Homework 4 (Due: 5/29th)

(1) Write a Matlab or Python program to measure the structural similarity (SSIM) of two images A and B. The sizes of A and B are equivalent.

$$\text{SSIM(A, B, c1, c2)}$$

where c1 and c2 are some adjust constants.

The Matlab or Python code should be handed out by NTUCool.   (20 scores)

(2) Suppose that the probabilities of Chinese characters can be modeled as

$$P[n] = \left(\exp(0.002) - 1\right)\exp(-0.002n) \qquad n = 1, 2, 3, \cdots\cdots, 80000$$

(a) Determine the entropy of the Chinese characters. (b) Estimate the range of the coding length if we use the Huffman code to encode 10^5 Chinese characters using binary numbers. (c) Estimate the range of the coding length if we use the arithmetic code to encode 10^5 Chinese characters using binary numbers.

(15 scores)

(3) Suppose that x is a complex number. What are the constraints of θ such that the multiplication of x and $\exp(j\,\theta)$ required only 2 real multiplications?

(10 scores)

(4) What is the complexity of the $M{\times}N{\times}P$-point 3D DFT? The deriving process should be given. (10 scores)

(5) How do we implement the 4-point DST-I with the lest number of nontrivial multiplications? The number of real multiplications should also be shown.

$$X[m] = \sum_{n=1}^{4} \sin\left(\frac{\pi}{5}mn\right)x[n] \qquad \begin{matrix} m = 1, 2, 3, 4 \\ n = 1, 2, 3, 4 \end{matrix} \qquad \text{(15 scores)}$$

$$\begin{bmatrix} X[1] \\ X[2] \\ X[3] \\ X[4] \end{bmatrix} = \begin{bmatrix} a & b & b & a \\ b & a & -a & -b \\ b & -a & -a & b \\ a & -b & b & -a \end{bmatrix} \begin{bmatrix} x[1] \\ x[2] \\ x[3] \\ x[4] \end{bmatrix} \qquad a = 0.5878, \quad b = 0.9511$$

(Hint: we can convert it into two 2x2 matrices.)

(6) Determining the numbers of real multiplications for the (a) 143-point DFT, (b) 195-point DFT, and the (c) 196-point DFT. (15 scores)

(7) Derive the transform matrices of the (a) forward and (b) inverse 5-point NTTs where the prime number M is 11 and the primitive root α should be as small as possible. (15 scores)

(Extra): Answer the questions according to your student ID number.

(ended with (2, 7), (3, 8), (4, 9), (0, 5))