Oblig 10 - FYS3140 - Ivar Haugerud

1a) $x^2y'' + xy' - 9y = 0 \Rightarrow y'' + \frac{1}{x}y' - \frac{9}{x^2}y = 0$

We can find a solution on the form $y(x) = x^s \sum_{n=0}^{\infty} a_n x^n$ using Fröbenius method, since the diff-eq is on the form $y'' + \frac{b(x)}{x}y' + \frac{c(x)}{x^2}y = 0$, let's calculate y's derivatives

$$y = \sum_{n=0}^{\infty} a_n x^{n+s}, \quad y' = \sum_{n=0}^{\infty} a_n (n+s) x^{n+s-1}, \quad y'' = \sum_{n=0}^{\infty} a_n (n+s)(n+s-1) x^{n+s-2}$$

And put this into our DE:

$$\sum_n a_n(n+s)(n+s-1)x^{n+s} + \sum_n a_n(n+s)x^{n+s} - \sum_n 9a_n x^{n+s} = 0$$

We match equal exponents, must always be zero, for $n=0$:

$$a_0 s(s-1) + a_0 s - 9a_0 = 0, \quad \text{use that } a_0 \neq 0$$

$$s^2 - s + s - 9 = 0$$

$$\underline{s = \pm 3}$$

Our two values of s differ by an integer. Therefore we use the lower one as this might give us the full solution.

For a general n with $s=-3$ we have

$$a_n(n-3)(n-4) + a_n(n-3) - 9a_n = 0$$

$$a_n(n^2 - 4n - 3n + 12 + n - 3 - 9) = 0$$

$$a_n(n^2 - 6n) = 0$$

$$a_n \cdot n(n-6) = 0 \quad \text{use } n \neq 0\text{, which we have solved already}$$

$$a_n(n-6) = 0$$

We see that the only coefficient which "survives" is $n=6$, which means $n+s = 6-3 = 3$, which is the solution we found for the indicial equation

$$\Rightarrow \underline{y(x) = c_1 x^{-3} + c_2 x^3}$$ with c_1 and c_2 being undetermined coefficients

1b) $3xy'' + (3x+1)y' + y = 0$

Use Frobenious method with $y(x) = \sum_{n=0}^{\infty} a_n x^{n+s}$

$\Rightarrow y'(x) = \sum_{n=0}^{\infty} a_n(n+s)x^{n+s-1}$, $y''(x) = \sum_{n=0}^{\infty} a_n(n+s)(n+s-1)x^{n+s-2}$

We insert this into DE

$$\sum_n 3a_n(n+s)(n+s-1)x^{n+s-2} + \sum_n (3x+1)a_n(n+s)x^{n+s-1} + \sum_n a_n x^{n+s} = 0$$

$$\sum_n 3a_n(n+s)(n+s-1)x^{n+s-1} + \sum_n 3a_n(n+s)x^{n+s} + \sum_n a_n(n+s)x^{n+s-1} + \sum_n a_n x^{n+s} = 0$$

Which must be zero for each matching exponent, we look at x^s terms by first multiplying with x

$$\sum_n 3a_n(n+s)(n+s-1)x^{n+s} + \sum_n 3a_n(n+s)x^{n+s+1} + \sum_n a_n(n+s)x^{n+s} + \sum_n a_n x^{n+s+1} = 0$$

For x^s we get no contribution from 2. and 4. term:

$3a_0(s-1)s + 0 + a_0 s - 0 = 0$

$a_0[3s(s-1) + s] = 0$, use that $a_0 \neq 0$

$3s^2 - 3s + s = 0$

$s(3s-2) = 0 \Rightarrow s_+ = \frac{2}{3}$, $s_- = 0$

These two do not differ by an integer $\Rightarrow$ two linearly independent solutions.

For $s = 0$ we have:

$3a_{n+1}(n+1)n + 3a_n \cdot n + a_{n+1}(n+1) + a_n = 0$

$a_{n+1}\left[3n(n+1) + (n+1)\right] + a_n\left[3n + 1\right] = 0$

$$a_{n+1} = \frac{-a_n(3n+1)}{(n+1)(3n+1)} = -\frac{a_n}{n+1}$$

This series will never terminate, but as we see the coefficients will get smaller factorically

1b) $a_0 = a_0$, $a_1 = -a_0$, $a_2 = -\frac{a_1}{2} = \frac{a_0}{2} = \frac{a_0}{2!}$

