ET6501
Homework #2

Date: March 20, 2005.
Due Date: April 17, 2005.   **Instructor: M. B. Lin**

Please note that **NO late homework** will be accepted.

1. Answer each the following questions:

   (a) Prove that if $a > b > 0$ and $c = a + b$, then $c \bmod a = b$.

   (b) Prove that if $a \mid b$ and $b \mid c$, then $a \mid c$.

2. Compute the values (d, x, y) that the call EXTENDED-EUCLID(899, 493) returns.

3. Prove that if a and b are any positive integers such that $a \mid b$, then

$$(x \bmod b) \bmod a = x \bmod a$$

   for any x. Prove, under the same assumptions, that

$$x = y \pmod{b} \text{ implies } x = y \pmod{a}$$

   for any integers x and y.

4. Prove that if p is prime and $0 < k < p$, then $p \mid \begin{pmatrix} p \\ k \end{pmatrix}$. Calculate that for all integers a, b and primes p,

$$(a + b)^p = a^p + b^p \pmod{p}$$

5. Answer the following questions:

   (a) Find all solutions to the equation $35x = 10 \pmod{50}$.

   (b) Find all solutions to the equations $x = 4 \pmod{5}$ and $x = 5 \pmod{11}$.