Models for Diode and BJT

Note that Q(V…) charge storage equations are only needed for Phase III, not Phase II. Also, the Q(V…) equations don't match with regular spice equations so the results for Phase III will not match exactly with spice. It's up to you to figure out whether your results are correct or not. Spice .model statements can be created to simulate the diodes and bjts. The format can be found in the HSPICE pdf manual which can be found on-line.

Diode Equations

The diode current I_D in terms of diode voltage V_D is given by

$$I_D = I(V_D) + \frac{\mathrm{d}}{\mathrm{d}t}(Q(V_D))$$

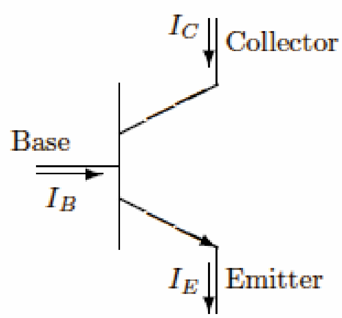
$$I(V_D) = I_S \left[\exp\left(\frac{V_D}{V_T}\right) - 1\right]$$

$$Q(V_D) = \tau I(V_D) + \begin{cases} C_j \frac{V_j}{1-m_j}\left[1 - \left(1 - \frac{V_D}{V_j}\right)^{1-m_j}\right] & V_D < f_c V_j \\ C_j V_D \frac{1.0 - f_c(1+m_j) + \frac{1}{2} m_j \frac{V_D}{V_j}}{(1-f_c)^{1+m_j}} + C_j V_j \left[\frac{1-(1-f_c)^{1-m_j}}{1-m_j} - f_c \frac{1-f_c\left(1+\frac{1}{2}m_j\right)}{(1-f_c)^{1+m_j}}\right] & V_D \geq f_c V_j \end{cases}$$

where $\tau = 2 \times 10^{-11}$, $I_S = 10^{-15}$, $V_T = 0.02585126075417$, $C_j = 10^{-14}$, $V_j = 0.8$, $f_c = 0.5$ and $m_j = 0.5$.

BJT Equations

The BJT is a three terminal device and an NPN BJT is shown below



The base collector voltage V_{BC} is defined as $V_B - V_C$ and the base emitter voltage V_{BE} is defined as $V_B - V_E$. The polarity of the currents for an NPN device are shown in the figure. The BJT equations are

$$I_C = I_{C_1} + I_{C_2} + \frac{\mathrm{d}}{\mathrm{dt}}(Q(V_C) - Q(V_{BC}))$$

$$I_B = \frac{I_{B_1}}{\beta_f} + \frac{I_{B_2}}{\beta_r} + \frac{\mathrm{d}}{\mathrm{dt}}(Q(V_{BC}) + Q(V_{BE}))$$

$$I_E = I_{C_1} + I_{C_2} + \frac{I_{B_1}}{\beta_f} + \frac{I_{B_2}}{\beta_r} - \frac{\mathrm{d}}{\mathrm{dt}}(Q(V_{BE}))$$

$$I_{C_1} = \frac{I_S}{Q_B}\left[\exp\left(\frac{V_{BE}}{V_T}\right) - \exp\left(\frac{V_{BC}}{V_T}\right)\right]$$

$$I_{C_2} = -\frac{I_S}{\beta_r}\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right]$$

$$I_{B_1} = I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right]$$

$$I_{B_2} = I_S\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right]$$

$$Q_B = \frac{1}{1 - \frac{V_{BC}}{V_{A_f}} - \frac{V_{BE}}{V_{A_r}}}$$

$$Q(V_C) = \begin{cases} C_{jcs}\frac{V_j}{1-m_j}\left[1 - \left(1 - \frac{V_C}{V_j}\right)^{1-m_j}\right] & V_C < f_c V_j \\ C_{jcs}V_C\frac{1.0 - f_c(1+m_j) + \frac{1}{2}m_j\frac{V_C}{V_j}}{(1-f_c)^{1+m_j}} + C_{jcs}V_j\left[\frac{1-(1-f_c)^{1-m_j}}{1-m_j} - f_c\frac{1-f_c\left(1+\frac{1}{2}m_j\right)}{(1-f_c)^{1+m_j}}\right] & V_C \geq f_c V_j \end{cases}$$

