Utility Function Approach to Portfolio Selection Problem

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Abstract. In this paper portfolio selection problem is studied under premise that there are two basic approaches to determine an optimal portfolio: (1) mean-variance approach implemented into the Markowitz model; and (2) the concept of utility, which in regard to modern portfolio theory is defined in terms of portfolio risk and return. This paper focuses on these two approaches showing how they apply to single-period investment problem and demonstrating how they work together to produce strong and useful pricing relationships. The modification of the utility function based on the Markowitz model is proposed and an algorithm for investment portfolio optimization problem is developed. A modified utility maximization model is formulated in terms of portfolio variance and standard deviation. An illustrative experimental case-study is performed via 'Moscow Exchange' Trading Organizer. The investment portfolio is optimized on the basis of the modified utility maximization model in real time. It is shown that the introduced *k*-coefficient in the utility objective function can be considered as the coefficient of risk aversion. It is concluded that in contrast to the traditional Markowitz model, the proposed approach based on the utility function takes into account the investor's risk preferences and extends our understanding of the location of the efficient frontier in consideration of the parameters set for the investor.

Keywords: Utility function · Markowitz model · Investment portfolio · Mean-variance approach · Risk aversion

1 Introduction

The problem of portfolio optimization remains a widely discussed issue in investment analysis. With the securities market development the investment portfolio analysis has become one of the most topical issues nowadays as is demonstrated by many publications on this subject (see e.g. [1–3]). There are different formulations and models

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developed to synthesize investment portfolios. The most popular approaches for the portfolio modelling include the Markowitz mean-variance approach, the Tobin extension of the Markowitz model, the Sharpe index model based on 'beta'-factor, the Capital asset pricing model, the Arbitrage pricing theory and their multiple modifications (see e.g. [1, 4, 5]).

The efficiency of the application of these models depends on the availability of the information for investor, diversity of the available investment assets and the level of development of securities market. However, the problem of determining the optimal portfolio on the efficient frontier remains an open question for all models [10, 11]. An investor needs a procedure for ranking random wealth levels. The present state of the art of investment portfolio management demonstrates that a utility function provides such a procedure [6, 7].

One strand of the literature is based on applying the exponential utility function with Arrow-Pratt risk-aversion index, which is constant for exponential utility independent of wealth [8]. The reason for choosing exponential utility is the assumption of its equivalency to quadratic utility and mean-variance analysis.

Another portfolio selection approach is based on the family of negative power utility functions, also called constant relative risk aversion [8]. It should be pointed out that such criteria depend on risk-aversion parameter and the initial wealth as well, so it is volatile and does not produce a unique value for portfolio selection.

One of the recent approaches is the use of utility functions with hyperbolic absolute risk aversion (HARA). One of the strong points of this approach is that such utility functions allow the derivation of a generalized two-funds separation theorem thus leading to sample capital market evaluation formulas, and to the generalization of the Sharpe ratio and the Treynor ratio as well [9].

Nevertheless, utility function approaches, though important, are rather subjective. The degree of investor's risk-aversion and the selection of utility function remain discussed questions. It is known that the most common used and efficient approaches for portfolio synthesis is the Markowitz model which presumes that the risk of the portfolio is measured by the variance of the portfolio return and depends on the correlation between the assets, measured by the covariance [5]. In this respect the Markowitz model can be reasonable for utility function construction in terms of mean-variance ratio optimization.

The aim of this paper is to formalize the utility function for portfolio selection problem based on the Markowitz model and to evaluate its efficiency relatively to the traditional mean-variance optimization approach.

The paper is organized as follows. The concept of utility, which is defined in terms of portfolio return and risk is formulated in the second section. The modification of the utility function based on the Markowitz model is proposed and an algorithm for investment portfolio optimization problem is developed. A modified utility maximization model is formulated in terms of portfolio variance and standard deviation. In the third section an illustrative experimental case-study is performed via 'Moscow Exchange' Trading Organizer. The investment portfolio is optimized on the basis of the modified utility maximization model. The results of model verification are discussed in the fourth section. The main findings of the research are summarized in the conclusion.

2 Methods

2.1 The Modified Utility Function Based on the Markowitz Model

In modern portfolio theory, the central place is occupied by the concept of utility, which is defined in terms of return and variance as follows [4]:

$$U = R_p - k\sigma_p^2 \tag{1}$$

where R_p is the expected return on the portfolio p, k is the coefficient that measures the degree of risk aversion of the investor, σ_p^2 is the variance of the portfolio p.

Such an objective function corresponds to the "expected value-variance" criterion used in decision-making theory as one of the criteria for evaluating a decision alternative [4].

