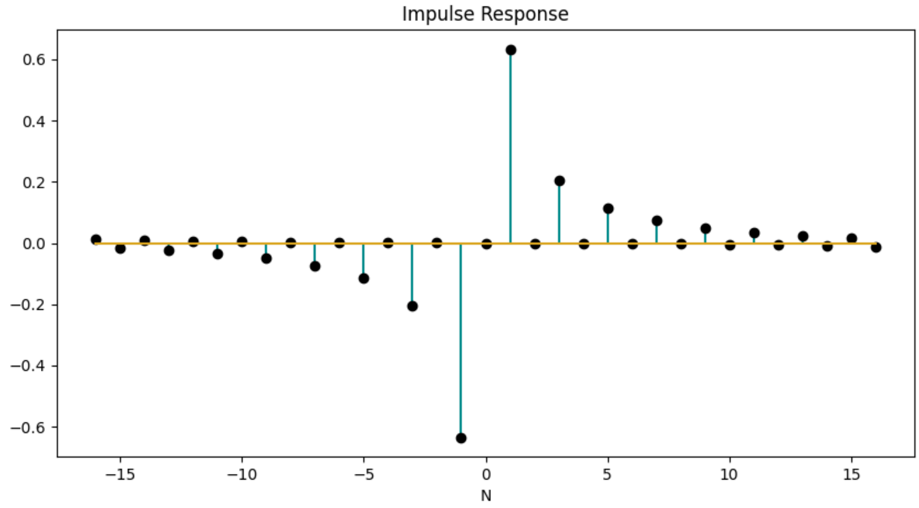
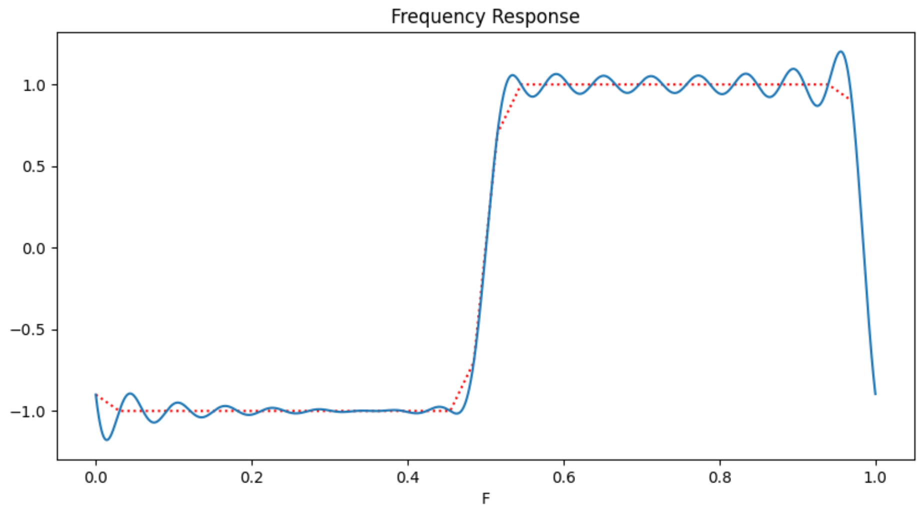
(1) Write a Matlab or Python code that uses the frequency sampling method to design a $(2k+1)$-point discrete Hilbert transform filter (k is an input parameter and can be any integer). (25 scores)

The transition band is assigned to reduce the error (unnecessary to optimize). (i) The impulse response and (ii) the imaginary part of the frequency response (DTFT of $r[n]$, see pages 113 and 114) of the designed filter should be shown in the homework. The code should be handed out by NTU Cool.





(2) Estimate the length of the digital filter if both the passband ripple and the stopband ripple are smaller than 0.01, the sampling interval Δ_t = 0.00005, and the transition band is from 5000Hz to 6000Hz. (10 scores)

$$\Delta t = 0.00005 \rightarrow f_s = 20000$$

$$\Delta F = (6000 - 5000)/20000 = 0.05$$

$$\delta_1 = 0.01, \; \delta_2 = 0.01$$

$$N = \frac{2}{3} \frac{1}{0.05} \log \frac{1}{10(0.01)^2} = 40 \; \#$$

(3) Why it is improper to use the method of $y[n]$ = IDFT(DFT($x[n]$)$H[m]$) for FIR filter design? (5 scores)

Sol:主要原因是 Large computation loading，時間複雜度達到 O(NlogN)，所以隨著 input 增加，計算量會非常龐大

(4) Derive the way to use the algorithm on page 58-61 to implement an odd symmetric filter with even length (i.e., type 4 on page 90). (10 scores)

When $h[n] = -h[N-1-n]$ and N is even

n^{th} term and $(n+1)^{th}$ term

$$-\sin(2\pi(n-\tfrac{1}{2})F) + \sin(2\pi(n+\tfrac{1}{2})F) = 2\sin(\pi F)\cos(2\pi nF) \quad \beta=\pi F,\ \alpha=2\pi nF$$

$$R(F) = \sin(\pi F)\sum_{n=0}^{k_1} s_1[n]\cos(2\pi nF)$$

$$= -\sum_{n=0}^{k_1}\tfrac{1}{2}s_1[n]\left(\sin(2\pi(n-\tfrac{1}{2})F)\right)+\sum_{n=0}^{k_1}\tfrac{1}{2}s_1[n]\sin(2\pi(n+\tfrac{1}{2})F)$$

$$= -\sum_{n=0}^{k_1}\tfrac{1}{2}s_1[n]\left(\sin(2\pi(n-\tfrac{1}{2})F)\right)+\sum_{n=1}^{k_1+1}\tfrac{1}{2}s_1[n-1]\left(\sin(2\pi(n-\tfrac{1}{2})F)\right)$$

