

## MATH 357 - PEER FEEDBACK FOR SET 2

Multiple computations on Set 2 involved periodic functions. We can often shorten computations by using periodicity. Sometimes, this may avoid a computation through a simple geometric argument. For example, on problem 5, integration by parts gave the term

$$f(t)\overline{e_n(t)}\Big|_0^1 \tag{1}$$

Noting that both  $f$  and  $\overline{e_n}$  are 1-periodic, we see their product is 1-periodic. So, the difference in boundaries of (1) reduces to 0.

On problem 6, we computed the real Fourier series of the sawtooth function. Many noticed that  $\phi$  was  $2\pi$ -periodic and odd. That it has both of these properties makes it's Fourier series have only sine terms:  $a_n = 0$  for all  $n$ .

$$\begin{aligned}\hat{\phi}_n &= \int_0^1 \phi(t)\overline{e_n(t)} dt \\ &= \int_{-\frac{1}{2}}^{\frac{1}{2}} \phi(t)\overline{e_n(t)} dt\end{aligned}$$

The real part of this integral is the product of an even function (cos) with the odd  $\phi$ , so it reduces to 0. Computing the Fourier coefficients from

$$a_n = \text{Re}(\hat{\phi}_n) \tag{2}$$

$$b_n = \text{Im}(\hat{\phi}_n) \tag{3}$$

or directly from the their definitions gives the same result. These instances show how properties of odd and even, and periodic functions can simplify or avoid unnecessary calculations.