As noted, variance and standard deviation are adequate measures of risk for symmetric distributions. Meanwhile, for distributions of portfolio returns that differ from normal, the utility function is formulated in a generalized form, taking into account the central moments of higher orders. In this case, the influence of the moments of higher orders on the value of the total utility is less significant than the influence of the moments of lower orders.

If skewness and kurtosis are important for an investor, then his utility function is formulated as follows:

$$U = R_p - k_1\sigma_p^2 - k_2 S_p^3 - k_3 S_p^4 \tag{2}$$

where, S_p^3, S_p^4 are the coefficients of skewness and kurtosis of the portfolio return p, k_1, k_2, k_3 are the coefficients of aversion to variance, skewness and excess, respectively. So, if a positive skewness of the distribution of returns ($S_p^3 > 0$) is preferable for an investor, then the contribution of asymmetry ($k_2 S_p^3$) increases the overall utility of the portfolio for a given investor, and, therefore, the coefficient of aversion to skewness should be set negative ($k_2 < 0$). Note that (1) is a special case of (2), which takes place for normal distributions ($S_p^3 = S_p^4 = 0$).

When choosing the parameter k, it is important to observe the investor's preferences and the requirement of non-negativity of the utility function: $U(k) \geq 0$. Thus, k is selected from the range: $k_{min} < k < k_{max}$, where k_{min} represents the lower bound of the values of k acceptable for the investor, depending on his preferences, k_{max} is the upper acceptable bound k, at which the value of the objective function is non-negative.

The prerequisites for the formulation of the modified Markowitz problem for portfolio selection are as follows [1, 10]:

1. Investors evaluate investment portfolios based on expected returns and standard deviations over the holding period.
2. Investors are never jaded, that is, other conditions being equal, they prefer a portfolio with a higher expected return.
3. Investors are rational, that is, other conditions being equal, they prefer a portfolio with less risk.

4. The portfolio consists only of risky assets, the risk measure of which is the standard deviation of return.
5. Investors assume a normal probability distribution of the expected return on assets. The portfolio return aggregates the returns on the securities included in it.
6. Private assets are infinitely divisible, that is, an investor can buy part of a share.
7. Taxes and transaction costs are immaterial.

The mean return of a portfolio of n assets is obtained as:

$$R_p = \sum\nolimits_{i=1}^{n} w_i \cdot \overline{R}_i, \tag{3}$$

where w_i is the weight of the asset i in the portfolio, $\overline{R}_i$ is the mean return on the asset i.

In respect that the risk of the portfolio is measured by the variance of the portfolio return and depends on the correlation between the assets measured by the covariance [Mark] the variance of a portfolio return is defined as [4]:

$$\sigma_p^2 = \sum\nolimits_{ij} w_i \cdot w_j \cdot Cov_{ij} \tag{4}$$

where w_i and w_j are the weights of the assets i and j in the portfolio, respectively, Cov_{ij} is the covariance between two assets i and j.

Hence, we modify the utility function (1) using (4) as follows:

$$U = \sum w_i \cdot \overline{R}_i - k \cdot \sum w_i \cdot w_j \cdot Cov_{ij} \tag{5}$$

The modified utility function can be formulated in terms of standard deviation as well. The initial prerequisites for the formulation of the utility function based on the Markowitz model will be prerequisites (1)–(7) and the following extra condition:
8. The distribution of returns is symmetric; the standard deviation is an adequate measure of risk.

In terms of standard deviation, the utility function (1) is assumed in the form:

$$U = \sum w_i \cdot \overline{R}_i - k \cdot \sqrt{\sum w_i \cdot w_j \cdot Cov_{ij}} \tag{6}$$

2.2 Utility Maximization Problem for Portfolio Selection

The main purpose of a utility function is to provide a systematic way to rank alternatives that captures the principle of risk aversion. The initial formulation of the Markowitz problem is to minimize (maximize) the variance (the mean return) of (on) the portfolio under fixed value R (σ^2) of the mean return (variance). That is [5]:

$$J_{Markowitz} = \sum\nolimits_{ij} w_i \cdot w_j \cdot Cov_{ij} \to min \tag{7}$$

subject to

$$\sum\nolimits_{i} w_i \cdot \overline{R_i} = R \tag{8}$$

$$\sum\nolimits_i w_i = 1$$

$$w_i \geq 0,\ i = \overline{1, n}$$

.

or

$$J_{Markowitz} = \sum w_i \cdot \overline{R}_i \rightarrow max \tag{9}$$

subject to

$$\sum\nolimits_{ij} w_i \cdot w_j \cdot Cov_{ij} = \sigma^2,\ i = \overline{1, n} \tag{10}$$

$$\sum\nolimits_i w_i = 1,\ i = \overline{1, n}.$$

$$w_i \geq 0,\ i = \overline{1, n}.$$

The solution of the problem yields the optimal weight coefficients w_i^* for the assets in the portfolio.

