

# ECE 552: Computational Techniques for Circuit Analysis - Fall 2010

## Project Report

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## 1 Introduction

### 1.1 Project description

In this project, a circuit simulator program is implemented. It can read SPICE like input netlist and perform three types of analysis, namely I) DC analysis, II) AC analysis and III) Transient analysis.

### 1.2 Description of Algorithms

The program accepts an input netlist file and outputs node voltages and branch currents. The information of nets, nodes and initial guess values will be read from the input file accordingly. Moreover, control parameters such as beginning, ending frequency and step size of AC analysis, finishing time and time step of transient analysis and the name of node that needs to be plot will be saved. The program determines which kind of analysis to be performed according to the input, and perform the analysis. Following are the details for three types of analysis.

#### 1.2.1 DC Analysis

Among the three types of analysis, DC analysis is the most important one since it will be used to determine the DC operating point for both AC analysis and transient analysis. If the nets are all linear devices, a MNA matrix will be built and solved. If there are non-linear devices, they will be linearized and Newton-Raphson iteration will be applied to iteratively solve for the node voltages. To overcome convergence issue, damped Newton-Raphson iteration is implemented. Note that capacitors and inductors are treated as open-circuited and short-circuited in DC analysis.

#### 1.2.2 AC Analysis

In AC analysis, the DC operating point of the circuits will be obtained first, using the DC analysis. AC voltage source are treated as short-circuited. Given a frequency value  $f$ , capacitors, inductors, diode and BJT are stamped according to  $f$ . The voltages of the nodes to be plot will be acquired and output to a file.

#### 1.2.3 Transient Analysis

Similar to AC analysis, DC analysis will be performed first to obtain the initial value at time  $t = 0$ . Then, Newton-Raphson iteration will be used to solve for node voltages for each successive time step. Backward Euler integration method and the Norton equivalent model is used in the project, since they will not introduce extra voltage sources. Furthermore, Backward Euler method is more accurate than Forward Euler and much simpler than Trapezoidal method.

### 1.3 Implementation Details

The program is implemented in C++. The nets and nodes are encapsulated as classes, where different devices can be modeled as subclasses of nets. C++ Standard Template Library is utilized in the program.

UMFPACK [1] is used to solve sparse matrix that contains complex number entries. Compressed row/column index storage method for sparse matrix is hard for human to read and operate on. UMFPACK supports a much simpler representation, i.e., Triplet arrays. Each element in the array is represented as a triplet  $(i, j, v)$ , where  $[i, j]$  is the index and  $v$  is the value. Duplicate index entry can exist in the array. UMFPACK provides a handy function that can help us transform the triplet form to the column index form. Hence, when stamping an element, we can simply add a triplet to the arrays. By using this method, the large.txt test case can be solved at around 0.01 second when run on a Intel Core II Duo 2.4 GHz machine.

During the implementation, the convergence issue occurs. Damped Newton-Raphson method must be used. The dampen value is set to 0.1.

Note that besides the circuit nodes, voltage source, vccs, cccs and ccvs will introduce extra rows and columns in the MNA matrix. Their index is stored in STL map class for  $O(1)$  access. Note that in my implementation, ground node is assigned a row and a column in the matrix for convenient. The triplet will not be inserted whenever  $i = 0$  or  $j = 0$ . Otherwise the MNA matrix will become singular. The matrix right hand side (RHS) will be set to 0 before solving.

## 2 Equations used in Analysis

This section details the equations derived for each type of analysis.

### 2.1 DC Analysis Equations for Diode

- $k^{th}$  iteration of Newton-Raphson method:

$$\begin{aligned}
 I_D = I(V_D) &= I_s \cdot (\exp(\frac{V_D}{V_T}) - 1) \\
 \frac{dI(V_D)}{dV_D} &= \frac{I_s}{V_T} \cdot \exp(\frac{V_D}{V_T}) \\
 I_k^D = f(V_k^D) &\approx f(V_D^{k-1}) + (V_D^k - V_D^{k-1}) \cdot f'(V_D^{k-1}) \\
 &= f(V_D^{k-1}) + (V_D^k - V_D^{k-1}) \cdot (\frac{I_s}{V_T} \cdot \exp(\frac{V_D^{k-1}}{V_T})) \\
 &= \overbrace{\frac{I_s}{V_T} \exp(\frac{V_D^{k-1}}{V_T}) V_D^k}^{G_{eq}^{k-1}} + \overbrace{I_s (\exp(\frac{V_D^{k-1}}{V_T}) - 1) - V_D^{k-1} \cdot (\frac{I_s}{V_T} \exp(\frac{V_D^{k-1}}{V_T}))}^{I_{eq}^{k-1}}
 \end{aligned}$$

- DC analysis matrix stamping:

$$\begin{matrix} & i & j \\ i & \begin{bmatrix} G_{eq}^{k-1} & -G_{eq}^{k-1} \\ -G_{eq}^{k-1} & G_{eq}^{k-1} \end{bmatrix} & \end{matrix} \times \begin{bmatrix} V_i^k \\ V_j^k \end{bmatrix} = \begin{bmatrix} -I_{eq}^{k-1} \\ I_{eq}^{k-1} \end{bmatrix}$$

### 2.2 DC Analysis Equations for BJT

Since the equations for NPN BJT are already given in the document of Phase II, only the equations for PNP BJT are given here.

- $k^{th}$  iteration of Newton-Raphson method:

