ET6501701
Homework #1

Date: February 28, 2024.
Due Date: March 20, 2024. **Instructor: M. B. Lin**

Please note that **NO late homework** will be accepted.

1. Prove that any sorting network on n inputs has depth at least $\lg n$.
2. Prove that the number of comparators in any sorting network is $\Omega(n \lg n)$.
3. Applying the following keys to the shuffling-based Batcher's odd-even merge network, answer the following questions:
   (a) Keys: A E Q S U Y E I N O S T.
   (b) Keys: 1 0 0 1 1 1 0 0 0 0 0 1 0 1 0 0.
4. Considering the sorting network SORTER[n], answer the following questions:
   (a) How many comparators are there in SORTER[n]?
   (b) Show that the depth of SORTER[n] is exactly $(\lg n)(\lg n + 1)/2$.
5. Give sorting networks for four, five, and six elements. Use as few comparators as possible.
6. The conversion of a Gray codeword into its binary equivalent can be carried out as in the following operations:

$$\begin{aligned} b_{n-1} &= g_{n-1} \\ b_i &= b_{i+1} \oplus g_i \qquad \text{where } 0 \le i \le n-2 \end{aligned}$$

   (a) Represent each b_i as a function of input g only, where $g = (g_{n-1} \cdots g_1 g_0)$.
   (b) Realize (a) with possible binary trees for each b_i. What is the time and space complexity of (a)?
   (c) Write each b_i as a prefix-sum expression.
   (d) Implement (c) with the Ladner-Fischer parallel-prefix network. Compare the time and space complexity with (a).
   (e) Implement (c) with the Brent-Kung parallel-prefix network. Compare the time and space complexity with (a) and (d).