$a_3 = -\frac{a_2}{3} = -\frac{a_0}{3\cdot 2\cdot 1} = -\frac{a_0}{3!}$, $a_4 = -\frac{a_3}{4} = \frac{a_0}{4!}$, $a_5 = -\frac{a_4}{5} = -\frac{a_0}{5!}$

$$\Rightarrow \quad \underline{a_n = (-1)^n \cdot \frac{a_0}{n!}}$$

Making our solution for $s=0$ to be:

$$Y(x) = \sum_n a_n x^n = \sum_n \frac{(-1)^n a_0}{n!} x^n = a_0 \sum_n \frac{(-x)^n}{n!}$$

$\underline{Y(x) = a_0 e^{-x}}$ Now we need to find the solution for $s = \frac{2}{3}$. We match equal exponents:

$$3a_{n+1}\left(n+\frac{2}{3}\right)\left(n+\frac{5}{3}\right) + 3a_n\left(n+\frac{2}{3}\right) + a_{n+1}\left(n+\frac{5}{3}\right) + a_n = 0$$

$$a_{n+1}\left[3\left(n+\frac{2}{3}\right)\left(n+\frac{5}{3}\right) + n + \frac{5}{3}\right] = -a_n\left(1 + 3\left(n+\frac{2}{3}\right)\right)$$

$$a_{n+1}\left[3n^2 + 7n + \frac{10}{3} + n + \frac{5}{3}\right] = -a_n(1+3n+2)$$

$$a_{n+1}\left[3n^2 + 8n + 5\right] = a_{n+1}\cancel{(n+1)}\left(n+\frac{5}{3}\right)\cancel{3} = -\cancel{3}a_n\cancel{(n+1)}$$

$$a_{n+1} = -\frac{a_n}{\left(n+\frac{5}{3}\right)}$$, the series never converges, but coefficients gets smaller

I do not recognize these coefficients, and was not able to write a_{n+1} independently. Our full solution is

$$Y(x) = a_0 e^{x} + x^{2/3} \sum_{n=0}^{\infty} b_n x^n$$

1c) $2xy'' - y' + 2y = 0$, Use Frö̃benius method

$$Y = \sum_n a_n x^{n+s}, \quad y' = \sum_n a_n (n+s) x^{n+s-1}, \quad y'' = \sum_n a_n (n+s)(n+s-1) x^{n+s-2}$$

Put into DE:

$$\sum_n 2a_n(n+s)(n+s-1)x^{n+s-1} - \sum_n a_n(n+s)x^{n+s-1} + \sum_n 2a_n x^{n+s} = 0$$

Match equal exponents for x^{s-1} and get no contributions from last term

$$2a_0(s-1)s - a_0 s + 0 = 0, \quad \text{use } a_0 \neq 0$$

$$2s(s-1) - s = 0$$

$$2s^2 - 2s - s = 2s^2 - 3s = s(2s-3) = 0$$

We find two solutions $S_+ = \frac{3}{2}$ and $s_- = 0$, which do not differ by an integer $\Rightarrow$ 2 linearly indep solutions.

For $s=0$ we have:

$$2a_{n+1}(n+1)n - a_{n+1}(n+1) + 2a_n = 0$$

$$a_{n+1}(n+1)[2n-1] = -2a_n$$

$$a_{n+1} = -\frac{2a_n}{(n+1)(2n-1)} =$$

$$a_{n+1} = -\frac{a_n}{(n+1)(n-\frac{1}{2})},$$ series never terminates, and I am not able to write a_{n+1} independently as a func of n

$$a_1 = 2a_0, \quad a_2 = -2a_0, \quad a_3 = \frac{4a_0}{9}, \quad a_4 = -\frac{2a_0}{45}$$

I do not see a pattern...

We do the same for $s = \frac{3}{2}$

4/6

1c) $2a_{n+1}\left(n+\frac{5}{2}\right)\left(n+\frac{3}{2}\right) - a_{n+1}\left(n+\frac{5}{2}\right) + 2a_n = 0$

$$a_{n+1}\left(n+\frac{5}{2}\right)\left[2\left(n+\frac{3}{2}\right)-1\right] = -2a_n$$

$$a_{n+1}\left(n+\frac{5}{2}\right)\left(2n+3-1\right) = -2a_n$$

$$a_{n+1}\left(n+\frac{5}{2}\right)(n+1)2 = -2a_n$$

$$a_{n+1} = \frac{-a_n}{(n+1)\left(n+\frac{5}{2}\right)}$$, series never terminates, alternating sign