$$\begin{aligned}
I_c(V_{BE}, V_{BC}) &= -I_S \left[ \left( 1 + \frac{V_{BC}}{V_{A_f}} + \frac{V_{BE}}{V_{A_r}} \right) \left( \exp\left(-\frac{V_{BE}}{V_T}\right) - \exp\left(-\frac{V_{BC}}{V_T}\right) \right) - \frac{1}{\beta_r} \left( \exp\left(-\frac{V_{BC}}{V_T}\right) \right) - 1 \right] \\
\frac{\partial I_C(V_{BE}^{k-1}, V_{BC}^{k-1})}{\partial V_{BE}} &= \frac{I_S}{V_T} \left( 1 + \frac{V_{BC}^{k-1}}{V_{A_f}} + \frac{V_{BE}^{k-1}}{V_{A_r}} \right) \exp\left(-\frac{V_{BE}^{k-1}}{V_T}\right) - \frac{I_S}{V_{A_r}} \left( \exp\left(\frac{V_{BE}^{k-1}}{V_T}\right) - \exp\left(\frac{V_{BC}^{k-1}}{V_T}\right) \right) \\
&= h_{C1}^{k-1} \\
\frac{\partial I_C(V_{BE}^{k-1}, V_{BC}^{k-1})}{\partial V_{BC}} &= -\frac{I_S}{V_T} \left( \exp\left(-\frac{V_{BE}^{k-1}}{V_T}\right) - \exp\left(-\frac{V_{BC}^{k-1}}{V_T}\right) \right) - \frac{I_S}{V_T} \left( 1 + \frac{V_{BE}^{k-1}}{V_{A_f}} + \frac{V_{BC}^{k-1}}{V_{A_r}} \right) \exp\left(-\frac{V_{BC}^{k-1}}{V_T}\right) \\
&\quad - \frac{I_S}{\beta_r V_T} \exp\left(-\frac{V_{BC}^{k-1}}{V_T}\right) = h_{C2}^{k-1} \\
I_C(V_{BE}^k, V_{BC}^k) &= I_C(V_{BE}^{k-1}, V_{BC}^{k-1}) + h_{C1}^{k-1} (V_{BE}^k - V_{BE}^{k-1}) + h_{C2}^{k-1} (V_{BC}^k - V_{BC}^{k-1}) \\
&= (h_{C1}^{k-1} + h_{C2}^{k-1}) V_B^k - h_{C2}^{k-1} V_C^k - h_{C1}^{k-1} V_E^k + \overbrace{I_C(V_{BE}^{k-1}, V_{BC}^{k-1}) - h_{C1}^{k-1} V_{BE}^{k-1} - h_{C2}^{k-1} V_{BC}^{k-1}}^{h_{C3}^{k-1}} \\
I_B(V_{BE}, V_{BC}) &= -I_S \left[ \frac{1}{\beta_f} \left( \exp\left(-\frac{V_{BE}}{V_T}\right) - 1 \right) + \frac{1}{\beta_r} \left( \exp\left(-\frac{V_{BC}}{V_T}\right) - 1 \right) \right] \\
\frac{\partial I_B(V_{BE}^{k-1}, V_{BC}^{k-1})}{\partial V_{BE}} &= \frac{I_S}{\beta_f V_T} \exp\left(-\frac{V_{BE}^{k-1}}{V_T}\right) = h_{B1}^{k-1} \\
\frac{\partial I_B(V_{BE}^{k-1}, V_{BC}^{k-1})}{\partial V_{BC}} &= \frac{I_S}{\beta_r V_T} \exp\left(-\frac{V_{BC}^{k-1}}{V_T}\right) = h_{B2}^{k-1} \\
I_B(V_{BE}^k, V_{BC}^k) &= I_B(V_{BE}^{k-1}, V_{BC}^{k-1}) + h_{B1}^{k-1} (V_{BE}^k - V_{BE}^{k-1}) + h_{B2}^{k-1} (V_{BC}^k - V_{BC}^{k-1}) \\
&= (h_{B1}^{k-1} + h_{B2}^{k-1}) V_B^k - h_{B2}^{k-1} V_C^k - h_{B1}^{k-1} V_E^k + \overbrace{I_B(V_{BE}^{k-1}, V_{BC}^{k-1}) - h_{B1}^{k-1} V_{BE}^{k-1} - h_{B2}^{k-1} V_{BC}^{k-1}}^{h_{B3}^{k-1}} \\
I_E(V_{BE}^k, V_{BC}^k) &= I_C(V_{BE}^k, V_{BC}^k) + I_B(V_{BE}^k, V_{BC}^k) \\
&= (h_{C1}^{k-1} + h_{C2}^{k-1} + h_{B1}^{k-1} + h_{B2}^{k-1}) V_B^k - (h_{B2}^{k-1} + h_{C2}^{k-1}) V_C^k - (h_{B1}^{k-1} + h_{C1}^{k-1}) V_E^k + h_{C3}^{k-1} + h_{B3}^{k-1}
\end{aligned}$$

- DC analysis matrix stamping:

$$\begin{array}{c} B \\ C \\ E \end{array} \begin{bmatrix} h_{B1}^{k-1} + h_{B2}^{k-1} & -h_{B2}^{k-1} & -h_{B1}^{k-1} \\ h_{C1}^{k-1} + h_{C2}^{k-1} & -h_{C2}^{k-1} & -h_{C1}^{k-1} \\ -(h_{B1}^{k-1} + h_{B2}^{k-1} + h_{C1}^{k-1} + h_{C2}^{k-1}) & h_{B2}^{k-1} + h_{C2}^{k-1} & h_{B1}^{k-1} + h_{C1}^{k-1} \end{bmatrix} \times \begin{bmatrix} V_B^k \\ V_C^k \\ V_E^k \end{bmatrix} = \begin{bmatrix} -h_{B3}^{k-1} \\ -h_{C3}^{k-1} \\ h_{B3}^{k-1} + h_{C3}^{k-1} \end{bmatrix}$$

## 2.3 AC Analysis

### 2.3.1 NPN

- Capacitor  $C_c$

$$C_c(V_c) = \frac{dQ(V_c)}{dV_c} = \begin{cases} C_{jcs} \left( 1 - \frac{V_c}{V_j} \right)^{-m_j} & V_c < f_c V_j \\ C_{jcs} \frac{1 - f_c(1 + m_j) + m_j \frac{V_c}{V_j}}{(1 - f_c)^{1 + m_j}} & V_c \geq f_c V_j \end{cases}$$

$$\frac{dC_c(V_c)}{dV_c} = \begin{cases} \frac{m_j C_{jcs}}{V_j} \left(1 - \frac{V_c}{V_j}\right)^{-m_j-1} & V_c < f_c V_j \\ \frac{m_j C_{jcs}}{V_j (1-f_c)^{1+m_j}} & V_c \geq f_c V_j \end{cases}$$

- Capacitor  $C_{BE}$

$$\begin{aligned} C_{BE}(V_{BE}, V_{BC}) &= \frac{dQ(V_{BE})}{dV_{BE}} = -\frac{\tau_f I_S}{V_{Ar}} \left[ \exp\left(\frac{V_{BE}}{V_T}\right) - 1 \right] + \frac{\tau_f I_S}{V_T} \exp\left(\frac{V_{BE}}{V_T}\right) \left(1 - \frac{V_{BC}}{V_{Af}} - \frac{V_{BE}}{V_{Ar}}\right) \\ &\quad + \begin{cases} C_{jbe} \left(1 - \frac{V_{BE}}{V_j}\right)^{-m_j} & V_{BE} < f_c V_j \\ C_{jbe} \frac{1-f_c(1+m_j)+m_j \frac{V_{BE}}{V_j}}{(1-f_c)^{1+m_j}} & V_{BE} \geq f_c V_j \end{cases} \\ C_{BE2}(V_{BE}, V_{BC}) &= \frac{dQ(V_{BE})}{dV_{BC}} = -\frac{\tau_f I_S}{V_{Af}} \left[ \exp\left(\frac{V_{BE}}{V_T}\right) - 1 \right] \\ \frac{\partial C_{BE}(V_{BE}, V_{BC})}{\partial V_{BE}} &= -\frac{2\tau_f I_S}{V_{Ar} V_T} \exp\left(\frac{V_{BE}}{V_T}\right) + \frac{\tau_f I_S}{(V_T)^2} \exp\left(\frac{V_{BE}}{V_T}\right) \left(1 - \frac{V_{BC}}{V_{Af}} - \frac{V_{BE}}{V_{Ar}}\right) \\ &\quad + \begin{cases} \frac{m_j C_{jbe}}{V_j} \left(1 - \frac{V_{BE}}{V_j}\right)^{-m_j-1} & V_{BE} < f_c V_j \\ \frac{m_j C_{jbe}}{V_j (1-f_c)^{1+m_j}} & V_{BE} \geq f_c V_j \end{cases} \\ \frac{\partial C_{BE}(V_{BE}, V_{BC})}{\partial V_{BC}} &= -\frac{\tau_f I_S}{V_{Ar} V_T} \exp\left(\frac{V_{BE}}{V_T}\right) \end{aligned}$$

