Power Narrowing of Quantum Spectral Line Profile

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Power broadening — the broadening of the spectral line profile of a two-state quantum transition as the amplitude of the driving field increases — is a well-known and thoroughly examined textbook phenomenon in spectroscopy. It typically occurs in continuous-wave driving when the intensity of the radiation field increases beyond the saturation intensity of the transition. In pulsed-field excitation, a linear power broadening occurs for a pulse of a rectangular temporal shape. Pulses with smooth shapes have been shown to exhibit much less power broadening, e.g. logarithmic for a Gaussian temporal shape. It has been predicted that pulse shapes which vanish in time in a polynomial manner should exhibit the opposite effect — power narrowing — in which the transition line profile *decreases* when the amplitude of the driving pulse increases. In this work, it is demonstrated experimentally for Lorentzian pulse shape and its powers on the IMB Quantum processor ibmq_manila. In some cases, we observe the reduction of the line width by a factor of 5 (?) when increasing the pulse area from π to 9π, thereby defying the power broadening paradigm.

Introduction Power broadening is one of the basic paradigms in spectroscopy [1]. Indeed, in continuous-wave driving of a two-state quantum transition, when the intensity of the driving radiation field increases beyond the saturation intensity of the transition, the line width increases in proportion to the square root of the intensity. This effect, which is detrimental to high-resolution spectroscopy, supplements other broadening mechanisms, such as natural broadening, Doppler broadening, collisional broadening, etc.

In pulsed-field excitation, power broadening is still ubiquitous, although its presence and extent depend on the shape of the excitation pulse and the measurement method [**?**], e.g. whether the signal is collected during the excitation or after it (post-pulse). In particular, the post-pulse spectral line depends very strongly on the pulse shape. To this end, it is well known that a linear power broadening occurs for a pulse of rectangular temporal shape. Pulses with smooth shapes have been shown to exhibit much less power broadening. Exponential pulse shapes show little or none broadening [**?**]. For instance, a Gaussian temporal shape exhibits logarithmic power broadening [**?**] while a hyperbolic-secant shape generates a spectral line profile which does not depend on the driving-field amplitude at all [**?**].

It has been predicted [49] that pulse shapes which vanish in time in a polynomial manner should exhibit the opposite effect — power narrowing — in which the transition line profile *decreases* when the amplitude of the driving pulse increases. The most prominent member of this family is the Lorentzian pulse shape, for which it has been predicted that the linewidth scales as $1/\Omega_0$, the inverse of the peak Rabi frequency. It has been predicted that power narrowing can be observed in pulses shaped like any power of Lorentzian, higher than $\frac{1}{2}$: the closer the power to $\frac{1}{2}$ the stronger the power narrowing effect.

In this work we produce and present evidence for power narrowing in pulses shaped like powers of Lorentzian, obtained with the use of ibmq_manila, one of IBM's open quantum processors. Some of the data in the demonstration is presented in the shape of 2-dimensional colour maps that show the transition probability as a function of both the detuning and pulse amplitude. One of these was made for the Lorentzian$^{3/4}$ to show a strong power narrowing pattern. Additionally, several different such colour maps for Lorentzian pulses with different pulse widths were made to show the gradual transition from power narrowing to power broadening, as the pulse is truncated closer to the amplitude point.

Excitation linewidth: Adiabatic condition

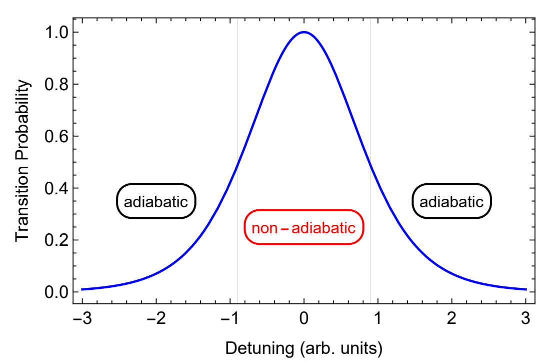

FIG. 1. A typical spectral profile of a two-state transition probability in pulsed excitation. For large detunings the evolution is nearly adiabatic and excitation is suppressed due to the effect of adiabatic population return. For small detunings — in the central part — the adiabatic condition is violated and high excitation is enabled. Therefore, the width of the spectral line profile is determined by the adiabatic condition.

