Ivan Lin
AMS361
Dr. Yuefan Deng
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Homework 6

Problem 1

6.1)

$$y''' + y'' + y' + y = 1 + e^{-x} + 2e^{2x}$$

y_c: $\lambda^3 + \lambda^2 + \lambda + 1 = 0$

$\lambda^2(\lambda+1) + (\lambda+1) = 0$

$(\lambda+1)(\lambda^2+1) = 0$

$\lambda = -1 \ , \ \pm i$

$y_1 = e^{-x}, \ y_2 = \text{cis}\,x \ , \ y_3 = \text{cis}-x$
$= \cos x - i\sin x$

y_0: $f(x) = 1 + e^{-x} + 2e^{2x}$

$y_T = A + Bxe^{-x} + Ce^{2x}$

$y_T' = Be^{-x} - Bxe^{-x} + 2Ce^{2x}$

$y_T'' = -2Be^{-x} + Bxe^{-x} + 4Ce^{2x}$

$y_T''' = 3Be^{-x} - Bxe^{-x} + 8Ce^{2x}$

$A = 1$, $B - 2B + 3B = 1$, $C + 2C + 4C + 8C = 2$
$B = 1/2$, $C = 2/15$

$y_{GS} = C_0e^{-x} + C_1\cos x + C_2\sin x +$
$1 + \frac{1}{2}xe^{-x} + \frac{2}{15}e^{2x}$

Problem 2

6.2) $y^{4} - y^{3} - y^{2} - y' - 2y = 4 + 3x + 2x^2 + x^3$

y_0: $\lambda^4 - \lambda^3 - \lambda^2 - \lambda - 2 = 0$ $\quad \frac{p}{q} = \pm 1, 2$

$\lambda = -1, 2$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & -1 & -1 & -2 \\ & & -1 & 2 & -1 & 2 \\ \hline 2 & 1 & -2 & 1 & -2 & 0 \\ & & 2 & 0 & 2 & \\ \hline & 1 & 0 & 1 & 0 & \end{array}$$

$\lambda^2 + 1 = 0$

$\lambda = \pm i$

$y_1 = e^{-x}$, $y_2 = e^{2x}$, $y_3 = \text{cis}\, x$, $y_4 = \cos x - i \sin x$

y_T: $A + Bx + Cx^2 + Dx^3 = y_T$ $\quad -2D = 1$

$B + 2Cx + 3Dx^2 = y'_T$ $\quad D = -1/2$

$2C + 6Dx = y''_T$ $\quad -3D - 2C = 2$

$6D = y'''_T$ $\quad \frac{3}{2} - 2C = 2$

$C = -\frac{1}{4}$

$-6D - 2C - B - 2A = 4$ $\quad -6D - 2C - 2B = 3$

$3 + 1/2 - 1/4 - 2A = 4$ $\quad 3 + 1/2 - 2B = 3$

$-2A = 3/4$ $\quad B = 1/4$

$A = -3/8$

$$y_{GS} = c_0 e^{-x} + C_1 e^{2x} + C_2 \cos x + C_3 \sin x$$
$$- 3/8 + 1/4\, x - 1/4\, x^2 - 1/2\, x^3$$

Problem 3

6.3) $x^3y''' + 6x^2y'' + 4xy' - 4y = 1 + 2x + 3x^2$

y_c: $\lambda(\lambda-1)(\lambda-2) + 6\lambda(\lambda-1) + 4\lambda - 4 = 0$

$\lambda^3 - 3\lambda^2 + 2\lambda + 6\lambda^2 - 6\lambda + 4\lambda - 4 = 0$

$\lambda^3 + 3\lambda^2 - 4 = 0$ $\qquad \frac{p}{q} = \pm 1, 2, 4$

$$\begin{array}{r|rrrr} 1 & 1 & 3 & 0 & -4 \\ & & 1 & 4 & 4 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

$\lambda^2 + 4\lambda + 4 = 0$

$(\lambda+2)^2 = 0$

$\lambda = 1, -2$

$y_1 = x^1,\ y_2 = x^{-2},\ y_3 = \ln x\, x^{-2}$

y_T: $f(x) = 1 + 2x + 3x^2$

$y_T = A + B\ln x\, x + Cx^2$

$y_T' = B + B\ln x + 2Cx$

$y_T'' = 2C$

$-4A = 1,\ A = -1/4$

$2x = 4x(B + B\ln x) - 4Bx\ln x$

$2x = 4xB \qquad B = 1/2$

$6 \cdot 2C + 4 \cdot 2C - 4C = 3$

$12C + 8C - 4C = 3$

$C = 3/16$

$y_{GS} = C_0x + C_1x^{-2} + C_2\ln x \cdot x^{-2}$

$- \frac{1}{4} + \frac{1}{2}\ln x \cdot x + \frac{3}{16}x^2$

Problem 4

6.4) $y''' - y'' + y' - y = e^x + \sin(2x) + \sin(3x) + e^{-4x}$

y_c: $\lambda^3 - \lambda^2 + \lambda - 1 = 0$

$(\lambda - 1)(\lambda^2 + 1) = 0$

$\lambda = 1, \pm i$

$y = e^x, (c_1 + c_2)\cos x + (c_1 - c_2)\sin x$

$y_T = Ae^x x + B\sin(2x) + C\sin(3x) + De^{-4x}$

$y_T' = Ae^x + Axe^x + 2B\cos(2x) + 3C\cos(3x) - 4De^{-4x}$

$y_T'' = 2Ae^x + Axe^x - 4B\sin(2x) - 9C\sin(3x) + 16De^{-4x}$

$y_T''' = 3Ae^x + Axe^x - 8B\cos(2x) - 27C\cos(3x) - 64De^{-4x}$

$3A - 2A + A - 0 = 1$

$2A = 1$

$A = 1/2$

$-8B + 4B + 2B - B = 1$

$-3B = 1$

$B = -1/3$

$C + 3C - 9C - 27C = 1$

$C = -1/16$

$-64D - 16D - 4D - D = 1$

$-85D = 1$

$D = -1/85$

$y_{GS} = C_0 e^x + C_1\cos(x) + C_2\sin(x)$
$+ \frac{1}{2}e^x - \frac{1}{3}\sin(2x) - \frac{1}{16}\sin(3x) - \frac{1}{85}e^{-4x}$

Problem 5

6.5) $x^2y'' - x(x+2)y' + (x+2)y = 0, \forall x > -2, y_1(x) = x$

$$y'' - \frac{x+2}{x} y' + \frac{x+2}{x^2} y = 0$$

$$y_2 = u(x)\, y_1(x)$$

$$\ln(u') = -\int a_1 + 2\frac{y_1'}{y_1}\, dx + C_0$$

$$= -\int \left(-\frac{x+\cancel{2}}{x}\right) + \cancel{2\frac{1}{x}}\, dx + C_0$$

$$= -\int -1\, dx + C_0$$

$$\ln(u') = x + C_0$$

$$u' = Ae^x$$

$$du = Ae^x dx$$

$$u = \int Ae^x dx = Ae^x$$

$$y_2 = y_1 u = Axe^x$$

$$y_{GS} = y_1 C_0 + y_2 C_1$$

$$y_{GS} = C_0 x + C_1 x e^x$$