LEGO Watt Balance

Special Report in Experimental Physics (FYS2150)

August Geelmuyden & Sebastian G. Winther-Larsen

University of Oslo

May 23, 2016

Abstract

As the kilogram is about to be redefined from 2018, we construct a LEGO version of the precise measurement device called the watt balance. The new definition of the kilogram will be realized through a fixed multiple of Planck's constant. This study is partially based on a LEGO watt balance design by NIST, with two systems for weight measurement; a shadow sensor and a coil system. We found the process to be rewarding, but the LEGO watt balance was much less capable than promised. The problems arose mostly because of friction in the coils, and additionally from noise in the light sensitive diode output voltage.

I. Introduction

> *It is mind boggling how precisely the Egyptians could compare masses. A lever and fulcrum is hard to beat!*
>
> J. Pratt, NIST

The International Prototype Kilogram (IPK)is the only mass on earth with zero uncertainty. The IPK is one of three cylinders forged in 1879 of a platinum alloy known as Pt-10lr consisting of 90% platium ond 10% iridium. Several copies of the IPK exists across the world, each one designated with the letter K and a number[1]. None of these "sister kilograms" have mass precisely equal to that of the IPK, their masses are calibrated and documented as offsett values. The masses of the carefully stored prototypes can drift for a variety of reasons, some known and some unknown. By definition, the error in the measured value of the IPK's mass is exactly zero; the IPK *is* the kilogram. However, changes in the IPK's mass can still be deduced by the changes in mass of it's official copies across the globe.

The stability of the IPK is crucial because the kilogram underpins much of SI system of measurement as it is currently defined and structured. The chunk of metal in a vault in france that is the IPK has served humanity well as a standard for mass. However, it is time for a change because we know that the IPK changes. The new definition proposed for the kilogram starts with the extremely precise agreed value for Planck's constant.

The device most commonly used to measure differences in mass in terms of quantum invariants is called a watt balance. Conceived by Bryan Kibble in 1975 [1], the watt balance is an apparatus that balances the weight of an object against an electromagnetic force generated by a current-carrying coil. It toggles between two measurement modes and indirectly compares electrical power and mechanical power, hence the name "watt balance".

This study describes the construction and operation of a simple watt balance built mainly from LEGO.

II. Theory

A watt balance is very similar to the well-known equal-arm balance in appearance. The equal-arm balance passively compares an unknown mass to a calibrated mass. The watt balance, on the other hand, *actively* compensates the unknown weight with a known force. There are two main modes of measurement; *velocity mode* and *force mode.*

1. Velocity mode

Velocity mode is based on the principle of Lorentz forces. Magnets that produce a magnetic field with

[1] K36 is kept at Kjeller in Norway, closely guarded by Justervesenet

magnetic flux density B are moved at a vertical speed v through a coil with wire length L.

The magnetic flux through the coil is given by the following integral.

$$\Phi_B = \iint \mathbf{B} \cdot \mathrm{d}\mathbf{A} \tag{1}$$

Faraday's Law of Induction states that a change in magnetic flux induces an electromotive force in a coil of wire.

$$V = N \frac{d\Phi_B}{dt} \tag{2}$$

where $d\Phi_B/dt$ is the rate of change of the flux Φ and N is the number of windings in the coil. Assuming the circumference of one winding is ℓ, related to the area swept by the magnetic field by $\mathbf{dA} = \ell \mathbf{dz}$. Additionally, if the magnetic field $\mathbf{B}$ points radially outwards then $d\Phi_B = BdA = B\ell dz$. It follows that

$$V = N\frac{d\Phi_B}{dt} = N\frac{d}{dt}B\ell dz = N\ell Bv \tag{3}$$

The total wire length is $L = \ell N$. In conclusion the induced voltage V is related to the velocity v by the factor BL as given by

$$V = BLv. \tag{4}$$

2. Force mode

The force mode is also based on Lorentz forces. The gravitational force on a mass m is counteracted by an electromagnetic force F generated by the current-carrying coil.