Hence, we formulate the utility maximization problem based on the Markowitz model using (5):

$$J_{(R-k\sigma^2)} = \sum w_i \cdot \overline{R}_i - k \cdot \sum w_i \cdot w_j \cdot Cov_{ij} \rightarrow max \tag{11}$$

subject to

$$\sum\nolimits_i w_i = 1,\ i = \overline{1, n} \tag{12}$$

$$w_i \geq 0,\ i = \overline{1, n}$$

$$\alpha_k \leq w_k \leq \beta_k,\ 0 \leq \alpha_k < 1,\ 0 < \beta_k \leq 1,\ k \in \overline{1, n},$$

where w_i is the weight of the asset i in the portfolio, $\overline{R}_i$ is the expected return on the asset i, k is the model parameter set in the range $k_{min} < k < k_{max}$; α_k and β_k are the boundaries of the interval of acceptable values of the asset weight k, for which there is a consideration of choosing certain weight values.

The solution to this problem yields weight coefficients $w^*_{(R-k\sigma^2)}$ for the assets in the portfolio.

In terms of standard deviation, we formulate the following utility maximization problem using (6):

$$J_{(R-k\sigma)} = \sum w_i \cdot \overline{R}_i - k \cdot \sqrt{\sum w_i \cdot w_j \cdot Cov_{ij}} \rightarrow max \tag{13}$$

subject to

$$\sum_i w_i = 1,\ i = \overline{1, n} \tag{14}$$

$$w_i \geq 0,\ i = \overline{1, n}.$$

$$\alpha_k \leq w_k \leq \beta_k,\ 0 \leq \alpha_k < 1,\ 0 < \beta_k \leq 1,\ k \in \overline{1, n}.$$

The solution to this problem produces weight coefficients $w^*_{(R-k\sigma)}$ for the assets in the portfolio.

The problems (11), (12) and (13), (14) are related to quadratic programming problems, since the objective function is quadratic and the constraints are linear [4]. This problem can be solved, for example, in MatLab and Excel software environment.

Solver Tool can be used to optimize portfolio in Excel. We will enter the portfolio utility into the target cell, and we will set the asset weights as the changing cells. In the constraints, we will set the sum of the asset weights, and constraints on the specific asset weights in the portfolio. To get an efficient set of portfolios, we will use the Visual Basic for Application add-in through recording a macros. The input parameters of the model are mean returns on the assets $\overline{R}_i$, obtained from series of moving stock returns R_{it}, variance-covariance matrix Cov_{ij}, assessing covariance between each pair of assets i and j, and risk aversion coefficient k acceptable for an investor.

2.3 Analysis of the *k*-Parameter in the Modified Utility Maximization Problem

In this section we estimate the influence of the coefficient k on the value of the objective utility function in problems (11), (12) and (13), (14). If k is set equal to zero, the objective function will reach the maximum possible value, which corresponds to the maximum expected return on the portfolio, and therefore to the one hundred percent share of investment in the asset with the maximum expected return (provided $\alpha_k = 0$, $\beta_k = 1$, $k = \overline{1, n}$). Obviously, a further increase of k-value will inevitably lead to a decrease in the objective function, since the desire to achieve a higher expected return will be constrained by the limitation of the second element of the objective utility function. Thus, we presume the dependence of the objective function on k as is conceptually shown in Fig. 1.

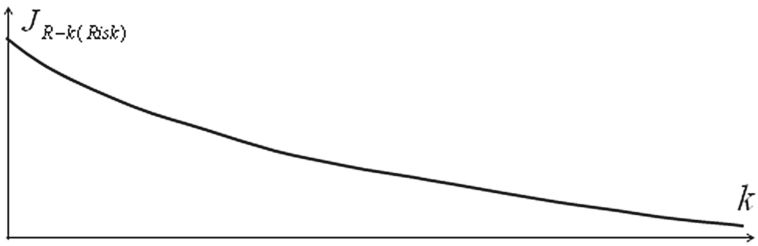

Fig. 1. Influence of the k coefficient on the value of the objective utility function.

