The Monte Carlo process

MONTE CARLO SIMULATIONS IN PYTHON

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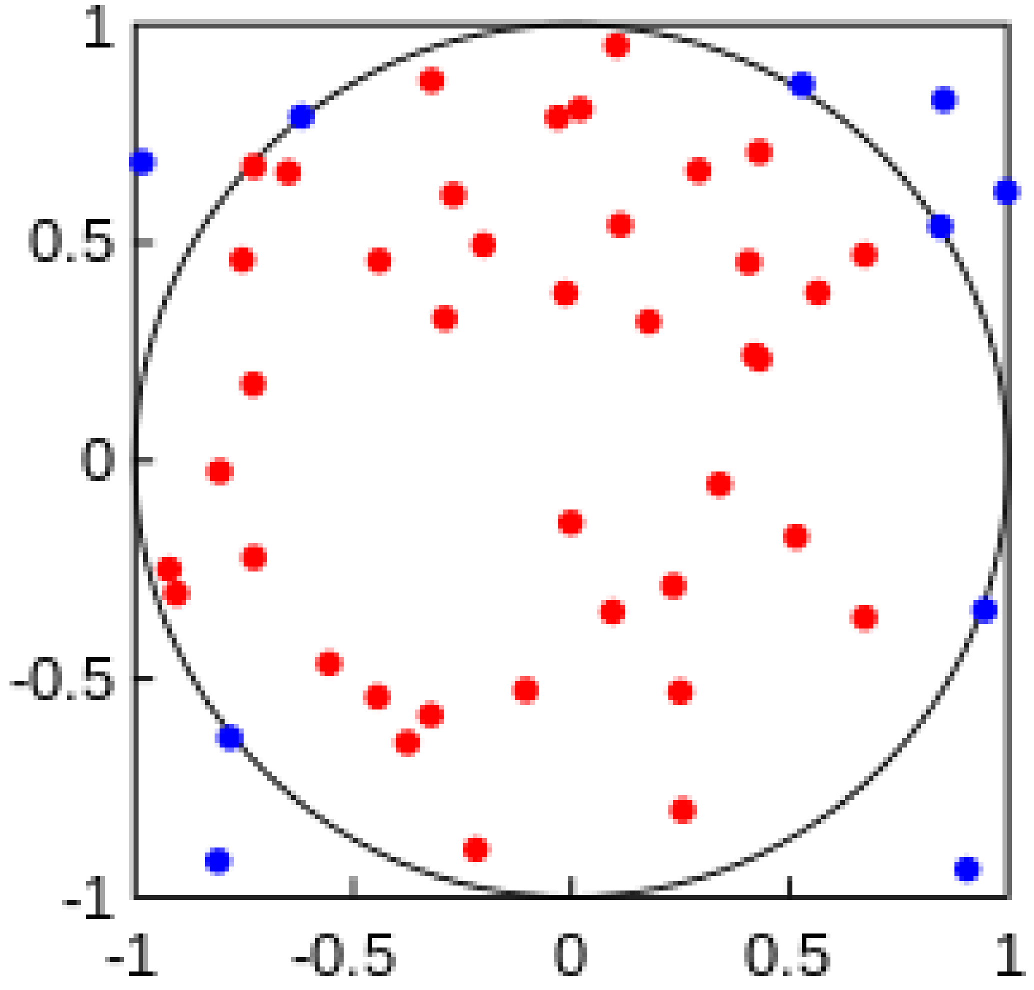
Simulation steps

1. Define the input variables and pick probability distributions for them
2. Generate inputs by sampling from these distributions
3. Perform a deterministic calculation of the simulated inputs
4. Summarize results

Calculating the value of pi

Generate random points (x, y) *where* x *and* y *are in the interval from -1 to 1.*



$$Area_{circle} = \pi$$

$$Area_{square} = 2 \times 2 = 4$$

$$\frac{Area_{circle}}{Area_{square}} = \frac{\pi}{4}$$

$$\frac{n_{red}}{n_{all}} = \frac{\pi}{4}$$

$$\pi = 4 \times \frac{n_{red}}{n_{all}}$$

Step 1

Define the input variables and pick probability distributions for them

- **Inputs:** the individual points represented by (x, y) coordinates
- **Probability distributions:** x and y follow uniform distributions from negative one to one.

```
circle_points = 0
square_points = 0
```

Step 2

Generate inputs by sampling from these distributions

Sample random x and y coordinate values distributed uniformly between -1 and 1:

```
for i in range(n):
    x = random.uniform(-1, 1)
    y = random.uniform(-1, 1)
```

Step 3

Perform deterministic calculation of the simulated inputs

Check whether each point lies within the circle: deterministic for given x and y

```
dist_from_origin = x**2 + y**2
```

If yes, add the point to `circle_points` ; always add the point to `square_points`

```
if dist_from_origin <= 1:
    circle_points += 1
square_points += 1
```

Step 4

Summarize the results to answer questions of interest

After many rounds of simulations, calculate the value of pi!

```
pi = 4 * circle_points/ square_points
```

All together now

```
n = 4000000
circle_points = 0
square_points = 0
for i in range(n):
    x = random.uniform(-1, 1)
    y = random.uniform(-1, 1)
    dist_from_origin = x**2 + y**2
    if dist_from_origin <= 1:
        circle_points += 1
    square_point += 1
pi = 4 * circle_points / square_points
print(pi)
```

```
3.142518
```

Let's practice!

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Generating discrete random variables