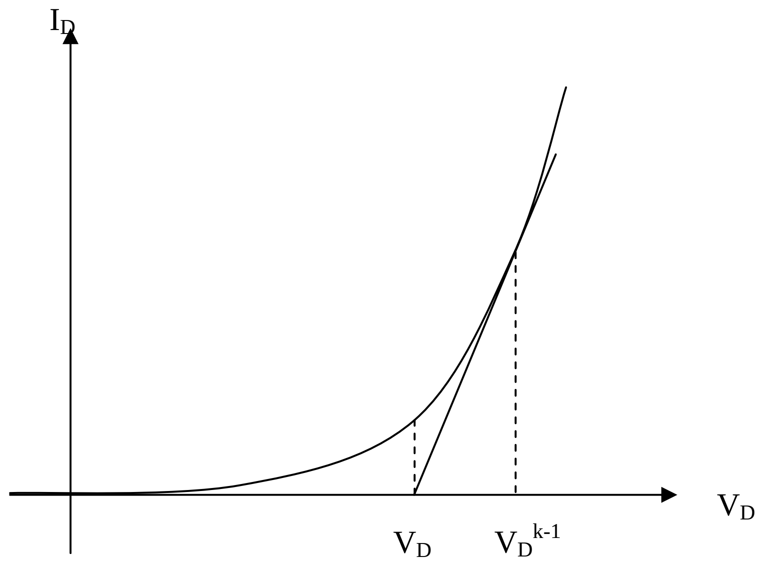
$$Q(V_{BE}) = \tau_f\frac{I_{B_1}}{Q_B} + \begin{cases} C_{jbe}\frac{V_j}{1-m_j}\left[1 - \left(1 - \frac{V_{BE}}{V_j}\right)^{1-m_j}\right] & V_{BE} < f_c V_j \\ C_{jbe}V_{BE}\frac{1.0 - f_c(1+m_j) + \frac{1}{2}m_j\frac{V_{BE}}{V_j}}{(1-f_c)^{1+m_j}} + C_{jbe}V_j\left[\frac{1-(1-f_c)^{1-m_j}}{1-m_j} - f_c\frac{1-f_c\left(1+\frac{1}{2}m_j\right)}{(1-f_c)^{1+m_j}}\right] & V_{BE} \geq f_c V_j \end{cases}$$

$$Q(V_{BC}) = \tau_r I_{B_2} + \begin{cases} C_{jbc}\frac{V_j}{1-m_j}\left[1 - \left(1 - \frac{V_{BC}}{V_j}\right)^{1-m_j}\right] & V_{BC} < f_c V_j \\ C_{jbc}V_{BC}\frac{1.0 - f_c(1+m_j) + \frac{1}{2}m_j\frac{V_{BC}}{V_j}}{(1-f_c)^{1+m_j}} + C_{jbc}V_j\left[\frac{1-(1-f_c)^{1-m_j}}{1-m_j} - f_c\frac{1-f_c\left(1+\frac{1}{2}m_j\right)}{(1-f_c)^{1+m_j}}\right] & V_{BC} \geq f_c V_j \end{cases}$$

where $\tau_r = 2 \times 10^{-10}$, $C_{jbe} = C_{jbc} = 10^{-11}$, $V_{A_f} = 100$, $V_{A_r} = 100$, $\beta_f = 100$, $\beta_r = 4$, $\tau_f = 2 \times 10^{-11}$, $I_S = 10^{-15}$, $V_T = 0.02585126075417$, $C_{jcs} = 2 \times 10^{-14}$, $V_j = 0.8$, $f_c = 0.5$ and $m_j = 0.33$. For a PNP BJT, multiply V_{BE} and V_{BC} with -1, use the above equations to compute the currents, and then multiply all the currents by -1.