$$\mathbf{F} = I \int d\ell \times \mathbf{B} \tag{5}$$

Where $d\ell$ points along a tangent to the edge of the coil. If the magnetic field vector $\mathbf{B}$ points radially outwards $\mathbf{F}$ is paralell to the direction that the pans of the balance beam will be moving. The expression above can then be simplified in the direction of interest and should equal the gravitational force

$$F_z = BLI = mg \tag{6}$$

3. Measuring a mass

A mass could be measured simply by operating in force mode, but a problem will arise when trying to measure B and L accurately. Velocity mode is therefore necessary as a means of calibration. By combining equations 4 and equation 6, cancelling out BL and rearranging a bit, one ends up with:

$$VI = mgv \implies m = \frac{VI}{vg} = \frac{BL}{g}I \tag{7}$$

This equation relates mechanical power to electrical power, thereby providing a way to measure a mass through electrical quantities. The power found in the numerator in the second to last term of equation 7 is the reason the balance is called a "watt balance". It is important to bear in mind that V and I are measured separately in the two modes described above. Thusly, the power $P = VI$ only exists virtually as a mathematical product.

III. Systems

A photography of the watt balance is shown in figure 1. One can see the similarity of the watt balance to a normal equal arm balance. The most important parts of the two main systems are magnified in figure 1. The shadow sensor system is indicated with the letter "A" and the coils with the letter "B".

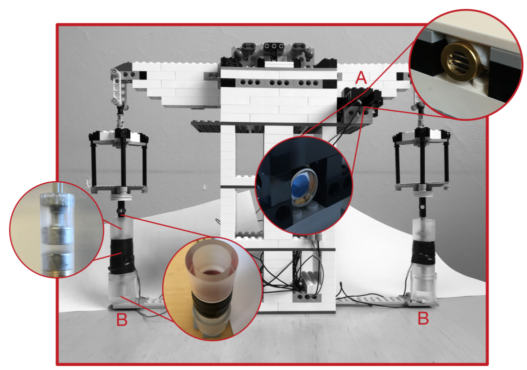

Figure 1: *The watt balance. Coils are indicated by "B" and shadow sensor by "A".*

The balance beam of the watt balance has a "knife's edge" radius of approximately 3.1mm and rests on a smooth surface. A description of the *shadow sensor* system and the *coil* system follows.

1. Shadow sensor

In order to obtain the vertical position of the coils it suffices to measure the tilt of the lever. Directing a line laser (LN60-650) towards a shadow sensor (PC50-7-TO8) in such a way that the lever partially blocks the laser ray, the tilt may be found using its proportionality to the voltage generated by the shadow sensor. An illustrative sketch of the system is in figure 2.

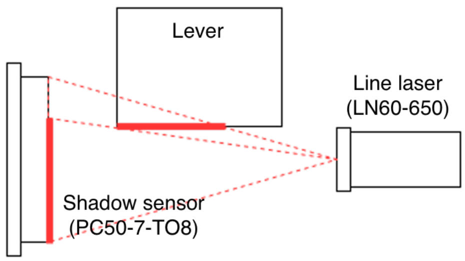

Figure 2: *The setup of the laser, lever and shadow sensor.*

The circuit diagram for the system is shown in figure 3, where the use of a voltage regulator (LM317) should be noted. The voltage regulator is designed to automatically maintain a constant voltage level. The purpose of including a voltage regulator in the circuit is to make sure that the intensity of the laser light is as stable as possible.

The shadow sensor system is highlighted in figure 1 with the letter "A". The magnified photography to the right displays the line laser and the magnified photography to the left displays the light sensitive diode.

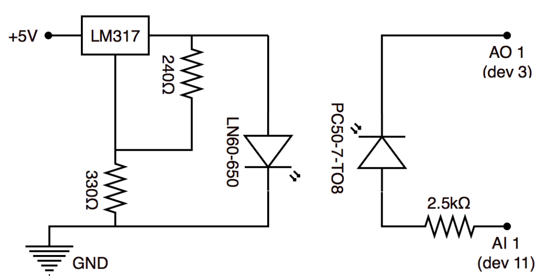

Figure 3: *The circuit for the shadow sensor system.*

Ideally, the voltage provided by the shadow sensor should be zero when the lever is horizontal. This would minimize the magnitudes of the measured voltages and hence allow for a narrower scope. A narrower scope makes it possible to exploit more of the channels when digitalizing the signal which in turn improves the precision of the measurements. Because of changing light intensity from the surroundings the shadow sensor will not produce zero voltage when the lever is balanced. Hence a balancing voltage, whose value is chosen during calibration of the apparatus, is applied to the shadow sensor circuit.