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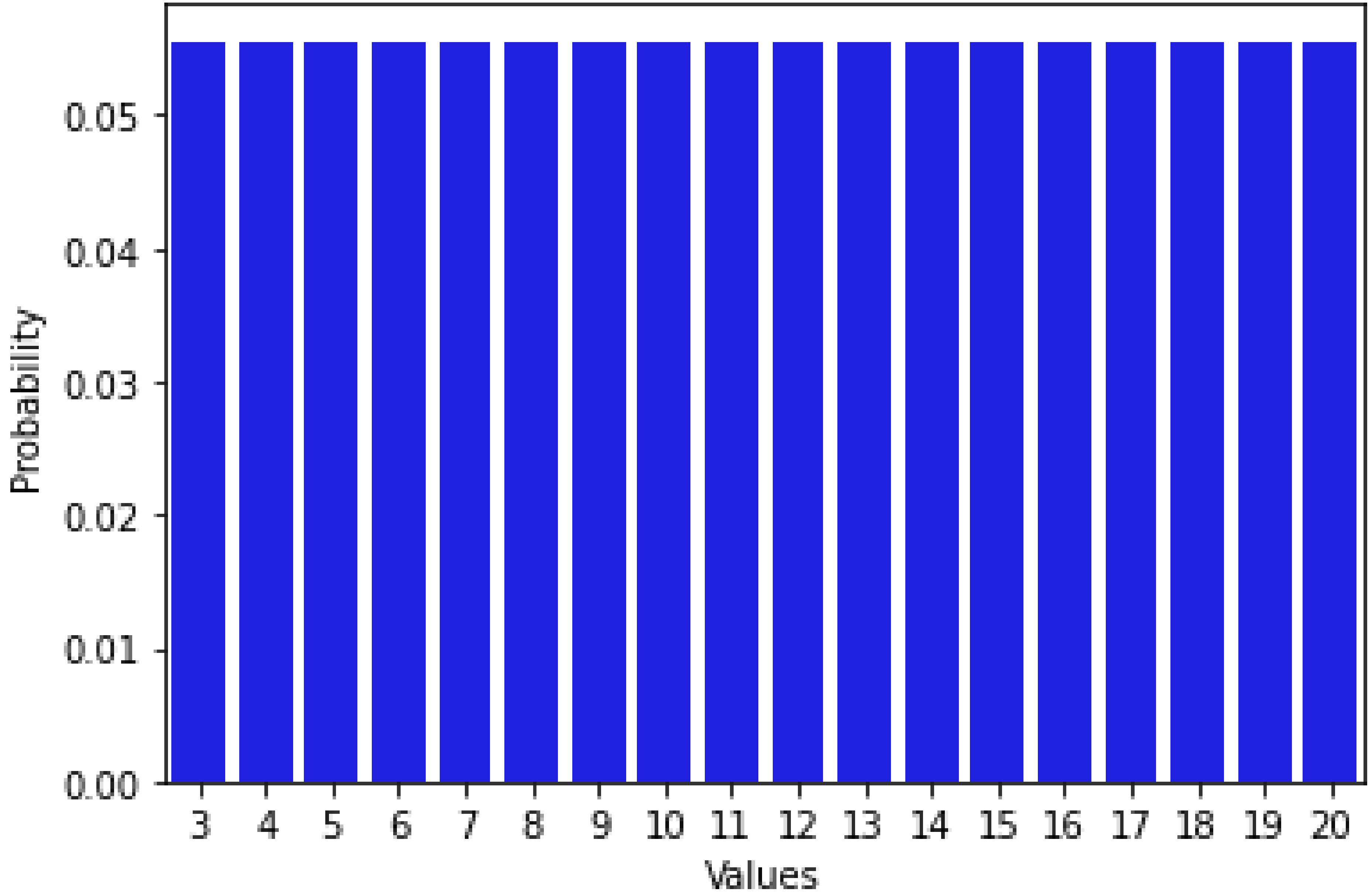
Required imports

```
import scipy.stats as st
import seaborn as sns
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
```

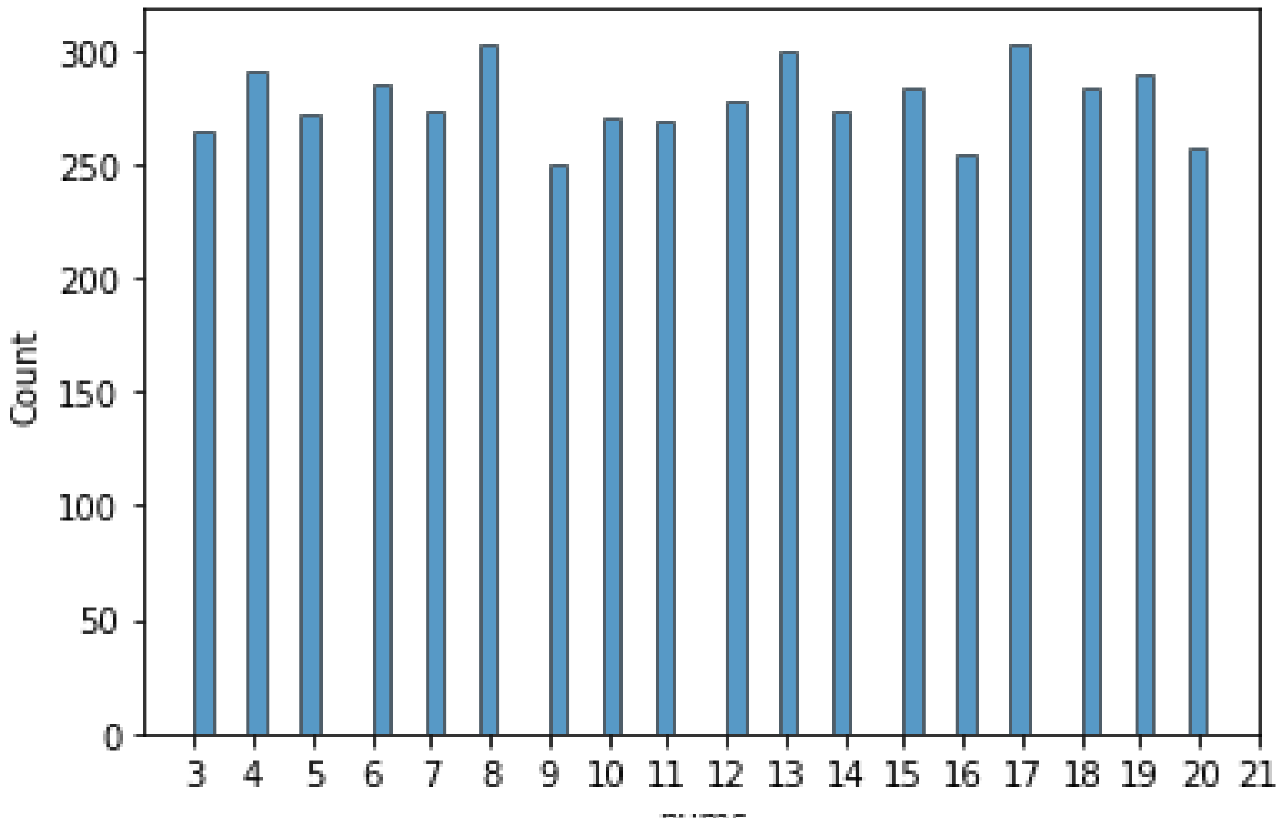
Discrete uniform distribution

Theoretical probability mass function (PMF):