To analyze the influence of the coefficient k on the parameters of the portfolio, let us trace how the shares of distribution in assets will change. It was found that zero k corresponds to a 100% investment in the most profitable asset. If we assume that as k increases, preference will continue to be given to the highest-yielding assets, we find that this would entail a sharp decrease in the objective function, since high-yielding assets entail a higher risk. Obviously, in order to avoid a sharp decline in the objective function, it will be necessary to reduce the portfolio risk, albeit at the expense of high returns. We conclude that with an increase in k, the share of investments will gradually redistribute from more profitable to less profitable assets, which will entail a decrease in the total return and risk of the portfolio. We forecast the dependance of the portfolio return on the k coefficient as is presented in Fig. 2. Thus, we consider the coefficient k as the degree of risk aversion of the investor.

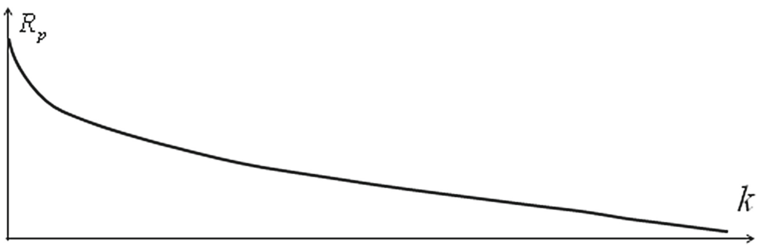

Fig. 2. Influence of the k coefficient on the portfolio return.

Note that it makes sense to consider the range of k values, $k \in [0; k']$, within which the utility, that is, the value of the objective function, is non-negative, that is $U(k) \geq 0$.

It is also advisable to distinguish the areas of values of k, in which the values of the portfolio parameters remain practically unchanged, and those areas for which there is a significant change in the values of the parameters of the portfolio. This emphasis is significant, since the changing values of return and risk describe the main part of the effective frontier of portfolios. Figure 3 shows the range of values of k and the corresponding sections of the efficient frontier.

It is notable that the major part of the efficient set corresponds to the interval $[k_1; k_2]$. Whereas on intervals $[0; k_1)$ and $(k_2; k']$ the parameters of the portfolio do not change substantially. Thus, the value of the parameter k should be selected from the interval $[k_1; k_2]$.

When setting the parameter k, it is important to take into account the investor's preference for risk. In this case, in order to take into account the investor's preferences regarding risk, it is advisable to subdivide the interval $[k_1; k_2]$ into a risk-aversion zone and a risk-preference zone. The zone of risk aversion corresponds to the values of k at which low values of standard deviation and return are obtained. The zone of risk preference will include the values of k, which correspond to high values of the standard deviation and portfolio return. These statements require experimental verification.

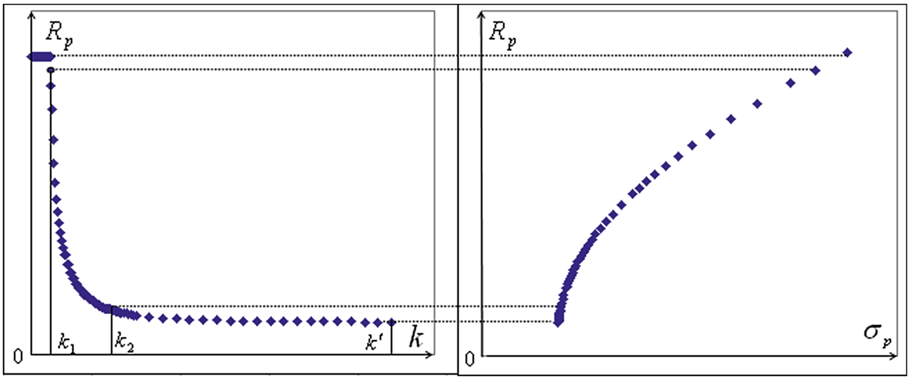

Fig. 3. Ranges of values of the k coefficient and sections of the efficient frontier.

3 Results

The 'Moscow Exchange' was chosen as the information basis for conducting this part of the experimental study as the oldest organizer and the leading trading platform for the securities market in Russia [12].

An important factor in the choice of the 'Moscow Exchange' was the open access to trading information; the prices that are formed on the 'Moscow Exchange' are a generally accepted guideline for investors making transactions with Russian stocks and depositary receipts. Trading information is broadcast to a huge number of consumers in Russia and abroad and is the basis for calculating the RTS and MICEX stock indices.

For further analysis and synthesis of the portfolio, diverse assets have been be selected that represent various sectors of the economy according to the chosen principle of branch diversification: electricity, oil and gas, financial sector, communications and telecommunications, metallurgical, air transportation, consumer sector. Preference is to be given to securities with good growth potential and liquidity.