2. Coils

The coils were fashioned on a lathe from plastic and 3000 windings of wire with a thickness of 0.1mm. When making a coil there will be a trade-off between the inductance and the resistance of the coil. More windings will result in a higher inductance, but also a higher resistance. High resistance can be a problem, because USB-powered data aquisition boxes have a limited capability to produce currents[2].

Two coils were made, each placed on opposites sides of the watt balance, as is shown in figure 1. One of the coils will function as the driving coil while the other will be used to read voltage changes as the balance beam moves. Through the driving coil, a known, preconfigured, sinusoidal signal can be sent in order to make measurements in velocity mode. In force mode a constant, but modifiable signal is necessary.

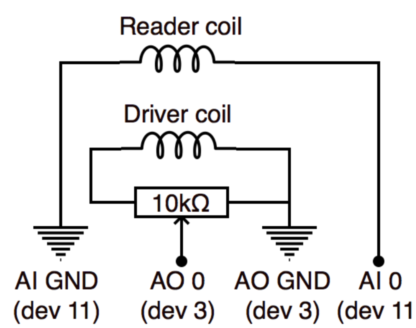

Figure 4: *The circuit for the coil-system.*

The coil system is highlighted in figure 1, indicated by the letter "B". Photographic enlargement of one of the coils with magnets removed is displayed here. The two neodymium magnets fastened to each end of the balance beam and placed withing the coils are facing each other with the opposing poles. This is to make sure that the magnetic field points radially outwards. The reasoning behind this configuration follows from equation 5. The cross product between the infinitesimal wire vector $\mathbf{d}\boldsymbol{\ell}$, which points along the wire, and the magnetic field $\mathbf{B}$, which points radially outwards, ensures that the Lorentz force from the coil with a current running through it is in the z-direction (up and down). A simulation of the magnetic field is shown in figure 5.

[2]More about the data aquisition system we use in the next section

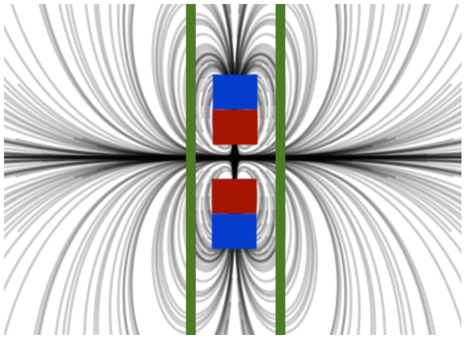

Figure 5: *Simulation of magnetic field in the coils*

3. Data aquisition

For data aquisition two National Instruments USB-6211 data aquistion (DAQ) devices were used. The DAQ boxes are named either "dev3" and "dev11". The names were somewhat arbitrarily chosen by the computer with which they were interacting. The different use of the two boxes in the circuits is clear in the circuit diagrams in figures 3 and 4.

In the original LEGO Watt Balance paper [3] which this study is based on, the authors recommends to use the LabJack U6 multifunction DAQ box. We found the NI-DAQ to be an adequate substitute.

IV. Calibration & Measurements

1. Shadow sensor

In order to find a relationship between the balance arm's z-directional displacement and the output voltage from the shadow sensor system, the following steps are followed to prepare for measurement and calibration.

1. The LEGO watt balance is placed on a flat, level surface a few meters away from a blackboard and a laser is fastened to the top of the arm.
2. The laser is shone upon the blackboard and the distance d from the pivot point of the balance to the wall, is measured.
3. The balance arm is aligned to the support tower. The balance should be fairly level and not rotated or skewed to one side of the support tower.
4. The coils are aligned with the corresponding magnets hanging down from the tip of the balance arms. If the magnets scrape against the interior of the coils, friction will cause problems for the measurement.

After these calibration steps the arm is positioned at a few different angles which are calculated by displacements x on the blackboard, given by the spot at which the laser shines, and the distance d to the blackboard. The balance angle is then determined by $\theta_i = (x_i - x_0)/d$. The coil height is found by multiplying the balance angle by the effective radius, $z_i = r_{eff}\theta_i$. Within a reasonable range, the voltage from the light sensitive diode in the shadow sensor system is a linear funtion of the coil height.

2. Velocity mode

The velocity mode measurement is the key for characterizing the electromagnetic properties of the balance, and is the first measurement step toward obtaining a mass value. The information from the calibrated shadow sensor, the procedure for which is described in the section above, is employed to find the time derivative of the z-displacement of the balance beam and find the velocity.