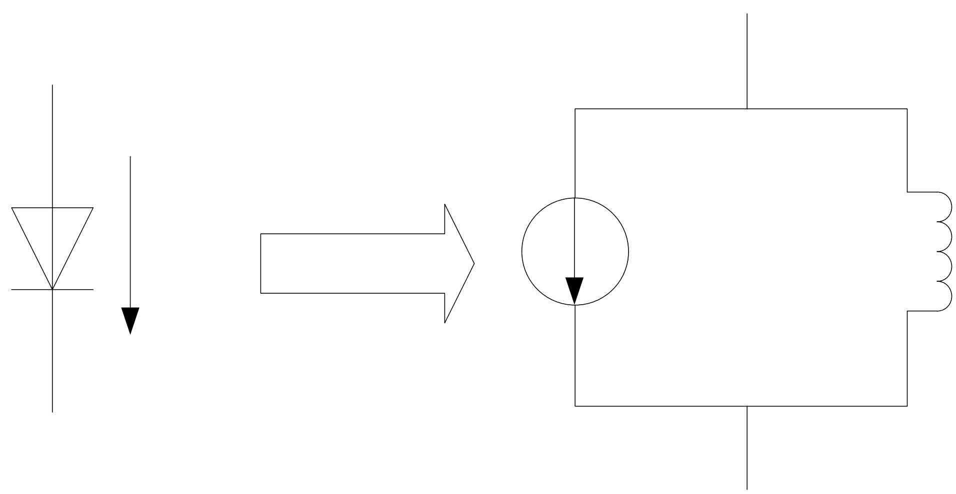
Diode:

$I_D = I(V_D) = I_S \cdot (\exp(V_D / V_T) - 1)$



$$
\begin{aligned}
I_D^k = f(V_D^k) &\approx f(V_D^{k-1}) + (V_D^k - V_D^{k-1}) \cdot f'(V_D^{k-1}) \\
&= f(V_D^{k-1}) + (V_D^k - V_D^{k-1}) \cdot (I_S \cdot \exp(V_D^{k-1} / V_T) / V_T) \\
&= G_{eq} \cdot V_D^k + I_{eq}
\end{aligned}
$$

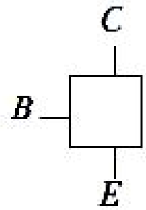
In which:
$G_{eq} = I_S \cdot \exp(V_D^{k-1} / V_T) / V_T$
$I_{eq} = I_S \cdot (\exp(V_D^{k-1} / V_T) - 1) - V_D^{k-1} \cdot (I_S \cdot \exp(V_D^{k-1} / V_T) / V_T)$



Initialization (guess a voltages) → linearization → Stamp matrices → Solve and obtain node voltage

BJT:

1. Consider the three terminal device shown below



The currents are defined positive going into the terminals. The constituent relationships are

$$I_C = g_1(V_B - V_E) + g_2(V_C - V_E),$$
$$I_B = \frac{I_C}{\beta},$$

and obviously due to KCL

$$I_E + I_C + I_B = 0.$$

Here g_1, g_2, and β are constants. Write the MNA stamp of this device.

