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Discerning Physical Properties of the Moon to Verify the Collision Theory

Abstract

In this paper, we identify the likelihood of the collision theory on the formation of the Earth's moon through optical means. This is done by using CCDs and a 14 inch Cassegrain Schmidt telescope to collect optical data of the Moon and a reference star, specifically Gamma Andromedae. With these images, we discern the radius of the Moon in order to find its density. We also use this data to find the luminosity of the Moon and ultimately its albedo. We compare the density of the Moon to that of the Earth's crust and upper mantle to verify that the Moon is likely composed of the same materials. We also compare the Moon's albedo to that of materials that constitute the Earth's mantle and find that the values are close. In order to approximate a minimum age of the Moon, we calculate the depths of a few craters in order to estimate the impactor size and the probable time period of the impact of such a size. Because the density of the Moon is very close to that of the Earth that would have been ejected into space, its albedo is close to the materials that probably constitute it, and the time period suggested by the crater sizes is consistent with Late Heavy Bombardment, we concluded that the collision theory is the most reasonable explanation for the Moon's presence.

1 Introduction

The purpose of this report is to verify the collision theory on the formation of the Moon which states that the Moon is composed of the material from a young, still-forming Earth that was released into space after a major collision about 4 billion years ago. We anticipate this theory to be correct, as opposed to the theory that the Moon was a satellite captured by Earth's gravity, which would suggest the Moon has a composition very different from that of Earth's crust and upper mantle. Although we do not have samples of Moon rock in order to verify the age of the Moon, we use data available through optical means to calculate crater depths and approximate the time period these impacts would have occured. We use this data also to calculate the radius, density, and albedo of the Moon which allow us to discern its composition.

2 Materials and Methods

2.a Finding the Radius

In order to find the radius of the Moon, we used the UNM campus observatory 14 inch Schmidt Cassegrain reflecting telescope and imaged the Moon using a CCD when it was 97% illuminated by the Sun on November 2nd. Because of the angular size of the Moon within this telescope, we could not look at the entire Moon within a single image. Instead, we calculated the chord length of several images of the Moon using SAO DS9. Using Photoshop CS, we projected the images of the Moon onto a circle and used a protractor to discern the angles that the chord lengths constituted as seen in *Figure 1*. From this, we were able to calculate the radius in pixels using *Equation 1*,

$$radius = \frac{chord\ length}{2sin(\theta/2)}$$

Equation 1

where θ is the angle in radians that contains the chord.

In order to find the radius in units of meters, we need to first find the angular resolution of the telescope using the following equation:

$$\left(\frac{Pixel\ size}{focal\ length}\right)206265'' = angular\ resolution$$

Equation 2

We were able to find the pixel size through the SAO DS9 data window and found the focal length of the telescope by following the instructions outlined in *Lab 2 Telescopes*, section 2 *Properties of Telescopes*. With this angular resolution, we could then convert the radius in pixels to arcseconds by multiplying the radius by the angular resolution. Once we have the length in arcseconds, we can find the radius in meters using the small angle formula,

$$D = \frac{\alpha * d}{206265''}$$

Equation 3

where *D* is the linear size of the object, α is the angular resolution in arcseconds, and *d* is the distance to the object in the same units as the linear size of the object. Each step of this process including the radius found for each image is recorded in *Table 1*.

The final value we calculated for the radius of the Moon is $1.753 * 10^6$ m. This has an error of 0.921% from the true value for the radius, which is $1.737 * 10^6$ m. Because we viewed the Moon when it wasn't entirely full, we would have expected our calculated value to be less than that of the real value. Sources of error for this include inaccurate distinctions for the edge of the moon that led to incorrect chord lengths and our precision for measuring the angle that contained the chord length was limited to degrees. If we had more precise tools, been able to image the Moon when it was full, and better distinguished the edge of the Moon, our answer would perhaps be closer to the true value.