- Capacitor  $C_{BC}$

$$\begin{aligned} C_{BC}(V_{BC}) &= \frac{dQ(V_{BC})}{dV_{BC}} = \frac{\tau_f I_S}{V_T} \exp\left(\frac{V_{BC}}{V_T}\right) + \begin{cases} C_{jbc} \left(1 - \frac{V_{BC}}{V_j}\right)^{-m_j} & V_{BC} < f_c V_j \\ C_{jbc} \frac{1-f_c(1+m_j)+m_j \frac{V_{BC}}{V_j}}{(1-f_c)^{1+m_j}} & V_{BC} \geq f_c V_j \end{cases} \\ \frac{dC_{BC}(V_{BC})}{dV_{BC}} &= \frac{\tau_f I_S}{(V_T)^2} \exp\left(\frac{V_{BC}}{V_T}\right) + \begin{cases} \frac{m_j C_{jbc}}{V_j} \left(1 - \frac{V_{BC}}{V_j}\right)^{-m_j-1} & V_{BC} < f_c V_j \\ \frac{m_j C_{jbc}}{V_j (1-f_c)^{1+m_j}} & V_{BC} \geq f_c V_j \end{cases} \end{aligned}$$

### 2.3.2 PNP

- Capacitor  $C_c$

$$\begin{aligned} Q(V_c) &= \begin{cases} -C_{jcs} \frac{V_j}{1-m_j} \left[ 1 - \left(1 + \frac{V_c}{V_j}\right)^{1-m_j} \right] & V_c > -f_c V_j \\ C_{jcs} V_c \frac{1-f_c(1+m_j)-m_j \frac{V_c}{2V_j}}{(1-f_c)^{1+m_j}} - C_{jcs} V_j \left[ \frac{1-(1-f_c)^{1-m_j}}{1-m_j} - f_c \frac{1-f_c(1+\frac{m_j}{2})}{(1-f_c)^{1+m_j}} \right] & V_c \leq -f_c V_j \end{cases} \\ C_c(V_c) &= \frac{dQ(V_c)}{dV_c} = \begin{cases} C_{jcs} \left(1 + \frac{V_c}{V_j}\right)^{-m_j} & V_c > -f_c V_j \\ C_{jcs} \frac{1-f_c(1+m_j)-m_j \frac{V_c}{2V_j}}{(1-f_c)^{1+m_j}} & V_c \leq -f_c V_j \end{cases} \\ \frac{dC_c(V_c)}{dV_c} &= \begin{cases} -\frac{m_j C_{jcs}}{V_j} \left(1 + \frac{V_c}{V_j}\right)^{-m_j-1} & V_c > -f_c V_j \\ -\frac{m_j C_{jcs}}{V_j (1-f_c)^{1+m_j}} & V_c \leq -f_c V_j \end{cases} \end{aligned}$$

- Capacitor  $C_{BE}$

$$Q(V_{BE}) = -\tau_f I_S \left[ \exp\left(\frac{V_{BE}}{V_T}\right) - 1 \right] \left( 1 + \frac{V_{BC}}{V_{Af}} + \frac{V_{BE}}{V_{Ar}} \right) + \begin{cases} -C_{jbe} \frac{V_j}{1-m_j} \left[ 1 - \left( 1 + \frac{V_{BE}}{V_j} \right)^{1-m_j} \right] & V_{BE} > -f_c V_j \\ C_{jbe} V_{BE} \frac{1-f_c(1+m_j)-m_j \frac{V_{BE}}{2V_j}}{(1-f_c)^{1+m_j}} - C_{jbe} V_j \left[ \frac{1-(1-f_c)^{1-m_j}}{1-m_j} - f_c \frac{1-f_c(1+\frac{m_j}{2})}{(1-f_c)^{1+m_j}} \right] & V_{BE} \leq -f_c V_j \end{cases}$$

$$C_{BE}(V_{BE}, V_{BC}) = \frac{dQ(V_{BE})}{dV_{BE}} = -\frac{\tau_f I_S}{V_{Ar}} \left[ \exp\left(-\frac{V_{BE}}{V_T}\right) - 1 \right] + \frac{\tau_f I_S}{V_T} \exp\left(-\frac{V_{BE}}{V_T}\right) \left( 1 + \frac{V_{BC}}{V_{Af}} + \frac{V_{BE}}{V_{Ar}} \right) + \begin{cases} C_{jbe} \left( 1 + \frac{V_{BE}}{V_j} \right)^{-m_j} & V_{BE} > -f_c V_j \\ C_{jbe} \frac{1-f_c(1+m_j)-m_j \frac{V_{BE}}{2V_j}}{(1-f_c)^{1+m_j}} & V_{BE} \leq -f_c V_j \end{cases}$$

$$C_{BE2}(V_{BE}, V_{BC}) = \frac{dQ(V_{BE})}{dV_{BC}} = -\frac{\tau_f I_S}{V_{Af}} \left[ \exp\left(-\frac{V_{BE}}{V_T}\right) - 1 \right]$$

$$\frac{\partial C_{BE}(V_{BE}, V_{BC})}{\partial V_{BE}} = \frac{2\tau_f I_S}{V_{Ar} V_T} \exp\left(-\frac{V_{BE}}{V_T}\right) - \frac{\tau_f I_S}{(V_T)^2} \exp\left(-\frac{V_{BE}}{V_T}\right) \left( 1 + \frac{V_{BC}}{V_{Af}} + \frac{V_{BE}}{V_{Ar}} \right) - \begin{cases} \frac{m_j C_{jbe}}{V_j} \left( 1 + \frac{V_{BE}}{V_j} \right)^{-m_j-1} & V_{BE} > -f_c V_j \\ \frac{m_j C_{jbe}}{V_j (1-f_c)^{1+m_j}} & V_{BE} \leq -f_c V_j \end{cases}$$

$$\frac{\partial C_{BE}(V_{BE}, V_{BC})}{\partial V_{BC}} = \frac{\tau_f I_S}{V_{Ar} V_T} \exp\left(-\frac{V_{BE}}{V_T}\right)$$

