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Homework 9a

Section 6.1 Problem 4

Build a generating function for a_r , the number of distributions of r identical objects into

a. Five different boxes with at most three objects in each box

$$\begin{aligned} r &= e_1 + e_2 + e_3 + e_4 + e_5 \\ 0 &\leq e_i \leq 3 \\ g(x) &= (1 + x + x^2 + x^3)^5 \\ g(x) &= (1 + x + x^2)^5 \end{aligned}$$

b. Three different boxes with between three and six objects in each box

$$\begin{aligned} r &= e_1 + e_2 + e_3 \\ 3 &\leq e_i \leq 6 \\ g(x) &= (x^3 + x^4 + x^5 + x^6)^3 \\ g(x) &= [x^3(1 + x + x^2 + x^3)]^3 \\ g(x) &= x^9(1 + x + x^2 + x^3)^3 \end{aligned}$$

c. Six different boxes with at least one object in each box

$$\begin{aligned} r &= e_1 + e_2 + e_3 + e_4 + e_5 + e_6 \\ e_i &\geq 1 \\ g(x) &= (x + x^2 + x^3 + \dots)^6 \\ g(x) &= [x(1 + x + x^2 + \dots)]^6 \\ g(x) &= x^6(1 + x + x^2 + \dots)^6 \end{aligned}$$

d. Three different boxes with at most five objects in the first box

$$\begin{aligned} r &= e_1 + e_2 + e_3 \\ e_1 &\leq 5, e_i \geq 0 \\ g(x) &= (1 + x + x^2 + x^3 + x^4 + x^5)(1 + x + x^2 + \dots)^2 \end{aligned}$$

Section 6.1 Problem 14

Find a generating function for a_r , the number of ways a roll of six distinct dice can show a sum r if

a. The first three dice are odd and the second three even

$$\begin{aligned}
r &= e_1 + e_2 + e_3 + e_4 + e_5 + e_6 \\
e_1, e_2, e_3 &\in \{1, 3, 5\}, e_4, e_5, e_6 \in \{2, 4, 6\} \\
g(x) &= (x + x^3 + x^5)^3 (x^2 + x^4 + x^6)^3
\end{aligned}$$

b. The i th die does not show a value of i

$$\begin{aligned}
r &= e_1 + e_2 + e_3 + e_4 + e_5 + e_6 \\
\{e_n | e_n &\in \{1, 2, 3, 4, 5, 6\}, e_n \neq n\} \\
g(x) &= (x^2 + x^3 + x^4 + x^5 + x^6)(x + x^3 + x^4 + x^5 + x^6)(x + x^2 + x^4 + x^5 + x^6)(x + \\
&x^2 + x^3 + x^5 + x^6)(x + x^2 + x^3 + x^4 + x^6)(x + x^2 + x^3 + x^4 + x^5)
\end{aligned}$$