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CSE 150 Foundations of Computer Science: Honors, Fall 2016
Assignment #4
Due Tuesday, November 29th, 2016

Problem 1:
I have twelve books.

- In how many ways can I line them up on a single shelf? - $12!$, *or* $479,001,600$ *ways*
- In how many ways can I choose seven of them and line them up on a single shelf? - $\frac{12!}{5!}$, *or* $3,991,680$ *ways*
- In how many ways can I choose seven to take to school? - $\frac{12!}{5!7!}$, *or* 792 *ways*

Problem 2:
An ogre has n captives to eat, one captive per day.

1. How many ways are there to make a menu for n days ? - $n!$ *ways*
2. How many ways are there to pick k captives to freeze them for winter season ? - $\frac{n!}{(n-k)!}$ *ways*
3. How many ways are there to buy n bottles of sauce for the captives, out of k kinds ? - $\frac{k!}{n!(k-n)!}$ *ways*

Problem 3:

- An ice-cream vendor sells eleven kinds of ice-cream. In how many different ways can I buy six cones, some or even all of which could be the same - $\frac{16!}{10!6!}$, *or* 8008 *ways*
- An ice-cream vendor sells six kinds of ice-cream. In how many different ways can I buy eleven cones, some or even all of which could be the same - $\frac{16!}{5!11!}$, *or* 4368 *ways*

Problem 4:

- There are 33 children, and they want to divide into three teams of eleven. In how many different ways can this be done? - $\frac{33!}{3!11!^3}$, *or* 2.2754499×10^{13} *ways*
- How many unique permutations exist for the letters in the 1980's band "BANANARAMA"? - $\frac{10!}{5!2!}$, *or* $15,120$ *ways*

Problem 5: Which number is bigger: the number of six-digit integers representable as a product of two three-digit integers, or the number of six-digit integers not representable in this form? - There are a greater number of six digit integers not representable as the product of two three digit integers. Since there are 900 unique 3 digit integers, there exists *at most* $\frac{900*900}{2}$, or $405,000$ possible unique products, some of which do not produce six digit products or produce the same product multiple times. There are 900,000 possible 6 digit integers, meaning less than half can be represented as the product of two 3 digit integers.

Problem 6: Among the number $1, 2, ..., 10^{10}$, are there more of those containing the digit 9 in their decimal notation, or those with no 9? - There are more integers in that range with a 9 then there are without. $\frac{1}{10}$ of the numbers in that range have a 9 in the 10^9 place. $\frac{1}{10}$ of the remaining numbers have a 9 in the 10^8 place. $\frac{1}{10}$ of the remaining numbers have a 9 in the 10^7 place. $\frac{1}{10}$ of the remaining numbers have a 9 in the 10^6 place... and so on and so forth... $\frac{1}{10}$ of the remaining numbers have a 9 in the 10^0 (ones) place. The formula to determine the number of numbers containing 9 is $\sum_{i=0}^{9} \frac{1}{10}(\frac{9}{10})^i$, which produces a number greater than $\frac{1}{2}$.

Problem 7: Professors and Keys

A group of five professors are setting a mathematics competition. When they go home at night, they leave their work in a room which has a certain number of locks on the door. Each professor has keys to some, but not all of the locks. In fact, any three professors will have enough keys between them to open the door, but any two professors will not have enough. What is the smallest number of locks needed, and how many keys will each professor have? Provide a proof or a clear explanation to get credit for this problem.

The first claim we can make is that each professor has the same number of keys.

Reasoning: Since ANY 3 professors can open the lock, each professor would have the same number of keys.

The second claim we can make is that there are 3 keys for each lock.

Reasoning: If there are 2 or fewer keys for a lock, a group of 3 professors can be formed who do not possess that key. In addition, since in any group of 3 professors, there exists at least one key that does not exist outside the group, there can't be 4 or more copies of the same key.

The third claim we can make is that there are 10 combinations of a group of 3 professors.

Reasoning: 5 choose 3 is 10.

We know that for any group of 3, there exists at least 1 key not possessed by the two other professors. Based on the third claim, this means there are at least 10 distinct locks. Furthermore, based on the second claim, we can determine there are at least a total of 30 keys. Using the first claim, we can determine that each of the 5 professors would have 6 keys.