Sampling from the discrete uniform distribution

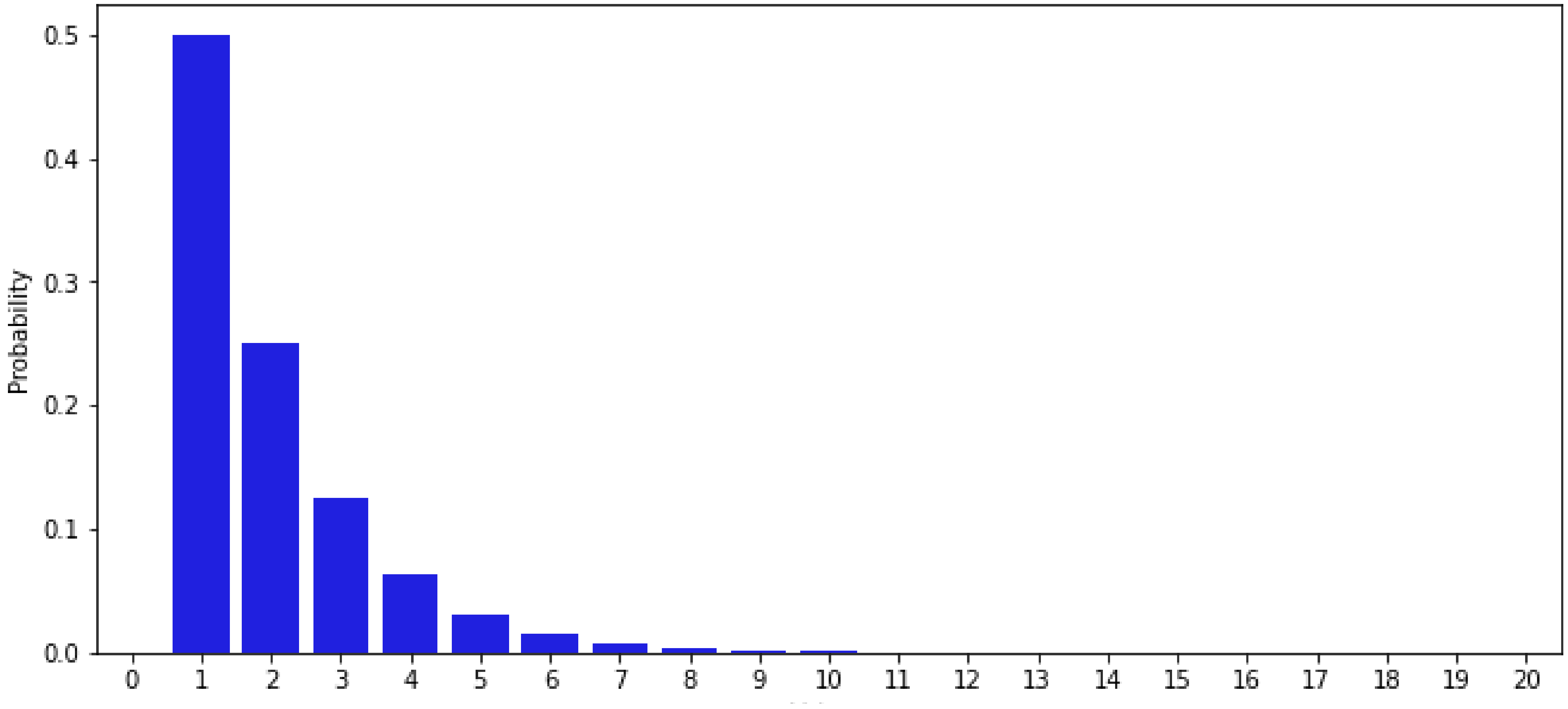
```
low = 3
high = 21
samples = st.randint.rvs(low, high, size=1000)
samples_dict = {"nums":samples}
sns.histplot(x="nums", data=samples_dict, bins=6, binwidth=0.3)
```



Geometric distribution

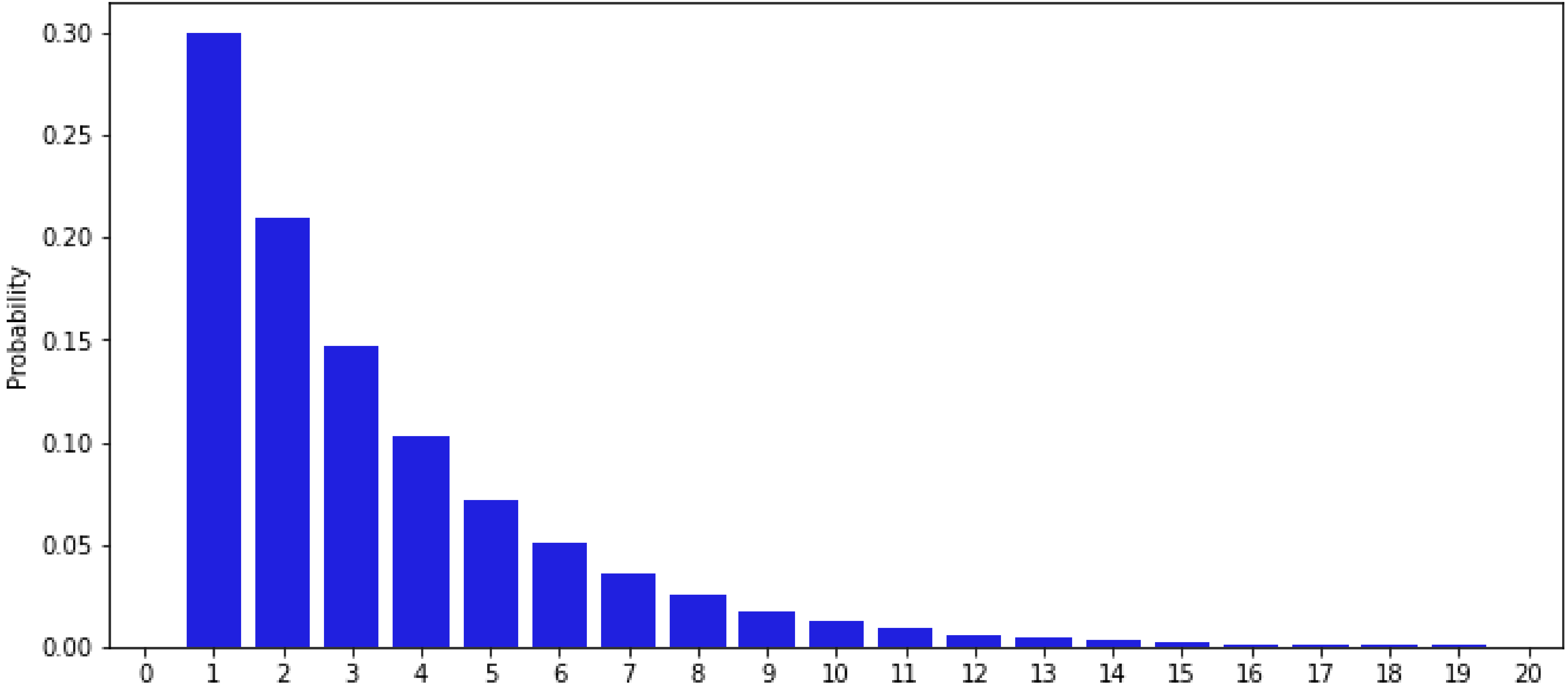
The probability distribution of the number of trials, X, needed to get one success, given the success probability, p.