To perform a watt balance experiment with the LEGO watt balance one outputs a sinusoidal signal through the driving coil and measurers the velocity with the reading coil. With the symmetrical design in this study either arm could be the driver. Both the induced voltage data from the coil and the voltage data from the shadow sensor is saved.The coil position can the be extracted from the shadow sensor voltage and the velocity is obtained by the simplest numerical derivative:

$$v_i = \frac{dz_i}{dt} \approx \frac{z_{i+1} - z_i}{\Delta t} \tag{8}$$

To obtain the factor $(BL)_v$ from equation 4, the induced voltage in the reader coil is regressed linearly against the velocity.

3. Force mode

In force mode the driving coil is used to apply an electromagnetic force to one arm of the balance. The magnitude of this force must be controlled somehow in order to balance the weight. The easiest way to achieve this magnitude control is with a potentiometer. Alternatively, the feedback of electromagnetic force can also be automated. For instance, the position of the balance beam can be detected by the shadow sensor and employed as the control variable for an analog or digital controller.

The goal is to continuously compare an offsett to a desired null position. Currents must be generated proportionally to this measured error, as the current I is the last piece needed in order to calculate the mass, as given by equation 7: $m = BLI/g$.

The mesasurement must be done in several steps, and for each step the balance must be reset to the null position. Here follows an example of how to make the measurement, beginning with no mass on any of the pans and the balance arm in null position.

1. A tare mass m_T is placed on one of the pans. Some current I_1 is necessary to maintain balance position. The following equation describes the weighing:

$$I_1(BL)_F = -m_T g \tag{9}$$

2. A calibrated mass m is added to the opposite pan. The current I_2 is required for balance. This weighing is described by the following equation:

$$I_2(BL)_F - mg = m_T g \tag{10}$$

Now it is sufficient to subtract equation 10 from equation 9 to get an estimate for $(BL)_F$,

$$mg = (I_2 - I_1)(BL)_F \tag{11}$$

$$\implies (BL)_F = \frac{mg}{I_2 - I_1} \tag{12}$$

However, it will be benefactory to perform more weighings for a more precise result.

3. Another weighing, with the calibrated mass removed, yields:

$$I_3(BL)_F = -m_T g \tag{13}$$

4. A second weighing, adding the calibrated mass, yields:

$$I_4(BL)_F - mg = -m_T g \tag{14}$$

5. Then one can remove the mass again and obtain:

$$I_5(BL)_F = m_T g \tag{15}$$

The weighing would continute in this fashion until one is satisfied. Using the the above equations, the current can be calculated quite precicely:

$$I = \frac{1}{3}(I_1 + I_3 + I_5) + \frac{1}{2}(I_2 + I_4) = \frac{mg}{(BL)_F} \tag{16}$$

It serves the purpose to restate equation 7 to see that every piece of the puzzle has been found and place. Through velocity and force mode in conjunction, the mass can be computed thusly:

$$m = \frac{BL}{g} I$$

V. Results

Without a driving force and with a reasonably balanced lever the voltage generated by the diode is measured to be 0.35V. Hence the balancing voltage on the diode is set to −0.35V. A precise measurement is not needed as the mean voltage produced by the diode will depend on the background light intensity.

When feeding the driver coil with a sinusoidal signal, the frequency should be chosen close to the natural frequency of the system, which was estimated to 0.65Hz. Hence a 0.65Hz sinusoidal signal with amplitude 0.3V was applied to the driving coil for a period of 15 seconds. The voltage produced by the diode during this time interval is shown in figure 6.

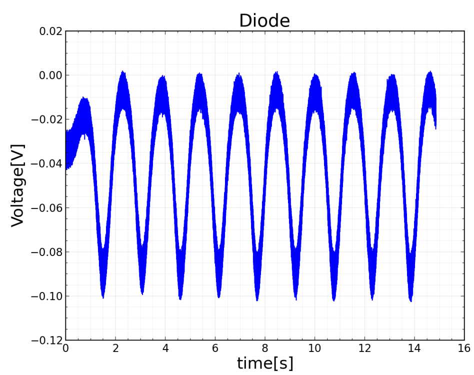

Figure 6: *Diode voltage with period of 15s at* 0.65Hz *and amplitude of* 0.3V.