- Capacitor  $C_{BC}$

$$Q(V_{BC}) = -\tau_f I_S \left[ \exp\left(-\frac{V_{BC}}{V_T}\right) - 1 \right] + \begin{cases} -C_{jbc} \frac{V_j}{1-m_j} \left[ 1 - \left( 1 + \frac{V_{BC}}{V_j} \right)^{1-m_j} \right] & V_{BC} > -f_c V_j \\ C_{jbc} V_{BC} \frac{1-f_c(1+m_j)-m_j \frac{V_{BC}}{2V_j}}{(1-f_c)^{1+m_j}} - C_{jbc} V_j \left[ \frac{1-(1-f_c)^{1-m_j}}{1-m_j} - f_c \frac{1-f_c(1+\frac{m_j}{2})}{(1-f_c)^{1+m_j}} \right] & V_{BC} \leq -f_c V_j \end{cases}$$

$$C_{BC}(V_{BC}) = \frac{dQ(V_{BC})}{dV_{BC}} = \frac{\tau_f I_S}{V_T} \exp\left(-\frac{V_{BC}}{V_T}\right) + \begin{cases} C_{jbc} \left( 1 + \frac{V_{BC}}{V_j} \right)^{-m_j} & V_{BC} < f_c V_j \\ C_{jbc} \frac{1-f_c(1+m_j)-m_j \frac{V_{BC}}{2V_j}}{(1-f_c)^{1+m_j}} & V_{BC} \geq f_c V_j \end{cases}$$

$$\frac{dC_{BC}(V_{BC})}{dV_{BC}} = -\frac{\tau_f I_S}{(V_T)^2} \exp\left(-\frac{V_{BC}}{V_T}\right) - \begin{cases} \frac{m_j C_{jbc}}{V_j} \left( 1 + \frac{V_{BC}}{V_j} \right)^{-m_j-1} & V_{BC} > -f_c V_j \\ \frac{m_j C_{jbc}}{V_j (1-f_c)^{1+m_j}} & V_{BC} \leq -f_c V_j \end{cases}$$

- Matrix Stamp:

$$\begin{matrix} & & B & & C & & E \\ \begin{matrix} B \\ C \\ E \end{matrix} & \left[ \begin{array}{ccc} (C_{BE}(V_{BE}, V_{BC}) + C_{BE2}(V_{BE}, V_{BC}) + C_{BC}(V_{BC})) \times 2\pi f & -(C_{BE2}(V_{BE}, V_{BC}) + C_{BC}(V_{BC})) \times 2\pi f & -C_{BE}(V_{BE}, V_{BC}) \times 2\pi f \\ -C_{BC}(V_{BC}) \times 2\pi f & (C_C(V_C) + C_{BC}(V_{BC})) \times 2\pi f & 0 \\ -(C_{BE}(V_{BE}, V_{BC}) + C_{BE2}(V_{BE}, V_{BC})) \times 2\pi f & C_{BE2}(V_{BE}, V_{BC}) \times 2\pi f & C_{BE}(V_{BE}, V_{BC}) \times 2\pi f \end{array} \right] \end{matrix}$$

## 2.4 Transient Analysis: $I_{eq}$ and $G_{eq}$ for capacitors in BJT

- Capacitor  $C_c$ :

$$I_{Ceq} = -\frac{C_c(V_c(t_{n+1}))}{\Delta t} V_c(t_n)$$

$$G_{Ceq} = \frac{C_c(V_c(t_{n+1}))}{\Delta t}$$

$k^{th}$  iteration of Newton-Raphson method from  $t_n$  to  $t_{n+1}$ :

$$\begin{aligned} I_{Ceq}^{I,k} &= -\frac{C_c(V_c^k(t_{n+1}))}{\Delta t} V_c(t_n) \\ &\approx -\frac{V_c(t_n)}{\Delta t} C_c(V_c^{k-1}(t_{n+1})) - \frac{V_c(t_n)}{\Delta t} \frac{dC_c(V_c^{k-1}(t_{n+1}))}{dV_c} (V_c^k(t_{n+1}) - V_c^{k-1}(t_{n+1})) \\ &= \underbrace{-\frac{V_c(t_n)}{\Delta t} \left\{ C_c(V_c^{k-1}(t_{n+1})) - \frac{dC_c(V_c^{k-1}(t_{n+1}))}{dV_c} V_c^k(t_{n+1}) \right\}}_{I_{Ceq}^{I,k-1}} - \underbrace{\frac{V_c(t_n)}{\Delta t} \frac{dC_c(V_c^{k-1}(t_{n+1}))}{dV_c} V_c^k(t_{n+1})}_{G_{Ceq}^{I,k-1}} \end{aligned}$$

$$\begin{aligned} I_{Ceq}^{G,k} &= G_{Ceq}^k V_c^k(t_{n+1}) \\ &= \frac{C_c(V_c^k(t_{n+1}))}{\Delta t} V_c^k(t_{n+1}) \\ &= \frac{C_c(V_c^{k-1}(t_{n+1}))}{\Delta t} V_c^{k-1}(t_{n+1}) \\ &\quad + \frac{1}{\Delta t} \left( \frac{dC_c(V_c^{k-1}(t_{n+1}))}{dV_c} V_c^{k-1}(t_{n+1}) + C_c(V_c^{k-1}(t_{n+1})) \right) (V_c^k(t_{n+1}) - V_c^{k-1}(t_{n+1})) \\ &= \underbrace{-\frac{(V_c^{k-1}(t_{n+1}))^2 dC_c(V_c^{k-1}(t_{n+1}))}{\Delta t dV_c}}_{I_{Ceq}^{G,k-1}} \\ &\quad + \underbrace{\frac{1}{\Delta t} \left( \frac{dC_c(V_c^{k-1}(t_{n+1}))}{dV_c} V_c^{k-1}(t_{n+1}) + C_c(V_c^{k-1}(t_{n+1})) \right) V_c^k(t_{n+1})}_{G_{Ceq}^{G,k-1}} \end{aligned}$$

$$\text{Let } I_{Ceq}^{k-1} = I_{Ceq}^{I,k-1} + I_{Ceq}^{G,k-1}, \quad G_{Ceq}^{k-1} = G_{Ceq}^{I,k-1} + G_{Ceq}^{G,k-1}.$$