Probability Mass Function, `p = 0.5`



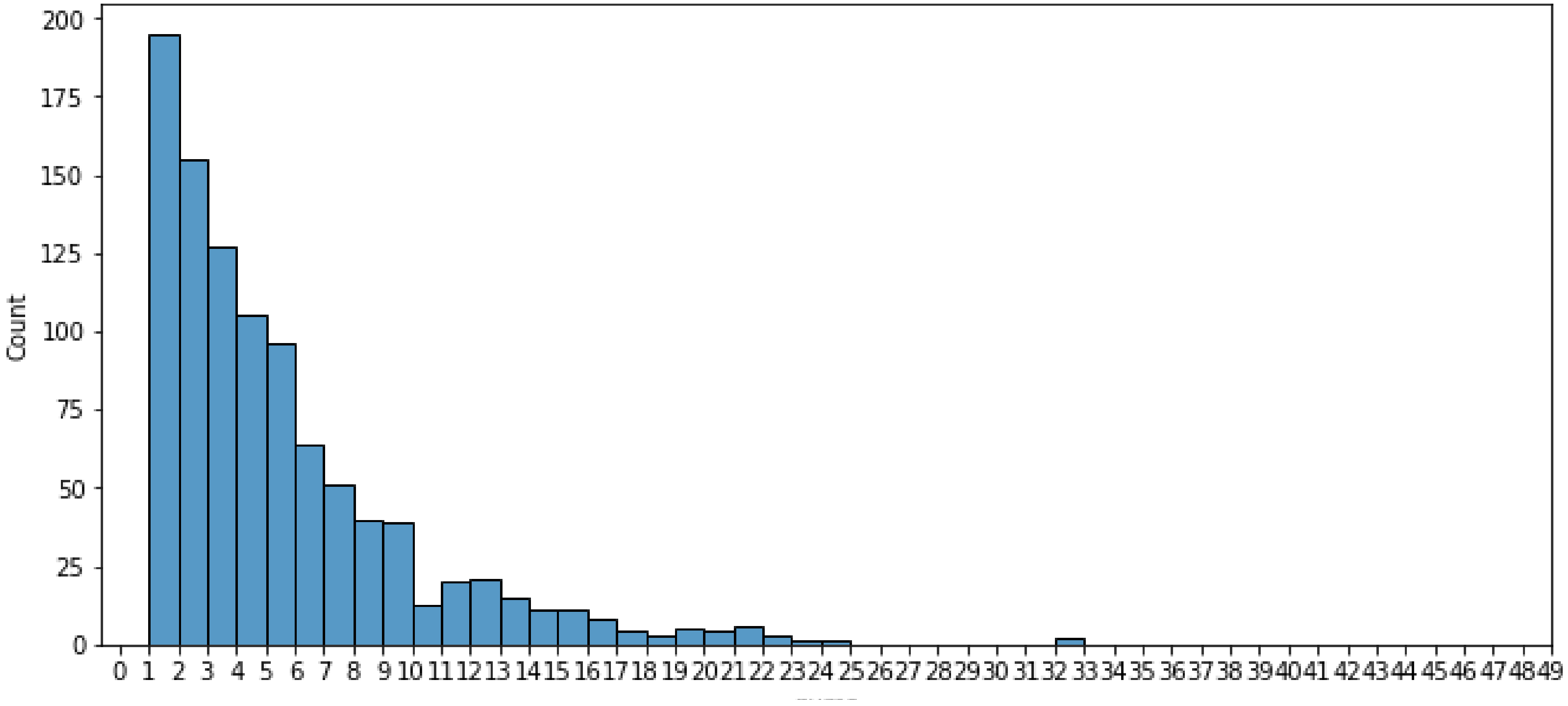
Geometric distribution

Probability Mass Function, `p = 0.3`



Sampling from geometric distribution

```
p = 0.2
samples = st.geom.rvs(p, size=1000)
samples_dict = {"nums":samples}
sns.histplot(x="nums", data=samples_dict)
```



More discrete probability distributions

- Poisson (`scipy.stats.poisson`)
  - Expresses the probability of a given number of events occurring in a fixed interval of time or space
- Binomial (`scipy.stats.binom`)
  - Expresses probability of the number of successes in a sequence of n independent experiments.
- And more!

[1] https://docs.scipy.org/doc/scipy/reference/stats.html#discrete-distributions

Let's practice!

MONTE CARLO SIMULATIONS IN PYTHON

Generating continuous random variables

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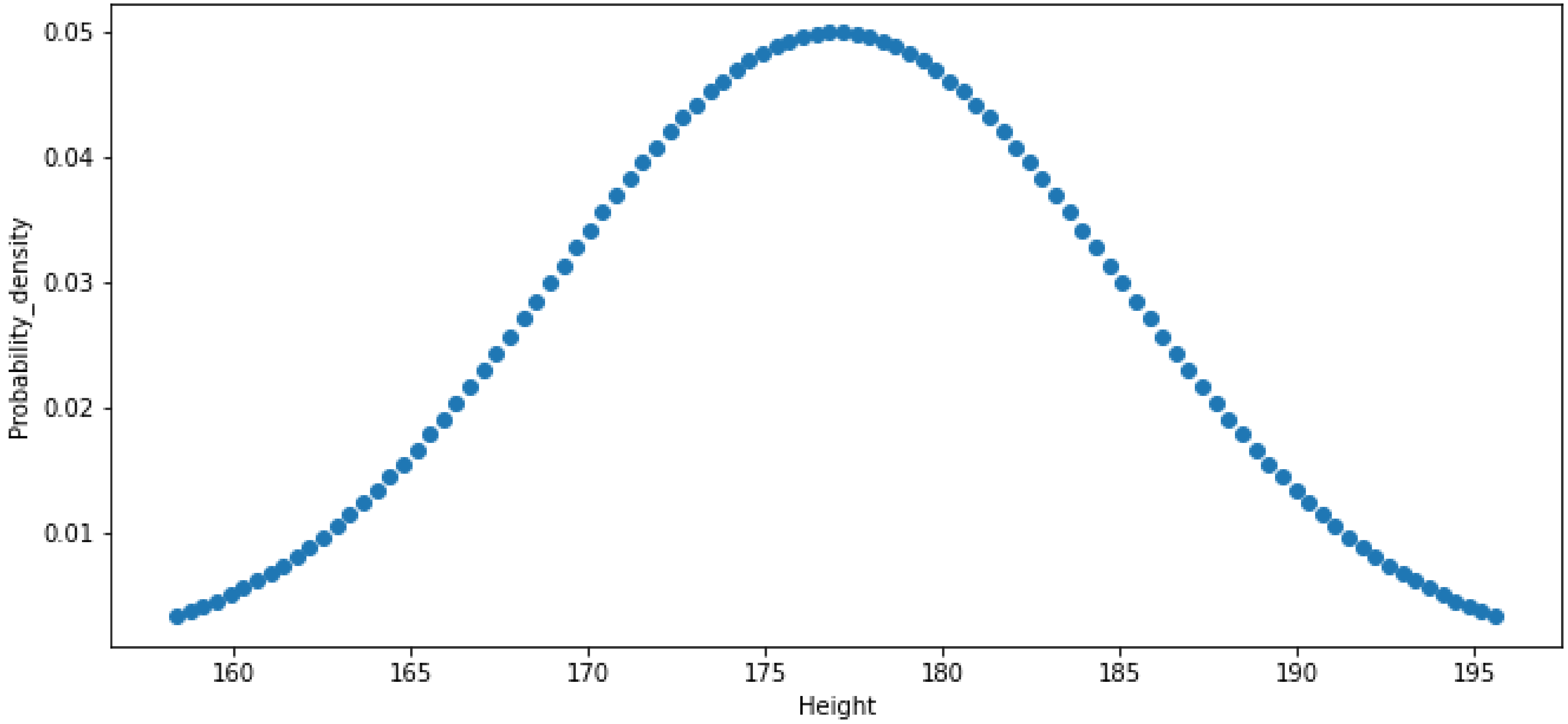
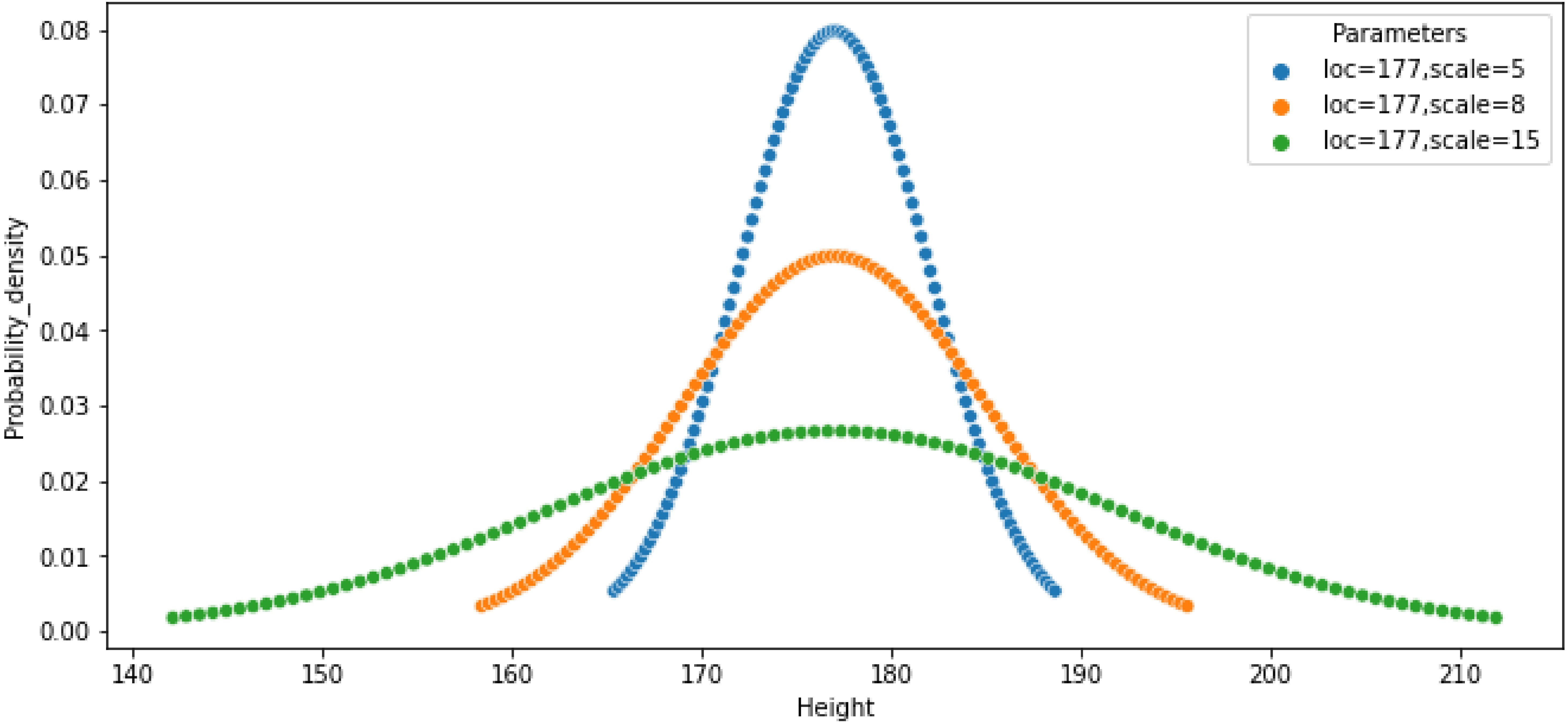
Normal distribution