The adiabatic condition is most heavily violated where $|\dot{\vartheta}(t)|$ is maximal. Due to symmetry, this happens in two points symmetric about the amplitude of the pulse – $t = \pm t_m$. In these points, we have a lower limit on $|\Delta|$, for which the adiabatic condition is satisfied – Δ_m. This Δ_m presents us with a range of detuning values for which the evolution is not adiabatic, which helps us quantify the linewidth of the transition line profile $P_{0\to1}(\Delta)$ – $\Delta_{\frac{1}{2}} \propto \Delta_m$. One can solve the adiabatic condition from

Eq. (**??**) as an equality at $t = t_m$ to find Δ_m:

$$\sqrt{\Omega^2(t_m) + \Delta_m^2} = \frac{2|\Delta_m \dot{\Omega}(t_m)|}{\Omega^2(t_m) + \Delta_m^2}. \quad (1)$$

One option is to have a pulse with large discontinuities (e.g. a rectangular pulse). The evolution close to the jumps at the initial and final times of the pulse is not adiabatic, namely $\dot{\Omega}(t) \xrightarrow{t \to t_{i,f}} \pm\infty$ and Eq. (**??**) is not satisfied. This implies that there will be transitions in the diabatic basis for larger Δ, as Ω_0 increases, and therefore power broadening would be observed.

However, with a certain class of pulses where the tail falls of as $f(t) \sim 1/t^n$ for $n > 1, |t| \to \infty$, we would have

$$t_m \sim \left(\frac{\Omega_0}{\Delta}\right)^{\frac{1}{n}}, \quad (2)$$

or

$$\Delta_m T \sim (\Omega_0 T)^{-\frac{1}{n-1}}, \quad (3)$$

where T is the pulse width. This shows that with increasing pulse amplitude, we get a narrower line width of the transition line profile, or in other words — power narrowing.

One candidate family of pulses that satisfy this requirement are pulses shaped as powers of the Lorentzian function

$$f_n(t) = \frac{1}{\left(1 + \left(\frac{t}{T}\right)^2\right)^N}, \quad (4)$$

which approaches $1/t^{2N}$ as $t \to \pm\infty$. The value of N should satisfy $2N > 1$ since above $n > 1$, so it can be any number larger that $1/2$. The detuning value at the critical point t_m after which adiabaticity is retained is

$$\Delta_m T = \left(\frac{(2N+1)^{\frac{2N+1}{2}}(2N-1)^{\frac{4N-1}{2}}}{(4N)^N \Omega_0 T}\right)^{\frac{1}{2N-1}}, \quad (5)$$

which is essentially proportional to $\Omega_0^{-\frac{1}{2N-1}}$. In case $2N - 1 > 0$, we observe power narrowing and the as it gets closer to 0 (N approaches $1/2$), the power narrowing becomes exponentially stronger.

Quantum processor specs In this work we use qubit 0 of ibmq_manila, one of the IBM Quantum Falcon r5.11L Processors [**?**]. It consists of five transmon qubits [**?**]. At the time of the demonstration the parameters of the zeroth qubit of the ibmq_manila system are calibrated as follows: the qubit frequency is 4.96229 GHz, with anharmonicity −0.34463 GHz. The T1 coherence time is 166.44 μs, the T2 coherence time is 116.53 μs and the readout assignment error is 3.14%.