- Capacitor  $C_{BE}$ :

$$I_{BEeq} = - \frac{C_{BE} (V_{BE} (t_{n+1}), V_{BC} (t_{n+1}))}{\Delta t} V_{BE} (t_n)$$

$$G_{BEeq} = \frac{C_{BE} (V_{BE} (t_{n+1}), V_{BC} (t_{n+1}))}{\Delta t}$$

$k^{th}$  iteration of Newton-Raphsonmethod from  $t_n$  to  $t_{n+1}$ :

$$\begin{aligned} I_{BEeq}^{I,k} &= - \frac{C_{BE} (V_{BE}^k (t_{n+1}), V_{BC}^k (t_{n+1}))}{\Delta t} V_{BE} (t_n) \\ &\approx - \frac{V_{BE}(t_n)}{\Delta t} C_{BE} (V_{BE}^{k-1} (t_{n+1}), V_{BC}^{k-1} (t_{n+1})) - \frac{V_{BE}(t_n)}{\Delta t} \frac{\partial C_{BE}(V_{BE}^{k-1}(t_{n+1}), V_{BC}^{k-1}(t_{n+1}))}{\partial V_{BE}} (V_{BE}^k (t_{n+1}) - V_{BE}^{k-1} (t_{n+1})) \\ &\quad - \frac{V_{BE}(t_n)}{\Delta t} \frac{\partial C_{BE}(V_{BE}^{k-1}(t_{n+1}), V_{BC}^{k-1}(t_{n+1}))}{\partial V_{BC}} (V_{BC}^k (t_{n+1}) - V_{BC}^{k-1} (t_{n+1})) \\ &= \overbrace{- \frac{V_{BE}(t_n)}{\Delta t} \left\{ C_{BE}(V_{BE}^{k-1}(t_{n+1}), V_{BC}^{k-1}(t_{n+1})) + \frac{\partial C_{BE}(V_{BE}^{k-1}(t_{n+1}), V_{BC}^{k-1}(t_{n+1}))}{\partial V_{BE}} V_{BE}^{k-1}(t_{n+1}) + \frac{\partial C_{BE}(V_{BE}^{k-1}(t_{n+1}), V_{BC}^{k-1}(t_{n+1}))}{\partial V_{BC}} V_{BC}^{k-1}(t_{n+1}) \right\}}^{I_{BEeq}^{I,k-1}} \\ &\quad - \overbrace{\frac{V_{BE}(t_n)}{\Delta t} \frac{\partial C_{BE}(V_{BE}^{k-1}(t_{n+1}), V_{BC}^{k-1}(t_{n+1}))}{\partial V_{BE}} V_{BE}^k (t_{n+1})}^{G_{BEeq}^{I,k-1}} \\ &\quad - \overbrace{\frac{V_{BE}(t_n)}{\Delta t} \frac{\partial C_{BE}(V_{BE}^{k-1}(t_{n+1}), V_{BC}^{k-1}(t_{n+1}))}{\partial V_{BC}} V_{BC}^k (t_{n+1})}^{G_{BEeq}^{I,k-1}} \\ I_{BEeq}^{G,k} &= G_{BEeq}^k V_{BE}^k (t_{n+1}) \\ &= \frac{C_{BE} (V_{BE}^k (t_{n+1}), V_{BC}^k (t_{n+1}))}{\Delta t} V_{BE}^k (t_{n+1}) \\ &\approx \frac{C_{BE} (V_{BE}^{k-1} (t_{n+1}), V_{BC}^{k-1} (t_{n+1}))}{\Delta t} V_{BE}^{k-1} (t_{n+1}) \\ &\quad + \frac{1}{\Delta t} \left( \frac{\partial C_{BE}(V_{BE}^{k-1}(t_{n+1}), V_{BC}^{k-1}(t_{n+1}))}{\partial V_{BE}} V_{BE}^{k-1} (t_{n+1}) + C_{BE} (V_{BE}^{k-1} (t_{n+1}), V_{BC}^{k-1} (t_{n+1})) \right) (V_{BE}^k (t_{n+1}) - V_{BE}^{k-1} (t_{n+1})) \\ &\quad + \frac{V_{BE}^{k-1} (t_{n+1})}{\Delta t} \frac{\partial C_{BE} (V_{BE}^{k-1} (t_{n+1}), V_{BC}^{k-1} (t_{n+1}))}{\partial V_{BC}} (V_{BC}^k (t_{n+1}) - V_{BC}^{k-1} (t_{n+1})) \\ &= \overbrace{- \frac{(V_{BE}^{k-1}(t_{n+1}))^2}{\Delta t} \frac{\partial C_{BE}(V_{BE}^{k-1}(t_{n+1}), V_{BC}^{k-1}(t_{n+1}))}{\partial V_{BE}} - \frac{V_{BE}^{k-1}(t_{n+1}) V_{BC}^{k-1}(t_{n+1})}{\Delta t} \frac{\partial C_{BE}(V_{BE}^{k-1}(t_{n+1}), V_{BC}^{k-1}(t_{n+1}))}{\partial V_{BC}}}^{I_{BEeq}^{G,k-1}} \\ &\quad + \overbrace{\frac{1}{\Delta t} \left( \frac{\partial C_{BE} (V_{BE}^{k-1} (t_{n+1}), V_{BC}^{k-1} (t_{n+1}))}{\partial V_{BE}} V_{BE}^{k-1} (t_{n+1}) + C_{BE} (V_{BE}^{k-1} (t_{n+1}), V_{BC}^{k-1} (t_{n+1})) \right)}^{G_{BEeq}^{G,k-1}} V_{BE}^k (t_{n+1}) \\ &\quad + \overbrace{\frac{V_{BE}^{k-1} (t_{n+1})}{\Delta t} \frac{\partial C_{BE} (V_{BE}^{k-1} (t_{n+1}), V_{BC}^{k-1} (t_{n+1}))}{\partial V_{BC}} V_{BC}^k (t_{n+1})}^{G_{BEeq}^{G,k-1}} \end{aligned}$$

Let  $I_{BEq}^{k-1} = I_{BEq}^{I,k-1} + I_{BEq}^{G,k-1}$ ,  $G_{BEq}^{BE,k-1} = G_{BEq}^{I_{BE},k-1} + G_{BEq}^{G_{BE},k-1}$ ,  $G_{BEq}^{BC,k-1} = G_{BEq}^{I_{BC},k-1} + G_{BEq}^{G_{BC},k-1}$ .

