Final Exam	Name:	
Linear Algebra	Class time: 12:00-12:50 pm MWF	100 points
Winter 2014	Instructor: Nate Iverson	Score:
4/28/2014	Section: 01	

Instructions: Show your work for full credit. Staple this sheet to the top of your work for each problem. Make sure each part is labelled and in the same order as this sheet. Good Luck :)

1. (4 points each) Rank Nullity Theorem
   (a) For a linear transformation $T : V \to W$ state the rank-nullity theorem.
   (b) If $T : \mathbb{R}^7 \to \mathbb{R}^3$ is a linear transformation that has $\dim \operatorname{null}(T) = 5$, what is $\operatorname{rank}(T)$?
   (c) Is it possible for a linear transformation $T : \mathbb{R}^7 \to \mathbb{R}^3$ to be one-to-one?
   (d) If $T : V \to W$ is a linear transformation with $\operatorname{rank}(T) = 4$ and $\dim \operatorname{null}(T) = 5$ find $\dim(V)$.

2. (6 points each) $T : \mathbb{R}^2 \to \mathbb{R}^4$ is defined by $T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a-b \\ b-a \\ 2a \\ a-3b \end{pmatrix}$
   (a) Show T is a linear transformation.
   (b) Find the standard matrix for T
   (c) Is T one-to-one? (Why?)
   (d) Is T onto? (Why?)

3. (60 points) Diagonalization
   (a) $A = \begin{pmatrix} -1 & 4 \\ 9 & -1 \end{pmatrix}$
      i. Find all the eigenvalues of A.
      ii. For each eigenvalue find a basis for the corresponding eigenspace.
      iii. Is A diagonizable? If yes, find diagonal matrix D, invertible matrix P and inverse P^{-1} such that $A = PDP^{-1}$. If no, state why not.
   (b) $B = \begin{pmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$
      i. Find all the eigenvalues of B.
      ii. For each eigenvalue find a basis for the corresponding eigenspace.
      iii. Is B diagonizable? If yes find diagonal matrix D, invertible matrix P and inverse P^{-1} such that $B = PDP^{-1}$. If no, state why not.