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ANALYSIS OF THE CURRENT–VOLTAGE CHARACTERISTIC OF SOLAR CELLS

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Abstract—The parameters for the generation–recombination current and diffusion current of a solar cell including series and shunt resistance are determined experimentally by a new method through applying equal current steps to the cell rather than voltage steps. This allows a simple evaluation of the generation–recombination current term in the presence of a low shunt resistance of the cell. In a second measuring cycle the series resistance and the diffusion current term of the cell are determined in a similar way. The presented method is a relative simple and low-cost analysis and it allows a quick and accurate on-line determination of the parameters of the current–voltage characteristic, especially for silicon solar cells.

NOTATION

- A, B constants evaluated by linear regression
- $I(N)$ impressed current [A]
- I_{01} saturation current for generation–recombination [A]
- I_{02} saturation current for diffusion [A]
- I_a current step [A]
- I_d voltage-dependent diffusion current [A]
- I_g voltage-dependent generation–recombination current [A]
- I_{sh} current through the shunt resistance [A]
- n_1 diode-quality factor for generation–recombination
- n_2 diode-quality factor for diffusion
- R_s series resistance [Ω]
- R_{sh} shunt resistance [Ω]
- $V(N)$ measured voltage [V]
- $V_f(N)$ forward voltage at current NI_a
- $V_r(N)$ reverse voltage at current NI_a
- V_f forward voltage [V]
- V_r reverse voltage [V]
- k Boltzmann's Constant ($8.62 \cdot 10^{-5}$ eV/K)
- q Electron charge
- T absolute temperature

1. INTRODUCTION

The current–voltage characteristics of solar cells described by the single-exponential equation, i.e. with only the diffusion current term taken into account, has been carried out using an analytical method by Picciano [1], and using numerical methods by Bryant and Glew [2] as well as by Braunstein *et al.* [3]. The double-exponential equation of solar cells, i.e. taking into account diffusion current and the recombination generation current of the space-charge region, is more closely related to the physical phenomena. There are two numerical methods which are usually employed. In the first method, the diode-quality factor for the recombination–generation current is assumed to be 2 and the diode-quality factor for the diffusion current is assumed as unity, so that the unknown parameters are reduced to four [4, 5]. In the second method, either the shunt resistance is neglected or the diode quality factor for the diffusion current is assumed as unity, and then in both cases the equation is solved numerically to determine five parameters [6, 7]. Both numerical methods are based on the modified least square fit for non-linear equations and require time consuming computer calculations. In this paper we develop a simple and rapid method for determining the parameters of the double-exponential equation for solar cells, including shunt and series resistance and diode-quality factors.

2. ANALYSIS OF THE CURRENT-VOLTAGE CHARACTERISTIC

According to the equivalent circuit of a solar cell, as shown in Fig. 1, the dark-current characteristic is given by [4–8]:

$$I = I_g + I_d + I_{sh}, \tag{1}$$

where I_g is the voltage-dependent generation–recombination current:

$$I_g = I_{01} \cdot \left[\exp\left(\frac{V - I \cdot R_s}{n_1 \cdot V_{th}}\right) - 1\right]. \tag{2}$$

I_d is the voltage-dependent diffusion current:

$$I_d = I_{02} \cdot \left[\exp\left(\frac{V - I \cdot R_s}{n_2 \cdot V_{th}}\right) - 1\right] \tag{3}$$

and I_{sh} is the current flowing over shunt resistance R_{sh}:

$$I_{sh} = \frac{V - I \cdot R_s}{R_{sh}} \tag{4}$$

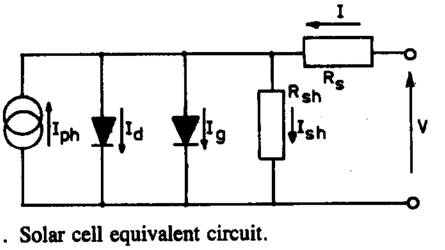

Fig. 1. Solar cell equivalent circuit.

I_{01} — the voltage dependence of I_{01}, especially for reverse voltages, is neglected — and I_{02} are constants and represent the saturation currents of the generation–recombination and diffusion process, respectively; n_1 and n_2 are the diode-quality factors for generation and diffusion, respectively. R_s is the series resistance and $V_{th} = q/(k \cdot T)$ is the thermal voltage.