• Capacitor  $C_{BC}$ :

$k^{th}$  iteration of Newton-Raphson method from  $t_n$  to  $t_{n+1}$ :

$$I_{BCeq} = -\frac{C_{BC}(V_{BC}(t_{n+1}))}{\Delta t} V_{BC}(t_n) i$$

$$G_{BCeq} = \frac{C_{BC}(V_{BC}(t_{n+1}))}{\Delta t}$$

$$\begin{aligned} I_{BCeq}^{I,k} &= -\frac{C_{BC}(V_{BC}(t_{n+1}))}{\Delta t} V_{BC}(t_n) \\ &\approx -\frac{V_{BC}(t_n)}{\Delta t} C_{BC}(V_{BC}^{k-1}(t_{n+1})) - \frac{V_{BC}(t_n)}{\Delta t} \frac{dC_{BC}(V_{BC}^{k-1}(t_{n+1}))}{dV_{BC}} (V_{BC}^k(t_{n+1}) - V_{BC}^{k-1}(t_{n+1})) \\ &= \overbrace{-\frac{V_{BC}(t_n)}{\Delta t} \left\{ C_{BC}(V_{BC}^{k-1}(t_{n+1})) - \frac{dC_{BC}(V_{BC}^{k-1}(t_{n+1}))}{dV_{BC}} V_{BC}^k(t_{n+1}) \right\}}^{I_{BCeq}^{I,k-1}} \\ &\quad - \overbrace{\frac{V_{BC}(t_n)}{\Delta t} \frac{dC_{BC}(V_{BC}^{k-1}(t_{n+1}))}{dV_{BC}} V_{BC}^k(t_{n+1})}^{G_{BCeq}^{I,k-1}} \end{aligned}$$

$$\begin{aligned} I_{BCeq}^{G,k} &= G_{BCeq}^k V_{BC}^k(t_{n+1}) \\ &= \frac{C_{BC}(V_{BC}^{k-1}(t_{n+1}))}{\Delta t} (V_{BC}^k(t_{n+1})) V_{BC}^k(t_{n+1}) \\ &\approx \frac{C_{BC}(V_{BC}^{k-1}(t_{n+1}))}{\Delta t} (V_{BC}^{k-1}(t_{n+1})) V_{BC}^{k-1}(t_{n+1}) \\ &\quad + \frac{1}{\Delta t} \left( \frac{dC_{BC}(V_{BC}^{k-1}(t_{n+1}))}{dV_{BC}} V_{BC}^{k-1}(t_{n+1}) + C_{BC}(V_{BC}^{k-1}(t_{n+1})) V_{BC}^{k-1}(t_{n+1}) \right) (V_{BC}^k(t_{n+1}) - V_{BC}^{k-1}(t_{n+1})) \\ &= \overbrace{-\frac{(V_{BC}^{k-1}(t_{n+1}))^2}{\Delta t} \frac{dC_{BC}(V_{BC}^{k-1}(t_{n+1}))}{dV_{BC}}}_{I_{BCeq}^{G,k-1}} \\ &\quad + \overbrace{\frac{1}{\Delta t} \left( \frac{dC_{BC}(V_{BC}^{k-1}(t_{n+1}))}{dV_{BC}} V_{BC}^{k-1}(t_{n+1}) + C_{BC}(V_{BC}^{k-1}(t_{n+1})) \right) V_{BC}^k(t_{n+1})}_{G_{BCeq}^{G,k-1}} \end{aligned}$$

Let  $I_{BCeq}^{k-1} = I_{BCeq}^{I,k-1} + I_{BCeq}^{G,k-1}$ ,  $G_{BCeq}^{k-1} = G_{BCeq}^{I,k-1} + G_{BCeq}^{G,k-1}$ .

• Matrix Stamping:



$k^{th}$  iteration of Newton-Raphson method from  $t_n$  to  $t_{n+1}$ :

$$\begin{matrix} B \\ C \\ E \end{matrix} \begin{bmatrix} \overset{B}{G_{BE_{eq}}^{BE,k-1} + G_{BE_{eq}}^{BC,k-1} + G_{BC_{eq}}^{k-1}} & \overset{C}{-G_{BE_{eq}}^{BC,k-1} - G_{BC_{eq}}^{k-1}} & \overset{E}{-G_{BE_{eq}}^{BE,k-1}} \\ -G_{BC_{eq}}^{k-1} & G_{C_{eq}}^{k-1} + G_{BC_{eq}}^{k-1} & 0 \\ -G_{BE_{eq}}^{BE,k-1} - G_{BE_{eq}}^{BC,k-1} & G_{BE_{eq}}^{BC,k-1} & G_{BE_{eq}}^{BE,k-1} \end{bmatrix} \times \begin{bmatrix} V_B^k(t_{n+1}) \\ V_C^k(t_{n+1}) \\ V_E^k(t_{n+1}) \end{bmatrix} = \begin{bmatrix} -I_{BE_{eq}}^{k-1} - I_{BC_{eq}}^{k-1} \\ -I_{C_{eq}}^{k-1} + I_{BC_{eq}}^{k-1} \\ I_{BE_{eq}}^{k-1} \end{bmatrix}$$

## 3 Experimental Results

### 3.1 Results of Phase I (Linear DC Analysis)

Table 1 in page 13 gives the output of phase I.

### 3.2 Results of Phase II (Nonlinear DC Analysis)

Table 2 in page 14 gives the output of phase II.

### 3.3 Results of Phase III (AC Analysis and Transient Analysis)

#### 3.3.1 bjt.txt

Figure 1 shows the result of bjt.txt.

#### 3.3.2 741\_amplifier.txt

Figure 2 shows the result of 741\_amplifier.txt.

#### 3.3.3 amp\_tran1.txt

Figure 3 shows the result of amp\_tran1.txt.

## References

- [1] T. A. Davis, "Umfpack, an unsymmetric-pattern multifrontal method." [Online]. Available: <http://www.cise.ufl.edu/research/sparse/umfpack/>

