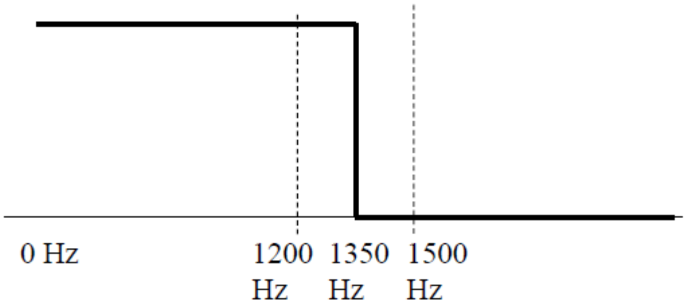
(1) Design a Mini-max **lowpass** FIR filter such that (40 scores)

① Filter length = 17, ② Sampling frequency f_s = 6000Hz,

③ Pass Band 0~1200Hz ④ Transition band: 1200~1500 Hz,

⑤ Weighting function: $W(F) = 1$ for passband, $W(F) = 0.6$ for stop band .

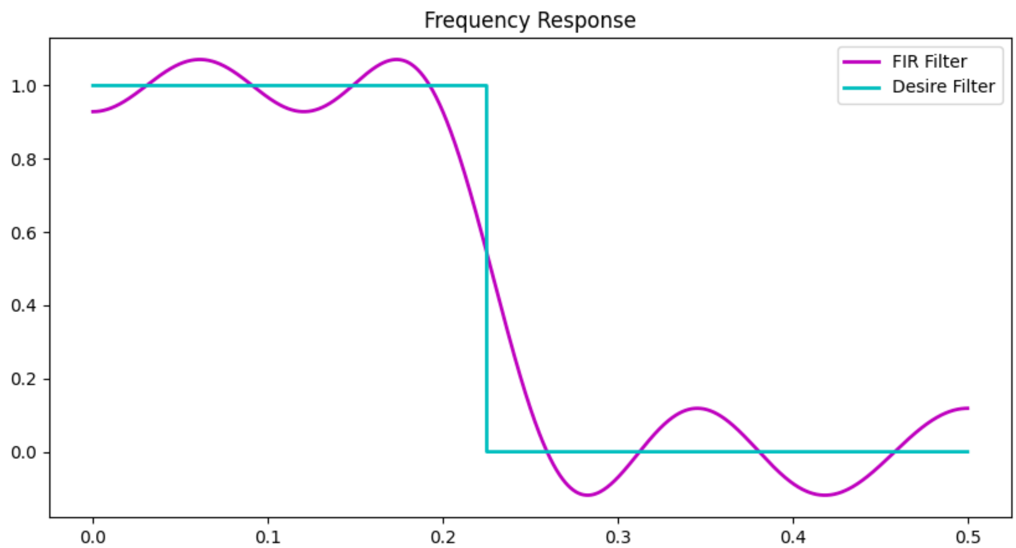
⑥ Set $\Delta = 0.0001$ in Step 5.



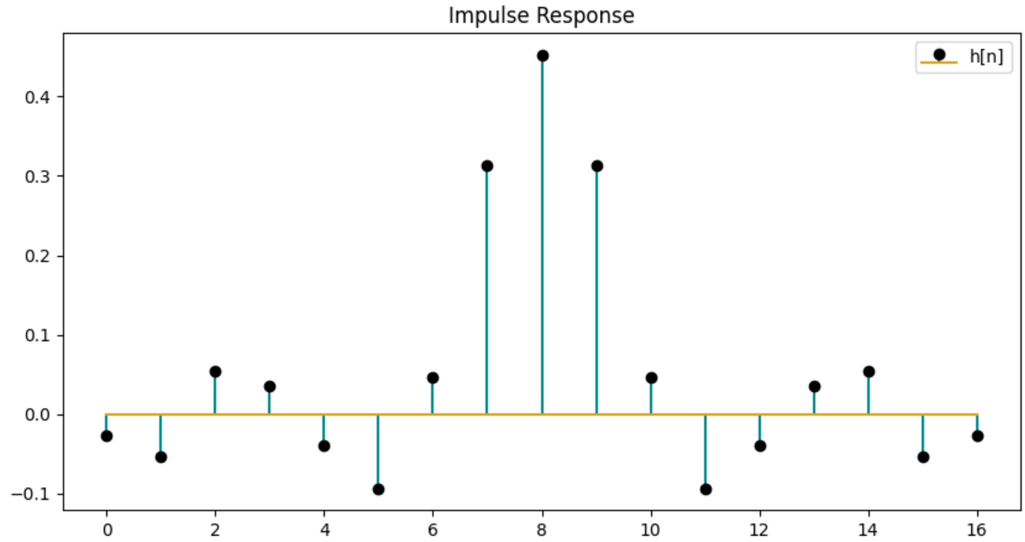
※ The code should be handed out by NTUCool, too.

Show (a) the frequency response, (b) the impulse response $h[n]$, and (c) the maximal error for each iteration.

a.



b.



c.

