CSE 150 – Foundations of Computer Science: Honors **Midterm Exam**
Michael Bender **Tuesday, November 1, 2016**

TEST VERSION: 1

Name: ______________________ ID #: ______________________

INSTRUCTIONS:

- Unless otherwise stated, your answers should be at most 1 or 2 sentences (excluding work.)
- This is a closed book, closed notes exam.
- Check to see that you have 14 pages including this cover and scratch pages.
- Read all the problems before starting work.
- Think before you write.
- If you leave a question blank or write just "I do not know," you get 25% automatically.
- Good luck!!

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty.
Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that if I am caught cheating (either receiving or giving unauthorized aid) I will get a "Q" grade for this course, and a letter will be sent to the Committee on Academic Standing and Appeals (CASA) requesting that an academic dishonesty notation be placed on my transcript. Further action against me may also be taken.

Signature [1] ______________________

Problem	Score	Maximum
signature		2
1		48
2		18
3		12
Total		80

[1] No "I dunno" points for leaving this blank. ☺

Problem 1. (48 points)

Write your choice on the left of the question number.

Part A

(In this part each question has a **single** correct answer. 4 points each.)

(1) Which of the following is a proposition:

A. A proposition.

B. Choose me.

C. Don't you want to choose the last option? -

D. Surely you're joking. - *Right answer. This statement can be true or false, which makes it a proposition. Other statements are imperative, interrogative, or fragments.*

E. Trust none of the above answers.

(2) Let S be an infinite set and suppose we have $S = S_1 \times S_2 \times S_3 \times \cdots \times S_n$, where $n \in \mathbb{N}$. Which of the following is correct?

A. At least one of the sets S_i is an infinite set. - *Right answer. The cartesian product of two finite sets is finite, which means for the product of a finite amount of sets to be infinite, one of the factors must be infinite*

B. At least one of the sets S_i is an finite set.

C. At most one of the sets S_i can be an infinite set.

D. At most one of the sets S_i is a finite set.

E. None of the above.

(3) If $f(x) = \lfloor x \rfloor$, over which of the following domains and co-domains is it bijective? (Answers in the form (Domain, Co-domain).)[2]

A. $(\mathbb{R}, \mathbb{R})$

B. $(\mathbb{Z}, \mathbb{N})$

C. $(\mathbb{Z}, \mathbb{Z})$ - *Right answer. Every element in* $\mathbb{Z}$ *maps to itself which means there exists a one-to-one relationship.*

D. $(\mathbb{Z}, \mathbb{Q})$

E. $(\mathbb{Q}, \mathbb{R})$

(4) Recall the block-unstacking problem from class. We start with a stack of n blocks, and the objective is to unstack them fully. The *unstacking operation* takes a stack of size s, and splits it into two stacks, of size x and $s - x$. The unstacking cost is $x(s - x) + 1$. What is the minimum cost to unstack all of the blocks?

A. $n^2 - 1$

B. $n(n - 2) + 2$

C. $n(n - 1)/2$

D. $n(n + 1)/2$

E. $(n+2)(n-1)/2$ - *Right answer. Solved by trial and error, process of elimination.*

[2]The notation $\lfloor x \rfloor$ (the "floor" of x) means the largest integer less than or equal to x. Thus, $\lfloor 2 \rfloor = 2$ and $\lfloor 1.9 \rfloor = 1$.

(5) If $a \equiv 0 \pmod 3$, $a \equiv 1 \pmod 4$, $a \equiv 2 \pmod 5$, which of the following could a equal?

A. 27

B. 256

C. 257

D. 357 - *Right answer. Simple plug in the choice, trial and error.*

E. 2357

(6) We say that a relation R is ***coreflexive*** if for any x and y:

$$(x, y) \in R \Rightarrow x = y.$$

Let $S = \{a, b, c\}$. Which of the following relations on S is **both** reflexive and coreflexive?

A. $\emptyset$

B. $\{(a, c), (b, c), (c, a), (c, b)\}$

C. $\{(a, a), (b, b), (c, c)\}$ - *Right answer. Both reflexive and coreflexive, whereas others choices are not coreflexive or not reflexive.*

D. $\{(a, a), (b, b)\}$

E. $\{(a, a), (b, b), (c, c), (a, c), (a, b)\}$

(7) Which of the following formulae correctly represents "The set A has exactly two elements"?

A. $\exists x, y \in A,\ x \neq y.$

B. $\forall x, y \in A,\ x \neq y.$

C. $\forall z \in A,\ \exists x, y \in A,\ (x \neq y) \wedge \big((z = x) \vee (z = y)\big).$

D. $\exists x, y \in A,\ (x \neq y) \wedge \big(\forall z \in A,\ (z \neq x) \Rightarrow (z = y)\big).$ - *Right answer. Others have cases that do not fulfill the statement.*

E. $\exists x, y \in A,\ \forall z \in A,\ (z = x) \vee (z = y).$

(8) Let T be the set of all reflexive binary relations on $\mathbb{N}$, the set of natural numbers. Then T is uncountable. We prove it by contradiction.