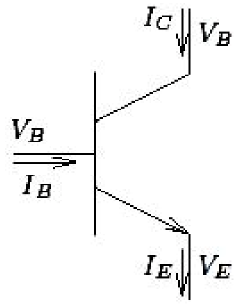
From the KCL equations, we have

$$I_B = \frac{g_1}{\beta}V_B + \frac{g_2}{\beta}V_C - \frac{(g_1 + g_2)}{\beta}V_E,$$
$$I_C = g_1 V_B + g_2 V_C - (g_1 + g_2)V_E,$$
$$I_E = -\frac{\beta + 1}{\beta}g_1 V_B - \frac{\beta + 1}{\beta}g_2 V_C + \frac{\beta + 1}{\beta}(g_1 + g_2)V_E,$$

and hence, the MNA stamp is

$$\begin{array}{c c} & \begin{array}{ccc} B & C & E \end{array} \\ \begin{array}{c} B \\ C \\ E \end{array} & \begin{bmatrix} \frac{g_1}{\beta} & \frac{g_2}{\beta} & \frac{-(g_1+g_2)}{\beta} \\ g_1 & g_2 & -(g_1 + g_2) \\ -\frac{\beta+1}{\beta}g_1 & -\frac{\beta+1}{\beta}g_2 & \frac{\beta+1}{\beta}(g_1 + g_2) \end{bmatrix} \end{array}.$$

2. A bipolar junction transistor is a three terminal device as shown below

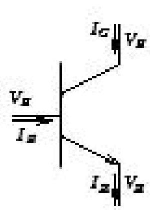


The base collector voltage V_{BC} is defined as $V_B - V_C$ and the base emitter voltage V_{BE} is defined as $V_B - V_E$. The polarity of the currents for an NPN device are shown in the figure. The BJT equations without charge storage are

$$
\begin{aligned}
I_C &= I_{C_1} + I_{C_2} \\
I_B &= \frac{I_{B_1}}{\beta_f} + \frac{I_{B_2}}{\beta_r} \\
I_E &= I_C + I_B \\
I_{C_1} &= \frac{I_S}{Q_B}\left[\exp\left(\frac{V_{BE}}{V_T}\right) - \exp\left(\frac{V_{BC}}{V_T}\right)\right] \\
I_{C_2} &= -\frac{I_S}{\beta_r}\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right] \\
I_{B_1} &= I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right] \\
I_{B_2} &= I_S\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right] \\
Q_B &= \frac{1}{1 - \frac{V_{BC}}{V_{A_f}} - \frac{V_{BE}}{V_{A_r}}}
\end{aligned}
$$

where V_T, V_{A_f}, V_{A_r}, β_f, β_r and I_S are constants. Develop analytical expressions for various entries in the MNA stamp of a BJT.

A bipolar junction transistor is a three terminal device as shown below The base collector voltage V_{BC} is defined



as $V_B - V_C$ and the base emitter voltage V_{BE} is defined as $V_B - V_E$. The polarity of the currents for an NPN

device are shown in the figure. The BJT equations without charge storage are

$$
\begin{aligned}
I_C &= I_{C1} + I_{C2} \\
I_B &= \frac{I_{B1}}{\beta_f} + \frac{I_{B2}}{\beta_r} \\
I_E &= I_C + I_B \\
I_{C_1} &= \frac{I_S}{Q_B}\left[\exp\left(\frac{V_{BE}}{V_T}\right) - \exp\left(\frac{V_{BC}}{V_T}\right)\right] \\
I_{C_2} &= -\frac{I_S}{\beta_r}\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right] \\
I_{B1} &= I_S\left[\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right] \\
I_{B2} &= I_S\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right] \\
Q_B &= \frac{1}{1 - \frac{V_{BC}}{V_{AF}} - \frac{V_{BE}}{V_{AR}}}
\end{aligned}
$$

where V_T , V_{AF} , V_{AR} , β_f , β_r and I_S are constants. Develop analytical expressions for various entries in the MNA stamp of a BJT.