2.b Finding the Mass and Density

Because we cannot find the true distance from the center of the Earth to the center of the Moon while simultaneously solving for its radius, we used a previously calculated average of both the Moon's distance and sidereal period as well as the mass of the Earth as recorded in *Universe* 10 Edition (Roger Freedman et. al). With this data, we can use Newton's modification of Kepler's law of orbits to discern the mass. Solving for the mass of the Moon, we got *Equation 4.b*:

$$T^2 = \frac{4\pi^2 a^3}{G(M_E + M_m)}$$

Equation 4.a

$$M_m = \frac{4\pi^2 a^3}{GT^2} - M_E$$

Equation 4.b

where M_M is the mass of the moon, M_E is the mass of the earth, a is the semimajor axis of the Moon's orbit around the Earth, and T is the period of the Moon's orbit. Once we had the mass and radius of the Moon, we could solve for its density through spherical geometry.

We found the mass of the Moon to be $7.542 * 10^{22}$ kg, 2.63% off from the true value of $7.349 * 10^{22}$ kg. Our calculated density was 3342 kg/m^3, which is just 0.06% off from true density of 3344 kg/m^3. Because we used predetermined values of the Moon's distance and sidereal period and the Earth's mass to calculate the mass, the source of error must be from round off error of the borrowed values. Because our percent error for density is so small- smaller than that for the mass- the error from both the radius and mass must have compensated for each other.

2.c Finding the Albedo

On November 2nd, we took images of the binary star system Gamma Andromedae (ɣ And) along with the Moon under a blue filter with exposure times ranging from 0.004 and 0.2 seconds. In order to calculate the albedo, we first calibrated the images of the Moon and ɣ And in Maxim DL by generating master biases, darks, and flats to distinguish the photons from the bodies of interest from any ambient photons. Although the general process for finding the brightness from each body is the same, because the Moon did not fit within the field of view as ɣ And did, there are some discrepancies in the procedure.

2.c.i Finding the Brightness of the Moon

In order to calculate the brightness of the Moon, we analyzed the calibrated images in SAO DS9. Within this software, we selected the portion of the image that contained the Moon, found the photons per pixel within the statistics window, and recorded this data for 20 images of the Moon in *Table 2*. Because the number of photons the CCDs were exposed to differed depending on the exposure time, we divided all of the values we collected for photons per pixel by the exposure time of each image.

In order to get the brightness of the entire moon, we needed to find the area in pixels of the Moon for each image. We did this by following a similar method described in section 2.b and shown in *Figure 1*; we projected the Moon section of interest onto a circle in Photoshop CS in order to find the angle that contains the chord length measured in SAO DS9 and used

Equation 2 in order to calculate the radius in pixels. Once we found the radius, we could find the area of the Moon in pixels. By multiplying the photons per pixel by the area of the Moon in pixels, we were able to calculate a total number of photons from the Moon as if the entire body were imaged. This process was repeated for all 20 images including different portions of the Moon in order to obtain a reasonable average.

2.c.ii Finding the Brightness of Gamma Andromedae

Using the same method described in 2.c.i, we found the photons per pixel for 15 images of ɣ And. However, because ɣ And fit within the field of view, we were able to use the statistics window which gave us the area of the selected region. Because the selected region only contained ɣ And, the area provided by the statistics window was the area of ɣ And. As in 2.c.i, we multiplied the photons per pixel by the area of the body in pixels in order to find the total number of photons from ɣ And as recorded in *Table 3*.

2.c.iii Finding a Scalar Between Photons and Flux

In order to find the flux from the Moon, we first needed to find a scalar that could describe the photons calculated in terms of Watts per squared meter. To do this, we used known values for ɣ And to calculate the flux from it. To calculate the flux, we found the luminosity and the distance from Earth of ɣ And from SIMBAD and used the following equation:

$$Flux = \frac{Luminosity}{4*\pi*D^2}$$

Equation 5

where *D* is the distance between ɣ And and the Earth. Because flux is related to the number of photons striking a unit of area, we can find a scalar conversion between the number of photons calculated in section 2.c.ii and the flux calculated here. The conversion is described as

$$p * \beta = Flux$$

Equation 6.a

$$\beta = \frac{Flux}{p}$$

Equation 6.b

where *p* is the number of photons and *β* is the scalar.

