Hands-on exercises 2: Equations of stellar structure

P. Käpylä, I. Milić

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These are mostly analytical exercises. But, as we deal with the real objects, sometimes we will want to calculate the physical values of our results. For that, it is very convenient to start a simple python session. We will also encourage you to do so :-)

Problem 1: Go through the derivation of the energy equation cast in the term of r as the independent variable. There is a hand-written cheat sheet to help with this. Based on the result of this show that the flux density (often also called just flux), i.e.:

$$F = \frac{dE}{dS\,dt} \tag{1}$$

in a stable, spherical star with a point-like energy source at the center scales like:

$$F(r) = F(R)\frac{R^2}{r^2}. \tag{2}$$

Convince yourself that, for a stellar atmosphere that is very thin, $F = const$ with height in the atmosphere.

Problem 2: Calculate the values of the dynamic, thermal and nuclear timescales for the Sun and for an example red giant (you can look up physical properties of your favorite red giant on google). This is an order of magnitude (OOM) estimation - so do not get too hung-up on the constants. Discuss once again their physical meaning.

Problem 3: Calculate the lower limit for the gas pressure in the center of the Sun.

Problem 4: Calculate the mean temperature of the Sun. Compare it with the effective temperature of the Sun.

Problem 5: Time permitting - discuss the boundary conditions for the equations of stellar structure, namely the values of temperature and pressure.

Useful physical constants

- $R_\odot = 696 \times 10^6\,\mathrm{m}$
- $M_\odot = 1.989 \times 10^{30}\,\mathrm{kg}$
- $L_\odot = 3.83 \times 10^{26}$ W

- $T_{\odot}^{\mathrm{eff}} = 5777\,\mathrm{K}$
- $1\,\mathrm{AU} = 1.496 \times 10^{8}\,\mathrm{km}$
- $c = 2.997 \times 10^{8}\,\mathrm{m/s}$
- $G = 6.674 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$
- $k = 1.38 \cdot 10^{-23}\ \mathrm{J/K}$
- $m_{\mathrm{H}} = 1.67 \cdot 10^{-27}\ \mathrm{kg}$