Ideally, this should stabilize to a perfect sinusoidal signal. Looking closely at the figure, however, one may find the bottom peaks to be sharper than the top peaks. This indicates that the lever is more rapidly accelerated in one direction than the other. Even worse is the signal produced by the reader coil, which is shown in figure 7. Although resonably periodic, the signal exhibits a clear bump when passing the mean voltage from high to low. A comparison with the voltage produced by the diode shows that this is exactly when the magnet reaches a maximal height in the reader coil.

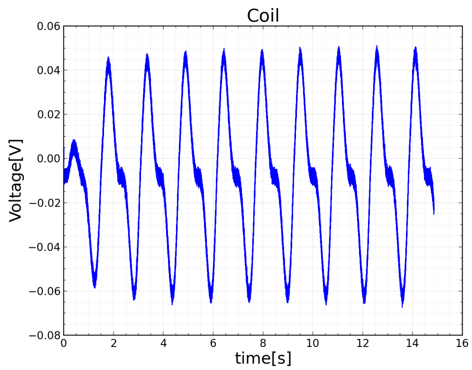

Figure 7: *Induced coil voltage with period of 15s at* 0.65Hz *and amplitude of* 0.3V*.*

Since the voltage induced in the reader coil is proportional to the vertical velocity of the magnet inside, the relation beween the voltage provided by the diode and the voltage provided by the coil should be similar to the relation between the position and velocity of the magnet in the reader coil. If the two voltages are plotted against each other, as shown in figure 8, the datapoints draws a path similar to the path in the phase space of the vertical position of the magnet. In the case of simple harmonic motion, the path will resemble a circle in phase space[3]. The tendency for the magnet to halt when it attains its maximal height is clearly seen in figure 8. Note that when the magnet reaches its top position, it seems to keep on moving upwards without inducing a voltage in the coil. This is seen from the horizontal segment in the far right of the figure. This is a strong indication that the magnet escapes the scope of the coil, where it can move without inducing voltage in the coil.

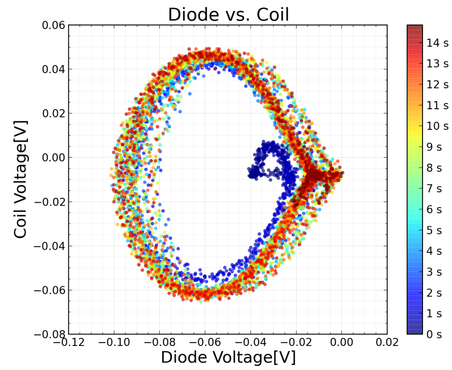

Figure 8: *Diode voltage versus induced coil voltage with a period of 15s at* 0.65Hz *and amplitude of* 0.3V*.*

1. Friction

To better understand the effects of friction in the system, a 0.5Hz sinusoidal signal with linearly decreasing amplitude is feeded to the driver coil. The induced voltage is shown as a function of the output amplitude in figure 9. Without the presence of friction, we would expect the amplitude of the induced voltage to be a linear function of the output amplitude. As indicated by the dotted red line, the data deviates from the linear trend when the output amplitude becomes smaller than about 0.35V. For all amplitudes smaller than 0.3V the lever did not move at all.

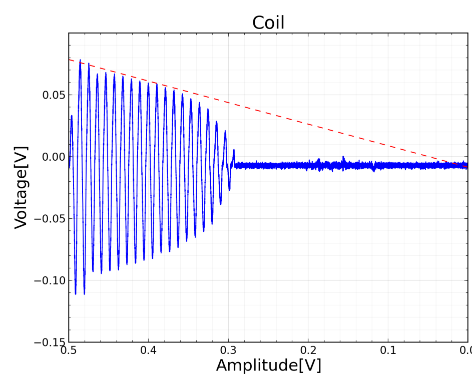

Figure 9: *Induced coil voltage from signal of linearly decreasing amplitude with* 0.5Hz *frequency.*

[3]As it is always possible to rescale the units of measurement, the path taken in phase space could also be an ellipse.

Taking only figure 9 into consideration one may be tempted to think that the ideal amplitude is somewhere around 0.4V. Recalling that the magnet may escape the scope of the coil at too high amplitudes, we may think that the lowest amplitude that respects the linear trend in figure 9 is the best choice.

Now we return to a signal whose frequency is the natural frequency of the system, 0.65Hz. We would then expect the linear trend to be respected for lower amplitudes. Hence the response of the systems is measured for the three different amplitudes 0.4V, 0.3V and 0.25V. The resulting signals from the reader coil is shown in figure 10.