2.c.iv Calculating the Flux of the Moon

Now that we had a scalar between the number of photons and flux, we can use *Equation 6.a* in order to find the flux from the Moon at the surface of the Earth. Using data from WolframAlpha, we found the distance from the Moon and Earth on November 2nd and subtracted the radius of each body from this value to get the distance from the surfaces of the Moon and Earth. Having found the flux from the Moon and its distance, we used *Equation 5* in order to calculate the luminosity of the Moon. Once the luminosity was found, we again used *Equation 5* to find the flux at the Moon's surface by using the Moon's radius, calculated in section 2.a, as the value for *D*.

2.c.v Calculating the Flux from the Sun and Albedo

Because the albedo of a material is the ratio of reflected light to incoming light, we needed to calculate the flux of the Sun at the Moon. We used information from *Universe* to find the luminosity of the Sun and WolframAlpha to find the distance from the Sun to the Moon on November 2nd. Because the distance from the Sun to the Moon is 85,000 times larger than the radius of the Moon and 213 times larger than the radius of the Sun, we did not make the distinction between the flux at the Moon's center versus its surface. Once we had the luminosity and distance of the Sun, we plugged these values into *Equation 5* to find the flux of the Sun at the Moon. With the flux of the Moon- the light reflected by the Moon- and the Sun's flux at the Moon- the incoming light- we calculated the albedo using the following equation:

$$\frac{Moon's\ Flux\ at\ its\ Surface}{Sun's\ Flux\ at\ the\ Moon}$$

Equation 7

The albedo we calculated from the averages for the Moon and ɣ And is 0.1021. The true average value for the albedo is 0.11, meaning a percent error of 7.55%. Because of the number of steps as well as the uncertainty in the data, our calculations for albedo were subject to more error than previous calculations. We did not take into consideration the possibility of gas in front of ɣ And that would obscure light from the star and affect our calculation of the relationship between photon count and anticipated flux. Not only this, but the distance between the Earth and the star varies by as much as 20 light years, so we were not able to use a precise value for the distance between the Earth and ɣ And, further affecting our calculation of the flux from the star and consequently the flux from the Moon. We also calculated the total photons from the Moon as if all parts of the Moon emit the same amount of light, which is not true. As in section 2.a, our ability to calculate the radius well was limited by our manual and angular precision. For more accurate results in the future, we should take the time to identify a more accurate distance to ɣ

And as well as be more meticulous about our calculation for the radius of the Moon and the amount of incoming light.

2.d Finding Crater Depths

In order to find the depth of Moon craters, we used images taken on the same telescope used for previous data collection. The images used were from November 28th, when the moon was waxing gibbous and 69% full. After deciding visually which craters we wanted to analyze, we referred to a map of the Moon and identified the craters as Walter, Werner, Geber, and Aliacensis (*Figure 2*).

In order to find the depths of the crater, we needed to finding the length of the shadow cast, the distance from the crater to the terminator, and the radius of the Moon. In order to find the distance from each crater to the terminator, we needed a whole image of the Moon (*Figure 3*) when it was 69% illuminated and in its waxing gibbous phase. We then used a physical ruler to measure the distance from each crater to the terminator, the distance from the edge of the moon to the terminator, and the length of the shadows in this reference image in terms of millimeters. To find the distance from the edge of the Moon to the terminator in meters, we multiplied the radius of the Moon calculated in part 2.a by the percent the Moon was illuminated on November 28th. Because we did not have the entire Moon imaged, we then multiplied this number by the fraction of the radius we did have imaged, which ended up being 0.7.

Now that we have the length from the edge to the terminator in meters and the scale for the portion of the image we have, as well as the length of the shadows, distance from the edge to the terminator, and distance from the craters to the terminator in millimeters, we can set up a series of ratios to solve for the length of the shadows and distance from the craters to the terminator in meters. To find the length of the shadows, we used the following ratio:

$$\frac{L}{R} = \frac{L'}{h}$$

Equation 8.a

$$L = \frac{L' * R}{h}$$

Equation 8.b

Where *L* is the length of the shadows in meters, *L'* is the length of the shadow measured in millimeters, *R* is the radius calculated in section 2.a and *h* is the height of *Figure 2* in millimeters.