The IBM Quantum processor is engaged by exerting pulse-level control using the Qiskit Pulse framework for Python. There are few limitations of the Qiskit Pulse framework, among which there are: (i) limits on the pulse duration and the total duration of the circuit; (ii) inability to use a time-dependent driving frequency and therefore detuning directly (but a workaround exists); (iii) discretisation of the Rabi frequency into small intervals of 2/9 ns each (possibly a hardware limitation). However, knowing these constraints, one can control the coupling, its phase, and the driving frequency and implement various quantum control models on one or more qubits.

Demonstration of the power narrowing phenomenon As we saw in Sec. **??**, the tail of the function needs to fall off as $1/t^n$ to observe power narrowing. However, in practice we cannot have infinite pulses and need to truncate the pulses somewhere. In this work, we move the truncation points of Lorentzian pulses to see how the adiabaticity induced by the long Lorentzian tail causes a gradual transition from power broadening (when we truncate right before/after the Lorentzian peak) to power narrowing (when we truncate the pulse sufficiently long after the main excitation). The truncation points are always symmetrical about the excitation peak. The strong power narrowing effects appear when we truncate the pulse at less than 1% of its maximum amplitude.

Additionally, the transition probability map of a single Lorentzian pulse with truncation sufficient time from the pulse peak was made. The transition line profiles at the maxima were also measured and their widths compared. Two supposedly symmetric off-resonance Rabi oscillations at detunings $\Delta = \pm\Delta_0$, where Δ_0 is close enough to the resonance so as to capture the damping of the higher order maxima w.r.t. the first one.

Finally, the transition probability of a lower power of a Lorentzian is measured and presented, by overcoming the Qiskit pulse duration limitations.

Figure 2 presents a gradual increase of the duration of the Lorentzian pulse. It aims to offer an insight into when the nonadiabatic effects become strong enough to distinguish power broadening instead of narrowing. For this purpose we use a Lorentzian of width 21.33 ns and decrease its duration by symmetrically clipping larger and larger parts of the tail. We start by truncating the pulse far from the maximum value, at a value of the Rabi frequency which is 0.5% of the maximum so the duration of the pulse is $T = 600.89$ ns or about 28 pulse widths. We can see the resulting transition probability map in the upper left of Fig. 2. For comparison, the shortest pulse, the one with a truncation height of 50% the maximum coupling (lower right in the figure), is of duration 42.67 ns or just 2 pulse widths.

By comparing the Lorentzians with different truncation times one can notice several trends. First, the maxima move to higher Rabi frequencies by decreasing the duration. As shown in Sec. **??**, the Rabi oscillations depend on the pulse area sinusoidally, which corresponds to the observed behaviour at $\Delta = 0$. Therefore, a shorter duration implies a smaller pulse area that can be compensated with an increase in Rabi frequency amplitude, as to Eq. (**??**).