The functional dependence of the total current I on the voltage V is determined by six parameters. In order to evaluate these parameters, the current–voltage characteristic is divided into three voltage region, so that certain simplification for eqn (1) are possible.

2.1 *Voltage region* I: *Reverse characteristic* $V \leqslant -8V_{th}$

For voltages $V \leqslant -8V_{th}$, i.e. $V \leqslant -0.2$ V the exponential terms in eqns (2) and (3) tend towards zero, so that $I_g = -I_{01}$ and $I_d = -I_{02}$. Furthermore in most solar cells, the saturation current I_{02} for the diffusion term is several orders of magnitude smaller than the saturation current I_{01} for the recombination–generation term. Therefore the characteristic is then mainly determined by the I_{sh} term. Because of the low total current I flowing in the reverse direction and the normally small values of the series resistance R_s ($R_s \leqslant 1\Omega$), the voltage drop $I \cdot R_s$ can also be neglected. Thus, for this voltage region, eqn (1) is reduced to:

$$I = I_g + I_{sh} = -I_{01} + \frac{V}{R_{sh}}. \tag{5}$$

By applying a current I with fixed current steps I_a in the reverse direction, i.e. $I(N) = N \cdot (-I_a)$, the differential resistance is determined for each pair of $V(N)$, $I(N)$ by:

$$R(N) = \frac{\Delta V}{\Delta I} = \frac{V(N+1) - V(N-1)}{2 \cdot I_a}. \tag{6}$$

Then, the maximum value of $R(N) = R_{\max}$ is evaluated; and a mean value for the shunt resistance R_{sh} is calculated as follows:

$$R_{sh} = \frac{1}{K - J + 1} \sum_{N=J}^{N=K} R(N)$$

where $R(J)$ is the first taken value from $R(N)$ and $R(K)$ is the last value of $R(N)$ which are located within an interval of $/R_{\max} - \Delta R/$. The value of ΔR can be selected and represents a certain deviation from $R_{\max}$, i.e. $\Delta R/R_{\max}$ should be 2–5%.

2.2 *Voltage region II*: $V \leqslant 15\ V_{th}$

For voltages $V \leqslant 15\ V_{th}$, *i.e.* $V \leqslant 0.4$ V, the diffusion current of a silicon solar cell can normally be neglected [6, 8, 9]. Furthermore, the voltage drop at the series resistance is much smaller than the terminal voltage V of the solar cell. In this case, the current–voltage characteristic is given by:

$$I = I_g + I_{sh} = I_{01} \exp\left(\frac{V}{n_1 \cdot V_{th}}\right) - I_{01} + \frac{V}{R_{sh}}. \tag{8}$$

By applying a current with N fixed steps in the forward and reverse direction of the cell, so that $V_f(1) \geqslant 4V_{th}$ and $V_r(1) \leqslant -8V_{th}$, eqn (8) can be written for the forward direction (index f) as:

$$I(N) = I_{01} \cdot \exp\left(\frac{V_f(N)}{n_1 \cdot V_{th}}\right) - I_{01} + \frac{V_f(N)}{R_{sh}}; \tag{9}$$

and for the reverse direction (index r) as:

$$-I(N) = -I_{01} + \frac{V_r(N)}{R_{sh}} \tag{10}$$

[see eqn (5)]. Because equal current steps are applied to the cell in forward and also in reverse direction, the forward and reverse current is given by:

$$I_f = I(N) = N \cdot I_a \quad \text{and} \quad I_r = -I(N) = N \cdot (-I_a),$$

the summation of eqns (9) and (10) then yields:

$$0 = I_{01} \exp\left(\frac{V_f(N)}{n_1 \cdot V_{th}}\right) - 2I_{01} + \frac{V_f(N) + V_r(N)}{R_{sh}} \tag{11}$$

and the subtraction of eqn (10) from eqn (9) yields:

$$2 \cdot I(N) = I_{01} \exp\left(\frac{V_f(N)}{n_1 \cdot V_{th}}\right) + \frac{V_f(N) - V_r(N)}{R_{sh}}. \tag{12}$$

