MATH 357 - PEER FEEDBACK FOR SET 6

A rotation map R_α is said to be periodic if there exists $n \in \mathbb{N}$ such that $R_\alpha^n = \mathrm{id}$, that is,

$$R_\alpha^n(x) = x, \text{ for all } x \in [0,1) \tag{1}$$

When grading this week's problem set, I noticed several solutions lacking the appropriate quantitiers for n and x in this definition. To prove that $\alpha \in \mathbb{Q}$ implies R_α, is periodic, we should produce an $n \in \mathbb{N}$ (independent of x) such that (1) holds. Any proof must explicitly clarify the quantitiers and their order.

In some situations, this led to confusion on part b of problem 2. The statement $\alpha \in \mathbb{R} \backslash \mathbb{Q}$ implies R_α is not periodic is true (and follows from problem 2, part a). However, R_α not periodic implies that for all $n \in \mathbb{N}$, there exists $x \in [0,1)$ with

$$R_\alpha^n(x) \neq x$$

But this is not the same as part b, since we switch the order of quantitiers ($\exists n \forall x$ in part a vs $\forall x \nexists n$ in part b).

There was some confusion on problem 4. Many approached the proof with an argument by contradiction. First, log rules were applied to derive the equation

$$b^{n_1} \cdot c^{n_2} = a^{n_3} \tag{2}$$

Then a contradiction came from applying the uniqueness of prime factorizations directly to (2). This general strategy is correct, but misses the detail that the n_i's are arbitrary integers and can be negative. For example, if n_3 were -1, the RHS of (2) would not be an integer, and thus doesn't have a prime factorization.

There is a simple fix to this argument. If n_1 is negative, we multiply (2) by a^{-n_1}. Repeating for each of n_2 and n_3, we get a similar version of (2). Notably, each exponent will be positive, so the powers of a, b, and c are integers. We have introduced the posibility that there are 0, 1, 2, or 3 factors on each side. When there are 1 or 2 factors on one side the modified equation will resemble

$$b^{|n_1|} \cdot c^{|n_2|} = a^{|n_3|} \tag{3}$$

with possibly different combinations of the factors. The argument procedes as before since each factor of (3) is an integer. This contradicts the uniqueness of prime factorizations since we get two different prime factorizations of

$$y := b^{|n_1|} \cdot c^{|n_2|}$$

$$y := a^{|n_3|}$$

by applying the prime factorizations of a, b, and c, and that they are pairwise coprime. When all the factors are on one side (the 0/3 case), we get

$$a^{|n_3|} \cdot b^{|n_1|} \cdot c^{|n_2|} = 1 \tag{4}$$

But this is clearly a contradiction since $b, c \neq 1$.