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Homework 3

Problem 1

3.1) $\frac{dP}{dt} = -KP^{-\frac{1}{2}} \cdot P$

$P(0) = 10000,\ P(17\text{ weeks}) = 188$

$$\frac{dP}{dt} = -KP^{\frac{1}{2}}$$

$$dP \cdot P^{-\frac{1}{2}} = -K\,dt$$

$$\int \frac{dP}{P^{1/2}} = \int_0^t -K\,dt$$

at $t=0 \rightarrow$ $2P^{\frac{1}{2}} = -Kt + c$

$$2\sqrt{10000} = 200 = c$$

at $t = 17$ weeks $\Rightarrow$ $2\sqrt{188} = -K(17) + 200$

$$K = \frac{200 - 2\sqrt{188}}{17}$$

at $P = 0 \Rightarrow$ $0 = -\left(\frac{200 - 2\sqrt{188}}{17}\right)t + 200$

$$-200 = -\left(\frac{200 - 2\sqrt{188}}{17}\right)t$$

$$3400 = (200 - 2\sqrt{188})t$$

$$\boxed{\frac{3400}{200 - 2\sqrt{188}} = t \text{ weeks}}$$

the fish count will never change if 10000 fish were still left

Problem 2

3.2) $\frac{dP}{dt} = -\alpha P(M-P)$

$$\frac{dP}{P(M-P)} = -\alpha dt$$

$$\int_{P_0}^{P} -\frac{dP}{P(M-P)} = \int_{t_0}^{t} \alpha dt$$

$$\int_{P_0}^{P} \frac{1}{M}\left(\frac{1}{P} - \frac{1}{P-M}\right) dP = \int_{t_0}^{t} \alpha dt$$

$$\frac{1}{M}\ln P - \frac{1}{M}\ln(P-M)\Big|_{P_0}^{P} = \alpha t\Big|_{t_0}^{t}$$

$$\ln\left(\frac{P}{P_0}\cdot\frac{P_0-M}{P-M}\right) = \alpha t M$$

$$\frac{P}{P-M}\cdot\frac{P_0-M}{P_0} = e^{\alpha t M}$$

$$\frac{P}{P-M} = \frac{P_0}{P_0-M}\, e^{\alpha t M}$$

$$\frac{P-M}{P} = 1-\frac{M}{P} = \frac{P_0-M}{P_0}e^{-\alpha t M} = \left(1-\frac{M}{P_0}\right)e^{-\alpha t M}$$

$$P = \frac{M}{1-\left(1-\frac{M}{P_0}\right)e^{-\alpha t M}}$$

$P \to \infty$ when $1-\left(1-\frac{M}{P_0}\right)e^{-\alpha t M} = 0$

given $P_0 > M$, $\alpha, M > 0$

$0 < \frac{M}{P_0} < 1$, $1-\frac{M}{P_0} > 0$

$$e^{-\alpha t M} = \frac{1}{1-\frac{M}{P_0}}$$

$$e^{\alpha t M} = 1-\frac{M}{P_0}$$

$$t = \frac{1}{\alpha M}\ln\left(1-\frac{M}{P_0}\right)$$

Problem 3

3.3) $F = F_g - F_R = ma$

$F_R = Kv^p$ since drag $\propto v$, $p=1$

$$mg - Kv = m\frac{dv}{dt} = mv\frac{dv}{dy}$$

$$\int_0^{h_1} dy = \int_0^{v_1} \frac{mv\,dv}{mg - Kv}$$

$v = \frac{mg}{K}$

$$\boxed{\begin{aligned} &\text{at } t=0, \\ &mg - Kv = 0 \\ &v_T = \frac{mg}{K} \end{aligned}}$$

$$h_1 = -\frac{v_T}{g}\int_0^{v_1}\left(1 + \frac{v_T}{v - v_T}\right)dv$$

Phase 2

$$mv\frac{dv}{dy} = mg - \beta Kv$$

$$\int_{h_1}^{H} dy = \int_{v_1}^{v_F} mv\frac{dv}{mg - \beta Kv}$$

$$(v_F - v_1) + \frac{v_T}{\beta}\ln\left|\frac{\beta v_F - v_T}{\beta v_1 - v_T}\right| = \frac{\beta g(H - h_1)}{v_T}$$

sys of eq for

open arms at: h_1

$$\begin{cases} (v_F - v_1) + \frac{v_T}{\beta}\ln\left|\frac{\beta v_F - v_T}{\beta v_1 - v_T}\right| = \frac{\beta g(H - h_1)}{v_T} \\ v_1 + v_T\ln\left|\frac{v_1}{v_T} - 1\right| = \frac{-gh_1}{v_T} \end{cases}$$

$$m\frac{dv}{dt} = mg - kv \quad \left(v_T = \frac{mg}{k}\right)$$

$$-\frac{v_T}{g}\int_0^{v_1} \frac{1}{v - v_T}\,dv = \int_0^{t_1} dt$$

$$-\frac{v_T}{g}\ln\left|\frac{v_1 - v_T}{v_T}\right| = t_1$$

$$-\frac{v_T}{g}\ln\left|\frac{v_1}{v_T} - 1\right| = t_1$$

Phase 2: $\quad m\frac{dv}{dt} = mg - \beta k v$

same process $\quad \int_{v_1}^{v_F} \frac{m\,dv}{mg - \beta k v} = \int_0^{t_2} dt$

$$t_2 = \frac{v_T}{\beta g}\ln\left|\frac{\beta v_1 - v_T}{\beta v_F - v_T}\right|$$

$$T = t_1 + t_2$$

$$= -\frac{v_T}{g}\ln\left|\frac{v_1}{v_T} - 1\right| + \frac{v_T}{\beta g}\ln\left|\frac{\beta v_1 - v_T}{\beta v_F - v_T}\right|$$

Disclaimer: Did not fully understand some of the integrations, so there are parts where I followed the guide posted by Dr. Deng

Problem 4

3.4) $V_0 = \frac{k}{m} L\left(1 + e^{\frac{k}{\mu} L}\right)$

$$m\frac{dv}{dt} = m\frac{dv}{dx}\cdot\frac{dx}{dt} = m\cdot v\frac{dv}{dx} = -kv^{\alpha}$$

$$dv\,v^{1-\alpha} = \frac{v\,dv}{v^{\alpha}} = -\frac{k}{m}dx$$

$$\int_{V_0}^{0} v^{1-\alpha} = \int_0^x -\frac{k}{m}dx$$

$$-\frac{1}{2-\alpha}V_0^{2-\alpha}\Big|_{V_0}^{0} = -\frac{k}{m}x$$

$$-\frac{1}{2-\alpha}V_0^{2-\alpha} = -\frac{k}{m}x$$

$$x = \frac{m}{k}\cdot\frac{V_0^{2-\alpha}}{2-\alpha}$$

block 1: resistance α v

$$x = \frac{m}{k}\cdot\frac{V_0^{2-1}}{2-1}$$

$$x = \frac{m}{k}V_0$$

block 2: resistance α $v^{3/2}$

$$x = \frac{m}{k}\cdot\frac{V_0^{3/2-1}}{3/2-1}$$

$$= \frac{m}{k}\cdot 2\cdot\sqrt{V_0}$$

block 3: resistance α v^2

$$x = \frac{m}{k}\cdot\frac{V_0^{2-2}}{2-2} = \frac{m}{k\cdot 0}$$

$$= \infty$$