Equation (11) or eqn (12) can be used to determine I_{01} and n_1; using eqn (11), one obtains:

$$\ln[-V_f(N) - V_r(N) + K(N)] = \ln(R_{sh} \cdot I_{01}) + \frac{1}{(n_1 \cdot V_{th})} \cdot V_f(N) \tag{13}$$

with

$$K(N) = \frac{2 \cdot \left(I(N) \cdot R_{sh} - V_f(N)\right)}{\left[\exp\left(\frac{V_f(N)}{2 \cdot V_{th}}\right) - 1\right]}.$$

Setting $y = \ln[-V_f(N) - V_r(N) + K(N)]$ and $x = V_f(N)$, eqn (13) can be written as:

$$y = A + Bx \tag{13a}$$

where A and B are constants for a given solar cell. They are determined by a linear least-square fit (linear regression):

$$A = \ln(I_{01} \cdot R_{sh}); \qquad I_{01} = \exp(A)/R_{sh} \tag{13b}$$

$$B = 1/(n_1 \cdot V_{th}); \qquad n_1 = 1/(B \cdot V_{th}). \tag{13c}$$

In an analog manner, eqn (12) can be applied to determine I_{01} and n_1, also using eqns (13b) and (13c). In this case, the value of y is given by $y = \ln[2 \cdot I(N) \cdot R_{sh} - V_f(N) + V_r(N)]$.

2.3 *Voltage region III: $V \geqslant 15\ V_{th}$*

If $I \geqslant I_g + I_{sh}$ holds, which can easily be proofed using the former results, then the current through the cell is given by:

$$I = I_d = I_{02} \cdot \exp\left(\frac{V - I \cdot R_s}{n_2 \cdot V_{th}}\right). \tag{14}$$

The series resistance R_s must be determined first, in order to evaluate I_{02} and n_2. In the literature several methods have been investigated. Most of them are based on illuminated current–voltage characteristics measurements [8, 10–12]. For sufficient accuracy in determining R_s, high illumination levels (several suns) are necessary. A further possibility for the determination of R_s has been given by Araujo [4] and Wolf *et al.* [6] and involves the method of minimizing the standard deviation for a given set of measured and calculated current values. Here we use the dark-characteristic measurement of a solar cell in order to evaluate R_s. With eqn (14), one obtains:

$$\ln I - \ln I_{02} = (V - I \cdot R_s)/(n_2 \cdot V_{th}). \tag{15}$$

Subtracting eqn (15) for two pairs of V and I, one obtains after some rearranging:

$$\frac{I_2 - I_1}{\ln(I_2/I_1)} = -\frac{n_2 \cdot V_{th}}{R_s} + \frac{1}{R_s} \cdot \frac{V_2 - V_1}{\ln(I_2/I_1)} \tag{16}$$

where $I_2 = I(N)$ and $I_1 = I(N - 1)$ are the applied currents and $V_2 = V(N)$ and $V_1 = V(N - 1)$ the measured voltages. Again, a least square fit (linear regression) can be made, with the variables $y = (I_2 - I_1)/\ln(I_2/I_1)$ and $x = (V_2 - V_1)/\ln(I_2/I_1)$. The series resistance is given from the gradient of the linear regression line:

$$R_s = 1/B \tag{17a}$$

or from the intersection of the regression line with the y-axis with $n_2 = 1$:

$$R_s = -n_2 \cdot V_{th}/A = -V_{th}/A. \tag{17b}$$

Consequently from one iteration process one obtains two values for R_s, one for $n_2 \neq 1$ [eqn (17a)] and a second one, assuming $n_2 = 1$ [eqn (17b)].

If the condition $I(N) \gg I_g(N) + I_{sh}(N)$ is not satisfied, then I_2 and I_1 are substituted by $I_2 = I(N) - I_g(N) - I_{sh}(N)$ and $I_1 = I(N - 1) - I_g(N - 1) - I_{sh}(N - 1)$ and R_S can be estimated with eqns (16)–(17b). The values for $I_g(N)$ and $I_{sh}(N)$ can be calculated with the known parameters for I_g and I_{sh}, with $V = V(N) - I(N)R_{so}$. For two measured pairs of V and I in the high-current region, one obtains from eqn (16) with $n_2 = 1$ and setting $R_s = R_{so}$:

$$R_{so} = \frac{V_2 - V_1}{I_2 - I_1} - V_{th}\frac{\ln(I_2/I_1)}{I_2 - I_1}. \tag{18}$$

The value of R_{so} can be considered as a rough approximation for R_s.