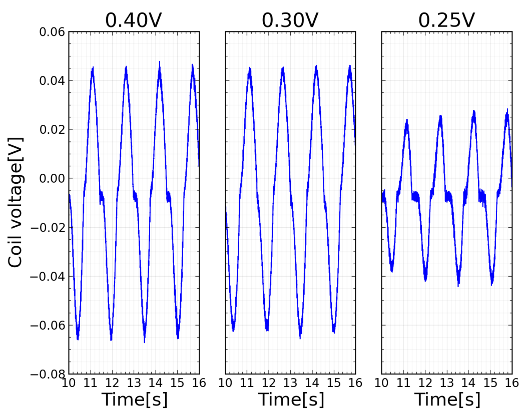

Figure 10: *Induced coil voltage with amplitudes* 0.4V, 0.3V *and* 0.25V *from a* 0.65Hz *signal.*

Out of the three signals, the 0.3V-amplitude signal seems to be the least problematic. Curiously, the problematic halting of the lever at its extremal positions occurs both at high and low amplitudes. If this effect is because the magnet spends too much time at or around maximal height, then why does it happen more at low amplitudes? Looking at the 0.25V-amplitude signal we see that the magnet halts both at its maximal and minimal height. If the joints connecting the pans to the lever are not moving freely, then we may imagine the magnets to touch the inside of the coil at extremal displacement. This effect will become more apparent for lower amplitudes as the force from the driver coil needs to get the magnet loose from the coil.

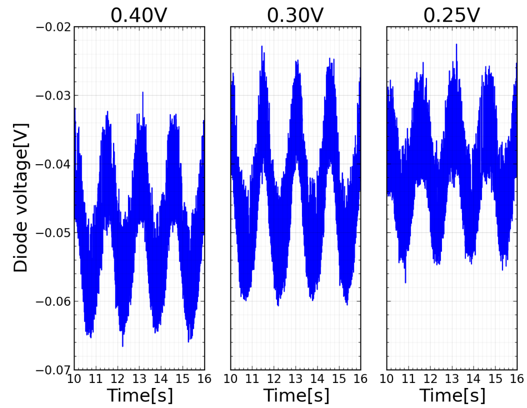

Figure 11: *Diode voltage for amplitudes* 0.4V, 0.3V *and* 0.25V *from a* 0.65Hz *signal.*

In figure 11 the measured signals from the diode is shown. Note that the mean voltage is different for all the amplitudes. This is strange as the three experiments were performed immediately after each other. One possible explanation is that the driver coil has a tendency to push the magnet in the reader coil below its resting position. This would cause the diode to recieve less light and thus produce a lower voltage.

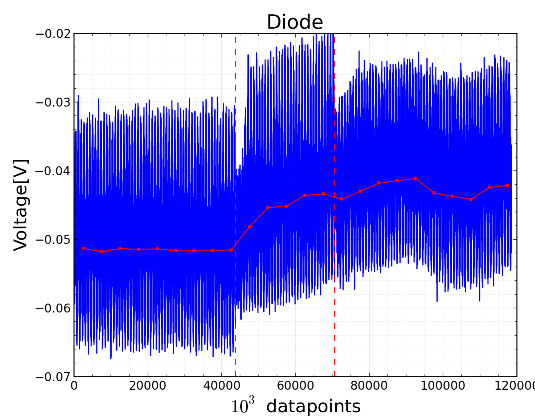

Figure 12: *Diode voltage for amplitudes* 0.4V, 0.3V *and* 0.25V *from a* 0.65Hz *signal. Respective periods are* 87s, 54s *and* 95s *and concatenated in chronological order with dotted, vertical red lines. The mean voltage, calculated from* 5000 *datapoints, is the red line.*

Alternatively, the different mean voltages are due to a varying intensity of the background light. In that case, the mean voltage produced by the diode should vary with time in each experiment. That this is indeed the case as seen in figure 12, where the three experiments, all consisting of about 60 seconds of measurement, are plotted together in chronological order. While the first experiment seems to experience a relatively constant background light intensity, the two succeeding measurements clearly exhibits a drift.

2. Noise

When looking upon figures 12 and 11 it is hard not to notice the noise of the diode. As we are only interested in the derivative of the voltage produced by the diode, this may be problematic. In figure 13 a part of figure 6 is show together with a 10-, 100- and 1000-fold magnification in the time domain.

