Ivan Lin
Dr. Esther Arkin
AMS301
4/18/17

Homework 9b

Section 6.2 Problem 8
Find the coefficient of x^{24} in $(x + x^2 + x^3 + x^4 + x^5)^8$

$g(x) = (x + x^2 + x^3 + x^4 + x^5)^8$
$g(x) = [x(1 + x + x^2 + x^3 + x^4)]^8$
$g(x) = x^8(1 + x + x^2 + x^3 + x^4)^8$
let $f(x) = (1 + x + x^2 + x^3 + x^4)^8$
coefficient of x^{24} in $g(x)$ is equal to coefficient of $x^{24-8} = x^{16}$ in $f(x)$
$f(x) = (1 + x + x^2 + x^3 + x^4)^8$
$f(x) = \frac{1-x^4}{1-x}^8$
$f(x) = (1 - x^4)^8 \frac{1}{1-x}^8$
$a = (1 - x^4)^8 = 1 - \binom{8}{1}x^4 + \binom{8}{2}x^8 + ... + \binom{8}{8}x^{32}$
$b = \frac{1}{1-x}^8 = 1 + \binom{8}{1}x + \binom{9}{2}x^2 + ... + \binom{23}{16}x^{16} + ...$
coefficient of x^{16} is $a_0b_{16} + a_1b_{15} + ... + a_{16}b_0$
$1\binom{23}{16} + \binom{8}{1}\binom{18}{12} + \binom{8}{2}\binom{15}{8} + \binom{8}{3}\binom{11}{4} + \binom{8}{4} * 1$

Section 6.2 Problem 14
Find the coefficient of x^{18} in $(1 + x^3 + x^6 + ...)^6$

$g(x) = (1 + x^3 + x^6 + ...)^6$
let $z = x^3$
$g(x) = (1 + z + z^2 + ...)^6$
$g(x) = \frac{1}{1-z}^6$
$g(x) = \frac{1}{(1-z)^6}$
$g(x) = 1 + \binom{6}{1}z + \binom{7}{2}z^2 + ... + \binom{11}{6}z^6 + ...$
coefficient of x^{18} is coefficient of z^6
$\binom{11}{6}$

Section 6.2 Problem 22
How many ways are there to get a sum of 25 when 10 distinct dice are rolled?

This question can be modelled as $e_1 + e_2 + ... + e_{10} = 25$ or alternatively the coefficient of x^{25} in $(1 + x^1 + x^2 + ... + x^6)^{10}$

$g(x) = (x^1 + x^2 + ... + x^6)^{10}$
$g(x) = [x(1 + x^1 + x^2 + ... + x^5)]^{10}$
$g(x) = x^{10} \frac{1-x^6}{1-x}^{10}$
coefficient of x^{25} is coefficient of x^{15} in $\frac{1-x^6}{1-x}^{10}$
$g(x) = (1 - x^6)^{10} \frac{1}{1-x}^{10}$
$a = (1 - x^6)^{10} = 1 - \binom{10}{1} x^6 + \binom{10}{2} x^{12} - ... + \binom{10}{10} x^{60}$
$b = \frac{1}{1-x}^{10} = 1 + \binom{10}{1} x + \binom{11}{2} x^2 + ... + \binom{24}{15} x^{15} + ...$
coefficient of x^{15}
$1\binom{24}{15} + \binom{10}{1}\binom{18}{9} + \binom{10}{2}\binom{12}{3}$