Ivan Lin
Dr. Esther Arkin
AMS301
2/10/17

Homework 3b

Section 2.2 Problem 4

Rules regarding Hamilton circuits:

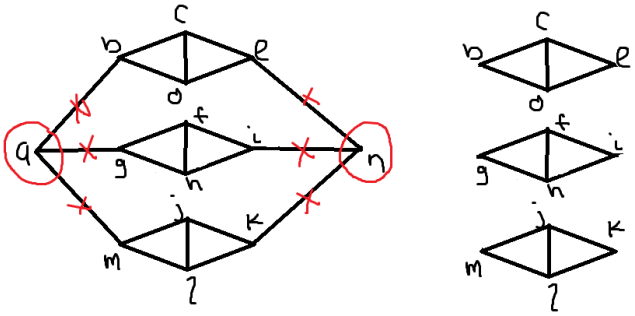
Rule 1: If a vertex x has degree 2, both of the edges touching x must be part of any Hamilton circuit.

Rule 2: No proper subcircuit can be formed when building a Hamilton circuit.

Rule 3: Once a Hamilton circuit is required to use 2 edges at a vertex x, all other (unused) edges touching x can be ignored (removed), as they cannot be used in a Hamilton circuit.

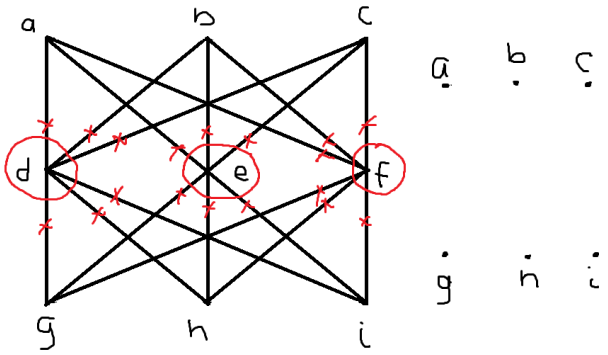
Theorem 1: If a graph G has a set S of $|S| = k$ vertices whose removal from G results in a graph G_S which has more than k connected components, then G does not have a Hamilton circuit.

e.



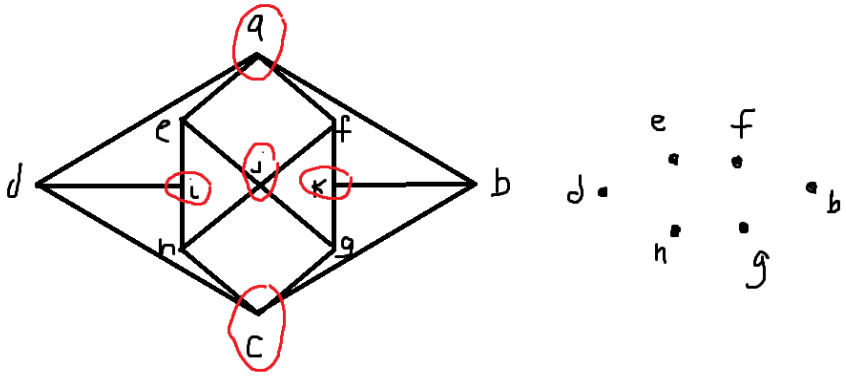
By removing a the set of vertices $\{a, n\}$, three components are formed - $\{b, c, e, d\}$, $\{g, f, i, h\}$, $\{m, j, k, l\}$. By theorem 1, since the size of the set of removed vertices is 2 and its removal forms more than 2 components, the graph does not have a Hamilton circuit.

f.



By removing a set of vertices $\{d, e, f\}$, six components are formed each consisting of a disconnected node $\{a\}$, $\{b\}$, $\{c\}$, $\{g\}$, $\{h\}$, $\{i\}$. By theorem 1, since the size of the set of removed vertices is 3 and its removal forms more than 3 components, the graph does not have a Hamilton circuit.

g.



Bt removing a set of vertices $\{a, j, c, i, k\}$, six components are formed each consisting of a disconnected node $\{d\}$, $\{e\}$, $\{h\}$, $\{f\}$, $\{g\}$, $\{k\}$. By theorem 1, since the size of the set of removed vertices is 5 and its removal forms more than 5 components, the graph does not have a Hamilton circuit.