In order to find the distance from each crater to the terminator in meters, we used the following relationship:

$$\frac{d}{t} = \frac{d'}{t'}$$

Equation 9.a

$$d = \frac{t * d'}{t'}$$

Equation 9.b

Where *d* is the distance from a crater to the terminator in meters, *d'* is the measured distance from a crater to the terminator in *Figure 3* in millimeters, *t* is the distance from the edge of the Moon to the terminator in meters, and *t'* is that same distance as measured on *Figure 3* in millimeters.

Now that we have the shadow lengths, crater-to-terminator distance, and radius of the Moon all in meters, we can calculate the height of each crater using:

$$H = L\left(\frac{d}{R}\right)$$

Equation 10

This entire process was done for each crater in three different images and the values were record in *Table 4,5,6,* and *7*.

Our crater depths ranged from 2706 to 4167 meters. While most of the percent errors were single digit, our percent error for the depth of Aliacensis was 26.87%. This error could have been the result of poor measurements within *Figure 3*. This error perhaps could have been avoided if we used the small angle formula to translate between angular size and linear size rather than using a ruler to measure the image and then translating to meters. Another thing that could have affected our data may have been the relationship between the amount of the Moon we had contained in our image relative to the actual amount of Moon visible when it is 69% illuminated and waxing gibbous.