$a_1 = -\frac{a_0}{5/2} = -\frac{2a_0}{5}$, $a_2 = \frac{-a_1}{2\cdot\frac{7}{2}} = \frac{2a_0}{35}$

$a_3 = -\frac{a_2}{3\cdot\frac{9}{2}} = -\frac{2a_2}{27} = -\frac{4a_0}{945}$

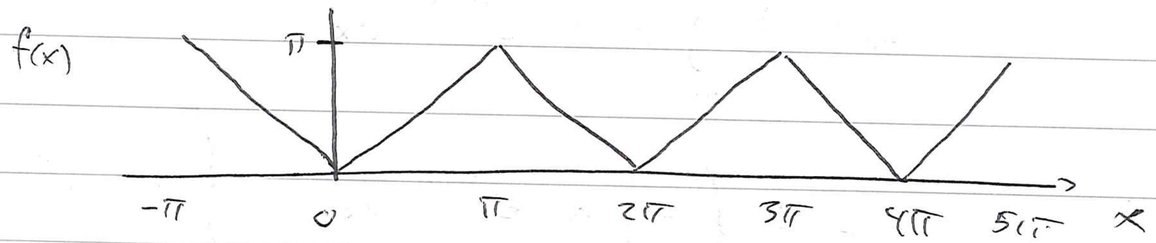
I do not see a pattern...

Our full solution is on the form

$$Y(x) = \sum_n a_n x^n + \sum_n b_n x^{n+3/2}$$

where $a_{n+1} = -\frac{a_n}{(n+1)(n-\frac{1}{2})}$, and $b_{n+1} = \frac{-b_n}{(n+1)(n+\frac{5}{2})}$

2a) $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$



Find fourier-coeffs

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\cos(nx)\,dx,$$ since f(x) is symmetric and cos(nx) is symmetric

$$= \frac{2}{\pi}\int_0^{\pi} f(x)\cdot\cos(nx)\,dx = \frac{2}{\pi}\int_0^{\pi} x\cos(nx)$$

$$= \frac{2}{\pi}\left(\left.\frac{x\cdot\sin(nx)}{n}\right|_0^{\pi} - \int_0^{\pi}\frac{\sin(nx)}{n}\,dx\right)$$

$$\boxed{\begin{aligned} &\int f'\cdot g\,dx = f\cdot g - \int g'\cdot f\,dx \\ &f' = \cos(nx) \qquad g = x \\ &f = \tfrac{1}{n}\sin(nx) \qquad g' = 1 \end{aligned}}$$

$$= \frac{2}{\pi}\left(\underbrace{\frac{\pi\sin(n\pi)}{n}}_{0} - 0 + \left[\frac{\cos(nx)}{n^2}\right]_0^{\pi}\right)$$

$$= \frac{2}{\pi}\left(\frac{\cos(n\pi)-1}{n^2}\right) = \begin{cases} -\frac{4}{\pi n^2} & \text{odd } n \\ 0 & \text{even } n \end{cases}$$

We find a_0 explicitly:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(0)\,dx = \frac{2}{\pi}\int_0^{\pi} x\,dx = \frac{1}{\pi}\left[x^2\right]_0^{\pi}$$

$$\underline{= \pi}$$

Since f(x) is symmetric we wont get sine contribution

$$\Rightarrow \underline{f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left(\cos(x) + \frac{\cos(3x)}{9} + \frac{\cos(5x)}{25} + \dots\right)}$$

2b) Same function $f(x) = \begin{cases} x & x \in [0, \pi] \\ -x & x \in [-\pi, 0] \end{cases}$

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, e^{-inx}\, dx$$

$$= \frac{1}{2\pi} \left(- \int_{-\pi}^{0} x\, e^{-inx}\, dx + \int_{0}^{\pi} x\, e^{inx}\, dx \right)$$

$$= \frac{1}{2\pi} \left(- \left\{ \left[\frac{x \cdot e^{-inx}}{-in} \right]_0^{\pi} - \int_0^{\pi} \frac{e^{-inx}}{-in}\, dx \right\} \right.$$

$$\left. + \left\{ \left[\frac{x\, e^{-inx}}{-in} \right]_0^{\pi} - \int_0^{\pi} \frac{e^{-inx}}{-in}\, dx \right\} \right)$$

$$\boxed{\begin{aligned} &\int f \cdot g' = f \cdot g - \int f' \cdot g \\ &f = x \qquad g' = e^{-inx} \\ &f' = 1 \qquad g = \frac{e^{-inx}}{-in} \end{aligned}}$$