Thus, six assets were selected that meet the listed characteristics of the following issuers: Public Joint stock company "ROSSETI" (ordinary share *RSTI*), Public Joint-Stock Company VTB Bank (ordinary share *VTBR*), Public Joint Stock Company "Mining and Metallurgical Company "NORILSK NICKEL" (ordinary share *GMKN*), Public Limited Liability Company Yandex N.V., shares of a foreign issuer (ordinary share *YNDX*), Public Joint Stock Company "Aeroflot-Russian Airlines" (ordinary share *AFLT*), Mobile TeleSystems Public Joint Stock Company (ordinary share *MTSS*) [12].

For the further analysis we selected one-year period. The price series of assets were transformed into series of moving returns with a smoothing window of one month. Monthly moving returns have been obtained for the one-year period, namely January, 01, 2020 – December 31, 2020. The initial parameters of the assets are presented in Table 1.

Variance-covariance matrix is given in Table 2.

Table 1. Initial statistic parameters

Asset	Asset parameters		
	$\overline{R}_i$	σ_i	σ_i^2
GMKN	1,023189	0,096822	0,009375
MTSS	1,006183	0,066658	0,004443
YNDX	1,062315	0,107592	0,011576
VTBR	1,058802	0,832589	0,693204
AFLT	1,002309	0,029359	0,000862
RSTI	1,035651	0,151427	0,022838

Table 2. Covariance matrix for the selected assets

Asset	Cov_{ij}					
	GMKN	MTSS	YNDX	VTBR	AFLT	RSTI
GMKN	0,009337	0,004135	0,002326	0,011534	0,000525	0,009864
MTSS	0,004135	0,004425	0,003937	0,009764	0,000862	0,007330
YNDX	0,002326	0,003937	0,011529	0,007151	0,000655	0,005283
VTBR	0,011534	0,009764	0,007151	0,690431	0,000995	0,024477
AFLT	0,000525	0,000862	0,000655	0,000995	0,000821	0,000875
RSTI	0,009864	0,007330	0,005283	0,024477	0,000875	0,022838

The portfolio optimization problem (7), (8) was solved using estimates of expected returns $\overline{R}_i$ from Table 1 and covariance matrix from Table 2. The solution of the $(w_i w_j Cov_{ij})$ - minimization problem yields the optimal weight coefficients w_i^* for portfolios from the efficient frontier, represented in Table 3.

In this part of the experiment, the portfolio is synthesized using the modified utility maximization models (11), (12) and (13), (14).

To solve utility maximization problems, the parameters k were set taking into account the nonnegativity of the objective function. For the corresponding k, weight distributions $w^*_{(R-k\sigma^2)}$, $w^*_{(R-k\sigma)}$, were obtained. It is notable that the obtained weight distributions $w^*_{(R-k\sigma^2)}$, and $w^*_{(R-k\sigma)}$ are close to each other: the difference in the minimum values of the weights does not exceed 0.4%, and the difference in the maximum values of the weights does not exceed 0.77%. The $w^*_{(R-k\sigma^2)}$ distribution is close to the distribution of the Markowitz problem: the difference in the minimum values of the weights does not exceed 0.02%, and the difference in the maximum values of the weights does not exceed 1.73%. The $w^*_{(R-k\sigma)}$ distribution is also close to the w^* distribution: the difference in the minimum values of the weights does not exceed 0.4%, and the difference in the maximum values of the weights does not exceed 1.73%.

