CSE 350 – Theory of Computation (Honors), Spring 2018

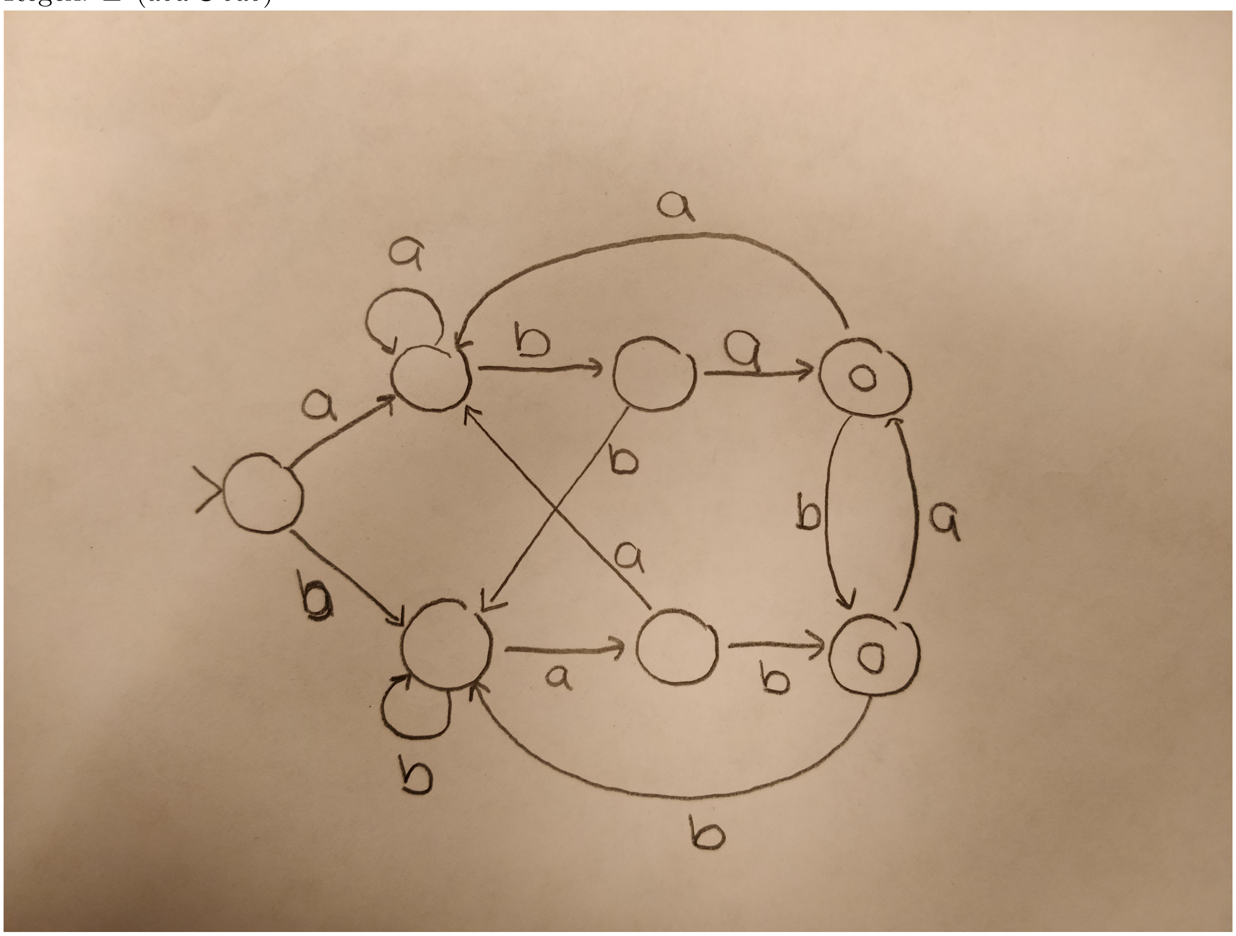
Assignment 2

Part A

Problem 1

Construct a regular expression for the language of strings on $\Sigma = \{a, b\}$ that end in "aba" or "bab". Construct a DFA for it.

Regex: $\Sigma^*(aba \cup bab)$



Problem 2

Let L be a regular language. Can you construct an NFA for L that has only one accepting state? Can you construct a DFA for L that has only one accepting state? Justify.

We can construct an NFA for any regular language L because NFAs are permitted to use empty string transitions between nodes. If the regular language has a form where there are multiple accepting states, all but one accepting states can be removed and instead an empty string can lead to the remaining accepting state.

We cannot construct a DFA for all regular languages. A simple counterexample would be the language containing the empty string or an odd number of as. The regular expression would be $e \cup a(aa)*$

Problem 3

For any string $w = w_1 w_2 \cdots w_n$, where each w_i is a character, the reverse of w, written w^R, is the string w in reverse order, $w_n \cdots w_2 w_1$. For any language A, let $A^R = \{w^R | w \in A\}$. Show that if A is regular, so is A^R.