3 Results

3.a 2.a Results

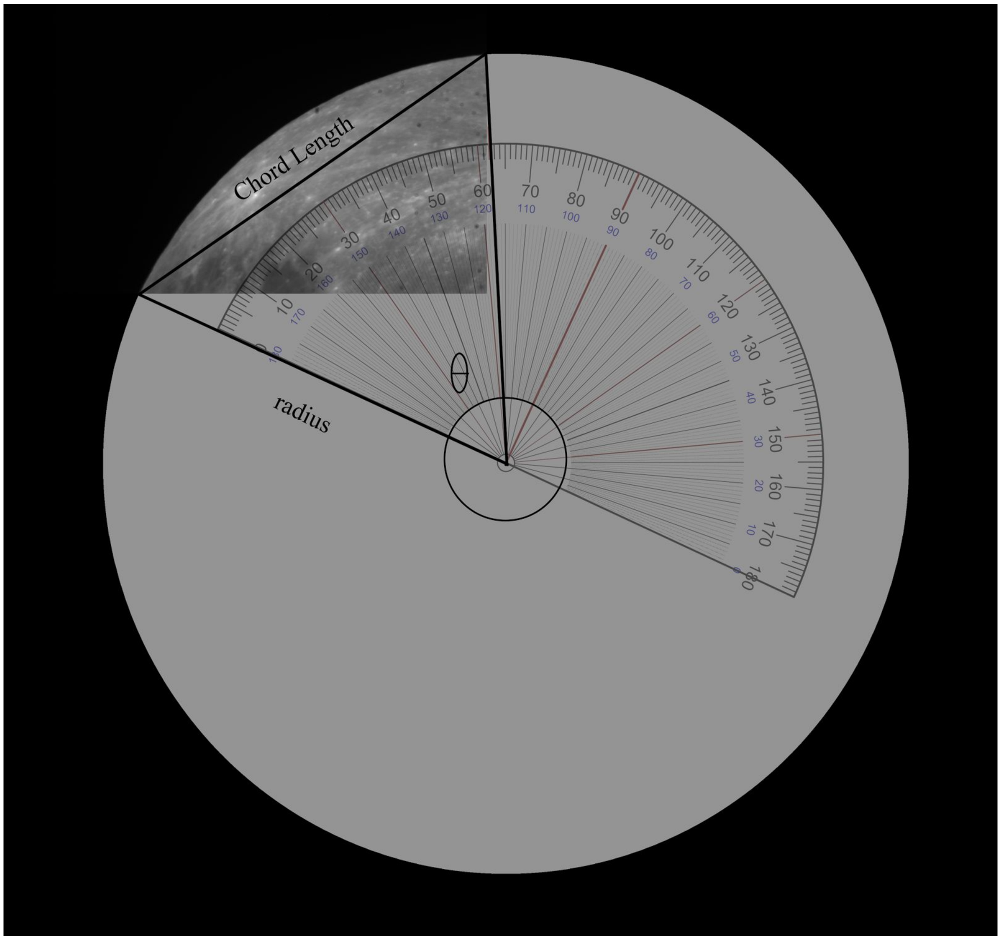

Figure 1

The Moon's Radius

File	Δx (pixels)	Δy (pixels)	Chord length (pixels)	Angle (radians)	Radius (pixels)	Radius (arcseconds)	Radius (meters)
1-1	1343	928	1632.431622	1.069	1,602.27	1,003.50	1.7514E+06
1-2	1318.074	947	1622.999713	1.07	1,591.67	996.86	1.7399E+06
2-1	1660	387.3292	1704.589073	1.099	1,631.93	1,022.08	1.7839E+06
2-2	1660	603.11	1766.165811	1.134	1,644.16	1,029.74	1.7972E+06
3-1	240.158	1250	1272.861291	0.84	1,560.80	977.53	1.7061E+06
3-2	297.5875	1250	1284.935142	0.831	1,592.38	997.31	1.7406E+06

Table 1

Finding the Angular Resolution of the Telescope:

Pixel size: 10.797 μm/pixel

Focal Length of the Telescope: 3556 mm

Angular Resolution of the Telescope (*Equation 2*): 0.6263 arcseconds/pixel

Average Calculated Radius: $1.753 * 10^6$ m

Real Radius: $1.737 * 10^6$ m

Percent Error: 0.921%

3.b 2.b Results

Mass of the Moon:

Average Distance Between Earth and Moon: $3.845 * 10^8$ m

Sidereal Period of the Moon's Orbit About Earth: 27.30 days = 2358720 s

Mass of the Earth: $5.972 * 10^{24}$ kg

Mass of the Moon (*Equation 4*): $7.542 * 10^{22}$ kg

Actual Mass of the Moon: $7.349 * 10^{22}$ kg

Percent Error: 2.63%

Average Density of the Moon: 3342 kg/m^3

Actual Average Density of the Moon: 3344 kg/m^3

Percent Error: 0.06%

3.c 2.c Results

The Moon's Brightness

File	Brightness/pixel^2	(Brightness/pixel^2) / Exposure time	Chord length (pixels)	Radius of Moon (pixels)	Area of Moon (pixels)	Total Brightness of Moon (photons)
001	15,704.97	392624.25	1632.43	1602.27	8065307.098	3.1666E+12
002	15,725.22	393130.45	1623.77	1576.475728	7807717.354	3.0695E+12

003	15,691.15	392278.85	1622.00	1574.757282	7790704.935	3.0561E+12
004	15,728.43	393210.775	1623.92	1576.621359	7809159.937	3.0706E+12
005	15,623.78	390594.575	1621.39	1577.227626	7815166.894	3.0526E+12
006	15,673.96	391849.025	1616.1	1546.507177	7513691.947	2.9442E+12
007	15,753.14	393828.55	1609.04	1562.174757	7666704.733	3.0194E+12
008	15,801.30	395032.6	1609.4	1562.524272	7670135.751	3.0300E+12
009	15,778.29	394457.3	1609.01	1562.145631	7666418.85	3.0241E+12
010	15,760.99	394024.725	1604.14	1557.417476	7620081.091	3.0025E+12
011	25,990.10	259901.03	1705	1578.703704	7829801.672	2.0350E+12
012	25,916.40	259164	1711.7	1584.907407	7891458.85	2.0452E+12
013	26,080.08	260800.81	1714.79	1641.732887	8467486.28	2.2083E+12
014	26,133.78	261337.81	1721.09	1590.656192	7948810.56	2.0773E+12
015	26,256.09	262560.91	1729	1590.616375	7948412.62	2.0869E+12
016	26,223.97	262239.69	1739.06	1589.634369	7938601.354	2.0818E+12
017	26,375.11	263751.14	1744	1595.608417	7998382.02	2.1096E+12
018	26,538.99	265389.86	1759.16	1584.828829	7890676.363	2.0941E+12
019	26,672.87	266728.74	1769.2	1593.873874	7981001.817	2.1288E+12
020	26,864.09	268640.89	1776.90	1586.517857	7907504.273	2.1243E+12