The collector current can be written as

$$
\begin{aligned}
I_C &= I_{C_1} + I_{C_2} \\
&= I_S\left[\frac{1}{Q_B}\left(\exp\left(\frac{V_{BE}}{V_T}\right) - \exp\left(\frac{V_{BC}}{V_T}\right)\right) - \frac{1}{\beta_r}\left(\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right)\right] \\
&= I_S\left[\left(1 - \frac{V_{BC}}{V_{AF}} - \frac{V_{BE}}{V_{AR}}\right)\left(\exp\left(\frac{V_{BE}}{V_T}\right) - \exp\left(\frac{V_{BC}}{V_T}\right)\right) - \frac{1}{\beta_r}\left(\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right)\right]
\end{aligned}
$$

This is a nonlinear equation and so we cannot obtain the MNA stamp directly as we did for simpler devices. Instead, we need to apply Taylor series expansion in order to linearize this equation. This means that at every Newton iteration the MNA stamp corresponding to the BJT is updated. This can be done as follows. First using the Taylor series expansion for two variables and neglecting h.o.t, we have at the k-th Newton iteration

$$
I_C^{(k)} = f(V_{BE}^{(k)}, V_{BC}^{(k)}) = f(V_{BE}^{(k-1)}, V_{BC}^{(k-1)}) + (V_{BE}^{(k)} - V_{BE}^{(k-1)})\left.\frac{\partial f}{\partial V_{BE}}\right|_{(V_{BE}^{(k-1)}, V_{BC}^{(k-1)})} + (V_{BC}^{(k)} - V_{BC}^{(k-1)})\left.\frac{\partial f}{\partial V_{BC}}\right|_{(V_{BE}^{(k-1)}, V_{BC}^{(k-1)})},
$$

where the functions evaluated at $(V_{BE}^{(k-1)}, V_{BC}^{(k-1)})$ are already known from the previous iteration. That is,

$$
f(V_{BE}^{(k-1)}, V_{BC}^{(k-1)}) = I_C^{(k-1)}
$$

$$
\begin{aligned}
\left.\frac{\partial f}{\partial V_{BE}}\right|_{(V_{BE}^{(k-1)}, V_{BC}^{(k-1)})} &= \frac{I_S}{V_T}\left(1 - \frac{V_{BC}^{(k-1)}}{V_{AF}} - \frac{V_{BE}^{(k-1)}}{V_{AR}}\right)\exp\left(\frac{V_{BE}^{(k-1)}}{V_T}\right) - \frac{I_S}{V_{AR}}\left[\exp\left(\frac{V_{BE}^{(k-1)}}{V_T}\right) - \exp\left(\frac{V_{BC}^{(k-1)}}{V_T}\right)\right] \\
&\doteq h_{C1}^{(k-1)}
\end{aligned}
$$

$$
\begin{aligned}
\left.\frac{\partial f}{\partial V_{BC}}\right|_{(V_{BE}^{(k-1)}, V_{BC}^{(k-1)})} &= -\frac{I_S}{V_{AF}}\left[\exp\left(\frac{V_{BE}^{(k-1)}}{V_T}\right) - \exp\left(\frac{V_{BC}^{(k-1)}}{V_T}\right)\right] - \frac{I_S}{V_T}\left(1 - \frac{V_{BC}^{(k-1)}}{V_{AF}} - \frac{V_{BE}^{(k-1)}}{V_{AR}}\right)\exp\left(\frac{V_{BC}^{(k-1)}}{V_T}\right) \\
&\quad - \frac{I_S}{\beta_r V_T}\exp\left(\frac{V_{BC}^{(k-1)}}{V_T}\right) \doteq h_{C2}^{(k-1)}.
\end{aligned}
$$