$$= \frac{1}{2\pi} \left(\cancel{- \frac{\pi e^{-in\pi}}{-in}} + \frac{1}{n^2} \left(e^{-in\pi} - e^{in0} \right) \right.$$

$$\left. + \cancel{\frac{\pi e^{-in\pi}}{-in}} + \frac{1}{n^2} \left(e^{-in\pi} - e^{0} \right) \right)$$

$$= \frac{1}{2\pi n^2} \left(2e^{-in\pi} - 2 \right) = \frac{1}{\pi n^2} \left(e^{-in\pi} - 1 \right)$$

$$= \begin{cases} -\frac{2}{\pi n^2} & \text{odd } n \\ 0 & \text{even } n \end{cases}$$

differ by a factor of 2, due to negative n. The sine terms will cancel due to n^2.

$c_n \sin(nx) + c_{-n} \sin(-nx) = \sin(nx)(c_n - c_{-n}) = \sin(nx)\left(-\frac{2}{\pi n^2} - [illegible]\right)$

$= 0. \Rightarrow$ No sine-terms

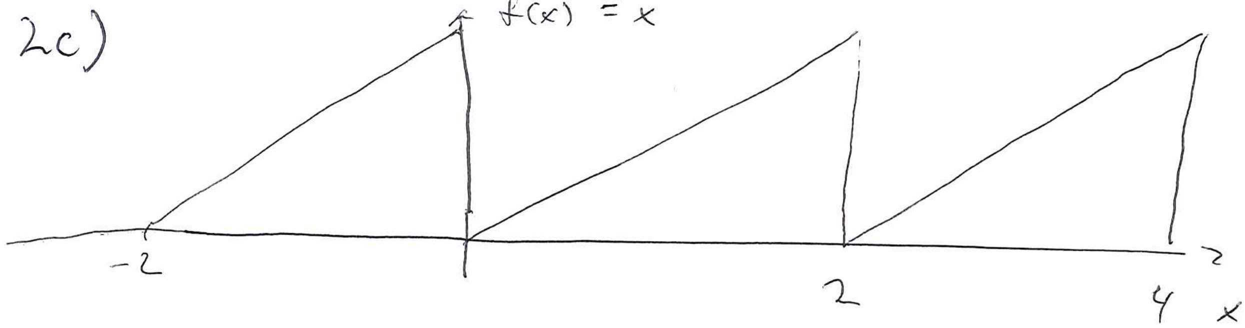
$c_n \cos(nx) + c_{-n} \cos(-nx) = \cos(nx)(c_n + c_{-n}) = \cos(nx)\left(-\frac{2}{\pi n^2} - \frac{2}{\pi n^2}\right)$

$= -\frac{4}{\pi n^2} \cos(nx)$ ✓ same as previous result.

$c_0 = a_0$ from previous task due to $c_0 = \int_{-\pi}^{\pi} f(x) = a_0$

$\Rightarrow \frac{a_0}{2} = \frac{\pi}{2} \Rightarrow$ Same series!

2c)



We want to find fourier coeffs on $[0,2]$
meaning $\ell = \frac{2}{2} = 1$

$$c_n = \frac{1}{2\ell}\int_0^{2\ell} f(x)\, e^{-in\pi x/\ell}\, dx = \frac{1}{2}\int_0^2 x\, e^{-in\pi x}\, dx$$

$$\boxed{\begin{aligned} &\int f' \cdot g = + f\cdot g - \int f \cdot g' \\ &f' = e^{-in\pi x}, \quad f = \frac{e^{-in\pi x}}{-in\pi} \\ &g = x, \quad g' = 1 \end{aligned}}$$

$$\Rightarrow 2c_n = \frac{x e^{-in\pi x}}{-in\pi}\Big|_0^2 - \int_0^2 \frac{e^{-in\pi x}}{-in\pi}\, dx$$

$$= \frac{2e^{-in2\pi}}{-in\pi} - \frac{1}{(-in\pi)^2}\left[e^{-in\pi x}\right]_0^2$$

$$= \frac{2}{-in\pi} + \frac{1}{n^2\pi^2}\left(e^{-in\pi 2} - e^{-in\pi\cdot 0}\right)$$