As soon as a proper value of R_s is found, I_{02} and n_2 may be determined. We start with eqns (1) and (4), and obtain:

$$\ln\{I(N) - I_g(N) - I_{sh}(N)\}$$
$$= \ln I_{02} + (V(N) - I(N) \cdot R_s/(n_2 \cdot V_{th})). \tag{19}$$

A linear regression [see eqn (13a)] can be carried out with the variables $y = \ln\{I(N) - I_g(N) - I_{sh}(N)\}$ and $x = V(N) - I(N) \cdot R_s$. This yields:

$$A = \ln I_{02}; \qquad I_{02} = \exp(A)$$

$$B = 1/(n_2 \cdot V_{th}); \qquad n_2 = 1/(B \cdot V_{th}). \tag{20b}$$

3. EXPERIMENTAL

Figure 2 shows the block diagram of the experimental arrangement. A personal computer (Hewlett Packard 86) is used to control the measurements and to process the measured data. As shown in Fig. 2 the current is applied by a programmable current source (Keithley 220) and the resulting voltage is measured by programmable digital multimeter (Keithley 195). The current source can apply a current from 1 pA up to 100 mA, with steps between 0.5 pA and 50 μA. In the effective voltage range (± 2 V) used, the digital multimeter has a resolution of 10 μV. A floppy-disc unit is connected to the computer by a HP-IB interface as an additional mass-storage unit. The sample is kept at constant temperature during measurement.

The first step in the measurement procedure is to determine the size of the smallest current step I_a. With the same number of steps in the reverse and forward direction, the pairs of values ($I(N)$, $V(N)$) are measured (region I and II). With these data the parameters of the shunt resistance R_{sh}, the generation–recombination current term I_{01} and the quality factor n_1 are calculated.

Following this, data are measured, starting at a current level I which satisfies the condition $I \geqslant I_g + I_{sh}$. The current I is here increased with a factor C, i.e. $I(N) = C \cdot I(N - 1)$, with $C = 1.1$–1.5 (region III). With these data and the results of the previous calculations for R_{sh}, I_{01} and n_1, the remaining parameters for the series resistance R_s, the diffusion current I_{02} and the diode quality factor for the diffusion current n_2 are determined. A simplified flow-chart diagram of the program is shown in Fig. 3.

4. RESULTS AND DISCUSSION

As examples only two cells of several investigated solar cells which had been differently processed will be discussed in the following. Cell A was a monocrystalline, 10 Ωcm resistivity, n^+–p–p^+ cell with an anti-reflection coating. Cell B was a polycrystalline, 2 Ωcm resistivity, n^+–p-cell without an anti-reflection coating. Both cells had an area of 4 cm^2. During measurement, the temperature was kept constant at

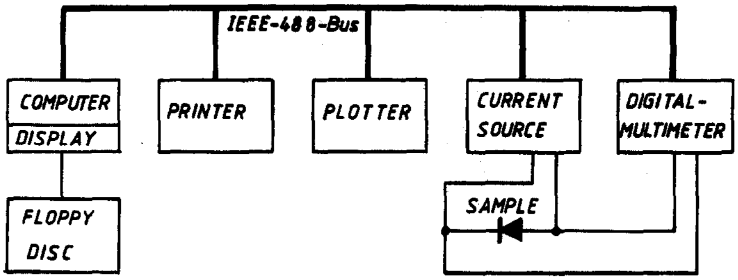

Fig. 2. Block diagram of the experimental arrangement.