Therefore, we get

$$
\begin{aligned}
I_C^{(k)} &= I_C^{(k-1)} + h_{C1}^{(k-1)}(V_{BE}^{(k)} - V_{BE}^{(k-1)}) + h_{C2}^{(k-1)}(V_{BC}^{(k)} - V_{BC}^{(k-1)}) \\
&= (h_{C1}^{(k-1)} + h_{C2}^{(k-1)})V_B^{(k)} - h_{C1}^{(k-1)}V_E^{(k)} - h_{C2}^{(k-1)}V_C^{(k)} + \underbrace{I_C^{(k-1)} - h_{C1}^{(k-1)}V_{BE}^{(k-1)} - h_{C2}^{(k-1)}V_{BC}^{(k-1)}}_{\doteq h_{C3}^{(k-1)}} \\
&= (h_{C1}^{(k-1)} + h_{C2}^{(k-1)})V_B^{(k)} - h_{C2}^{(k-1)}V_C^{(k)} - h_{C1}^{(k-1)}V_E^{(k)} + h_{C3}^{(k-1)}.
\end{aligned}
$$

Similarly for the base current, we have

$$
\begin{aligned}
I_B &= \frac{I_{B1}}{\beta_f} + \frac{I_{B2}}{\beta_r} \\
I_B &= I_S\left[\frac{1}{\beta_f}\left(\exp\left(\frac{V_{BE}}{V_T}\right) - 1\right) + \frac{1}{\beta_r}\left(\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right)\right]
\end{aligned}
$$

and at the k-th iteration we have

$$
I_B^{(k)} = g(V_{BE}^{(k)}, V_{BC}^{(k)}) = g(V_{BE}^{(k-1)}, V_{BC}^{(k-1)}) + (V_{BE}^{(k)} - V_{BE}^{(k-1)})\left.\frac{\partial g}{\partial V_{BE}}\right|_{(V_{BE}^{(k-1)}, V_{BC}^{(k-1)})} + (V_{BC}^{(k)} - V_{BC}^{(k-1)})\left.\frac{\partial g}{\partial V_{BC}}\right|_{(V_{BE}^{(k-1)}, V_{BC}^{(k-1)})},
$$

where

$$
g(V_{BE}^{(k-1)}, V_{BC}^{(k-1)}) = I_B^{(k-1)}
$$

$$
\left.\frac{\partial g}{\partial V_{BE}}\right|_{(V_{BE}^{(k-1)}, V_{BC}^{(k-1)})} = \frac{I_S}{V_T\beta_F}\exp\left(\frac{V_{BE}^{(k-1)}}{V_T}\right) \doteq h_{B1}^{(k-1)}
$$

$$
\left.\frac{\partial g}{\partial V_{BC}}\right|_{(V_{BE}^{(k-1)}, V_{BC}^{(k-1)})} = \frac{I_S}{V_T\beta_R}\exp\left(\frac{V_{BC}^{(k-1)}}{V_T}\right) \doteq h_{B2}^{(k-1)},
$$

and therefore, we get

$$
\begin{aligned}
I_B^{(k)} &= I_B^{(k-1)} + h_{B1}^{(k-1)}(V_{BE}^{(k)} - V_{BE}^{(k-1)}) + h_{B2}^{(k-1)}(V_{BC}^{(k)} - V_{BC}^{(k-1)}) \\
&= (h_{B1}^{(k-1)} + h_{B2}^{(k-1)})V_B^{(k)} - h_{B1}^{(k-1)}V_E^{(k)} - h_{B2}^{(k-1)}V_C^{(k)} + \underbrace{I_B^{(k-1)} - h_{B1}^{(k-1)}V_{BE}^{(k-1)} - h_{B2}^{(k-1)}V_{BC}^{(k-1)}}_{\doteq h_{B3}^{(k-1)}} \\
&= (h_{B1}^{(k-1)} + h_{B2}^{(k-1)})V_B^{(k)} - h_{B2}^{(k-1)}V_C^{(k)} - h_{B1}^{(k-1)}V_E^{(k)} + h_{B3}^{(k-1)}.
\end{aligned}
$$