Table 3. Portfolio parameters and weight distribution w_i^*

Portfolio return	Standard deviation	Weights of the assets					
		GMKN	MTSS	YNDX	VTBR	AFLT	RSTI
1,0410	0,008535	0,00000	0,00000	0,00000	0,00000	0,0173	0,9827
1,0400	0,006863	0,00000	0,00000	0,00000	0,00000	0,11831	0,88169
1,0390	0,005423	0,00000	0,00000	0,00000	0,00000	0,21941	0,78059
1,0380	0,004214	0,00000	0,00000	0,00000	0,00000	0,32052	0,67948
1,0370	0,003237	0,00000	0,00000	0,00000	0,00000	0,42172	0,57828
1,0360	0,002492	0,00000	0,00000	0,00000	0,00000	0,52273	0,47727
1,0350	0,001978	0,02683	0,00000	0,00000	0,00000	0,5577	0,41547
1,0340	0,001696	0,07572	0,00000	0,00000	0,00000	0,52285	0,39038
1,0330	0,001604	0,12021	0,02903	0,00000	0,00000	0,48914	0,36162
1,0320	0,001540	0,16475	0,04703	0,00000	0,00000	0,4554	0,33282
1,0310	0,001480	0,20921	0,06512	0,00000	0,00000	0,42172	0,30408
1,0300	0,001424	0,32608	0,10768	0,02489	0,00000	0,31901	0,22234
1,0290	0,001371	0,32484	0,09755	0,07761	0,00000	0,28996	0,21004
1,0280	0,001321	0,32362	0,08742	0,13033	0,00000	0,2609	0,19775
1,0270	0,001274	0,32236	0,07728	0,18305	0,00000	0,23185	0,18546
1,0260	0,001230	0,32117	0,06715	0,23576	0,00000	0,20277	0,17315
1,0250	0,001190	0,31996	0,05701	0,28847	0,00000	0,1737	0,16086
1,0240	0,001153	0,31864	0,04688	0,34122	0,00000	0,14468	0,14858
1,0230	0,001119	0,31741	0,03675	0,39394	0,00000	0,11563	0,13628
1,0220	0,001088	0,31616	0,02662	0,44666	0,00000	0,08657	0,12399
1,0210	0,001060	0,31492	0,01649	0,49937	0,00000	0,05752	0,1117
1,0200	0,001034	0,31369	0,00635	0,5521	0,00000	0,02846	0,0994
1,0190	0,001011	0,31094	0,00000	0,60461	0,00000	0,00000	0,08445
1,0180	0,000991	0,28597	0,00000	0,65878	0,00000	0,00000	0,05525

The dependence of the parameters of the portfolio and the objective utility function on the parameter k is analyzed. The dependences of the parameters of the portfolio R_p, σ_p, σ_p^2 and objective utility functions $(R_p - k\sigma_p^2)$, $(R_p - k\sigma_p)$ on the k coefficient are shown in Figs. 4, 5, 6, 7, 8 and 9.

According to the results of the experimental study, presented in Figs. 4, 5, 7, 8 the return and variance (standard deviation) curves may be seen as consisting of three parts: interval k, on which the risk and return values change insignificantly; interval k, at which portfolio risk and return change considerably; interval k, at which the parameters of the portfolio tend to a constant value. In this case, the objective function decreases from k,

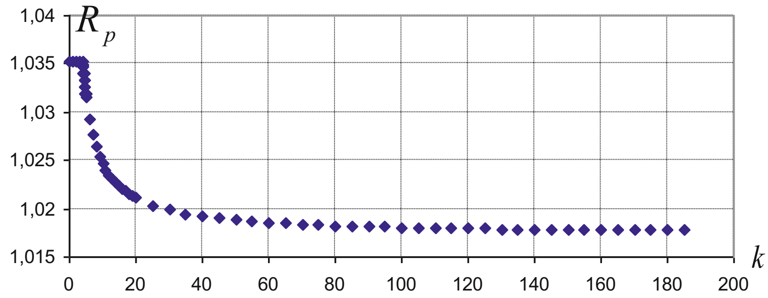

Fig. 4. Dependence of the portfolio return R_p on k coefficient for the $(R_p - k\sigma_p^2)$-maximization model

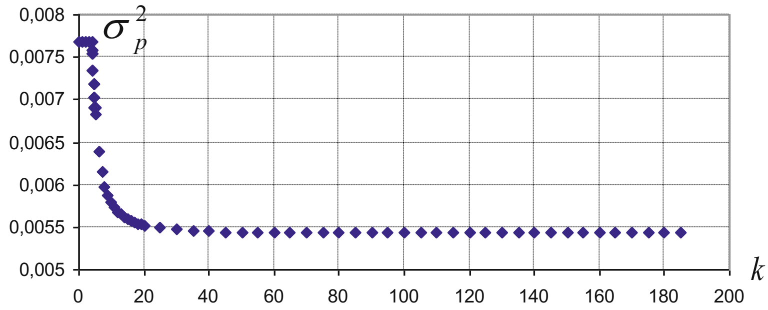

Fig. 5. Dependence of the portfolio variance σ_p^2 on k coefficient for the $(R_p - k\sigma_p^2)$-maximization model

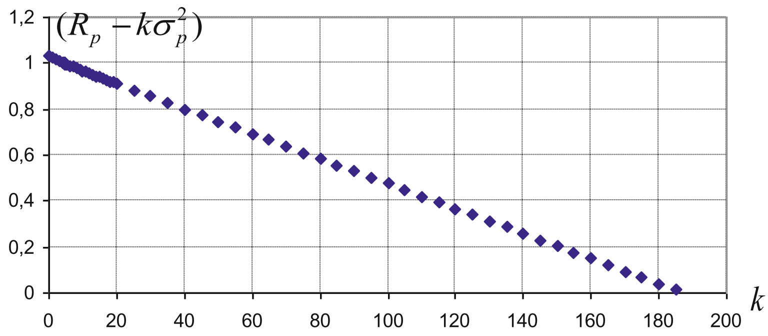

Fig. 6. Dependence of the objective $(R_p - k\sigma_p^2)$-utility function on k coefficient.

which confirms the assumptions made in Sect. 2. Thus, the coefficient k can be regarded as the coefficient of risk aversion.