Table 2

Gamma Andromedae's Brightness

File	Brightness/pixel^2	$\frac{\text{(Brightness/pixel}^2\text{)}}{\text{Exposure time}}$	Area of Aɣ (pixels)	Total Brightness of ɣ And
001	15149.893	216427.0429	28	6.0600E+06
002	16021	228871.4286	29	6.6373E+06
003	16547.375	236391.0714	32	7.5645E+06
004	13051.303	186447.1857	33	6.1528E+06
005	14685.692	209795.6	26	5.4547E+06
006	28004.25	400060.7143	8	3.2005E+06
007	16996.045	242800.6429	22	5.3416E+06
008	26832.778	383325.4	9	3.4499E+06
009	23655.182	337931.1714	11	3.7172E+06
010	18716.933	267384.7571	15	4.0108E+06
011	15251.333	217876.1857	18	3.9218E+06
012	19569.417	279563.1	12	3.3548E+06

013	18168.5	259550	16	4.1528E+06
014	22437.889	320541.2714	9	2.8849E+06
015	18348.091	262115.5857	11	2.8833E+06

Table 3

Calculated Average Number of Photons from the Moon: $2.5713 * 10^{12}$ photons

Calculated Average Number of Photons from ɣ And: $4.5858 * 10^{6}$ photons

Flux from ɣ And at Earth:

Distance between Earth and ɣ And: 350 ly = $3.311 * 10^{18}$ m

Luminosity of ɣ And: 2000☉ = $7.656 * 10^{29}$ W

Flux at Earth (*Equation 5*): $5.557 * 10^{-9}$ W/m^2

β (*Equation 6.b)*: $1.212 * 10^{-15}$ W/(photons*m^2)

Reflected Light from Moon:

Flux of the Moon at Earth (*Equation 6.a*): 0.003116 W/m^2

Distance from the Earth to the Moon on November 2nd: $3.722 * 10^{8}$ m

Luminosity of the Moon (*Equation 5*): $5.425 * 10^{15}$ W

Flux at the Moon's surface (*Equation 5*): 140.5 W/m^2

Incoming Light from Sun:

Luminosity of the Sun: $3.828 * 10^{26}$ W

Distance from the Sun to the Moon on November 2nd: $1.488 * 10^{11}$ m

Flux from Sun at the Moon (*Equation 5*): 1376 W/m^2

Calculated Albedo (*Equation 7*): 0.1021

Real Albedo Average: 0.11

Percent error: 7.55%

3.d Results 2.d

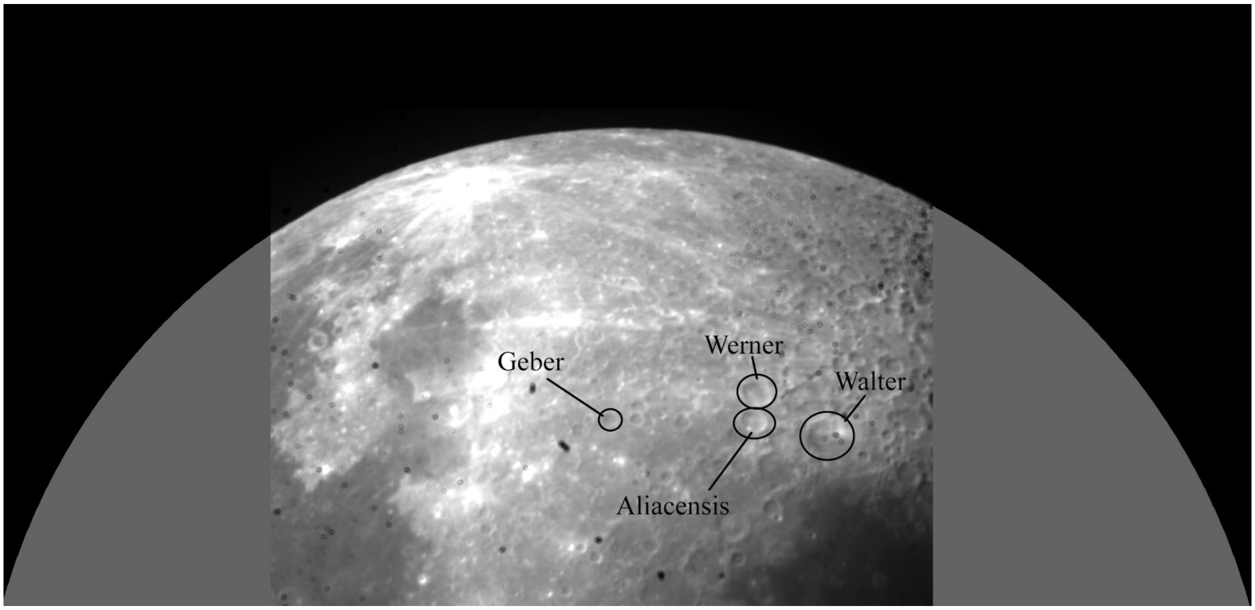

Figure 2

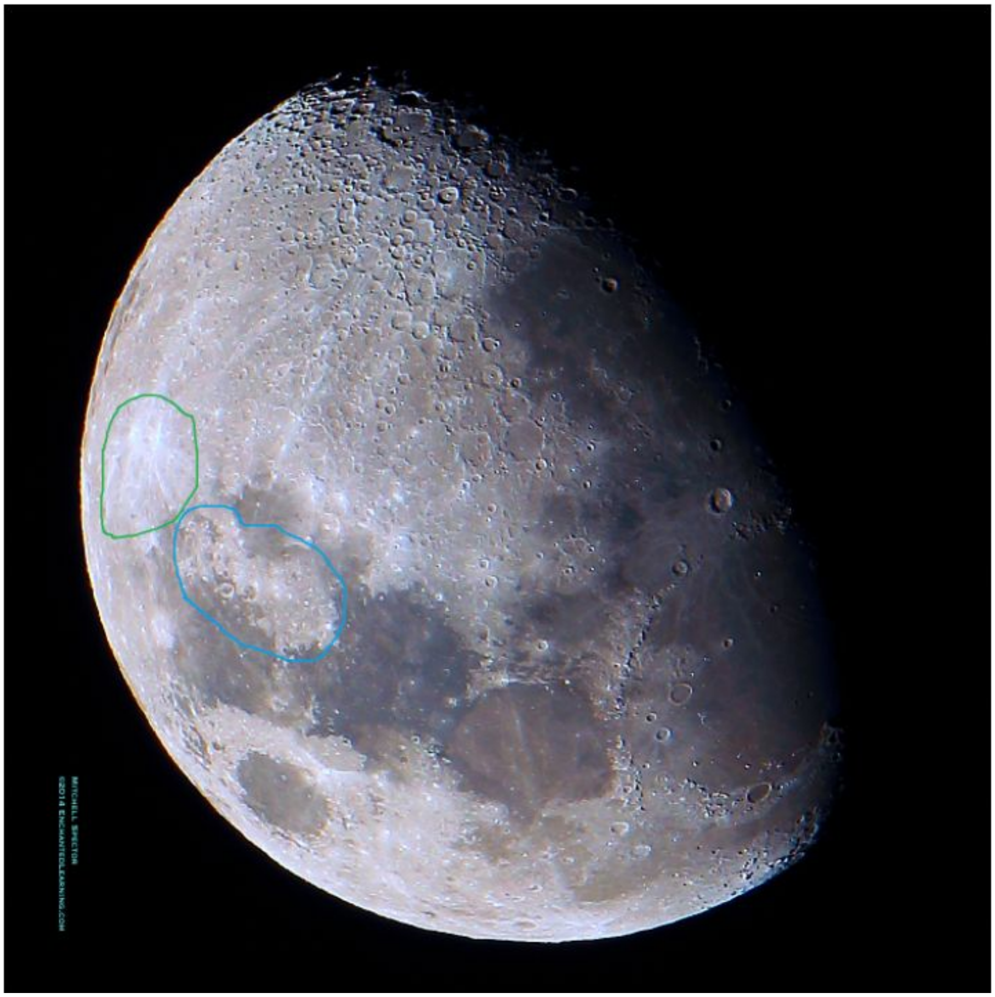

Figure 3

Walter Crater Depth

File	Shadow size (mm)	Shadow size (m)	Length to terminator (m)	Crater depth (m)
001	2.3	22990.2	331609.1	4356.433
009	2.1	20991.05	331609.1	3977.613
015	2.2	21990.62	331609.1	4167.023