$$= -\tfrac{1}{2}s_1[0]\sin(\pi F)-\sum_{n=1}^{k_1}\tfrac{1}{2}(s_1[n]-s_1[n-1])\sin(2\pi(n-\tfrac{1}{2})F)+\tfrac{1}{2}s_1[k_1]\sin(2\pi(k_1+\tfrac{1}{2})F)$$

$$= -\tfrac{1}{2}s_1[1]\sin(\pi F)-\sum_{n=2}^{k-\frac{1}{2}}\tfrac{1}{2}(s_1[n]-s_1[n-1])\sin(2\pi(n-\tfrac{1}{2})F)+\tfrac{1}{2}s_1[k-\tfrac{1}{2}]\sin(2\pi kF)$$

$$s[1] = -\tfrac{1}{2}s_1[1]$$

$$s[n] = -\tfrac{1}{2}(s_1[n]-s_1[n-1]) \text{ for } n=2,3,\ldots,k-\tfrac{1}{2}$$

$$s[k+\tfrac{1}{2}] = \tfrac{1}{2}s_1[k-\tfrac{1}{2}]$$

$$err(F) = [R(F)-H_d(F)]W(F)$$

$$= \left[\sin(\pi F)\sum_{n=0}^{k_1} s_1[n]\cos(2\pi nF)-H_d(F)\right]W(F)$$

$$= \left[\sum_{n=0}^{k_1} s_1[n]\cos(2\pi nF)-\csc(\pi F)H_d(F)\right]\sin(\pi F)W(F)$$

$\rightarrow$ $H_d(F)$ 換成 $\csc(\pi F)H_d(F)$

$W(F)$ 換成 $\sin(\pi F)W(F)$

k 換成 $k-\frac{1}{2}=\frac{N}{2}-1$

(5) Suppose that $x[n] = 1+\sin(n)$. (a) What is the Hilbert transform of $x[n]$?
(b) What is the analytic function corresponding to $x[n]$? (10 scores)

a. $x[n] = 1 + \sin(n) = 1 + \sin(2\pi n f_0) \quad f_0 = \frac{1}{2\pi}$

$$X(F) = \delta(F) + \frac{1}{2j}\delta(F-1) - \frac{1}{2j}\delta(F+1)$$

$$X_H(F) = -\frac{1}{2}\delta(F-1) - \frac{1}{2}\delta(F+1)$$

$$X_H[n] = -\frac{1}{2}\exp(j\,2\pi n) - \frac{1}{2}\exp(-j\,2\pi n)$$

$$= -\cos(2\pi n)$$

b. $X_a[n] = (1+\sin(n)) + j(-\cos(2\pi n))$

(6) Among the following filters: (i) the Notch filter (ii) the Hilbert transform, (iii) the matched filter, (iv) the difference, (v) the Kalman filter, (vi) the particle filter, and (vii) the Wiener filter,

(a) Which filters are suitable for edge detection? (b) Which filters are suitable for prediction? (10 scores)

Sol:

a. (ii),(iv)

b. (v),(vi)

(7) (a) What are the <u>two main advantages</u> of the minimum phase filter? (b) Compared to the equalizer, what are the <u>two main advantages</u> of the cepstrum to deal with the multipath problem? (10 scores)

Sol:

a.1.Minimum phase filter 是確保了 foeword 是 stable 且 causal。

2.也確保了在 inverse 是 stable 且 causal，且能讓能量 concentrating 在 n=0 附近。

b.1.不須測量不同 path 的 delay，直接由 cepstrum 濾除雜訊。

2.相比之下 Equalizer H(z)可能 unstable 且通常是 dynamic response。

(8) If the z-transform of $h[n]$ is $H(z) = \dfrac{1+z^{-1}-1.5z^{-2}+z^{-3}}{1-0.3z^{-1}-0.4z^{-2}}$

(a) Determine the cepstrum of $h[n]$.

(b) Convert the IIR filter into the minimum phase filter. (20 scores)

(a) $H(z) = \dfrac{1+z^{-1}-1.5z^{-2}+z^{-3}}{1-0.3z^{-1}-0.4z^{-2}}$

$$= \frac{(1-(-2)z^{-1})(1-(0.5+\frac{i}{2})z^{-1})(1-(0.5-\frac{i}{2})z^{-1})}{(1-0.8z^{-1})(1-(-0.5)z^{-1})}$$

$$h[n] = \begin{cases} \log(1)=0, & n=0 \\ -\frac{(-2)^n}{n}-\frac{(\frac{1+i}{2})^n}{n}-\frac{(\frac{1-i}{2})^n}{n}+\frac{(0.8)^n}{n}+\frac{(-0.5)^n}{n}, & n>0 \\ 0, & n<0 \end{cases}$$

(b) $n<0$ 為0

$\because$ minimum phase filter為

$$\frac{(1-(-2)z^{-1})(1-(0.5+\frac{i}{2})z^{-1})(1-(0.5-\frac{i}{2})z^{-1})}{(1-0.8z^{-1})(1-(-0.5)z^{-1})}$$

$$= \frac{(z+0.5)(z-1+i)(z-1-i)}{-0.4(z-1.25)(z+2)}$$

(Extra): Answer the questions according to your student ID number.
(ended with (4, 9), (0, 5), (1, 6), (2, 7))

when $f = -10000$
$m = ?$

$$-10{,}000 = m \frac{40{,}000}{400{,}000} - 40{,}000$$

$$30{,}000 = \frac{m}{10}$$

$$300{,}000 = m \ \#$$