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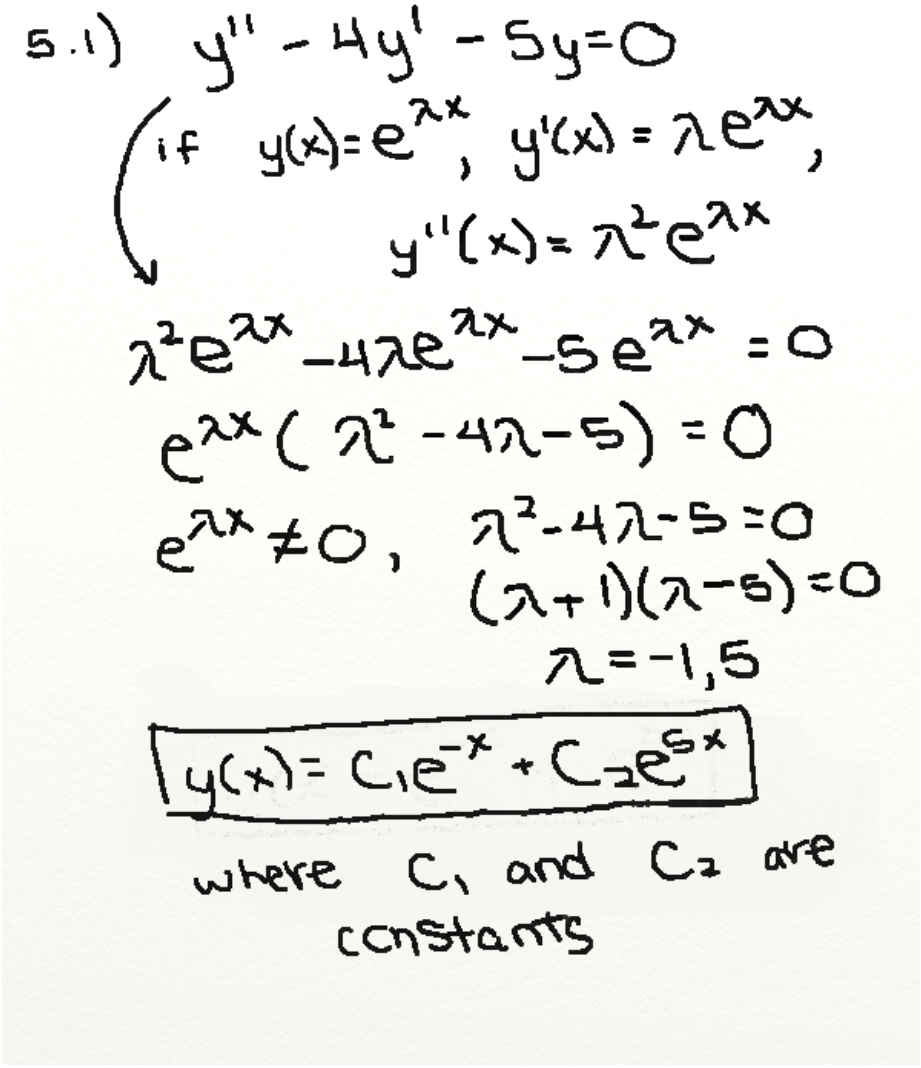
Homework 5

Problem 1



Problem 2

5.2) $y'' - y' - 6y = 0$

if $y(x) = e^{\lambda x}$, $y'(x) = \lambda e^{\lambda x}$

$y''(x) = \lambda^2 e^{\lambda x}$

$\lambda^2 e^{\lambda x} - \lambda e^{\lambda x} - 6e^{\lambda x} = 0$

$e^{\lambda x}(\lambda^2 - \lambda - 6) = 0$

$e^{\lambda x} \neq 0$, $\lambda^2 - \lambda - 6 = 0$

$(\lambda + 2)(\lambda - 3) = 0$

$\lambda = -2, \lambda = 3$

$$\boxed{y(x) = C_1 e^{-2x} + C_2 e^{3x}}$$

where C_1 and C_2 are constants

Problem 3

5.3) $x^2y'' + 2xy' - 2y = 0$

$y = x^r$, $y' = rx^{r-1}$, $y'' = r(r-1)x^{r-2}$

$x^2 r(r-1)x^{r-2} + 2xrx^{r-1} - 2x^r = 0$

$r(r-1)x^r + 2x^r r - 2x^r = 0$

$x^r(r^2 - r + 2r - 2) = 0$

$x^r(r^2 + r - 2) = 0$

$x^r \neq 0$

$r^2 + r - 2 = 0$

$(r+2)(r-1) = 0$

$r = -2, 1$

$y_1 = x^{-2}$, $y_2 = x^1$

$y = C_1 x^{-2} + C_2 yx$

where C_1, C_2 are constants

Problem 4

5.4)

$$(D-2)^3(D-1)^2(D^2-6D-7)y(x)=0 \qquad D=\frac{d}{dx}$$

$$(D-2)^3(D-1)^2(D+1)(D-7)y=0$$

$\lambda=2$: $y_1=e^{2x}C_1$, $y_2=xe^{2x}C_2$, $y_3=x^2e^{2x}C_3$

$\lambda=1$: $y_4=e^{x}C_4$, $y_5=xe^{x}C_5$

$\lambda=-1$: $y_6=e^{-x}C_6$

$\lambda=7$: $y_7=e^{7x}C_7$

$$y(x)=C_1e^{2x}+C_2xe^{2x}+C_3x^2e^{2x}+C_4e^{x}+C_5xe^{x}+C_6e^{-x}+C_7e^{7x}$$

where $C_1, C_2, \ldots, C_6$ are constants

Problem 5

5.5) $y''' + y'' + y' + y = 0$

$y(x) = e^{\lambda x}$, $y'(x) = \lambda e^{\lambda x}$,

$y''(x) = \lambda^2 e^{\lambda x}$, $y'''(x) = \lambda^3 e^{\lambda x}$

$\lambda^3 e^{\lambda x} + \lambda^2 e^{\lambda x} + \lambda e^{\lambda x} + e^{\lambda x} = 0$

$e^{\lambda x}(\lambda^3 + \lambda^2 + \lambda + 1) = 0$

$e^{\lambda x} \neq 0$ $\quad \lambda^3 + \lambda^2 + \lambda + 1 = 0$

$\lambda^2(\lambda+1) + (\lambda+1) = 0$

$(\lambda+1)(\lambda^2+1) = 0$

$\lambda = -1 \quad \lambda = \pm i$

$y_1 = e^{-x} C_0 \quad y_2 = C_1 e^{ix} \quad y_3 = C_2 e^{-ix}$

$y(x) = C_0 e^{-x} + C_1 e^{ix} + C_2 e^{-ix}$

where C_0, C_1, C_2 are constants