Bell-shaped and centered at the mean (or loc); width defined by standard deviation (or scale)

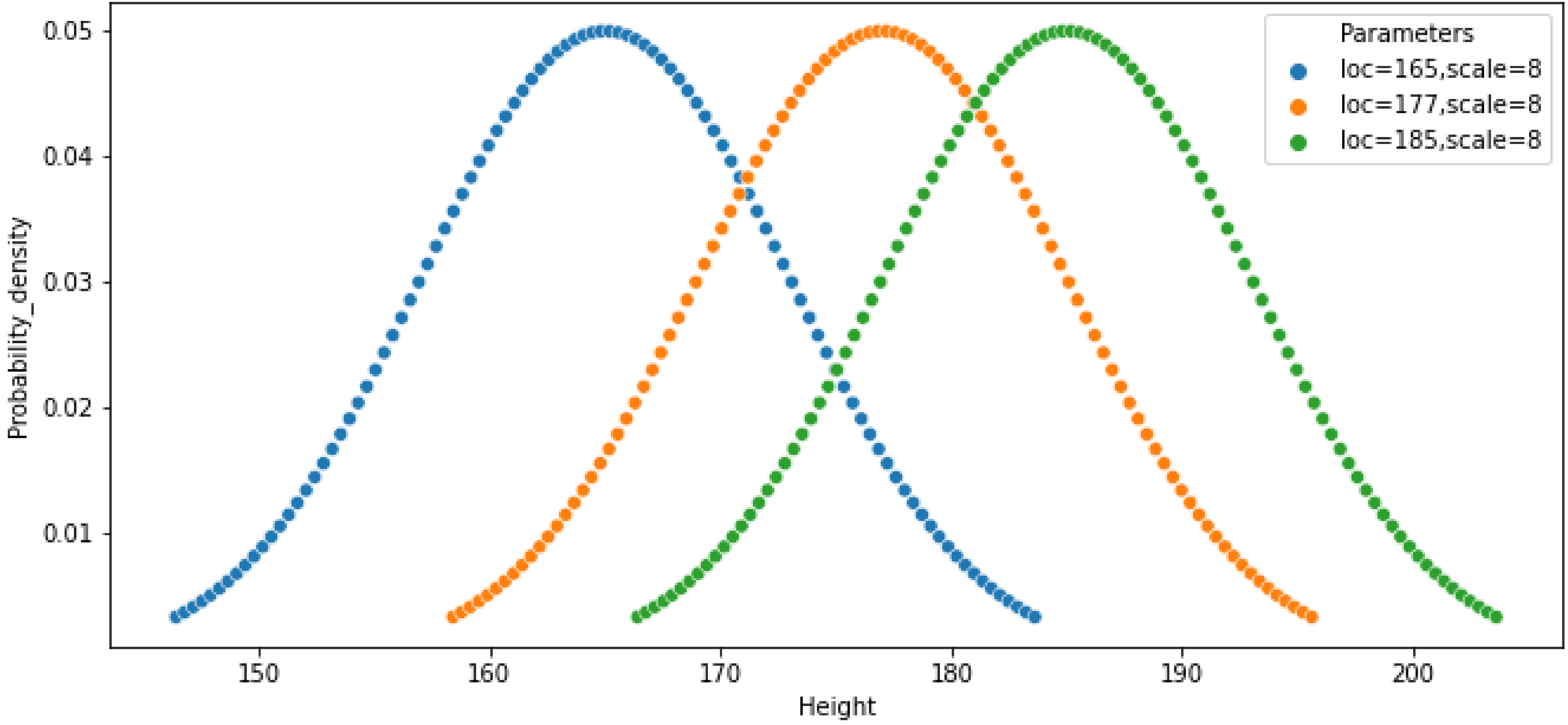
The heights of American adult males are normally distributed:



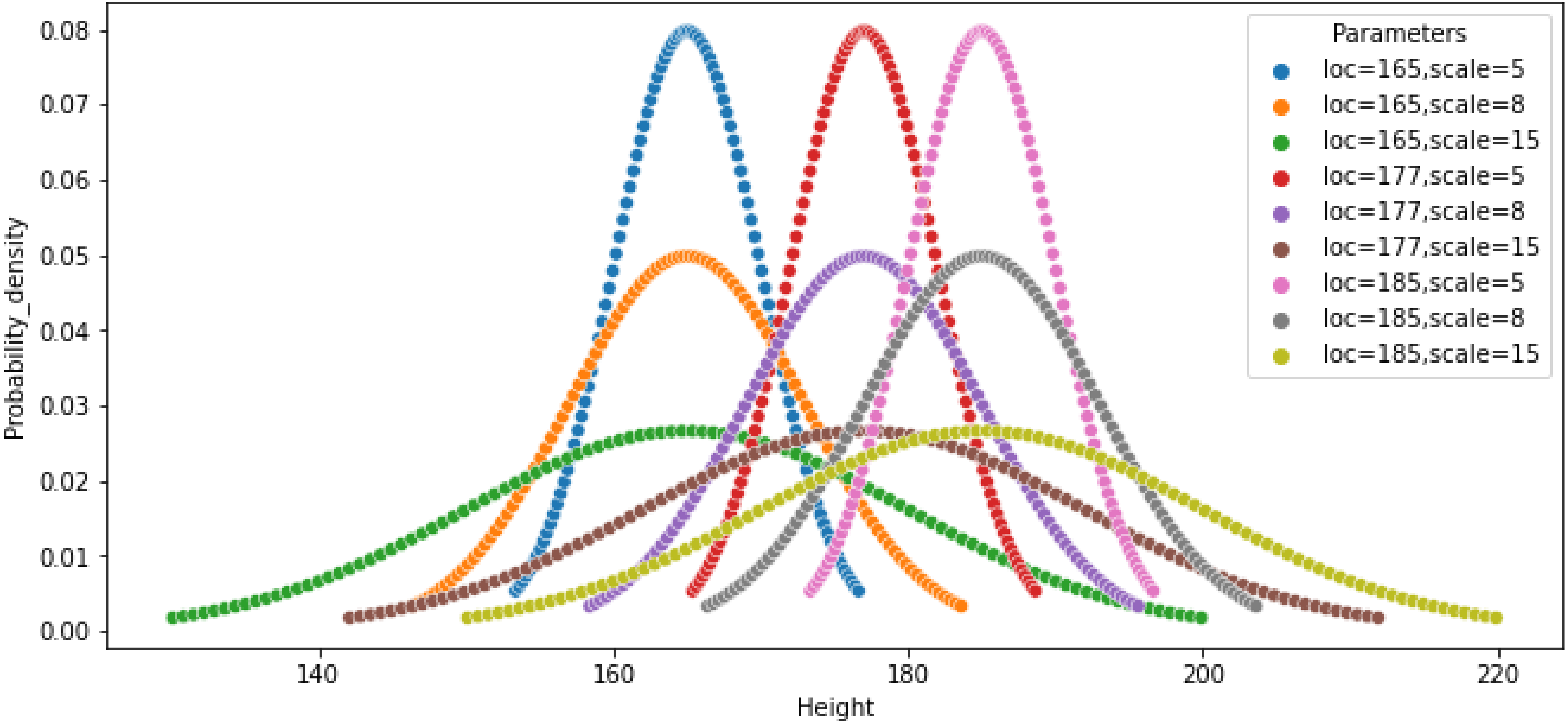
Changing the scale (standard deviation)



Changing the loc (mean)



Changing both scale and loc



Sampling from normal distributions

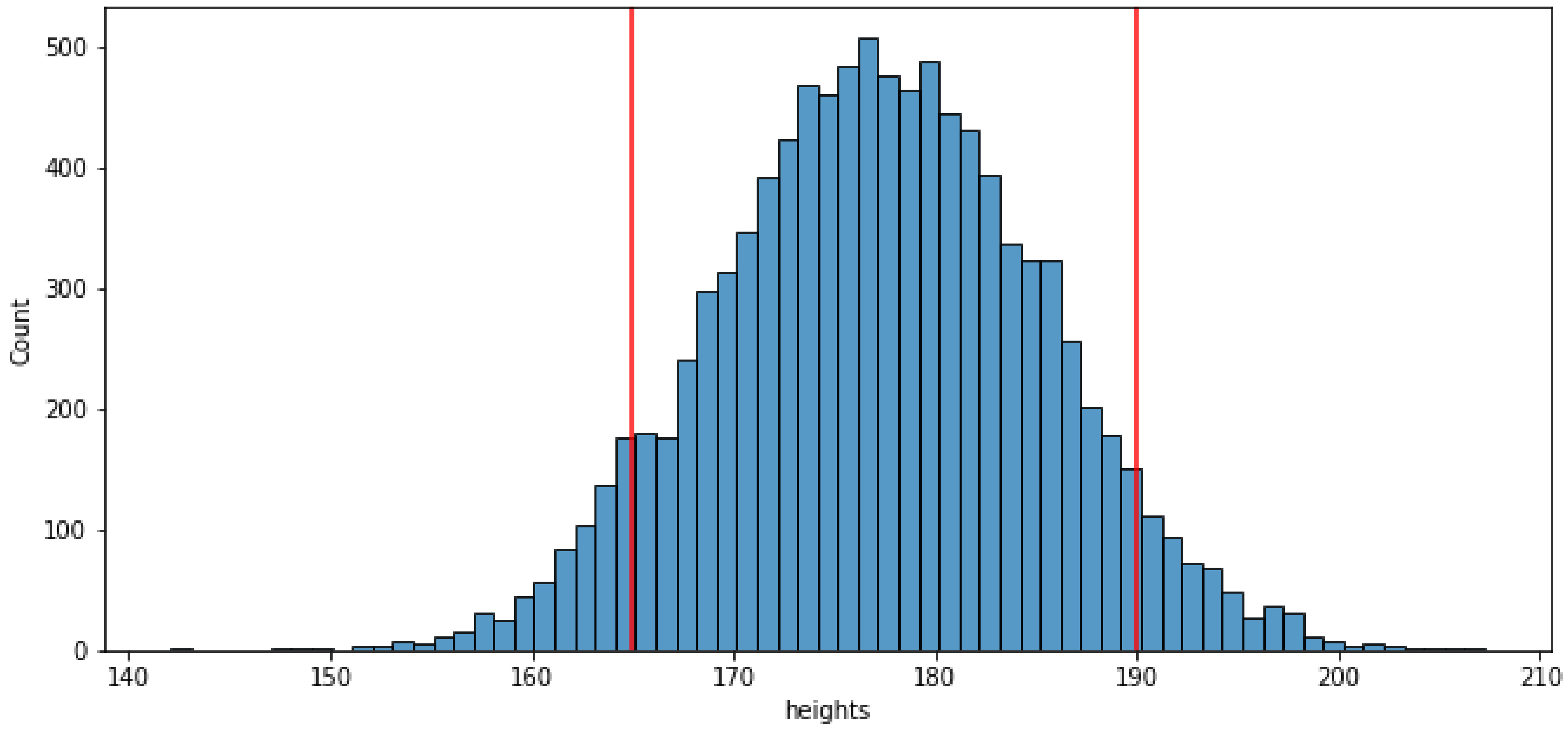
- Normally distributed US adult male heights; mean = 177 cm; standard deviation = 8 cm
- What's the percentage of people with a height either above 190 or below 165 cm?

```
heights = st.norm.rvs(loc=177, scale=8, size=10000)
qualified = (heights < 165) | (heights > 190)
print(np.sum(qualified) * 100/10000)
```

```
12.28
```