$$2c_n = \frac{2}{-in\pi} + \frac{1}{n^2\pi^2}(1-1) = \frac{2i}{n\pi}$$

$$\boxed{c_n = \frac{i}{n\pi}}$$

We have to calculate c_0 seperately: $c_0 = \frac{1}{2}\int_0^2 x\, dx = \frac{1}{2}\cdot\frac{1}{2}\left[x^2\right]_0^2$

$$= \frac{1}{4}\cdot 4 = 1$$

We see that c_n is anti-symmetric wrt n, meaning all cosine-terms will cancel, and sine terms will increase by a factor of two:

$$f(x) = c_0 + \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/\ell} = 1 + 2\sum_{n=1}^{\infty} \frac{i}{n\pi}\, i\sin(n\pi x)$$

$$\boxed{f(x) = 1 - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi x)}{n}}$$

[Husker ikke hvor dette kom [illegible]

We have from the previous task that

$$f(x) = x^2 = \sum_{n=0}^{\infty} \underbrace{\frac{\cos(n\pi)}{n^2\pi^2}}_{a_n} \cos(2n\pi x)$$

where $a_n = \begin{cases} \frac{1}{n^2\pi^2} & \text{even } n \\ -\frac{1}{n^2\pi^2} & \text{odd } n \end{cases}$ and $a_0 = \frac{1}{6}$

We insert this into our result:

$$\sum_{n=0}^{\infty} |a_n|^2 = \sum_{n=1}^{\infty} \frac{1}{n^4\pi^4} + |a_0|^2 = |a_0|^2 + \frac{1}{\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{1}{\pi}\int_{-1/2}^{1/2} x^4\,dx$$

$$\Rightarrow \sum_{n=1}^{\infty}\frac{1}{n^4} = -\pi^4|a_0|^2 + \pi^3\int_{-1/2}^{1/2} x^4\,dx$$

$$= -\pi^4 \cdot \frac{1}{12^2} + \pi^3\left[\frac{1}{5}x^5\right]_{-1/2}^{1/2}$$

$$= -\frac{\pi^4}{12^2} + \frac{\pi^3}{5}\cdot\left(\left(\frac{1}{2}\right)^5 - \left(-\frac{1}{2}\right)^5\right)$$

$$= -\frac{\pi^4}{12^2} + \frac{2\pi^3}{5}\cdot\left(\frac{1}{16}\right)$$

We have found $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(2n\pi x)$ [Husker ikke hvor dette hører hjemme]

We write this as and remember $\frac{1}{2}$ for a_0 $\longrightarrow$ $= \sum_{n=0}^{\infty} a_n \cos(2n\pi x)$

We square each side

$$f(x) \cdot f^*(x) = |f(x)|^2 = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_n a_m^* \cos(2n\pi x) \cos(2m\pi x)$$

We integrate both sides over one period, which in our case was $[-\frac{1}{2}, \frac{1}{2}]$

$$\int_{-\frac{1}{2}}^{1/2} |f(x)|^2 dx = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_n a_m^* \underbrace{\int_{-1/2}^{1/2} \cos(2n\pi x) \cos(2m\pi x)\, dx}$$

$$\int_{-1/2}^{1/2} |f(x)|^2 dx = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_n a_m^* \delta_{m,n}$$

$$\begin{cases} 0 & \text{if } m \neq n \\ 1 & \text{if } m = n \neq 0 \\ 2 & \text{if } m = n = 0 \end{cases}$$

this factor get's cancelled by $\frac{a_0}{2}$

$$\int_{-1/2}^{1/2} |f(x)|^2 dx = \sum_{n=0}^{\infty} |a_n|^2$$

$$\Rightarrow \sum_{n=0}^{\infty} |a_n|^2 = \int_{-1/2}^{1/2} |f(x)|^2 dx.$$

9.33

2d) $f(x) = x^2$ in $x \in [-\frac{1}{2}, \frac{1}{2}]$

Since $f(x) = f(-x)$ this is an even function. Therefore we know that the coefficients are

$$f(x) = \begin{cases} a_n = \frac{2}{\ell} \int_0^{\ell} f(x) \cos\left(\frac{n\pi x}{\ell}\right) dx \\ b_n = 0 \end{cases}$$

where $\ell = \frac{1}{2}$, let's calculate a_n

$$a_n = \frac{2}{1/2} \int_0^{1/2} x^2 \cos\left(\frac{n\pi x}{1/2}\right) dx$$

$$= 4 \int_0^{1/2} x^2 \cos(2n\pi x)\, dx$$

Solve this through integration by parts:

$$\boxed{\int f \cdot g' = f \cdot g - \int f' \cdot g}, \quad \boxed{f = x^2,\ f' = 2x \\ g' = \cos(2n\pi x),\ g = \frac{\sin(2n\pi x)}{2n\pi}}$$

$$\int_0^{1/2} x^2 \cos(2n\pi x)\, dx = x^2 \cdot \frac{\sin(2n\pi x)}{2n\pi}\bigg|_0^{1/2} - \int_0^{1/2} 2x \cdot \frac{\sin(2n\pi x)}{2n\pi}\, dx$$

$$= \underbrace{\frac{\frac{1}{4}\sin(n\pi)}{2n\pi}}_{0} - 0 - \frac{1}{n\pi} \int_0^{1/2} x \cdot \sin(2n\pi x)\, dx$$

$$\boxed{f = x,\ f' = 1 \\ g' = \sin(2n\pi x), \\ g = -\frac{\cos(2n\pi x)}{2n\pi}}$$

$$= -\frac{1}{n\pi}\left(\left[-x \cdot \frac{\cos(2n\pi x)}{2n\pi} \right]_0^{1/2} - \int_0^{1/2} -\frac{\cos(2n\pi x)}{2n\pi}\, dx \right) = -\frac{1}{n\pi}\left(-\frac{1}{2}\frac{\cos(n\pi)}{2n\pi} + \frac{1}{(2n\pi)^2}\Big[\sin(2n\pi x)\Big]_0^{1/2} \right) = \frac{\cos(n\pi)}{4n^2\pi^2} + 0$$

$$\Rightarrow \underline{a_n = \frac{\cos(n\pi)}{n^2\pi^2}} \Rightarrow a_n = \begin{cases} \frac{1}{n^2\pi^2} & \text{even } n \\ -\frac{1}{n^2\pi^2} & \text{odd } n \end{cases}$$

We also need to calculate the average value :

$$a_0 = 4 \int_0^{1/2} x^2 \, dx = 4 \cdot \frac{1}{3} \cdot \left[x^3 \right]_0^{1/2}$$

$$= \frac{4}{3} \cdot \frac{1}{8} = \frac{1}{6}$$

Making our series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{1/2}\right)$$

$$= \frac{1}{12} + \frac{1}{\pi^2}\left(-\cos(2\pi x) + \frac{1}{4}\cos(4\pi x) - \frac{1}{9}\cos(6\pi x) + \dots\right.$$

2e) We want to solve $\sum_{n=1} \frac{1}{n^4}$ using Parseval's theorem, which states that

$$\sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{1}{2L}\int_{-L}^{L} |f(x)|^2\,dx$$

In problem 9.9 we found the coefficients for $f(x)=x^2$ in $x\,[-\frac{1}{2},\frac{1}{2}]$, where we found $a_n = \frac{\cos(n\pi)}{n^2\pi^2}$ and $b_n = 0$ and $a_0 = \frac{1}{12}$, since $\cos(n\pi) = \pm 1$ we get

$$|a_n|^2 = \left(\frac{1}{n^2\pi^2}\right)^2 = \frac{1}{\pi^4 n^4}$$

$$\sum_{n=0}^{\infty} |a_n|^2 = |a_0|^2 + \sum_{n=1}^{\infty} \frac{1}{\pi^4 n^4} = \left|\frac{1}{12}\right|^2 + \frac{1}{\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^4}$$

which has to be equal to

$$\frac{1}{2L}\int_{-L}^{L} x^4\,dx = \frac{1}{2\cdot\frac{1}{2}}\int_{-\frac{1}{2}}^{\frac{1}{2}} x^4\,dx = \frac{1}{5}\left(\left(\frac{1}{2}\right)^5 - \left(-\frac{1}{2}\right)^5\right)$$

$$= \frac{2}{5}\left(\frac{1}{2}\right)^5 = \frac{1}{5}\cdot\left(\frac{1}{2}\right)^4 = \frac{1}{16\cdot 5} = \frac{1}{80}$$

$$\frac{1}{80} = \frac{1}{144} + \frac{1}{\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^4}$$

$$\sum_{n=1}^{\infty}\frac{1}{n^4} = \pi^4\left(\frac{1}{80} - \frac{1}{144}\right) = \frac{\pi^4}{180}$$

This is wrong by a factor of 2, I am not sure why... I tried to solve this using a_n not c_n, and therefore have a sum from 0 to ∞ instead of $-\infty$ to ∞, maybe thats the reason.