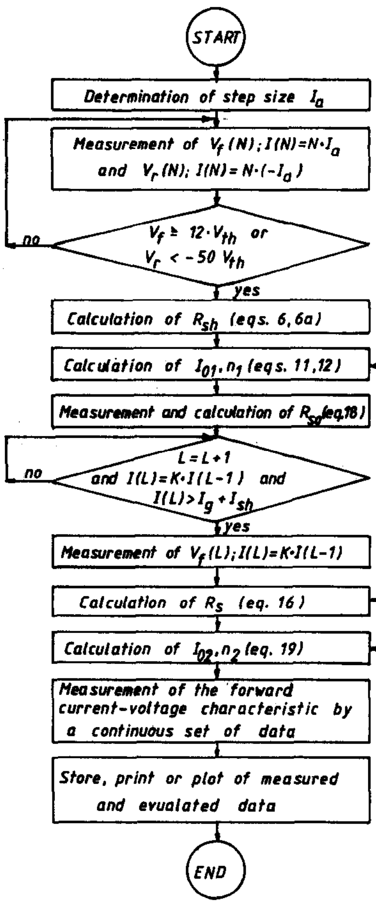

Fig. 3. Simplified flow chart diagram of the computer program.

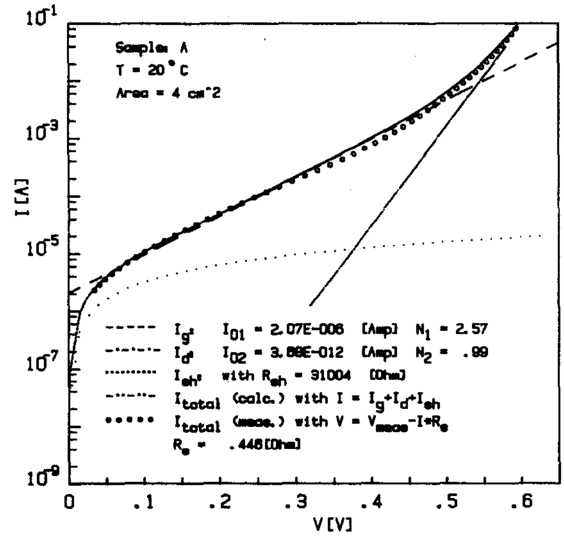

Fig. 4. Current–voltage characteristic of Cell A.

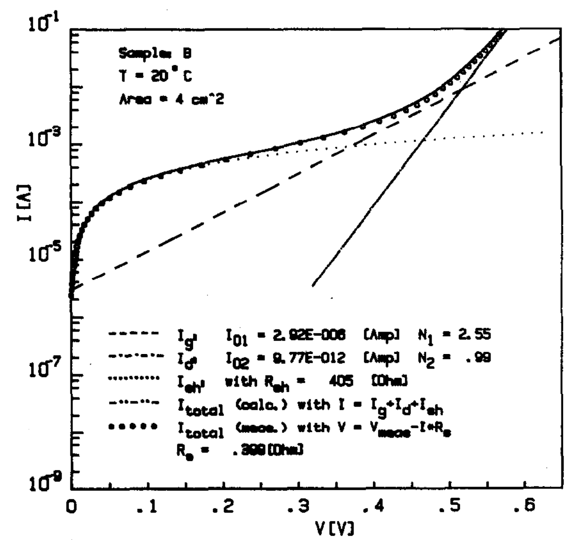

Fig. 5. Current–voltage characteristic of Cell B.

$T = 20°C$. Figures 4 and 5 show the analyzed currents I_g, I_d, I_{sh} and $I_{tot} = I_g + I_d + I_{sh}$ as well as the measured current–voltage characteristic. These figures illustrate the good agreement between the calculated and measured characteristics. In Table 1 all analyzed values of the solar cell parameters are given.

To confirm the results obtained for the six parameters to describe the current-voltage characteristic of a cell, additional measurements under illuminated condition were made, i.e. the dependence of the open-circuit voltage V_{oc} on the short circuit current I_{sc} of the cell under varying illumination level were measured and these V_{oc} and I_{sc} values were compared with those computed from the I–V characteristic using the analyzed parameters. Under illumination of the cell a photocurrent I_{ph} (Fig. 1) is generated. This photocurrent has to be added to eqn (1). For $V = V_{oc}$ and $I = O$ (open circuit