Plotting simulation results

```
heights_dict = {"heights":heights}
sns.histplot(x="heights", data=heights_dict)
plt.axvline(x=165, color="red")
plt.axvline(x=190, color="red")
```



More continuous probability distributions

- Continuous Uniform distribution (`st.uniform`)
  - The continuous analog of the discrete uniform distribution
- Exponential distribution (`st.expon`)
  - The continuous analog of the geometric distribution

[1] https://docs.scipy.org/doc/scipy/tutorial/stats/continuous.html

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Generating multivariate random variables

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Sampling from multivariate distributions

Multinomial distribution

- Variables each follow binomial distribution
- Probabilities of these variables sum to one

Example: simulating the results of flipping a biased coin

```
scipy.stats.multinomial.rvs()
```

Sampling from multivariate distributions

Multivariate normal distribution

- Variables each follow normal distribution
- Variables can be correlated with each other or not

Example: simulating price and demand

```
scipy.stats.multivariate_normal.rvs()
```

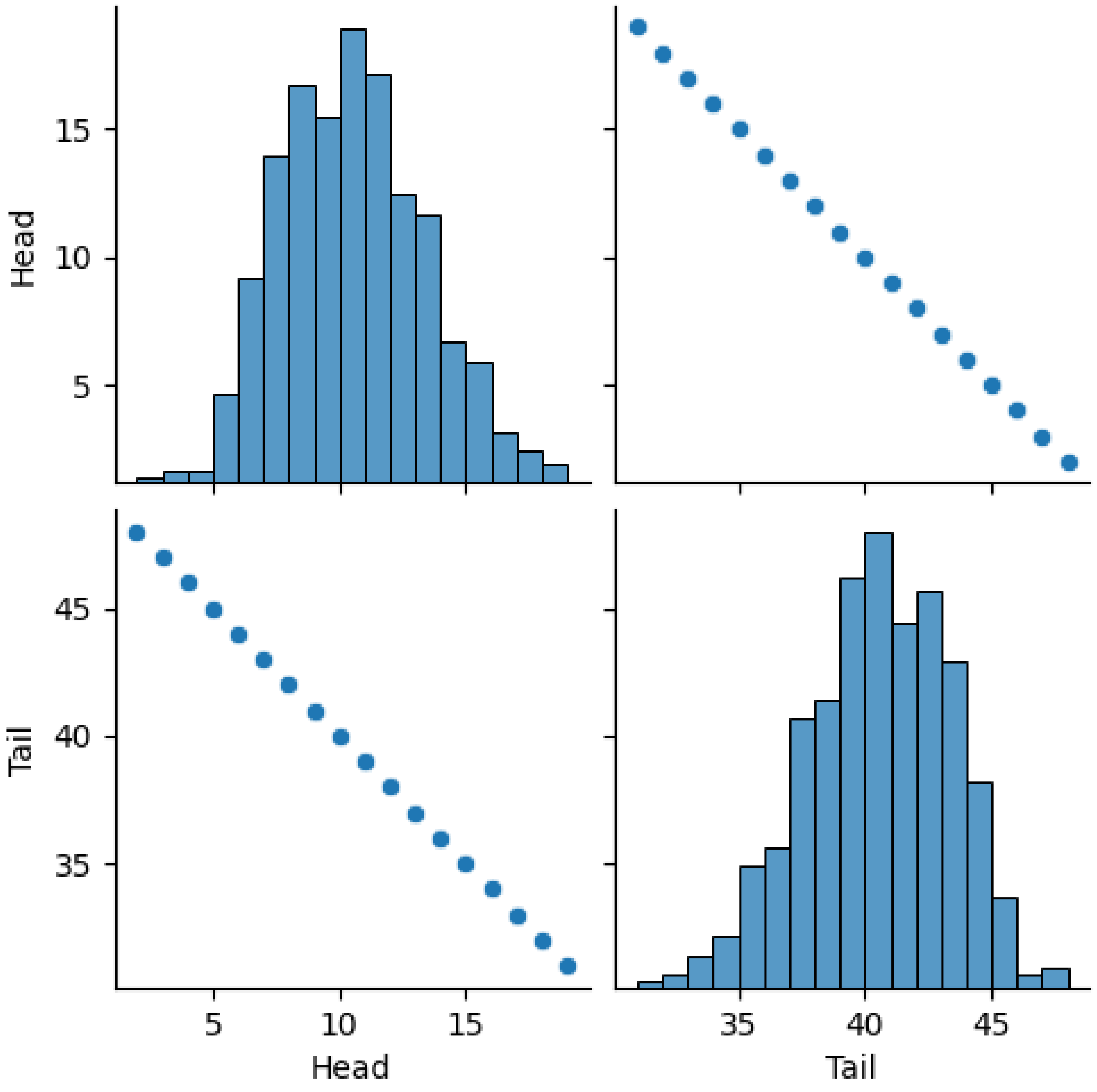
Sampling from multinomial distributions

Simulation: `.rvs(n, p, size)`

- n: `50`
- p: `[0.2, 0.8]`
- size: `500`

```
results = st.multinomial.rvs(50,
    [0.2, 0.8], size=500)

df_results=pd.DataFrame(
    {"Head":results[:, 0],
     "Tail":results[:, 1]})
sns.pairplot(df_results)
```



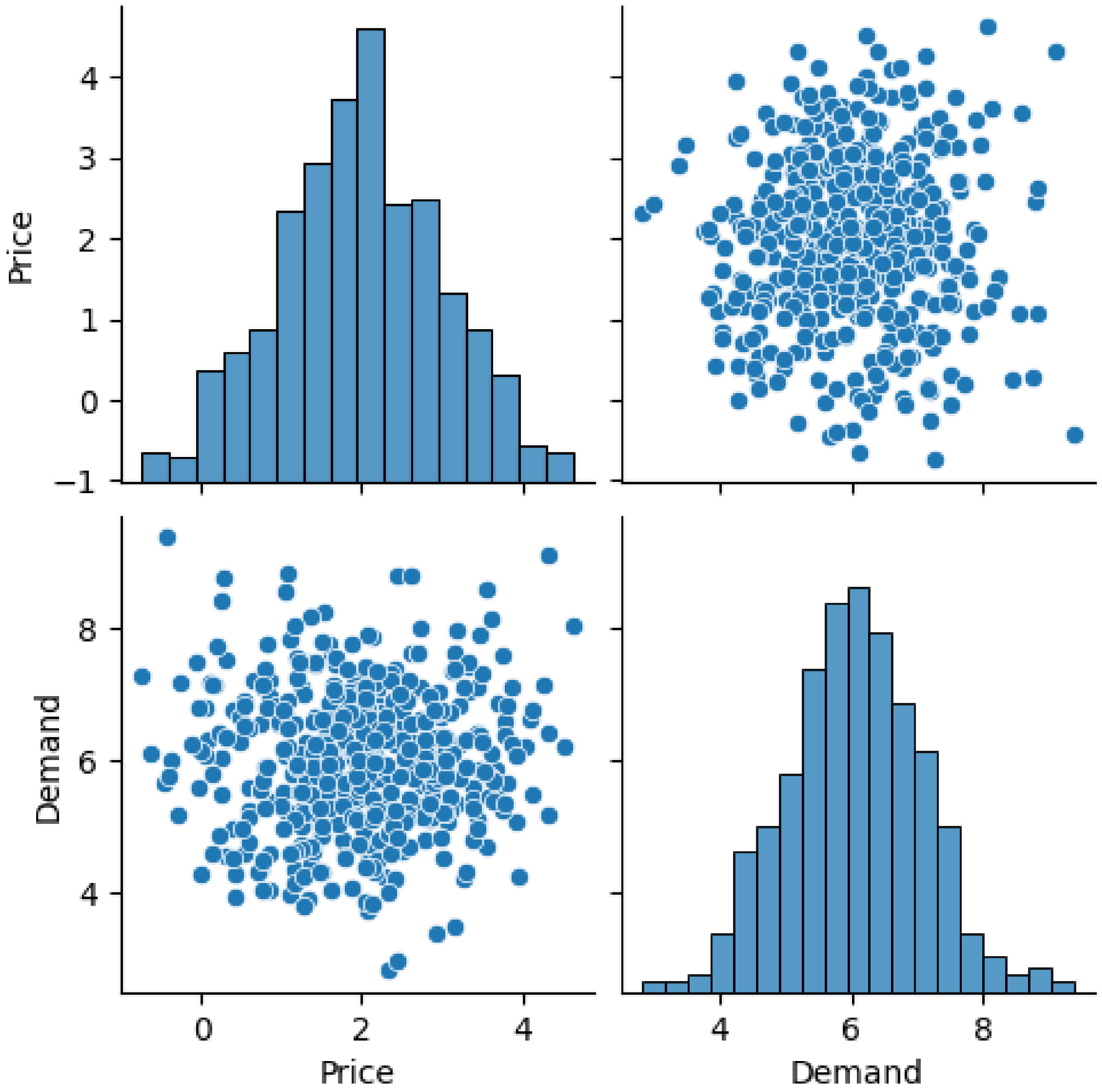
Sampling from multivariate normal distributions

