MATH 357 - PEER FEEDBACK FOR SET 2

Multiple computations on Set 2 involved periodic functions. We can often shorten computations by using periodicity. Sometimes, this may avoid a computation through a simple geometric argument. For example, on problem 5, integration by parts gave the term

$$f(t)\overline{e_n(t)}\Big|_0^1 \tag{1}$$

Noting that both f and $\overline{e_n}$ are 1-periodic, we see their product is 1-periodic. So, the difference in boundaries of (1) reduces to 0.

On problem 6, we computed the real Fourier series of the sawtooth function. Many noticed that ϕ was 2π-periodic and odd. That it has both of these properties makes it's Fourier series have only sine terms: $a_n = 0$ for all n.

$$\begin{aligned}\hat{\phi}_n &= \int_0^1 \phi(t)\overline{e_n(t)}\ dt \\ &= \int_{-\frac{1}{2}}^{\frac{1}{2}} \phi(t)\overline{e_n(t)}\ dt\end{aligned}$$

The real part of this integral is the product of an even fuction (cos) with the odd ϕ, so it reduces to 0. Computing the Fourier coefficients from

$$a_n = \mathrm{Re}(\hat{\phi}_n) \tag{2}$$

$$b_n = \mathrm{Im}(\hat{\phi}_n) \tag{3}$$

or directly from the their definitions gives the same result. These instances show how properties of odd and even, and periodic functions can simplify or avoid unecessary calculations.