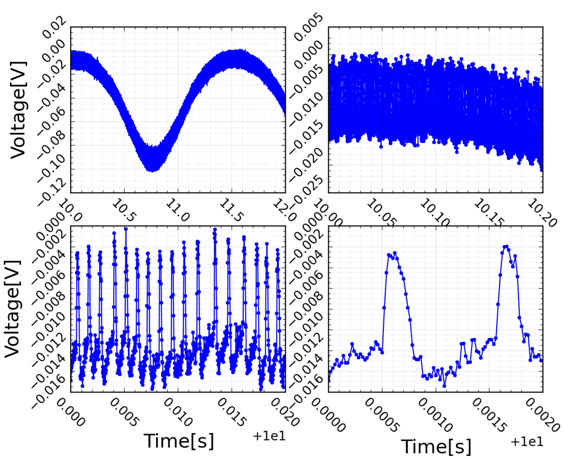

Figure 13: *The time interval from* 10s *to* 12s *of figure 6 together with a* 10-*,* 100- *and* 1000-*fold magnification in the time domain.*

This amount of noise means that the signal would have to go through many layers of smoothing filters in order to yield reasonable derivatives. Such a function ended up shifting nad smearing the signal, thus making the determination of the constant BL very inaccurate. This, together with the problems caused by friction and the non-harmonic response, makes it impossible to perform a velocity mode measurement.

Additionally, when trying to perform the force mode measurement as described in the previous section, the watt balance proved unable to support masses above 10g. The force of the driver coil was not strong enough to lift the mass. Moreover, when a weight was put on the balance the magnet left the scope of the coil making it unable to lift the magnet even if it would have been strong enough.

This catastrophe serves as a death sentence for the watt balance described in this report.

VI. Improvements

As the section above is testimony of, our LEGO watt balance proved to be absolutely horrible at what it was designed to do. It was supposed to measure small masses at very high precision, but we were unable to make it past the calibration stage. Here is a summary of the main improvements necessary to make the LEGO watt balance operational and an outline of the potential of a watt balance.

Firstly, the coils should be wider and longer. Making the coils wider will ensure that the magnets does not rub againts the inside of the coil, causing friction. By extending the length of the coil one overcomes the problem of the magnetic field exiting the coil. Moreover, more windings in the coil would be favourable.

Secondly, the joints connecting the balance beam to the pans of the watt balance should be optimized. Regrettably, a somewhat thorough scouring of the web ended in no unearthing of a LEGO part that is superior to the one we are using now. Therefore, some ingenuity and imagination is needed. However, replacing the joints is not of highest priority.

Thirdly, the light sensitive diode should be replaced. The light sensitive diode we used, as recommended by NIST [3] proved vastly inferior for its purpose. The most obvious explanation is that the diode we used was defective. As we had no alternative diode, the actuality of this matter could not be uncovered. Regardless, an improvement of the design would be to use a four quadrant photo detector instead[4]. A four quadrant photo detector will also circumvent the problem with drift in the photo diode voltage, as a difference and not an absolute voltage is measured.

Figure 14 shows a “real” watt ballance built at NIST in Gaithersburg, Maryland. The next sections describes two quantum effects that can be employed

[4]QP50-6-TO8 is a four quadrant photodiode available from Mouser Electronics

by a watt balance in order to make measurements in increments of Planck's constant.

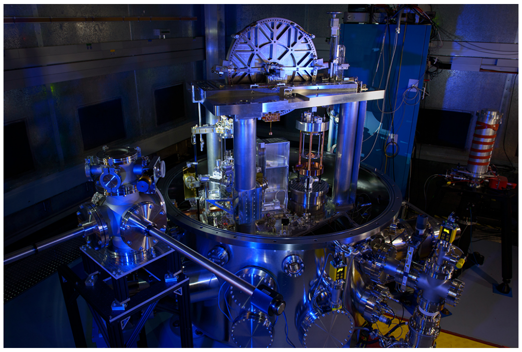

Figure 14: *The Watt balance of NIST in Gaithersburg, MD*

1. Josephson effect

The Josephson effect is a macroscopic quantum phenomenon where a current flows without any voltage applied. The direct current Josephson effect is when a direct current crosses an insulator in the absence of any electromagnetic field, because of quantum tunneling.

