2.3) In this task we will use

$$f(a) \cdot 2\pi i = \oint \frac{f(z)}{z-a}\,dz$$

to solve the integrals

a) $\oint_\Gamma \frac{\sin z\,dz}{2z-\pi}$ we see that $\frac{\sin z}{2z-\pi}$ diverges at $z = \frac{\pi}{2}$, which is inside the circle Γ. We will use Cauchy-integral formula; with $a = \frac{\pi}{2}$, and $f(z) = \sin(z)$

$$\oint_\Gamma \frac{\sin z}{2z-\pi}\,dz = \frac{1}{2}\oint \frac{\sin(z)}{z-\frac{\pi}{2}}\,dz = \frac{1}{2}\cdot 2\pi i \sin\left(\frac{\pi}{2}\right)$$

$$= \underline{\pi i}$$

b) Since the singularity is at $z = \frac{\pi}{2} > 1$ our function is analytic inside Γ

$\xRightarrow{}$ the integral is $\underline{\underline{0}}$

↑
Cauchy's theorem

2.3) c) $\oint_\Gamma \frac{\sin(2z)}{6z-\pi}\,dz = \frac{1}{3}\oint_\Gamma \frac{\sin(2z)}{2z-\frac{\pi}{3}}\,dz$

$u = 2z$
$du = 2\,dz$

$= \frac{1}{3}\oint_\Gamma \frac{\sin(u)}{u-\frac{\pi}{3}}\frac{du}{2} = \frac{1}{6}\oint_\Gamma \frac{\sin(u)}{u-\frac{\pi}{3}}$

$= \frac{1}{6}\cdot 2\pi i \sin(\pi/3) = \frac{\pi i}{3}\sqrt{\frac{3}{4}} = \underline{\frac{\pi i}{2\sqrt{3}}}$

We use this method since the integrand diverges at $z = \frac{\pi}{6} < 1$

d) The integrand diverges at $z = \ln(2) < 2$,
so we use Cauchy integral formula

$\oint_\Gamma \frac{e^{2z}}{z-\ln(2)}\,dz = 2\pi i\, e^{2\cdot\ln(2)} = 2\pi i\,(2)^2 = \underline{8\pi i}$

In these task we do not have to care about the shape of the curves in any of these problems. We only have to care about if the ~~cu~~ closed curve contains singularities or not.