Table 4

Average calculated depth: 4167.023 m

Real depth: 4100 m

Percent error: 1.63%

Aliacensis Crater Depth

File	Shadow size (mm)	Shadow size (m)	Length to terminator (m)	Crater depth (m)
001	1.45	14493.82	378981.8	3138.797
009	1	9995.737	378981.8	2164.687
015	1.3	12994.46	378981.8	2814.093

Table 5

Average calculated depth: 2705.859 m

Real depth: 3700 m

Percent error: 26.87%

Wener Crater Depth

File	Shadow size (mm)	Shadow size (m)	Length to terminator (m)	Crater depth (m)
001	2.2	21990.62	363190.9	4563.882
009	1.7	16992.75	363190.9	3219.972
015	2	19991.47	363190.9	4148.984

Table 6

Average calculated depth: 3977.613 m

Real depth: 4200 m

Percent error: 5.29%

Geber Crater Depth

File	Shadow size (mm)	Shadow size (m)	Length to terminator (m)	Crater depth (m)
001	1.2	11994.88	611897.7	4194.082
009	0.8	7996.59	611897.7	2796
015	1.1	10995.31	611897.7	3844.575

Table 7

Average calculated depth: 3611.57 m

Real depth: 3500 m

Percent error: 3.19%

4 Discussion

What we found is that the density of the Moon is very close to the average densities of the Earth's upper mantle and crust and is nearly equal to the density of the upper mantle. Because the mantle constitutes so much more mass of the Earth than the crust, it makes sense that the density of the Moon would be closer to that of the mantle than the crust. This suggests that the Moon did in fact form from the material of young Earth. Because of this, we can expect the albedo of the moon to be close to that of the materials which make up the Earth's mantle. The mantle is constituted primarily of silicates but also includes nickel and magnesium oxide with some iron. This composition closely resembles S-type asteroids which are composed of metallic nickel-iron and magnesium- silicates. The albedo of such asteroids is between 0.10 and 0.22 and so agrees with our data and verifies the probable composition of the Moon. M-type asteroids which are composed of nickel and iron also have an albedo between 0.1 and 0.2 but lack the silicate composition that is characteristic of the Earth's mantle. While this might suggest that the Moon is an asteroid captured by Earth's gravity, their densities do not agree with the Moon's; S-type asteroids have a density of 2,710 kg/m^3 and M-type asteroids have a density of 5,320 kg/m^3. Because of this, we find it reasonable to conclude that the collision theory of the Moon's formation is valid and likely.

Furthermore, the average depth of the craters was about 3616 meters. Since the transient cavity is typically a quarter to a third of the crater's diameter, the average diameter for the craters is 904 to 1205 meters. Craters of this magnitude tend to be about 10 times the size of their impactors, so the average diameter of the meteoroid that created these craters, and by visual approximation many of the other craters in this region, is about 90 to 120 meters in diameter.

These values are consistent with those associated with the average impactors from the Late Heavy Bombardment (LHB) which affected many bodies within the inner solar system, including the Earth. Because we can see evidence of the effects of the LHB on the Moon, an event which ended about 3.8 billion years ago, we can conclude that the Moon must be at least 3.8 billion years old, and should be much older since the pattern of impact is not indicative of collision with molten material.

Although the averages of our data provided results that supported our hypothesis, it is important to acknowledge that these averages came from fairly drastic ranges of values. Calculating the sample variance showed that values for flux deviated from the average to the same order of magnitude in both the Moon and ɣ And data. Possible remedies for this have been discussed in section 2 of this paper but, in general, many of these errors could have been avoided simply by being more meticulous about our manual measurements and being more aware of the uncertainty in the data that we have to use from databases. Although we know the percent errors of our values, it's uncertain how much the discrepancies in our methods affect our calculations. Despite this, the general results of our work, as well as the work of others, still implies the legitimacy of the collision theory.

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