And finally for the emitter current is given by

$$
\begin{aligned}
I_E^{(k)} &= I_C^{(k)} + I_B^{(k)} \\
&= (h_{C1}^{(k-1)} + h_{C2}^{(k-1)} + h_{B1}^{(k-1)} + h_{B2}^{(k-1)})V_B^{(k)} - (h_{C2}^{(k-1)} + h_{B2}^{(k-1)})V_C^{(k)} - (h_{C1}^{(k-1)} + h_{B1}^{(k-1)})V_E^{(k)} + h_{C3}^{(k-1)} + h_{B3}^{(k-1)}.
\end{aligned}
$$

Therefore, the BJT MNA stamp is

$$
\begin{array}{cc}
\begin{bmatrix}
\vdots & \vdots & \vdots \\
(h_{C1}^{(k-1)} + h_{C2}^{(k-1)}) & -h_{C2}^{(k-1)} & -h_{C1}^{(k-1)} \\
\vdots & \vdots & \vdots \\
(h_{B1}^{(k-1)} + h_{B2}^{(k-1)}) & -h_{B2}^{(k-1)} & -h_{B1}^{(k-1)} \\
\vdots & \vdots & \vdots \\
-(h_{C1}^{(k-1)} + h_{C2}^{(k-1)} + h_{B1}^{(k-1)} + h_{B2}^{(k-1)}) & (+h_{C2}^{(k-1)} + h_{B2}^{(k-1)}) & (+h_{C1}^{(k-1)} + h_{B1}^{(k-1)}) \\
\vdots & \vdots & \vdots
\end{bmatrix}
\begin{bmatrix}
\vdots \\ V_B^{(k)} \\ \vdots \\ V_C^{(k)} \\ \vdots \\ V_E^{(k)} \\ \vdots
\end{bmatrix}
-
\begin{bmatrix}
\vdots \\ -h_{C3}^{(k-1)} \\ \vdots \\ -h_{B3}^{(k-1)} \\ \vdots \\ +h_{C3}^{(k-1)} + h_{B3}^{(k-1)} \\ \vdots
\end{bmatrix}
&
\begin{array}{c}
\\ C \\ \\ B \\ \\ E \\ \\
\end{array}
\\
\begin{array}{ccc}
\qquad B \qquad\qquad & \qquad\qquad C \qquad\qquad & \qquad E \qquad
\end{array}
&
\end{array}
$$

You have to work on PNP device yourself.

Input format:

diode: dxxx anode-node cathode-node
bjt: qxxx collector-node base-node emitter-node polarity

ex. dty 2 0
qty out b 0 npn

An initial guess will be given to you in the following spice format:

.nodeset v(node1)=voltage1 v(node2)=voltage2 i(vsrc1)=current1 …

OR

.nodeset v(node1)=voltage1
+ v(node2)=voltage2
+ i(vsrc1)=current1
…

Example:
.nodeset v(supply)=5 v(cbias)=4.28 v(out)=0.092 v(b)=0.716 v(input)=0.9 i(vcc)=-2e-3 i(vin)=-1.83e-5

Output format:

You should output the initial voltage of each node and then your iteration procedure and finally your result. Note that there shouldn't be many iterations(definitely <20) otherwise it means your solution is not converging properly. The output should be similar to the following format:

Input nodes:
voltage at 0=***
voltage at 1=***
voltage at 2=***

Iteration begins:
iteration: 1, difference =***
iteration: 2, difference =***
iteration: 3, difference =***
Total number of iterations: 3

Result:
voltage at 0=***
voltage at 1=***
voltage at 2=***

*** is real number in the format of *.******E*** with 7 effective digits and in scientific notation

Difference is computed in the following way:
Vold is the vector of node voltages obtained by previous iteration, Vnew is the vector of node voltages obtained by current iteration.
Difference = $((Vnew - Vold)^T(Vnew - Vold))^{1/2}$
Iteration should terminate when the difference is smaller than 1E-10

You can calculate the difference using just the vector of voltages or also including the vector of currents. It shouldn't have any effect on your output because the reported result is only to 7 digits while the iteration is performed until the relative difference is accurate to 10 digits.