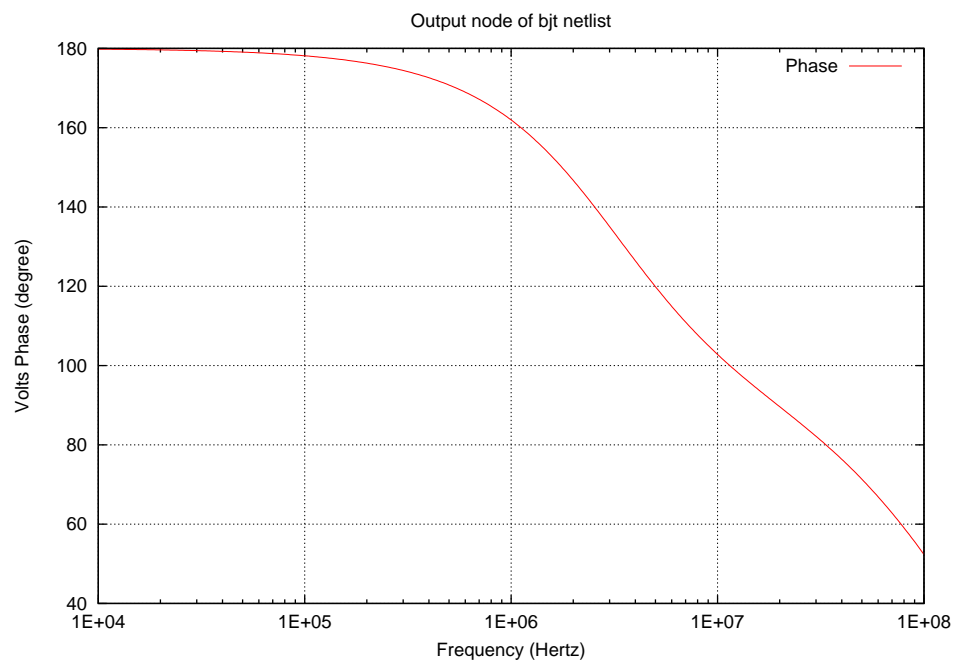
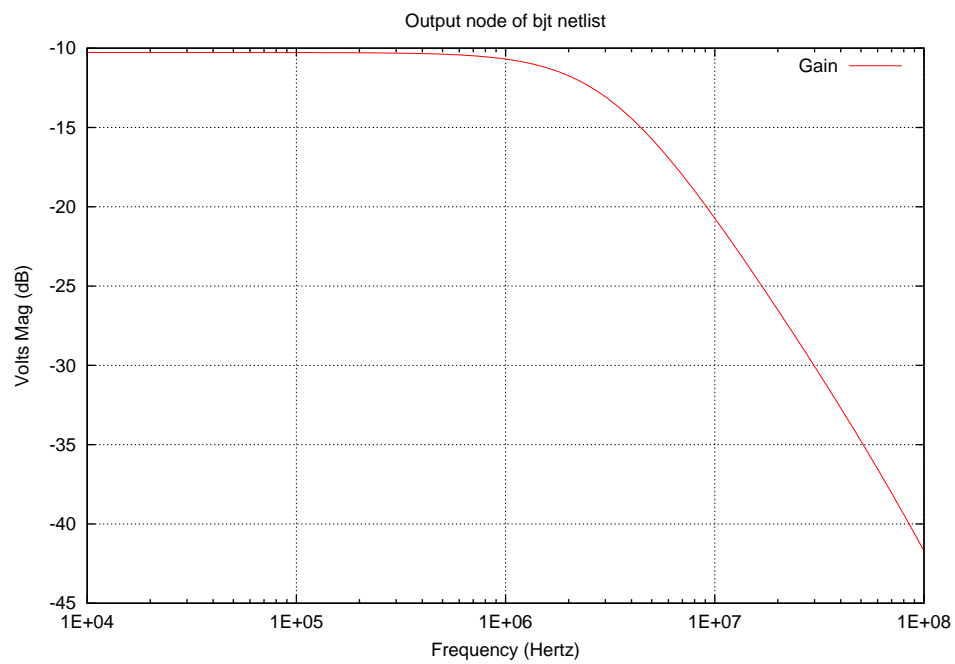


Figure 1: Gain and phase plot of node 'out'.

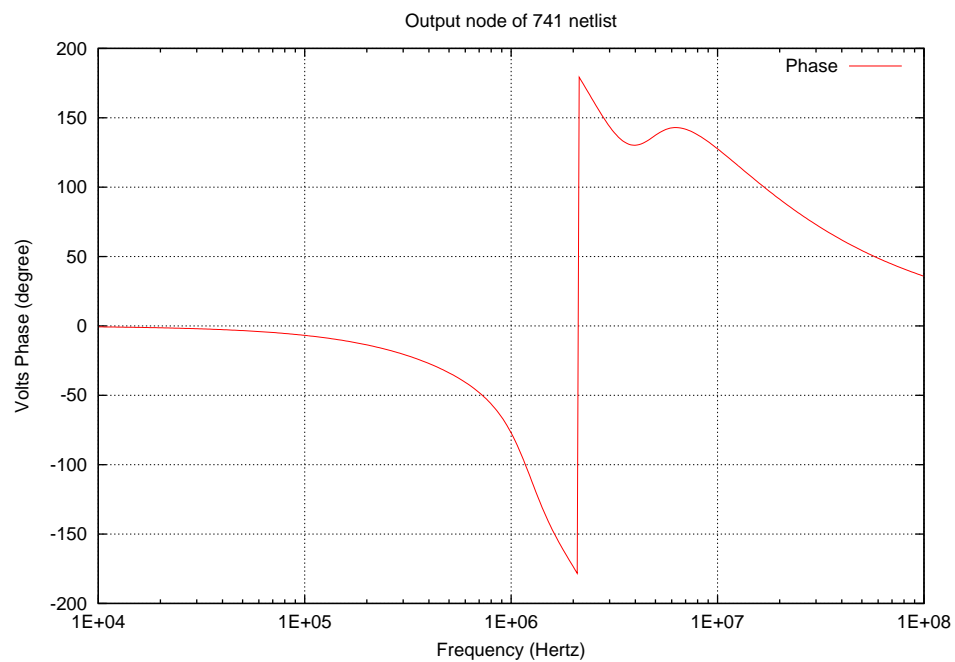
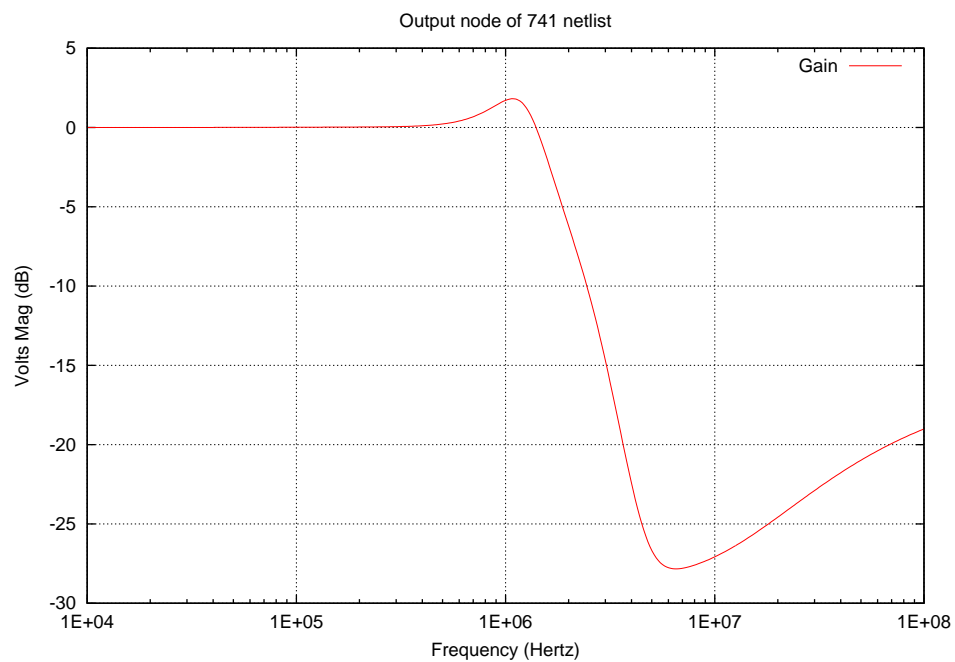


Figure 2: Gain and phase plot of node '22'.

