Luby’s Algorithm for Maximal Independent Set

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What is Maximal Independent Set (MIS)?

Input: undirected graph.

Output: **Maximal** collection of vertices I subject to the restriction that **no pair of vertices in I are adjacent**.

Maximal independent set

Maximum independent set

General MIS algorithm

Algorithm 1: A high level description of the algorithm

Input: Graph $G = (V, E)$
Output: A maximal independent set I

1 $G' \leftarrow (V', E') \leftarrow (V, E)$;
2 $I \leftarrow \emptyset$;
3 **while** G' *is not empty* **do**
4 | Select an independent set $I' \subseteq V'$ in G';
5 | $I \leftarrow I \cup I'$;
6 | $Y \leftarrow I' \cup N(I')$;
| // $N(I')$ is the set of neighbors of vertices in I'
7 | $G' \leftarrow$ induced subgraph on $V' \setminus Y$;
8 **end**
9 **return** I;

The Select Step

- The select step can be implemented on a PRAM so that its execution time is small
- The number of executions of the body of the while loop before G’ is empty is very small.

Strong Scaling Results - not connected

80000 vertices, 640000 edges

Weak Scaling Results - not connected

80000 vertices, 640000 edges

Given a graph, find a set of vertices such that:

- No two are adjacent
- The set is *maximal* (no more vertices can be added)

Fundamental problem in parallel graph algorithms