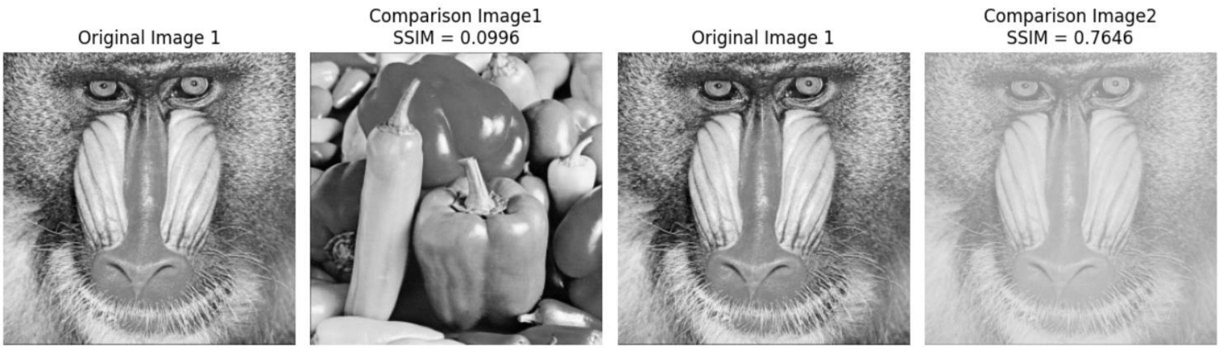
(1) Write a Matlab or Python program to measure the structural similarity (SSIM) of two images A and B. The sizes of A and B are equivalent.

SSIM(A, B, c1, c2)

where c1 and c2 are some adjust constants.

The Matlab or Python code should be handed out by NTUCool. (20 scores)

Sol:



左邊是 Result1，右邊是 Result2，SSIM 算出來右邊的相似度比較高

(2) Suppose that the probabilities of Chinese characters can be modeled as

$$P[n] = (\exp(0.002) - 1)\exp(-0.002n) \qquad n = 1, 2, 3, \cdots\cdots, 80000$$

(a) Determine the entropy of the Chinese characters. (b) Estimate the range of the coding length if we use the Huffman code to encode 10^5 Chinese characters using binary numbers. (c) Estimate the range of the coding length if we use the arithmetic code to encode 10^5 Chinese characters using binary numbers.

(15 scores)

Sol:

(a) entropy $= \sum_{j=1}^{J} P(S_j) \ln \frac{1}{P(S_j)}$ P: probability

$= \sum_{n=0}^{80000} (e^{0.002} - 1)e^{-0.002n} \ln \frac{1}{(e^{0.002}-1)e^{-0.002n}}$

=7.2172 #

(b) $\text{ceil}(\text{N}\frac{entropy}{lnk}) \le \text{b} \le \text{floor}(\text{N}\frac{entropy}{lnk} + \text{N})$

$= \text{ceil}(10^5\frac{7.2172}{ln2}) \le \text{b} \le \text{floor}(10^5\frac{7.2172}{ln2} + 10^5)$

=> $1041222 \le \text{b} \le 1141221$ #

(c) $\text{ceil}(N\frac{entropy}{lnk}) \leq b \leq \text{floor}(N\frac{entropy}{lnk} + \log_k 2 + 1)$

$= \text{ceil}(10^5\frac{7.2172}{ln2}) \leq b \leq \text{floor}(10^5\frac{7.2172}{ln2} + 1 + 1)$

$=> 1041222 \leq b \leq 1041223$ #

(3) Suppose that x is a complex number. What are the constraints of θ such that the multiplication of x and $\exp(j\ \theta)$ required only 2 real multiplications?

(10 scores)

Sol:

$\exp(j\theta) = \overset{c}{\cos\theta} + j\overset{d}{\sin\theta}$, $x = a + jb$

if $\cos\theta = \sin\theta \Rightarrow \theta = \frac{\pi}{4} \Rightarrow c = d$

$$\begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} c & -c \\ c & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} c & c \\ c & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 0 & -2c \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

1 MUL + 1 MUL = 2 MUL ※

if $\theta = \frac{\pi}{6}$, $\cos\theta$ or $\sin\theta \Rightarrow$ 其中一個為 0

$$\begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} c & c \\ c & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} 0 & -c \\ -c & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

1 MUL + 1 MUL = 2 MUL ※

$\therefore \theta = \frac{i}{6}\pi$ or $\theta = \frac{i}{4}\pi$ where $i \in \mathbb{N}$

(4) What is the complexity of the $M \times N \times P$-point 3D DFT? The deriving process should be given. (10 scores)

Sol:

Complexity of the 1-D N-point DFT=O(NlogN)

Complexity of the 2-D MN-point DFT:

M(NlogN)+N(MlogM)=MN(logN+logM)=MNlog(MN)

Complexity of the 3-D MNP-point DFT:

PM(NlogN)+PN(MlogM)+MN(PlogP)=MNP(logN+logM+logP)

=MNPlog(MNP)

⇨ O(MNPlog(MNP))

(5) How do we implement the 4-point DST-I with the lest number of nontrivial multiplications? The number of real multiplications should also be shown.