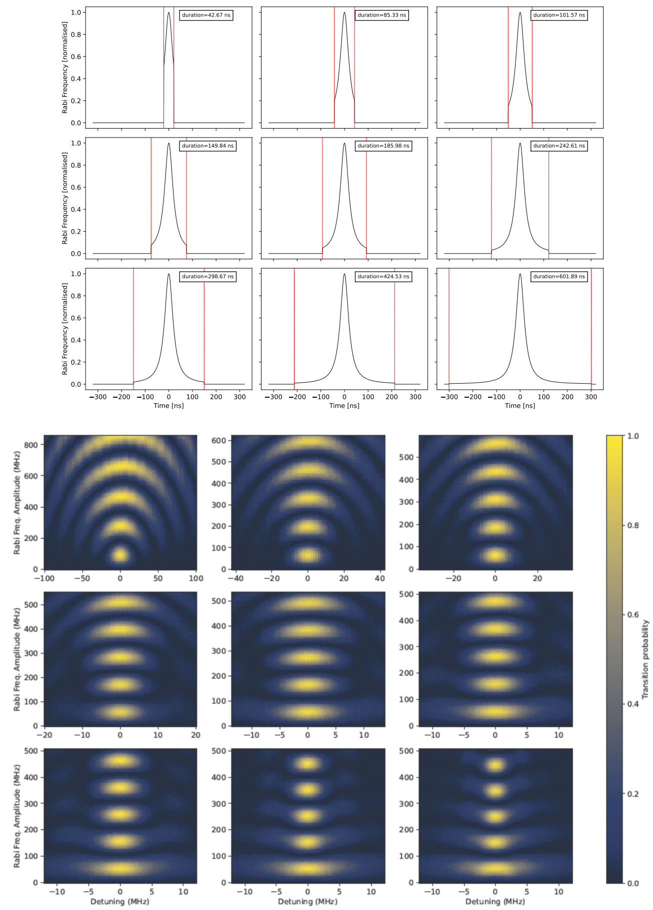

FIG. 2. The evolution of a truncated Lorentzian pulse from power narrowing (top left) to power broadening (bottom right) with the shifting of truncation point closer and closer to pulse peak is shown in the 9 plots. From top to bottom and left to right the truncation points are located at 0.5%, 1%, 2%, 3%, 5%, 7.5%, 15%, 20% and 50% of the maximum coupling value.

Another interesting observation is that the first excitation line profile shows no significant line width change versus the detuning. However, the similarities end there, all the other peaks exhibit serious broadening with duration contraction. While the first peak is about 20 MHz wide, the fourth one expands from a width of approximately 10 MHz in the 0.5%-height case to roughly 100 MHz in the 50%-height case, without taking the wave-like spill into lower amplitude ranges into account. This effect is a result of the transition between power narrowing and power broadening.

A. Lorentzian ($N = 1$) pulse power narrowing

A natural choice for a function with polynomially vanishing wings is the Lorentzian. The excitation pulses were shaped using the Qiskit Pulse framework. The Lorentzians used for this demonstration had a pulse

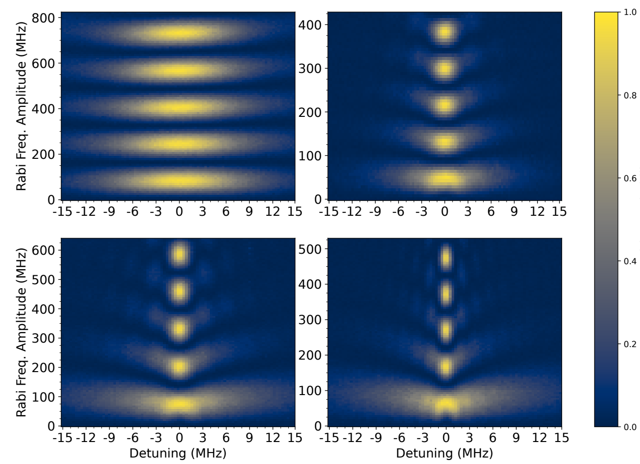

FIG. 3. Caption

width $\sigma = 21.33$ ns and the duration was $T = 600.89$ ns. These produce a Rabi frequency magnitude 0.5% of the amplitude at the truncation points. One could expect that the tail was clipped far enough from the maximal point to realise power narrowing, as demonstrated in Fig. 2.

On Fig. **??** we can see a two-dimensional colour map that captures the first 5 maxima of the Rabi oscillations and their vanishing with detuning. Five different maxima are indeed visible at Rabi frequency values that correspond to pulse areas $\pi, 3\pi, 5\pi, 7\pi$ and 9π. There is a noticeable decrease in the linewidth of the 2^{nd} maximum compared to the first one by an estimated factor of almost 2. The narrowing from the 1^{st} to the 5^{th} maximum is however much stronger, with a decrease roughly by a factor of 3.5. This narrowing still comes short of the predicted narrowing $\Delta \propto \Omega_0^{-1}$ which suggests a width reduction by a factor of 3 between the π and 3π area peaks.

