$\Delta \quad \nabla$

$\nabla^2 \qquad f(t,x,y)$

$$\frac{df}{dt} = c^2\left(\frac{d^2f}{dx^2} + \frac{d^2f}{dy^2}\right)$$

$$f_t = c^2\nabla^2 f$$

domain: $0 < x < a \qquad t > 0$

$0 < y < b$

b. condition:

$f(0,y,t) = 0 = f(a,y,t)$

$f(x,0,t) = f(x,b,t) = 0$

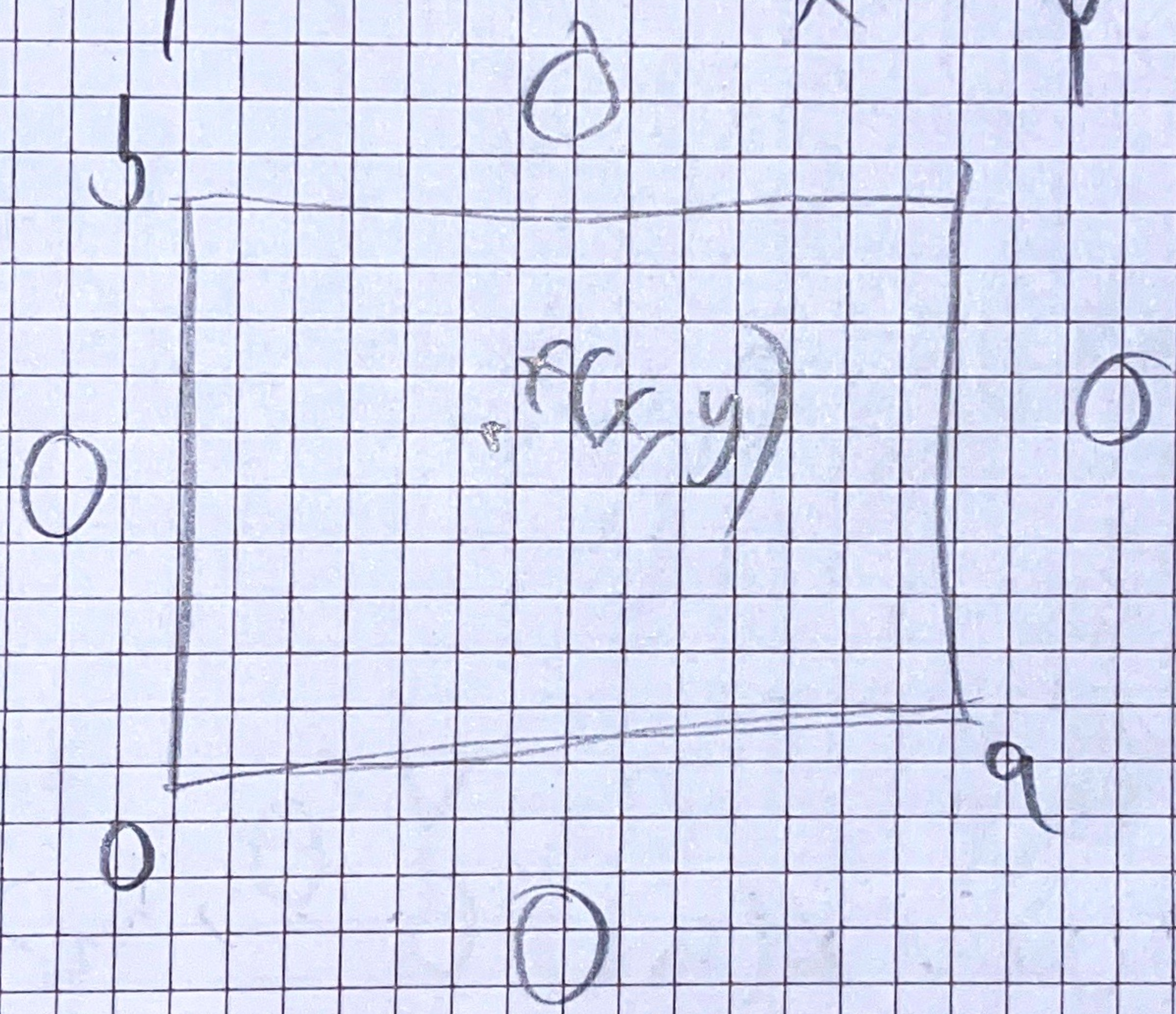
other:

$f(x,y,0) = f_2(x,y)$

Seperation:

$$T'XY = c^2(TX''Y + TXY'')$$

$$\frac{T'}{T} = c^2\left(\frac{X''}{X} + \frac{Y''}{Y}\right) = k_1^2$$



over tid blir alt 0 gitt initialbetingelsene.

$$\frac{T'}{c^2T} = k_1^2 \qquad \frac{X''}{X} = k_1^2 - \frac{Y''}{Y}$$

$$\frac{X''}{X} = k_2^2 = k_1^2 - \frac{Y''}{Y}$$

$$X'' - k_2^2 X = 0 \qquad X(0) = 0$$
$$X(a) = 0$$

$$X'' e^{\pm k_2 x} = k_2^2 X e^{\pm k_2 x}$$
$$X = C_1 e^{k_2 x} + C_2 e^{-k_2 x}$$

For tidlig $Y'' = (-k_2^2 + k_1^2) Y \qquad Y(0) = 0$

$Y(b) = 0$

$$C_3 e^{(ik_2 + k_1)y} + C_4 e^{-(ik_2 + k_1)y}$$

$$X(0) = X(a) = 0 \qquad k_2 > 0$$
$\Downarrow \qquad X(x) = 0$

$$\cancel{C_1 + C_2 = 0} \qquad k_2 = 0$$
$X(x) = 0$

$$X = C_1 \cos(ik_2 x) + C_2 \sin(ik_2 x) \quad k_2 < 0$$

fra $X(0)=0$ $X(0) = C_1 = 0 \qquad k_2 \in \mathbb{C} \quad k_2 = i$

$X(a) = 0 \qquad k_2 a = \pi n \qquad k_2 = \frac{\pi n}{a} \qquad k_2^2 = -\left(\frac{\pi n}{a}\right)$

$$Y_{\mathcal{Y}}'' + (k_2^2 - k_1^2) Y = 0$$

$$k_2^2 = -\left(\frac{\pi n}{a}\right)^2$$

$$Y'' - \left(\left(\frac{\pi n}{a}\right)^2 + (k_1)^2\right) Y = 0$$

Fra X ODE ser vi at

$$-\theta^2 = \left(\left(\frac{\pi n}{a}\right)^2 + k_1^2\right) < 0 \qquad \theta \in \mathbb{C}$$

$Y'' + \theta^2 Y = 0$ gjør som for X

$$\theta = \frac{m\pi}{b} \quad \text{og} \quad Y_m(y) = \sin(\theta y)$$

$$= C_4 \sin\left(\frac{m\pi}{b} y\right)$$

$$-\theta^2 - \left(\frac{\pi n}{a}\right)^2 = k_1^2$$

$$-\left(\left(\frac{m\pi}{b}\right)^2 + \left(\frac{\pi n}{a}\right)^2\right) = k_1^2$$

$$k_2^2 = -\left(\frac{n\pi}{a}\right)^2$$

$$k_1^2 = -\left(\left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{a}\right)^2\right)$$

T, då:

$$T' - k_1^2 c^2 T = 0$$

ODE

$$\to T_{m,n}(t) = C_1 e^{k_1^2 c^2 t}$$

$$T_{m,n}(t) = C_1 e^{\left(-\left(\left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{a}\right)^2\right) c^2 t\right)}$$

~~ab~~ ~~T~~ när $t \to \infty$

$$T \Rightarrow 0$$

Sammensetting

$$f_{n,m}(x,y,t) = X_n Y_m T_{m,n}$$

kombinerer C_n-er

jeg orker ikke skrive

$$= C \sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{m\pi}{b}y\right)\cdot T$$

normaliser

Summer løsninger:

$$f(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} f_{n,m}(x,y,t)\cdot C_{n,m}$$

$$f_{n,m}(x,y,0) = f_{2_{n,m}}(x,y)$$

ortogonal funksjon til f_2

$$\phi_{n,m}(x,y) = \sum\sum \sin\left(\frac{n'\pi x}{a}\right)\sin\left(\frac{m'\pi}{b}\right)$$

$$\iint \quad = 0 : n \neq n' \quad m \neq m'$$

$$\frac{ab}{4} : (n = n'), \quad m = m'$$

Ser på en periode

$$a_{n,m} = \frac{4}{ab}\int_0^a\int_0^b f_2(x,y)\,\phi_{n,m}(x,y)\,dy\,dx$$

$$f_{n,m}(x,y,t)$$

$$= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\frac{4}{ab}\int_0^a\int_0^b f_2(x,y)_{n,m}^2\,dy\,dx\right)\cdot\sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{m\pi}{b}y\right)$$

$$\cdot\, e^{-\left(\left(\frac{n\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2\right)c^2t}$$