Ivan Lin
Dr. Esther Arkin
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# Homework 7a

Section 5.1 Question 6

How many ways are there to pick a man and a woman who are not husband and wife from a group of $n$ married couples.
There are $n(n-1)$ or $n^2 - n$ ways of picking a man and woman who are not husband and wife. There are $n$ men to choose from for the male and $n-1$ women to choose from who are not his wife.

Section 5.1 Question 14

How many different numbers can be formed by various arrangements of the six digits $1, 1, 1, 1, 2, 3$?
There are $\frac{6!}{4!}$ or 30 different numbers that can be formed by arrangements of those six digits. There are 6 digits to order, so that's 6! arrangments. However, there are 4 duplicates of 1 and since we're only interested in different numbers, so we divide by 4! to account for arrangements that yield the same number due to the duplicates.

Section 5.1 Question 18

(a) How many different license plates involving three letters and two digits are there if the three letters appear together either at the beginning or end of the license?
There are $26^3 * 2 * 10^2$ or 3515200 different license plates that can be formed in such a way. There are $26^3$ ways of choose three ordered letters. However, they appear together at either the beginning or the end of the license plate, meaning that the amount must be multiplied by 2 to represent the two possible locations the grouping of letters can appear. That is then multiplied by $10^2$ to account for the possible choices for the two digits.

(b) How many license plates involving one, two, or three letters and one, two, or three digits are there if the letters must appear in a consecutive grouping?
*Note: I do not understand exactly what the question is asking. I take it to mean that we must consider all possible combinations of letter and digit amounts (i.e. 1 letter + 1 digit, 1 letter + 2 digits, 2 letters + 1 digit, etc.)*
If there are $[(26^1*2+10^1)+(26^2*2+10^1)+(26^3*2+10^1)]+[(26^1*3+10^2)+(26^2*3+10^2)+(26^3*3+10^2)]+[(26^1*4+10^3)+(26^2*4+10^3)+(26^3*4+10^3)]$ possible license plates. For a license plate with $m$ letters and $n$ numbers, there are $26^m$ possible letter permutations, multiplied by $n+1$ places the group of letters can occur among the digits, multiplied by $10^n$ permutations of digits, so

there would be $26^m * (n+1) * 10^n$ possible licenses. I found this term for all possible combinations of 1, 2, and 3 letters paired with 1, 2, and 3 digits.

Problem A Consider a decimal sequence of length 8 (decimal meaning digits 0,1,2,...,9 may appear). Each of the following parts is independent of the others.
(a). How many such sequences start and end with at least two 3s?
$10^4$ or 10000 of the sequences start and end with at least two 3's. If at least the first two and last two digits must be 3, then the remaining 4 or fewer digits are free to be chosen from the digits from 0 to 9, so the answer is $10^4$.

(b). How many such sequences have exactly 2 different digits appearing (e.g. 05550005)?
$\frac{10*9}{2*1} * (2^8 - 2)$ such sequences exist. A combination of two digits must be chosen to appear, and the order in which the two are selected do not matter, so $\frac{10*9}{2*1}$ accounts for the different possible digits. The $2^8 - 2$ term accounts for different arrangements the two digits in an 8-digit number. 2 is subtracted to account for the outcome where all of the numbers are the same, since exactly 2 digits must appear.

(c). How many such sequences have the digit 7 appearing at most 3 times?
$9^5 * \frac{8!}{5!3!} + 9^6 * \frac{8!}{6!2!} + 9^7 \frac{8!}{7!1!} + 9^8 \frac{8!}{8!1!}$ sequences have the digit 7 appearing 3 or fewer times. If the digit 7 occurs $n$ times in a sequence, we find the sum of the sequences where $n = 0, 1, 2, 3$. There are $8 - n$ places that can be one of the nine digits besides 7, so the term includes $9^{8-n}$, and it is multiplied by the possible arrangements of those digits, $\frac{8!}{n!(8-n)!}$. The result is there are $9^{8-n} \frac{8!}{n!(8-n)!}$ sequences where the digit 7 occurs exactly $n$ times. The answer is the sum of terms where $n < 3$.