GM}{R^2}, \tag{1}$$

which can be integrated from $R$ to the point where $p$ vanishes

$$p_R = \frac{GM}{R^2} \int_R^\infty \rho dr. \tag{2}$$

- Furthermore, the optical depth of the photosphere, characterised by $T_{\rm eff}$, is of the order on unity and thus $\int_R^\infty \kappa \rho dr = \overline{\kappa} \int_R^\infty \rho dr$, where $\overline{\kappa}$ is the mean opacity in the photosphere.
- Taking $\overline{\kappa} = \kappa(R)$ and assuming it to be a power law in $\rho_R$ and $T_{\rm eff}$ gives:

$$\kappa_0 \rho_R^a T_{\rm eff}^b \int_R^\infty \rho dr = 1. \tag{3}$$

# Hayashi zone and pre-main-sequence phase

- Combining Eqs. (2) and (3) gives:

$$p_R = \frac{GM}{R^2 \kappa_0} \rho_R^{-a} T_{\mathrm{eff}}^{-b}. \tag{4}$$

- Yet another relation between the thermodynamic quantities at $R$ is given by the equation of state, here taken to be ideal gas equation:

$$p_R = \frac{\mathcal{R}}{\mu} \rho_R T_{\mathrm{eff}}. \tag{5}$$

- Finally, the temperature at $R$ is related to the luminosity via

$$L = 4\pi R^2 T_{\mathrm{eff}}^4. \tag{6}$$

- Now we have four equations that describe the surface of the star: Eqs.(**??**) (with Eqs. (**??**) and (**??**)), (4), (5), and (6)

# Hayashi zone and pre-main-sequence phase

- These read in logarithmic form:

$$
\begin{aligned}
n \log p_R &= (n-1)\log M + (3-n)\log R + (n+1)\log \rho_r + \text{const.} && (7)\\
\log p_R &= \log M - 2\log R - a\log \rho_r - b\log T_{\rm eff} + \text{const.} && (8)\\
\log p_R &= \log \rho_R + \log T_{\rm eff} + \text{const.} && (9)\\
\log L &= 2\log R + 4\log T_{\rm eff} + \text{const.} && (10)
\end{aligned}
$$

- Eliminating $\log R$, $\log \rho_R$, and $\log p_R$ yields:

$$
\begin{aligned}
\log L &= A\log T_{\rm eff} + B\log M + \text{const.} && (11)\\
A &= \tfrac{(7-n)(a+1)-4-a+b}{0.5(3-n)(a+1)-1}, \quad B = -\tfrac{(n-1)(a+1)+1}{0.5(3-n)(a+1)-1}. && (12)
\end{aligned}
$$

- This relation traces the *Hayashi track* in the HR diagram. These should not be interpreted as evolutionary tracks but rather as an asymptote.

# Hayashi zone and pre-main-sequence phase

- We assume for simplicity that $a = 1$ which is reasonably accurate, such that

$$A = \frac{9-2n+b}{2-n}, \quad B = -\frac{2n-1}{2-n}. \tag{13}$$

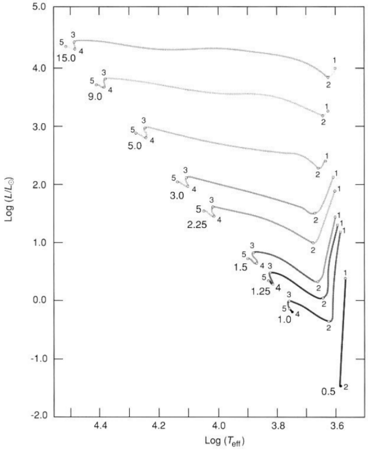
  $b$ varies much more but is usually positive
- Dynamical stability requires that $n < 3$ and therefore the polytropic index is limited to the range $1.5 \leq n < 3$.
- For $b \approx 4$ and $n = 1.5$ we find that $A = 20$. This means that the Hayashi track is almost vertical in the $(\log T_{\mathrm{eff}}, \log L)$ plane.
- As a function of mass the tracks are stacked near each other and higher mass leads to a shift toward higher temperatures because $A$ and $B$ have opposite signs.
- The slope changes with composition that can be associated with an effective polytropic index.

# Hayashi zone and pre-main-sequence phase

- The signifigance of the Hayashi track can be seen from considering $\overline{\gamma}$ which is an average value of $\gamma = \frac{d \ln p}{d \ln \rho}$ over the whole star. $\overline{\gamma}_{\rm a}$ is the corresponding adiabatic index.
- For a fully convective star $\overline{\gamma} = \overline{\gamma}_{\rm a}$.
- If any part of the star is radiative with $\gamma < \gamma_{\rm a}$, then $\overline{\gamma} < \overline{\gamma}_{\rm a}$. Correspondingly, the average polytropic index $n > n_{\rm a}$ where $n_{\rm a}$ is the adiabatic polytropic index defining the Hayashi track.
- If $\overline{\gamma} > \overline{\gamma}_{\rm a}$, the situation is unstable and therefore such state is "forbidden". In practise in such a situation, convection in the star would very quickly restore near-adiabaticity by transporting any excess heat to the surface because a very small superadiabaticity is enough to transport massive amounts of energy (homework!).

