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CSE150 - Honors Foundations of Computer Science Fall 2016

<center>Homework 1a</center>

<u>Problem 1</u> Write $P \implies Q$ using $\vee$ and $\sim$. Show that your two representations are equivalent.

$$(P \implies Q) \iff (\sim P \vee Q)$$

| $P$ | $Q$ | $\sim P$ | $P \implies Q$ | $\sim P \vee Q$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

<u>Problem 2</u> Prove that the propositional formulas

$$P \vee Q \vee R$$

and

$$(P \wedge \sim Q) \vee (Q \wedge \sim R) \vee (R \wedge \sim P) \vee (P \wedge Q \wedge R)$$

are equivalent.

| $P$ | $Q$ | $R$ | $\sim P$ | $\sim Q$ | $\sim R$ | $P \wedge \sim Q$ | $Q \wedge \sim R$ | $R \wedge \sim P$ | $P \vee Q \vee R$ |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F | F | F | T |
| T | T | F | F | F | T | F | T | F | F |
| T | F | T | F | T | F | T | F | F | F |
| F | T | T | T | F | F | F | F | T | F |
| T | F | F | F | T | T | T | F | F | F |
| F | F | T | T | T | F | F | F | T | F |
| F | T | F | T | F | T | F | T | F | F |
| F | F | F | T | T | T | F | F | F | F |

| $(P \wedge \sim Q) \vee (Q \wedge \sim R) \vee (R \wedge \sim P) \vee (P \wedge Q \wedge R)$ | $P \vee Q \vee R$ |
|---|---|
| T | T |
| T | T |
| T | T |
| T | T |
| T | T |
| T | T |
| T | T |
| F | F |

<u>Problem 3</u> (a) Write the biconditional ($\Leftrightarrow$) using only implies ( $\implies$ ) and and ($\wedge$). Prove that the new version is equivalent.

$$(P \Leftrightarrow Q) \iff (P \implies Q) \wedge (Q \implies P)$$

| $P$ | $Q$ | $P \implies Q$ | $Q \implies P$ | $P(\implies Q) \wedge (Q \implies P)$ | $P \Leftrightarrow Q$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | F | F | F | F |
| F | T | T | T | F | F |
| F | F | T | T | T | T |

(b) Write it using only $\vee$ and $\sim$. Show your derivation.

$$(P \Leftrightarrow Q) \iff (P \implies Q) \wedge (Q \implies P)$$
$$\text{given } A \iff \sim (P \implies Q), B \iff \sim (Q \implies P)$$
$$(P \implies Q) \wedge (Q \implies P) \iff (\sim A \wedge \sim B)$$
$$(\sim A \wedge \sim B) \iff \sim (A \vee B) \text{ by De Morgan's Laws}$$
$$\sim (A \vee B) \iff \sim (\sim (P \implies Q) \vee \sim (Q \implies P))$$
$$\text{since } (P \implies Q) \iff (\sim P \vee Q)$$
$$\sim (\sim (P \implies Q) \vee \sim (Q \implies P)) \iff \sim (\sim (\sim P \vee Q) \vee \sim (\sim Q \vee P))$$