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Iteration 1 - Maximal Error: 0.20056979196292035
Iteration 2 - Maximal Error: 0.226271521374527
Iteration 3 - Maximal Error: 0.31123624376473447
Iteration 4 - Maximal Error: 0.49951410150514736
Iteration 5 - Maximal Error: 0.08466982879340709
Iteration 6 - Maximal Error: 0.07123793676848646
Iteration 7 - Maximal Error: 0.07120728523467912
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(2) How do we implement $y[n] = x[n] *(0.8^n u[n] + 0.5^n u[n])$ efficiently where * means convolution and $u[n]$ is the unit step function? (10 scores)

$Z[n] = 0.8^n u[n] + 0.5^n u[n]$, for $n \geq 0$,否則 $Z[n]=0$

$$Z(z) = \sum_{n=-\infty}^{\infty} Z[n] z^{-n} = \sum_{n=0}^{\infty} (0.8^n z^{-n} + 0.5^n z^{-n}) = \sum_{n=0}^{\infty} 0.8^n z^{-n} + \sum_{n=0}^{\infty} 0.5^n z^{-n}$$

$$= \frac{1}{1-0.8z^{-1}} + \frac{1}{1-0.5z^{-1}} = \frac{2 - 1.3z^{-1}}{(1-0.8z^{-1})(1-0.5z^{-1})}$$

$$Y(z) = X(z)Z(z) = X(z) \frac{2 - 1.3z^{-1}}{(1-0.8z^{-1})(1-0.5z^{-1})}$$

$$(1-0.8z^{-1})(1-0.5z^{-1})Y(z) = X(z)(2 - 1.3z^{-1})$$

$$\Big\Downarrow Z^{-1}$$

$$Y[n] - 1.3Y[n-1] + 0.4Y[n-2] = 2X[n] - 1.3X[n-1]$$

$$Y[n] = 1.3Y[n-1] - 0.4Y[n-2] + 2X[n] - 1.3X[n-1] \quad \#$$

(3) (a) What are the two main advantages of the Fourier transform (FT)? (b)What are the two main problems to implement the FT? (10 scores)

Sol:

a1. 可以用來spectrum analysis

a2. convolution to multiplication

b1. not real operation

b2. irrational number multiplication

(4) Suppose that $x[n] = y(0.002n)$ and the length of $x[n]$ is 2000. If $X[m]$ is the FFT of $x[n]$, which frequencies do (a) $X[200]$ and (b) $X[1600]$ correspond to? (10 scores)

$$f_s = \frac{1}{0.002} = 500\,\text{Hz} \qquad N = 2000$$

$$\text{(a)}\ X[200] \rightarrow f = 200 \times \frac{500}{2000} = 50\,\text{Hz}\ \#$$

$$\text{(b)}\ X[1600] \rightarrow \frac{N}{2} = 1000,\ 1600 > 1000$$

$$\therefore f = 1600 \times \frac{500}{2000} - 500 = -100\,\text{Hz}\ \#$$

(5) Why (a) the step invariance method and (b) the bilinear transform can reduce or avoid the aliasing effect in IIR filter design? (10 scores)

sol:

(a)因為step invariance透過捲積->積分方式將高頻的能量壓下，所以能夠"reduce" 在高頻產生的aliasing effect

(b) 因為藉由tan inverse function將−∞~∞的訊號mapping到-fs/2~fs/2，所以the bilinear transform can fully avoid the aliasing effect in IIR filter design

(6) (a) Which of the following filters are usually even? (b) Which of the following filters are usually odd? (i) Notch filter; (ii) highpass filter; (iii) edge detector; (iv) integral; (v) differentiation 4 times; (vi) particle filter; (vii)matched filter. (10 scores)

sol:

a. (i) Notch filter (ii) highpass filter (v) differentiation 4 times

b. (iii)edge detector (iv) integral

(7) Use the MSE method to design the 7-point FIR filter that approximates the highpass filter of *Hd*(F) = 1 for |*F*| < 0.25 and *Hd*(F) = 0 for 0.25 < |*F*| < 0.5. (15 scores)

$$S[0] = \int_{-\frac{1}{2}}^{\frac{1}{2}} H_d(F)dF = 0.5$$

$$S[n] = 2\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\pi nF)\, H_d(F)\, dF$$

$$S[1] = 2\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos(2\pi F) H_d(F) dF$$

$$= 2\int_{-0.25}^{0.25} \cos(2\pi F) dF$$

$$= 2\left[\sin(2\pi F)\frac{1}{2\pi}\right]_{-0.25}^{0.25}$$

$$= \frac{1}{\pi}\left[\sin(0.5\pi) - \sin(-0.5\pi)\right]$$

$$= \frac{2}{\pi}\sin(0.5\pi) = 0.6366$$

$$S[2] = 2\int_{-0.25}^{0.25} \cos(4\pi F) dF$$

$$= 2\left[\sin(4\pi F)\frac{1}{4\pi}\right]_{-0.25}^{0.25}$$

$$= \frac{1}{\pi}\sin(\pi) = 0$$

$$S[3] = 2\int_{-0.25}^{0.25} \cos(6\pi F) dF$$

$$= 2\left[\sin(6\pi F)\frac{1}{6\pi}\right]_{-0.25}^{0.25}$$

$$= \frac{2}{3\pi}\sin(1.5\pi) = -0.2122$$

$$S[n] = 2\, r[n]$$

$\Rightarrow$

n	0	1	2	3
S[n]	0.5	0.6366	0	-0.2122
r[n]	0.5	0.3183	0	-0.1061

$$h[0] = -0.1061 = h[6]$$

$$h[1] = 0 = h[5]$$

$$h[2] = 0.3183 = h[4]$$

$$h[3] = 0.5 \quad \#$$

(Extra): Answer the questions according to your student ID number.
Q: Suppose that $x[n] = y(n/6000)$ and the length of $x[n]$ is 30000.
If f=-150 Hz, m=?(Hint:m>=0).

$$-150 = m\frac{6000}{30000} - 6000$$

$$\frac{1}{5}m = 5850$$

$$m = 29250 \ \#$$