# Hayashi zone and pre-main-sequence phase

- Stars from from contrating gas clouds (molecular clouds) through dynamical collapse. These clouds are (parsecs) and fragment in the process.
- Most of the gas in such clouds is in the form of molecular hydrogen ($H_2$). The collapse happens in dynamical timescale $\tau_{\text{dyn}} \propto \rho^{-1.2}$.
- Gradually the $H_2$ molecules are dissociated, after which hydrogen and later helium start to be ionised. These processes use up most of the energy from continuing collapse and the temperature stays nearly constant.
- Finally the ionisation is nearly complete and the temperature starts to increase and a hydrostatic equilibrium is restored. The object is now a protostar.




Credits: Iben (1965), Astrophys. J., 141

# Hayashi zone and pre-main-sequence phase

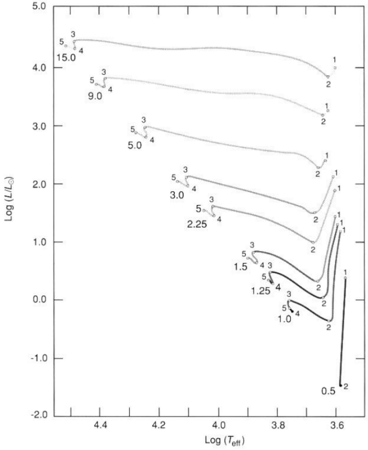
- Estimate of protostellar radius can be obtained by assuming that all of the gravitational energy is spent to dissociate $H_2$ and ionize H and He. Then,

$$\alpha \frac{GM^2}{R_{\rm ps}} \approx \frac{M}{m_{\rm H}} \left( \frac{X}{2}\chi_{\rm H_2} + X\chi_{\rm H} + \frac{Y}{4}\chi_{\rm He} \right), \quad (14)$$

where $\chi_{\rm H_2} = 4.5$ eV, $\chi_{\rm H} = 13.6$ eV, and $\chi_{\rm He} = 79$ eV.

- Taking $Y \approx 1 - X$ and $\alpha = \frac{1}{2}$ gives

$$\frac{R_{\rm ps}}{R_\odot} \approx \frac{50}{1 - 0.2X} \frac{M}{M_\odot}. \quad (15)$$



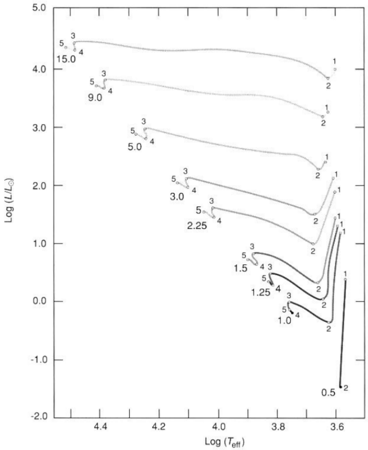

Credits: Iben (1965), Astrophys. J., 141

# Hayashi zone and pre-main-sequence phase

- Recalling the average temperature from virial theorem and inserting the estimate for $R_{\rm ps}$ with $X = 0.7$ gives:

$$\overline{T} = \frac{\alpha}{3}\frac{\mu}{k}\frac{GMm_{\rm H}}{R_{\rm ps}} \approx 6 \cdot 10^4 \ \mathrm{K}. \qquad (16)$$

  Note that the temperature is independent of $M$.
- At this starting point on the Hayashi track the star is fully convective and the gas is still opaque.
- Contraction continues until all of the gas is ionized. The opacity drops first in the interior and the convection zone recedes. $T_{\rm eff}$ starts to rise slowly.
- Nuclear reactions start gradually when core temperature increases and increase the luminosity. Evolutionary track is complicated by ignition of different branches of hydrogen burning.

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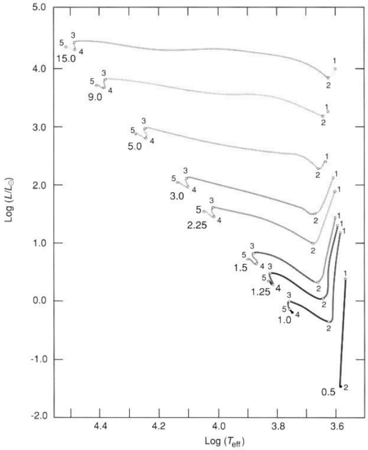
# Hayashi zone and pre-main-sequence phase

**Table 9.1** Evolutionary lifetimes (years)

| $M/M_\odot$ | *1–2* | *2–3* | *3–4* | *4–5* |
|---|---|---|---|---|
| 15 | 6.7(2) | 2.6(4) | 1.3(4) | 6.0(3) |
| 9 | 1.4(3) | 7.8(4) | 2.3(4) | 1.8(4) |
| 5 | 2.9(4) | 2.8(5) | 7.4(4) | 6.8(4) |
| 3 | 2.1(5) | 1.0(6) | 2.2(5) | 2.8(5) |
| 2.25 | 5.9(5) | 2.2(6) | 5.0(5) | 6.7(5) |
| 1.5 | 2.4(6) | 6.3(6) | 1.8(6) | 3.0(6) |
| 1.25 | 4.0(6) | 1.0(7) | 3.5(6) | 1.0(7) |
| 1.0 | 8.9(6) | 1.6(7) | 8.9(6) | 1.6(7) |
| 0.5 | 1.6(8) | | | |

*Note:* powers of 10 are given in parentheses.

Credits: Prialnik.

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- The time that stars spend in the PMS phase depends strongly on the mass.

# Main-sequence phase

- TBA

# Main-sequence phase

- TBA

# Main-sequence phase

- TBA

# Red Giant phase

- TBA

# Red Giant phase

- TBA

# Red Giant phase

- TBA