Homework 1 (Due: March 19th)

(1) Find the solutions of the following nonlinear DEs. (30 scores)

(a) $y''(x)y'(x) = 1, \quad y'(0) = 0$

(b) $y''(x) = -3y'(x)y^2(x), \quad y(1) = 2^{-1/2}, \quad y'(1) = -2^{-3/2}$

(c) $y''(x) = \exp(y(x)), \quad , y(0) = 0 \quad y'(0) = \sqrt{2}$

(2) Solve the following PDEs. (40 scores)

(a) $x^2 \dfrac{\partial}{\partial x} u(x, y) = y \dfrac{\partial}{\partial y} u(x, y)$

(b) $\dfrac{\partial^2}{\partial x^2} u(x, y) = u(x, y) + \dfrac{\partial}{\partial y} u(x, y) \qquad 0 < x < 2, \quad y > 0,$

$u(0, y) = u(2, y) = 0, \quad u(x, 0) = \cos(\pi x)\sin(2\pi x)$

(c) $\dfrac{\partial^2}{\partial x^2}u(x,y)+\dfrac{\partial^2}{\partial y^2}u(x,y)=0$ $\qquad 0<x<1,\quad 0<y<1,$

$\qquad u(0,y)=u(1,y)=u(x,0)=0,\quad u(x,1)=1-2|x-1/2|$

(d) $\quad (x+1)\dfrac{\partial}{\partial x}u(x,y)=\dfrac{\partial}{\partial y}u(x,y)+\cos y$

(3) Solve the following 1$^{\text{st}}$ order nonlinear DE numerically. <u>Plot the result $y(x)$</u>. The Matlab (or Python) code should also be handed out.

(30 scores)

$$\dfrac{\partial y(x)}{\partial x}=5\cos\left(-\tfrac{1}{5}|xy|\right),\quad y(0)=0,\quad 0\le x\le 10,\quad x_{n+1}-x_n=0.01$$

(a) By Euler's method.

(b) By modified Euler's method.

(c) By the RK4 method.