The interval values of R_p, σ_p^2 and R_p, σ_p for different k-intervals of risk aversion coefficient for models $\left(R_p - k \cdot \sigma_p^2\right)$, $(R_p - k \cdot \sigma_p)$, are shown in Table 4 and Table 5, respectively.

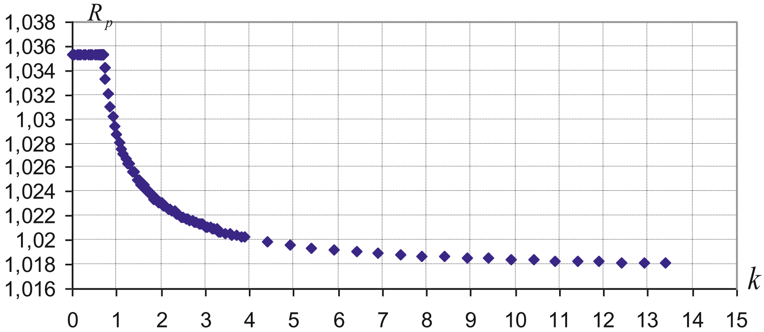

Fig. 7. Dependence of the portfolio return R_p on k coefficient for the $(R_p - k\sigma_p)$-maximization model

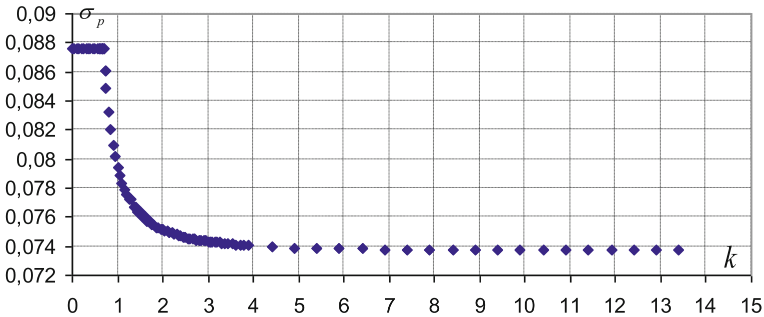

Fig. 8. Dependence of the portfolio risk σ_p on k coefficient for the $(R_p - k\sigma_p)$-maximization model

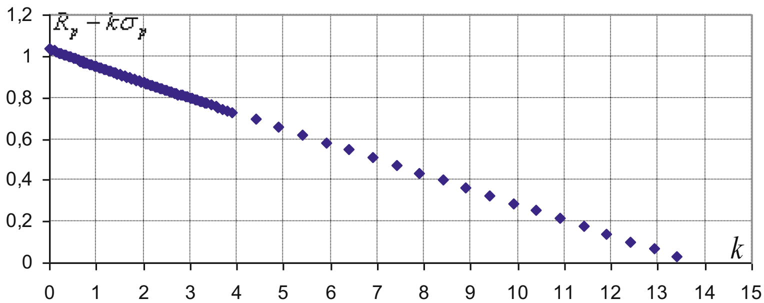

Fig. 9. Dependence of the objective $(R_p - k\sigma_p)$-utility function on k coefficient.

We recommend to set the coefficient k from the interval within which there is a significant change in the parameters of the portfolio. According to the results of the experimental study, these interval values for k are evaluated as [0.71; 3.1], [4.01; 20] for models $(R_p - k \cdot \sigma_p)$, $(R_p - k \cdot \sigma_p^2)$, respectively.

Table 4. Interval values of R_p and σ_p^2 for $\left(R_p - k \cdot \sigma_p^2\right)$-model

k-interval	Portfolio parameters	
	R_p	σ_p^2
[0; 4]	1.0353263	0,007675682
[4.01; 20]	[1.021131, 1.03495]	[0.005523, 0.007581]
[21; 185]	[1.017734, 1.020364]	[0.005428, 0.005488]

Table 5. Interval values of R_p and σ_p for $\left(R_p - k \cdot \sigma_p\right)$-model

k-interval	Portfolio parameters	
	R_p	σ_p
[0; 0.7]	1,0353263	0,087611
[0.71; 3.1]	[1.021095, 1.034242]	[0.074304, 0.087611]
[3.15; 13.4]	[1.018112, 1.020911]	[0.073697, 0.074244]

It should be pointed out that the portfolio parameters found for different k, corresponding to the maximum utility, form an efficient set similar to the Markowitz efficient frontier as shown in Fig. 10.

