Bellman Equation

1. Bellman Equation

Definition 1.1 (Bellman Eqation).

..

$$\begin{split} & \boldsymbol{v_{\pi}(s)} = \mathbb{E}[R_{t+1}|S_t = s] + \gamma \mathbb{E}[G_{t+1}|S_t = s], \\ & = \sum_{a \in \mathcal{A}} \pi(a|s) \sum_{r \in \mathcal{R}} p(r|s, a) + \gamma \sum_{a \in \mathcal{A}} \pi(a|s) \sum_{s' \in \mathcal{S}} p(s'|s, a) v_{\pi}(s') \\ & = \sum_{a \in \mathcal{A}} \pi(a|s) \left[\sum_{r \in \mathcal{R}} p(r|s, a) r + \gamma \sum_{s' \in \mathcal{S}} p(s'|s, a) v_{\pi}(s') \right], \forall s \in \mathcal{S} \end{split} \tag{1.1}$$

2. Bellman Optimal Eqation

By Eq. (1.1),...

$$v(s) = \max_{\pi(s) \in \prod(s)} \sum_{a \in \mathcal{A}} \pi(a|s) \left(\sum_{r \in \mathcal{R}} p(r|s, a)r + \gamma \sum_{s' \in \mathcal{S}} p(s'|s, a)v(s') \right), \quad \forall s \in \mathcal{S}$$

$$= \max_{\pi(s) \in \prod(s)} \sum_{a \in \mathcal{A}} \pi(a|s)q(s, a), \qquad \forall s \in \mathcal{S}$$

$$(2.1)$$

By Eq. (2.1)...

3. Case: Shortest Path of Islands

```
1 #let csv1 = csv("islands.csv")
2 #figure(
3 tableq(csv1, 9, inset: 0.31em),
4 caption: "Geographic Info of Islands",
5 supplement: "Table",
6 kind: table,
7 )
```

Table 1 Geographic Info of Islands

culture	population	contact	total_tools	mean_TU	lat	lon	lon2	logpop
Malekula	1100	low	13	3.2	-16.3	167.5	-12.5	7.00
Tikopia	1500	low	22	4.7	-12.3	168.8	-11.2	7.31
Santa Cruz	3600	low	24	4	-10.7	166	-14	8.19
Yap	4791	high	43	5	9.5	138.1	-41.9	8.47
Lau Fiji	7400	high	33	5	-17.7	178.1	-1.9	8.91
Trobriand	8000	high	19	4	-8.7	150.9	-29.1	8.99
Chuuk	9200	high	40	3.8	7.4	151.6	-28.4	9.13
Manus	13000	low	28	6.6	-2.1	146.9	-33.1	9.47
Tonga	17500	high	55	5.4	-21.2	-175.2	4.8	9.77
Hawaii	275000	low	71	6.6	19.9	-155.6	24.4	12.52

Qooklet Example