Fig. **??** shows horizontal slices of the first 5 maxima on Fig. **??** along the detuning. We can estimate the full width at half maximum (FWHM) to be around 9 MHz for the first excitation (marked with thick red points) and about 4.5 MHz for the second one (marked with yellow stars). In a similar manner we can notice that the third maximum (marked with green crosses) the FWHM is around 3.5 MHz. Regardless of their significance, these decreases in linewidth are smaller than the theoretically predicted ones. The anticipated narrowing is by a factor of 3 between the first and second maximum, and by a factor of 5/3 between the second and third maximum. The difference in the width between the fourth and fifth maxima is too small to be estimated. This suggests that there could be a hardware limit to how narrow the transition line profiles can become, and it could interfere with the theoretical narrowing estimates.

As evident from Eq. (5), powers of Lorentzian lower

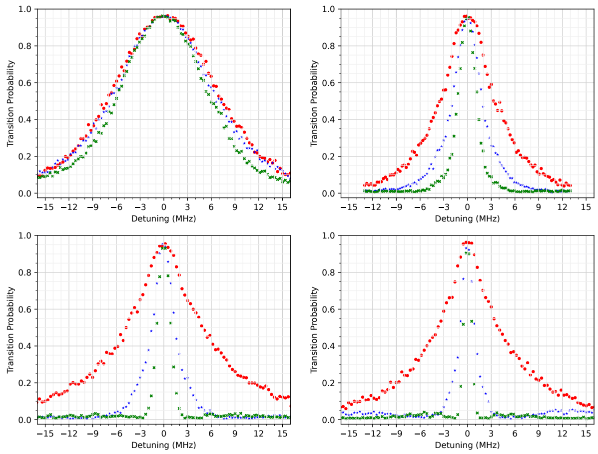

FIG. 4. Caption

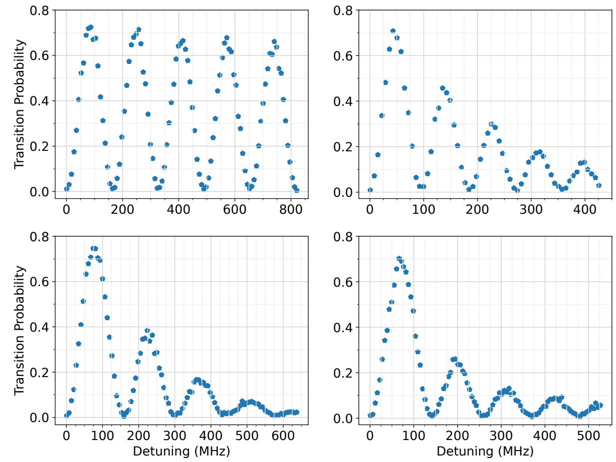

FIG. 5. Caption

or equal to 1/2 have a divergent area and thus they cannot be used to demonstrate power narrowing. However, theory predicts that any power of a Lorentzian $1/2 < n < 1$ would exhibit stronger power narrowing than a plain Lorentzian. Thus, a function with a stronger power narrowing tendency than the Lorentzian was chosen: Lorentzian$^{3/4}$. The main condition to observe power narrowing is the tail falling off as $1/t^n$ for $n > 1$, so the duration of the Lorentzian$^{3/4}$ needed to be longer than that of the Lorentzian, so as to clip the tail far from the amplitude. For the same pulse width $\sigma = 21.33$ ns, the duration was increased to 1112.89 ns, or almost twice. On the other hand, due to the heavier tail, the pulse area for the same pulse width was larger, so a smaller drive amplitude range was exploited — from maximum amplitude ≈ 500 MHz to a bit above 300 MHz.

The power spectrum of the Lorentzian$^{3/4}$ pulse is shown in Fig. **??**. There, one can notice a greater effect than seen in plain Lorentzian pulses, as the first maximum overshadows the rest. However, a possible experimental $\bigcap$-shaped artefact appears right below the π-area island. Nevertheless, the artefact does not seem to impact the linewidth of the maximum.

Conclusions

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