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Homework 2b

Problem 5

Show that each function $f : \mathbb{N} \implies \mathbb{N}$ has the listed properties.

1. $f(x) = 2x$ (one-to-one but not onto)
$f'(x) = \frac{x}{2}$

There exists only one y for any x, thus it is injective. There exists $f(x)$ in $\mathbb{N}$ that do not have a corresponding x in $\mathbb{N}$, when $f(x)$ is odd and does not divide evenly into 2. This means the function is not surjective.

2. $f(x) = x + 1$ (one-to-one but not onto)
$f'(x) = x - 1$

There exists only one y for any x, thus it is injective. However, depending on the definition of $\mathbb{N}$ and whether it includes 0, the smallest element of the domain has no corresponding y value in $\mathbb{N}$

3. $f(x) =$ if x is odd then $x - 1$ else $x + 1$ (bijective)

When x is odd, $f(x) = x - 1$ is even, $x = \{1, 3, 5, ...\}, f(x) = \{0, 2, 4, ..\}$, and when x is even, $f(x) = x + 1$ is odd $x = \{0, 2, 4, ...\}, f(x) = \{1, 3, 5, ...\}$. The domain and range cover all $\mathbb{N}$. The function is bijective.

Problem 6

Show that the product $(a + bi)(c + di)$ of two complex numbers can be evaluated using just three real number multiplications. You may use a few extra additions.

$(a + bi)(c + di)$
$ac + adi + cbi - bd$
$ac - bd + adi + cbi$

At this point, I tried several different methods, but failed to get the answer. I then looked up the answer and subsequently put my palm to my face in an act of frustration.

$ac - bd + i[(a - b)(d - c) + ac + bd]$

Problem 7

Given a function $f : A \implies A$. An element $a \in A$ is called a fixed point of f if $f(a) = a$. Find the set of fixed points for each of the following functions.

1. $f : A \implies A$ where $f(x) = x$.
$\forall a \in \mathbb{R} \implies a \in A$

2. $f : \mathbb{N} \implies \mathbb{N}$ where $f(x) = x + 1$
$\varnothing$

3. $f : \mathbb{N}_6 \implies \mathbb{N}_6$ where $f(x) = 2x \bmod 6$.
$\{0\}$

4. $f : \mathbb{N}_6 \implies \mathbb{N}_6$ where $f(x) = 3x \bmod 6$.
$\{0, 3\}$

Problem 8 Let $f(x) = x^2$ and $g(x, y) = x + y$. Find compositions that use the functions f and g for each of the following expressions.

1. $(x + y)^2$
$f(g(x, y))$

2. $x^2 + y^2$
$g(f(x), f(y))$

3. $(x + y + z)^2$
$f(g(x, y), z)$

4. $x^2 + y^2 + z^2$
$g(g(f(x), f(y)), f(z))$