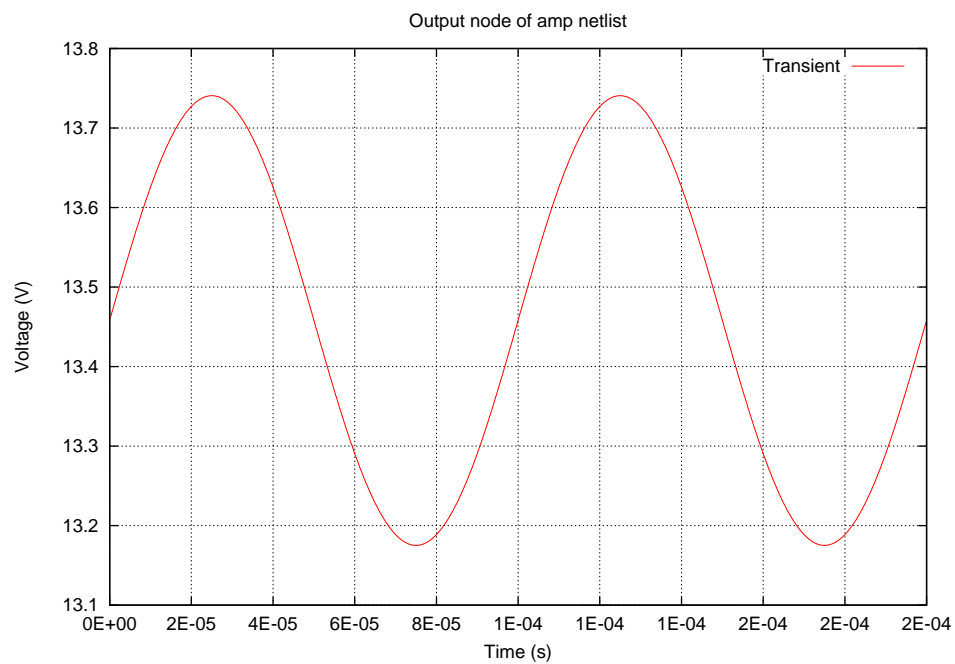
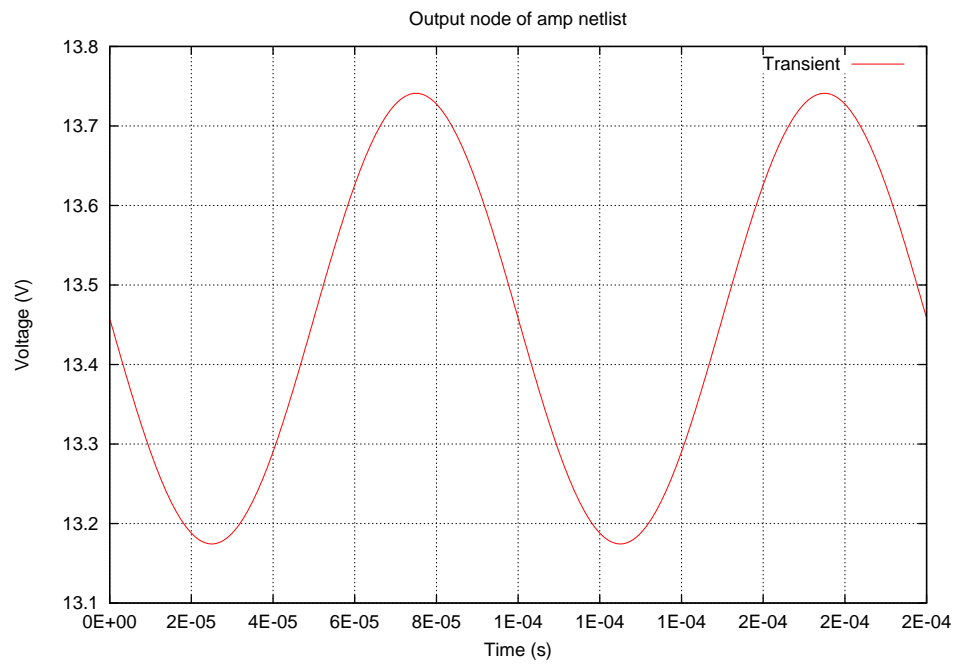


Figure 3: Voltage plot of node 'out1' and 'out2'.

```

* representative
voltage at node a = 1.000000e+00
voltage at node b = 6.000000e-01
voltage at node c = 1.600000e+00
voltage at node d = 1.701538e+01
voltage at node e = -3.040000e+01
voltage at node f = -1.600000e+01
voltage at node g = -2.200000e+00
voltage at node h = -1.075513e+03
current through source v1 = -2.000000e-01
current through source e1 = 2.569231e+00
current through source h1 = 8.962609e+01
current through source f1 = -1.600000e+00

* large
run-time: 0.01593
machine clock frequency: 2.26 GHz Intel Core 2 Duo
voltage at node 1 = 1.999980e-10
voltage at node 2 = 5.999940e-10
voltage at node 3 = 1.199988e-09
voltage at node 4 = 1.999980e-09
voltage at node 5 = 2.999970e-09
voltage at node 6 = 4.199958e-09
voltage at node 7 = 5.599944e-09
voltage at node 8 = 7.199928e-09
voltage at node 9 = 8.999910e-09
voltage at node 10 = 1.099989e-08
voltage at node 11 = 1.319987e-08
voltage at node 12 = 1.559984e-08
voltage at node 13 = 1.819982e-08
voltage at node 14 = 2.099979e-08
voltage at node 15 = 2.399976e-08
voltage at node 16 = 2.719973e-08
voltage at node 17 = 3.059970e-08
voltage at node 18 = 3.419966e-08
voltage at node 19 = 3.799962e-08
voltage at node 20 = 4.199958e-08

* error
Error: dependent voltage source [v2] of cccs [f1] not present!
Error: dependent voltage source [v3] of ccvs [h1] not present!

```

Table 1: Phase I output.

```

* Diode:

Iteration begins:
iteration: 1, difference = 3.287877e-03
iteration: 2, difference = 1.884439e-04
iteration: 3, difference = 6.766938e-07
iteration: 4, difference = 8.683831e-12
Total number of iterations: 4

Result:
voltage at      0 = 0.000000e+00
voltage at      1 = 2.000000e+00
voltage at      2 = 7.030988e-01
current through source v1 = -6.484506e-04

* BJT:

Iteration begins:
iteration: 1, difference = 9.000161e-01
iteration: 2, difference = 6.440751e-04
iteration: 3, difference = 8.155846e-06
iteration: 4, difference = 1.286492e-09
iteration: 5, difference = 9.566619e-16
Total number of iterations: 5

Result:
voltage at      0 = 0.000000e+00
voltage at supply = 5.000000e+00
voltage at cbias = 4.286034e+00
voltage at out = 9.198898e-02
voltage at b = 7.161394e-01
voltage at input2 = 9.000000e-01
voltage at input1 = 9.000000e-01
current through source vcc = -2.021662e-03
current through source vin = -1.838606e-05
current through source vin2 = -1.838606e-05

```

Table 2: Phase II output.