Assume we have a enumeration $T = \{R_1,\ R_2,\ \dots\}$. Then we create a relation R, such that $(i, i) \in R,\ \forall i \in \mathbb{N}$. And for all i and j where $i \neq j$, $(i, j) \in R \iff (i, j) \notin R_{|i-j|}$. Then $R \notin T$ but R is reflexive. So T is uncountable.

Which of the following judgments is correct?

A. The conclusion is wrong.

B. The proof is wrong because the assumed enumeration is not clear.

C. The proof is wrong because R is not even reflexive.

D. The proof is wrong because R could still equal some $R_k \in T$.

E. The proof is right. - *Right answer. The proof is correct.*

Part B

(In this part each question may have **one or more** correct answers. 4 points each.
If your answer is a **nonempty strict subset** of the correct answer, you get 2.5 points.)

(1) Let R_1 and R_2 be two binary relations.

R_1 is said to be ***contained*** in R_2 if for any x and y, $(x, y) \in R_1 \Rightarrow (x, y) \in R_2$.

The relation "is contained in" (which is a relation on relations) is:

A. Reflexive - *Right answer. All elements in a set is an element of that same set*

B. Symmetric

C. Transitive - *Right answer. Diagrammed out, if A is contained in B and B is contained in C, there exists no elements in A not in C, so A is contained in C.*

(2) Let $A = \Big\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\Big\}$. Which of the following are (is) true?

A. $\Big\{\{\{\emptyset\}\}\Big\} \in P(A)$.

B. $A \times \{\emptyset\} = \{\emptyset\}$.

C. $A \times \emptyset = \emptyset$. - *Right answer. A set times the empty set produces the empty set.*

(3) Let A be an infinite set and B be a countable set. Which pair(s) of sets of the following must have a bijection?

A. A and B

B. A and $P(B)$

C. A and $A \cap B$

D. A and $A \cup B$ - *Right answer. A is infinite and $A \cup B$ is also infinite, so a bijection can exist. Special cases exist for other answers such that there could exist no bijection.*

E. A and $A \times B$

(4) Which of the following statements are (is) true?

A. $\mathbb{N} \times \mathbb{Q}$ is countable. - *Right answer. $\mathbb{N}$ and $\mathbb{Q}$ are both countable and the cartesian product of two countable sets is countable.*

B. The set of all binary relations on a countably infinite set is countable.

C. $P(\mathbb{Q})$ is countable.

D. The union of countably many countable sets can be uncountable.

E. The intersection of finitely many uncountable sets can be countably infinite. - *Right answer. The intersection of a countably infinite set with itself will also be countably infinite.*

Problem 2. (18 points) For each of the following statements about sets, state whether it is always true (provide an **example**), **sometimes** true (provide an **example** and **counterexample**), or **never** true (provide a **counterexample**).

(1) $S \in P(P(S))$ **always** **sometimes**✓ **never**

Example: S = $\underline{\emptyset}$

Counterexample: S = $\underline{\{1\}}$

(2) $P(S \cap T) = P(S) \cap P(T)$ **always**✓ **sometimes** **never**

Example: S = $\underline{\{1\}}$

T = $\underline{\{2\}}$

Counterexample: S =

T =

(3) $P(S - T) = P(S) - P(T)$ **always** **sometimes** **never**✓

Example: S =

T =

Counterexample: S = $\underline{\{1\}}$

T = $\underline{\{1\}}$

Problem 3. (12 points)

Let's do some counting. Given a finite set S, where $|S| = n$, fill in the blanks below and give a **one-sentence** justification for each.

(1) Total number of binary relations on S: $\underline{2^{n^2}}$

There are n^2 ordered binary pairs in $S \times S$ since very element in the first set is paired with every element in the second set, resulting in n^2 pairs, meaning there are 2^{n^2} possible relations containing some subset of $S \times S$.

(2) Number of reflexive binary relations on S: $\underline{2^{n^2-n}}$

Since reflexive binary relations must contain all pairs (x, x) where $x \in S$, there are n pairs that must be in the reflexive binary relation and $n^2 - n$ pairs that may optionally be in a reflexive binary relation, making the set of possible different binary relations 2^{n^2-n}

(3) Number of symmetric binary relations on S: $\underline{2^{\frac{n(n+1)}{2}}}$

Since any symmetric binary relation that contains (S_1, S_2) also contains (S_2, S_1), the number of possible pairs that can be in a symmetric relation is equal to the number of (S_1, S_2) where $S_1 = S_2$ plus the half the number of pairs (S_1, S_2) where $S_1 \neq S_2$ (since the other half consists of ordered pairs (S_2, S_1) that must also appear by necessity if its inverse appears), which in sum is equal to $\frac{n(n+1)}{2}$, meaning there are $2^{\frac{n(n+1)}{2}}$

Scratch Paper

Scratch Paper

Scratch Paper

Scratch Paper