$$X[m]=\sum_{n=1}^{4}\sin\left(\frac{\pi}{5}mn\right)x[n] \qquad \begin{matrix} m=1,2,3,4 \\ n=1,2,3,4 \end{matrix} \qquad \text{(15 scores)}$$

$$\begin{bmatrix} X[1] \\ X[2] \\ X[3] \\ X[4] \end{bmatrix} = \begin{bmatrix} a & b & b & a \\ b & a & -a & -b \\ b & -a & -a & b \\ a & -b & b & -a \end{bmatrix} \begin{bmatrix} x[1] \\ x[2] \\ x[3] \\ x[4] \end{bmatrix} \qquad a=0.5878, \quad b=0.9511$$

(Hint: we can convert it into two 2x2 matrices.)

Sol:

$$\begin{bmatrix} X[1] \\ X[3] \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} x[1]+x[4] \\ -x[2]-x[3] \end{bmatrix} \Rightarrow \text{Case4} \Rightarrow \text{3MULs}$$

$$\begin{bmatrix} X[2] \\ X[4] \end{bmatrix} = \begin{bmatrix} b & -a \\ a & b \end{bmatrix} \begin{bmatrix} x[1]-x[4] \\ -x[2]+x[3] \end{bmatrix} \Rightarrow \text{Case4} \Rightarrow \text{3MULs}$$

共6個MULs

(6) Determining the numbers of real multiplications for the (a) 143-point DFT, (b) 195-point DFT, and the (c) 196-point DFT. (15 scores)

Sol:

a.143=11*13

MUL143=11MUL13+13MUL11=11*52+13*40=1092

b.195=13*15

MUL195=13MUL15+15MUL13=13*40+15*52=1300

c.196=4*49

MUL196=4MUL49+49MUL4

=4*(7MUL7+7MUL7+3*36)+49*0

=4*332=1328

(7) Derive the transform matrices of the (a) forward and (b) inverse 5-point NTTs where the prime number M is 11 and the primitive root α should be as small as possible. (15 scores)

Sol:

$M=11,\ N=5,$

$g=2:\ g^1=2\ (\text{mod } 11)\quad g^2=4\ (\text{mod } 11)\quad g^3=8\ (\text{mod } 11)\quad g^4=5\ (\text{mod } 11)$

$g^5=10\ (\text{mod } 11)\quad g^6=9\ (\text{mod } 11)\quad g^7=7\ (\text{mod } 11)\quad g^8=3\ (\text{mod } 11)$

$g^9=6\ (\text{mod } 11)\quad g^{10}=1\ (\text{mod } 11)$

$\alpha=g^2=4 \Rightarrow g^2,\ g^4,\ g^6,\ g^8$

$\alpha,\ \alpha^2,\ \alpha^3,\ \alpha^4$

$4,\ 5,\ 9,\ 3$

(a) forward : NTT

$$W=\begin{bmatrix} W^0 & W^0 & W^0 & W^0 & W^0 \\ W^0 & W^1 & W^2 & W^3 & W^4 \\ W^0 & W^2 & W^4 & W^6 & W^8 \\ W^0 & W^3 & W^6 & W^9 & W^{12} \\ W^0 & W^4 & W^8 & W^{12} & W^{16} \end{bmatrix} \Rightarrow NTT=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 4 & 5 & 9 & 3 \\ 1 & 5 & 3 & 4 & 9 \\ 1 & 9 & 4 & 3 & 5 \\ 1 & 3 & 9 & 5 & 4 \end{bmatrix}$$

(b) Inverse : INTT

$N^{-1} \times 5 \equiv 1 \pmod{11}$

$N^{-1} = 9$

$4^{-1} = 3 \pmod{11}$

$\alpha^{-1} = \beta = 3 \quad \beta^{1} = 3 \pmod{11} \; \beta^{2} = 9 \pmod{11} \; \beta^{3} = 5 \pmod{11}$

$\beta^{4} = 4 \pmod{11} \; \beta^{5} = 1 \pmod{11}$

$$W^{-1} = N^{-1} \begin{bmatrix} w^0 & w^0 & w^0 & w^0 & w^0 \\ w^0 & w^1 & w^2 & w^3 & w^4 \\ w^0 & w^2 & w^4 & w^6 & w^8 \\ w^0 & w^3 & w^6 & w^9 & w^{12} \\ w^0 & w^4 & w^8 & w^{12} & w^{16} \end{bmatrix} \Rightarrow \text{INNT} = 9 \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 5 & 4 \\ 1 & 9 & 4 & 3 & 5 \\ 1 & 5 & 3 & 4 & 9 \\ 1 & 4 & 5 & 9 & 3 \end{bmatrix}$$

$$\Rightarrow \text{INNT} = \begin{bmatrix} 9 & 9 & 9 & 9 & 9 \\ 9 & 5 & 4 & 1 & 3 \\ 9 & 4 & 3 & 5 & 1 \\ 9 & 1 & 5 & 3 & 4 \\ 9 & 3 & 1 & 4 & 5 \end{bmatrix}$$

EXTRA:

MUL400 = 16MUL25+25MUL16

=16*148+25*20

=2868 #