If A is of the form $(k, \Sigma, s, \delta, F)$, the DFA A_R would be of the form $(k, \Sigma, s_R, \delta_R, F_R)$.

s_R is a state that combines all the accepting states in F.
$F_R = s$
$\delta_R = \{((x,z),y) | \forall((y,z),x) \in \delta \wedge x, y \neq s\} \cup \{((s_R,z),x) | \forall((x,z),y) \in \delta, y \in F\} \cup \{((y,z),s_R) | \forall((x,z),y) \in \delta, x \in F\}$

Essentially the DFA where the accepting and starting states have been switched and transitions have reversed directions.

Problem 4

Let

$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

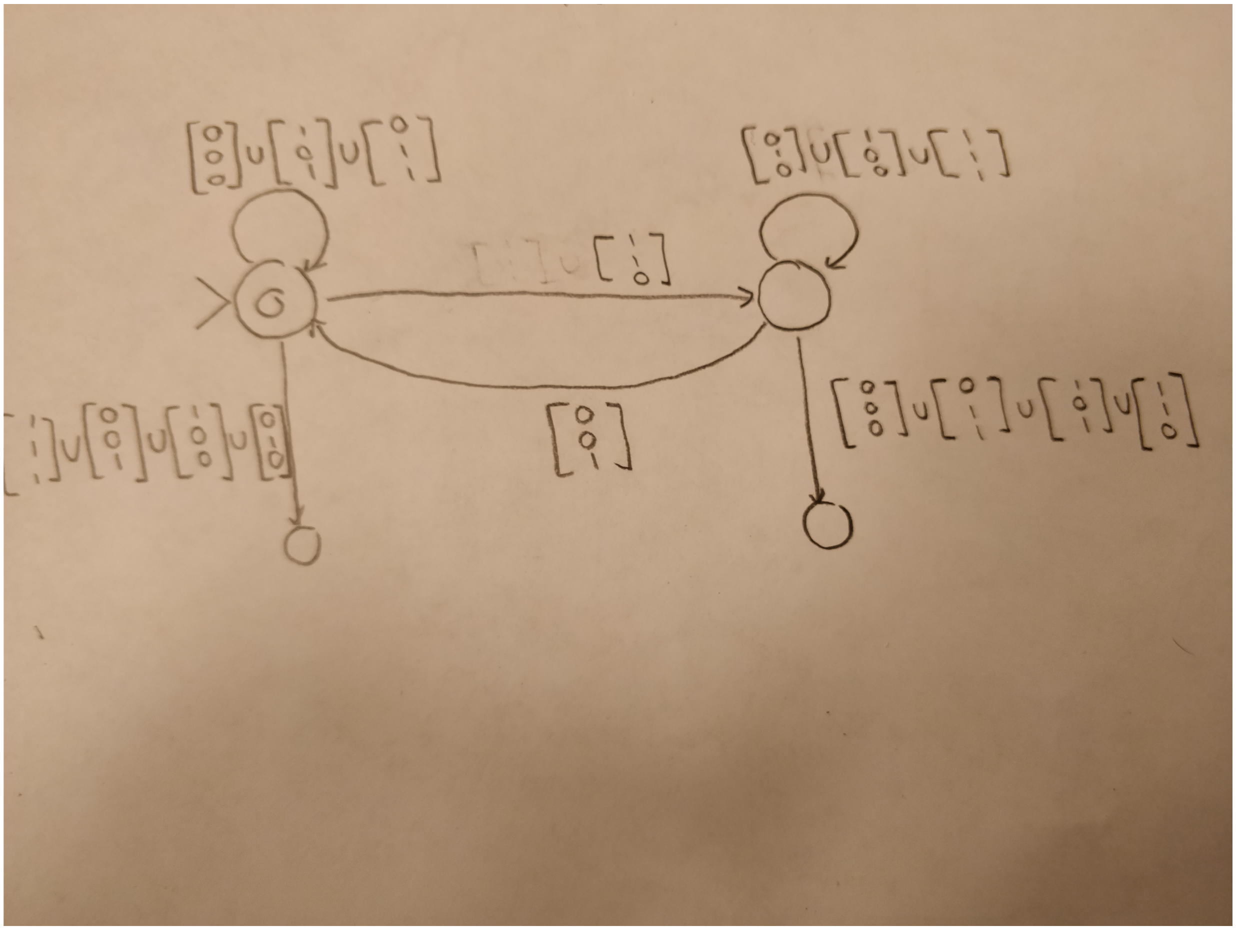
Σ_3 contains all size 3 columns of 0s and 1s. A string of symbols in Σ_3 gives 3 rows of 0s and 1s. Consider each row to be a binary number and let

$$B = \{w \in \Sigma_3^* \,|\, \text{the bottom row of } w \text{ is the sum of the top two rows}\}.$$

For example,

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in B, \text{ but } \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \notin B.$$

Show that B is regular. (Hint: Working with B^R is easier. You may assume the result proven in problem 3.)



Above is the DFA for B^R, which means that B is also regular.

* Credit to Andy Liang for correcting an issue with the DFA