Homework 2 (Due: April 16th)

(1) Solve the following PDEs. (30 scores)

(a) $$\frac{\partial u(x,y,z)}{\partial x} + y\frac{\partial u(x,y,z)}{\partial y} + z^2\frac{\partial u(x,y,z)}{\partial z} = 0$$

(b) $$\frac{\partial u(x,y,z)}{\partial x} + \frac{\partial u(x,y,z)}{\partial y} + \frac{\partial u(x,y,z)}{\partial z} = x + y + z$$

(c) $$\frac{\partial^2 u(x,y,t)}{\partial x^2} + \frac{\partial^2 u(x,y,t)}{\partial y^2} = \frac{\partial u(x,y,t)}{\partial t} \qquad 0 \le x \le 2, \quad 0 \le y \le 2, \quad t \ge 0$$

$$u(0,y,t) = u(2,y,t) = u(x,0,t) = u(x,2,t) = 0$$

$$u(x,y,0) = \left(2x - x^2\right)\left(2y - y^2\right)$$

(2) Solve the steady temperature $u(r, \theta)$ in a fan-shape plane where

$$0 \le r \le 1, \quad 0 \le \theta \le \pi/3,$$

$$u(1,\theta) = \sin\left(6\theta\right) + \sin\left(12\theta\right), \quad 0 < \theta < \pi/3$$

$$u(r,0) = 0, \quad u(r,\pi/3) = 0, \quad 0 < r < 1$$

(10 scores)

(3) Solve the steady temperature $u(r, z)$ in a cylinder region where

$$0 \le r \le 1, \quad -1 \le z \le 1,$$

$$u(1, z) = z \ \text{ for } 0 < z < 1, \quad u(1, z) = 2 - z \ \text{ for } 1 < z < 2,$$

$$u(r, 0) = 0, \ \ u(r, 2) = 0 \ \ 0 < r < 1$$

Suppose that $u(r, z)$ is independent of θ. (10 scores)

(4) Solve the following PDE by the 1-sided Laplace transform. (10 scores)

$$\frac{\partial u(x,t)}{\partial x} + \frac{\partial u(x,t)}{\partial t} = 1, \quad x > 0, \quad t > 0$$

$$u\left(0,t\right) = t^2 + t, \quad u\left(x,0\right) = 0$$

(Hint): When solving the ODE of x, the constant may be a function of s.

(5) (a) Convert 1, x, and x^2 into an orthonormal function set for $x \in [0, 4]$.

(b) Approximate $\min(x, 4\text{-}x)$ by a 2nd order polynomial with the least mean square error for $x \in [0, 4]$. (20 scores)

(6) Suppose that there is a set of five 'discrete' basis.

$$b_k[n] = n^k \qquad n = -6, -5, \cdots, 5, 6$$
$$k = 0, 1, 2, 3, 4$$

(a) Write a code to use the Gram-Schmidt method to convert $b_k[n]$ (k = 0~4) into an orthonormal set.

(b) Write a code to use the Gram-Schmidt method to convert $b_k[n]$ (k = 0~4) into an orthonormal set if the weight is $w[n] = 1 - |n|/7$.

The codes should be handed out by NTUCool. (20 scores)