Simulation: `.rvs(mean, size)`

- mean: `[2, 6]`
- size: `500`

```
results=st.multivariate_normal.rvs(
    mean=[2, 6], size=500)

df_results=pd.DataFrame(
    {"Price":results[:, 0],
     "Demand":results[:, 1]})
sns.pairplot(df_results)
```



Covariance matrix

- Captures the variance and covariances of variables
- Definition using two random variables x and y:

$$
\begin{array}{ccc}
 & x & y \\
x & \multicolumn{2}{c}{\multirow{2}{*}{$\begin{bmatrix} var(x) & cov(x,y) \\ cov(x,y) & var(y) \end{bmatrix}$}} \\
y & &
\end{array}
$$

Example:

```
df_historical.cov()
```

```
|        | Price     | Demand   |
|--------|-----------|----------|
| Price  | 0.920545  | -0.85578 |
| Demand | -0.855780 | 0.98417  |
```

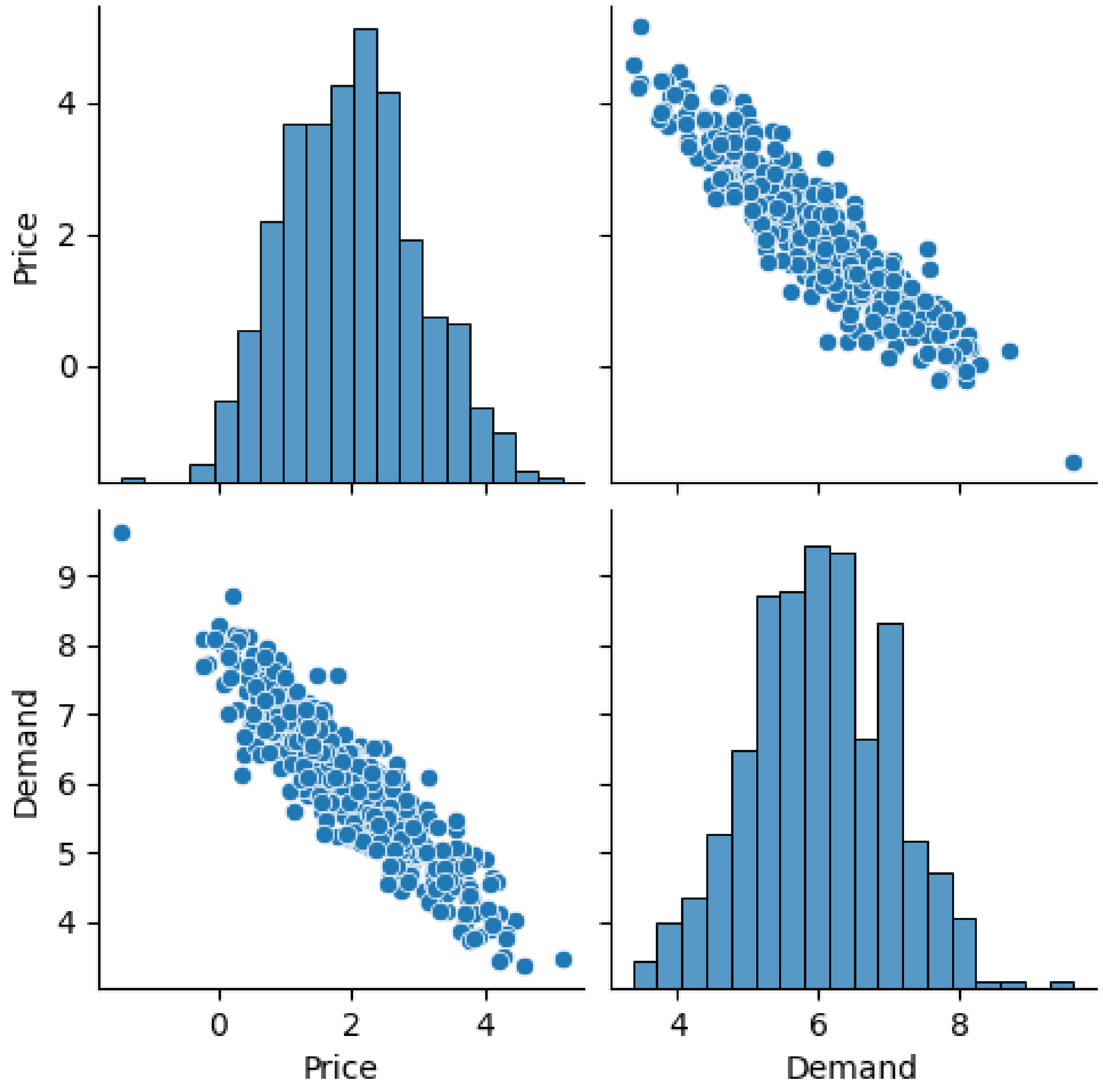
Multivariate normal sampling with defined covariance

Simulation: `.rvs(mean, size)`

- mean: `[2, 6]`
- size: `500`
- cov: `np.array([[1, -0.9], [-0.9, 1]])`

```
cov_mat = np.array([[1,-0.9], [-0.9,1]])
results = st.multivariate_normal.rvs(
    mean=[2,6], size=500, cov=cov_mat)

df_results = pd.DataFrame(
    {"Price":results[:,0],
     "Demand":results[:,1]})
sns.pairplot(df_results)
```



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