The alternating current Josephson effect can be observed in a Josephson junction, which consists of two superconductors separated by a weak link. A bias current is forced through this junction and a voltage will develop across the junction if the junction is exposed to an electrical field with the frequency f:

$$V = \frac{h}{2e} f \equiv K_J^{-1} f \tag{17}$$

K_J is called the Josephson constant after Brian Josephson who predicted the effect i 1962 [4].

With enough of the junctions described above, one has a Josephson voltage standard. This is, in effect, a digitally adjustable battery. With approximately 250000 junctions immersed in liquid helium one can produce any voltage up to 10V with an uncertainty of $1nV$.

2. Quantum Hall effect

When a current-carrying plate is placed in a magnetic field, the current carriers in the plate will move to one side of the plate. When there is a sufficient surplus of current carriers on this one side of the plate the current will be carried in a regular manner again. Consequently, one can measure a voltage difference across the width of the plate, transverse to the current and perpendicular to the magnetic field. This voltage is referred to as the Hall voltage and the effect is called the Hall effect.

The quantum hall effect is a quantum mechanical version of the Hall effect occurring in two-dimensional arrays of electrons or other current carriers. The Hall resistance as a ratio between the Hall voltage and the current becomes quantized to

$$R_H = \frac{V_H}{I} = \frac{1}{i}\frac{h}{e^2} \equiv \frac{1}{i} R_K \tag{18}$$

where $R_K = h/e^2$ is called the von Klitzing constant after Klaus von Klitzing who discovered the effect in 1980 [5].

A quantum Hall resistance standard can be fashioned by application of the Quantum Hall effect and scaling with a cryogenic current comparator.

3. Measuring Planck's constant

Using a Josephson voltage standard and a quantum Hall resistance standard together enables one to build a watt balance with a minuscule relative measurement uncertainty. The current is simply found by equating Ohm's law: $I = V/R$.

However, precise measurements of the current I is difficult. Instead of measuring $P = VI$, the current I is driven through a precisely calibrated resistor R, producing a voltage drop V_R, yielding $P = VV_R/R$. By measuring both voltages compared to the Josephson voltage standard, their values can be expressed in terms of a frequency and the Josephson constant.

$$P = \frac{VV_R}{R} = Kf_1f_2 \frac{h}{2e}\frac{h}{2e}\frac{e^2}{h} = \frac{Kf_1f_2}{4} h \tag{19}$$

K is a constant indicating the number of junctions used in the Josephson voltage standard. Combining this expression with equation 7 yields:

$$h = \frac{4}{Kf_1f_2} mgv \rightarrow m = \frac{Kf_1f_2}{4}\frac{h}{gv} \tag{20}$$

VII. Closing remarks

The kilogram is arguably the most important of the SI units. All the other SI units except kelvin depends directly, or indirectly, upon the definition of the kilogram. Therefore it is about time to introduce a more stable and precise definition of the kilogram in terms of Planck's constant.

This study has presented an interesting glimpse into the field of metrology, realized by the construction of the LEGO watt balance. The process has been a wonderful change from the regular laboratory work. We have faced new and interesting challenges by designing the apparatus on our own – we are grateful to be given such an oppourtunity!

As it sadly tured out, the LEGO watt balance did not live up to our expectations. In fact, it was uable to measure any mass at all. The coil system was plagued by a non-harmonic response to harmonic output signals as well as friction in the coil interior. Furthermore, noise in the voltage from the light sensitive diode in the shadow sensor system makes treatment and further use of the data incredibly difficult[5].

References

[1] Kibble, B., A Measurement of the Gyromagnetic Ratio of the Proton by the Strong Field Method, *Atomic Masses and Fundamental constants* vol. 5, edited by Sanders, H. and Wapstra, A.H., pp. 454-551, New York: Plenum, 1976.

[2] Squires, G. L. *Practical Physics*, Cambridge: Cambridge UP, 1985.

[3] Chao, L.S., Schlamminger, S., Newell, D.B., Pratt, J.R., Seifert, Zhang, X., Sineriz, G., Liu, M., and Haddad, D,. A LEGO Watt Balance: An Apparatus to determine a mass based on the new SI, *American Journal of Physics* 83.11 pp. 913-922, 2015.

[4] Josephson, B.D., Possible new effects in superconductive tunnelling, *Physics Letters 1* 7 pp. 251-253, 1962.

[5] von Klitzing, K., Dorda, G., and Pepper, M. Realisation of a resistance standard based on fundamental constants. *Physical Review Letters* 45 p. 494 1980.

[5] Make sure to keep away from LEGO watt balances in the future