Table 1. Calculated parameters of the investigated cells

PARAMETERS	CELL A	CELL B
R_{sh}	31 000 Ohm	400 Ohm
I_{01} ; n_1 according to eq. 11	$1.91 \cdot 10^{-6}$A ; 2.47	$2.92 \cdot 10^{-6}$A ; 2.55
" eq. 12	$2.07 \cdot 10^{-6}$A ; 2.57	$2.68 \cdot 10^{-6}$A ; 2.74
R_s according to eq. 16	0.446 Ohm	0.399 Ohm
" eq. 16 (n_2 = 1)	0.449 "	0.357 "
" ref. [4]	0.489 "	0.398 "
I_{02} according to eq. 19	$3.68 \cdot 10^{-12}$ A	$9.77 \cdot 10^{-12}$ A
" ref. [4]	$4.98 \cdot 10^{-12}$ A	$8.22 \cdot 10^{-12}$ A
n_2 according to eq. 19	0.99	0.99

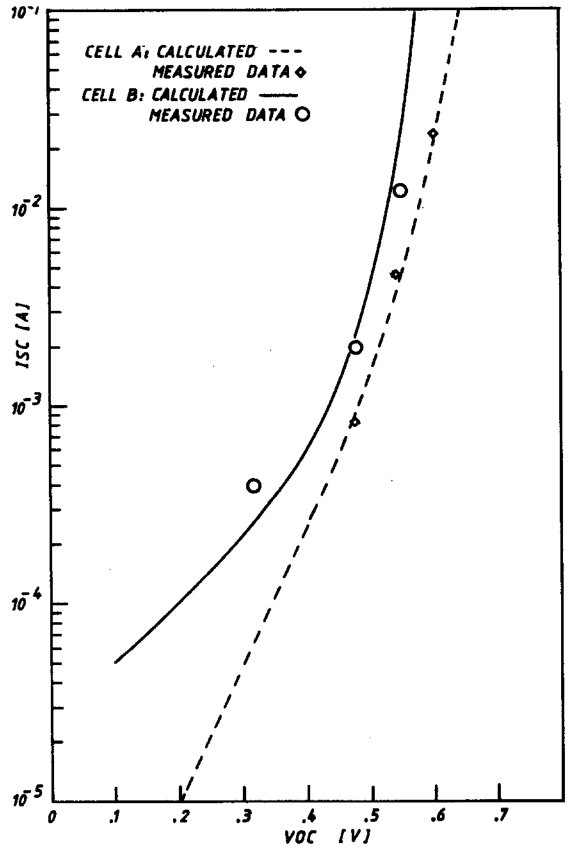

Fig. 6. V_{oc}-I_{sc}-characteristic of cells A and B calculated with the evaluated parameters and compared with measured values.

condition) one yields:

$$0 = I_{01}\left[\exp\left(\frac{V_{oc}}{n_1 \cdot V_{th}}\right) - 1\right] + I_{02}\left[\exp\left(\frac{V_{oc}}{n_2 V_{th}}\right) - 1\right] + \frac{V_{oc}}{R_{sh}} - I_{ph} \quad (21)$$

and for $V = 0$ and $I = I_{sc}$ (short current condition) one gets

$$I_{sc} = I_{01}\left[\exp\left(\frac{-I_{sc} \cdot R_s}{n_1 V_{th}}\right) - 1\right] + I_{02}\left[\exp\left(\frac{-I_{sc} \cdot R_s}{n_2 V_{th}}\right) - 1\right]\frac{I_{sc} R_c}{R_{sh}} - I_{ph}. \quad (22)$$

From eqns (21) and (22) the relation between I_{sc} and V_{oc} of a dark current analyzed cell can be calculated and compared with measured data for I_{sc} and V_{oc} of the cell.

As shown in Fig. 6 there is an excellent agreement between measured and calculated data for the open circuit voltage V_{oc} and the short circuit current I_{sc}.

5. CONCLUSIONS

The procedure shown to analyze the I–V characteristic of a solar cell is suitable to determine the parameters of the generation–recombination and diffusion current including the shunt and series resistance of the cell. By this method time-consuming calculations are not necessary and the measurements as well as the analysis of the characteristic are made on-line. This is possible because current steps are applied to the cell rather than voltage steps. By this technique it is further possible to determine the parameters by linear regression. The dark current characteristic of the investigated silicon solar cells could be well described by the model, which may be further proofed by the measured and calculated $I_{sc} - V_{oc}$ relation of the cells.

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