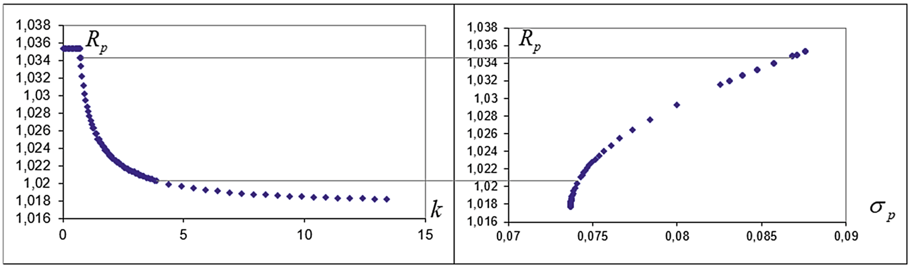

Fig. 10. Mean-standard deviation diagram for portfolios optimized on the basis of $(R_p - k\sigma_p)$-utility function for k-coefficient.

4 Discussions

It has been shown that an alternative to the risk-minimization approach is the objective utility function maximization. In Markowitz's problem, modified taking into account the utility function, the maximum value of this function corresponds to the trade-off between an increase in the expected return and a decrease in risk, specified by the value of the coefficient k. Unlike the risk minimization problem, the utility maximization problem does not require the introduction of an additional constraint on the expected portfolio return.

Another difference in the statement of the utility maximization problem is the introduced parameter k, which we consider as the degree of investor's aversion to risk: low values of k correspond to high values of portfolio return and risk; setting a higher value of k entails lower profitability and risk. It is advisable to set the parameter k from a range of values $k_{min} < k < k_{max}$, which makes it possible to take into account the investor's preferences regarding risk and to comply with the condition of non-negativity of the objective function for different measures of risk. We recommend choosing a value of k from the interval that corresponds to the main part of the effective set. For the case study under consideration these interval k-values have been determined as [0.7; 3.1], [4.01; 20] for models $(R_p - k \cdot \sigma_p)$, $(R_p - k \cdot \sigma_p^2)$, respectively.

It has been shown that the portfolio parameters found for different k, corresponding to the maximum utility, form an efficient set similar to the Markowitz efficient frontier. In contrast to the traditional Markowitz model, the proposed approach based on the utility function allows us to take into account the investor's risk preferences and extends our understanding of the location of the efficient frontier, taking into account the parameters set for the investor.

Note that the utility maximization problem is formulated depending on the accepted risk measure. For symmetric distributions, the problem is formulated as $(R_p - k\sigma_p^2)$ - or $(R_p - k\sigma_p)$-maximization. To take into account the left-sided deviations of asymmetric distributions, this approach should be developed for alternative risk measures taking into account non-normal distributions and substantial downside tail risk.

5 Conclusion

In this paper an alternative to the risk-minimization approach based on the objective utility function maximization is proposed.

The utility function is modified using the Markowitz model for portfolio risk assessment. It has been shown that the maximum value of this function corresponds to the trade-off between an increase in the expected return and a decrease in risk, specified by the value of the coefficient k. Unlike the risk minimization problem, the utility maximization problem does not require the introduction of an additional constraint on the expected portfolio return.

The results of experimental study demonstrate that the k coefficient can be considered as the coefficient of risk aversion: an increase in k leads to a decrease in the risk and return of the portfolio. The weight distributions obtained when solving the risk minimization problems are close to the distributions obtained when solving the problems of maximizing the objective utility function using the appropriate risk measures. It is recommended to set the coefficient k from the interval within which there is a significant change in the parameters of the portfolio.

To the best of our knowledge the portfolio parameters found for different k, corresponding to the maximum utility, form an efficient set similar to the Markowitz efficient frontier. In contrast to the traditional Markowitz model, the proposed approach based on the utility function allows us to take into account the investor's risk preferences and extends our understanding of the location of the efficient frontier, taking into account the parameters set for the investor.

It should be pointed out that the obtained results best apply to symmetric portfolio distributions. It remains to be settled if the excellency of utility maximization approach based on the Markowitz model holds for non-normal distributions setting, when asset and portfolio returns might exhibit substantial downside tail risk. This issue is left for further research.

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