Quantifying Systemic Risk in Europe

A Bayesian Approach to Map Financial Contagion Through CDS Spreads

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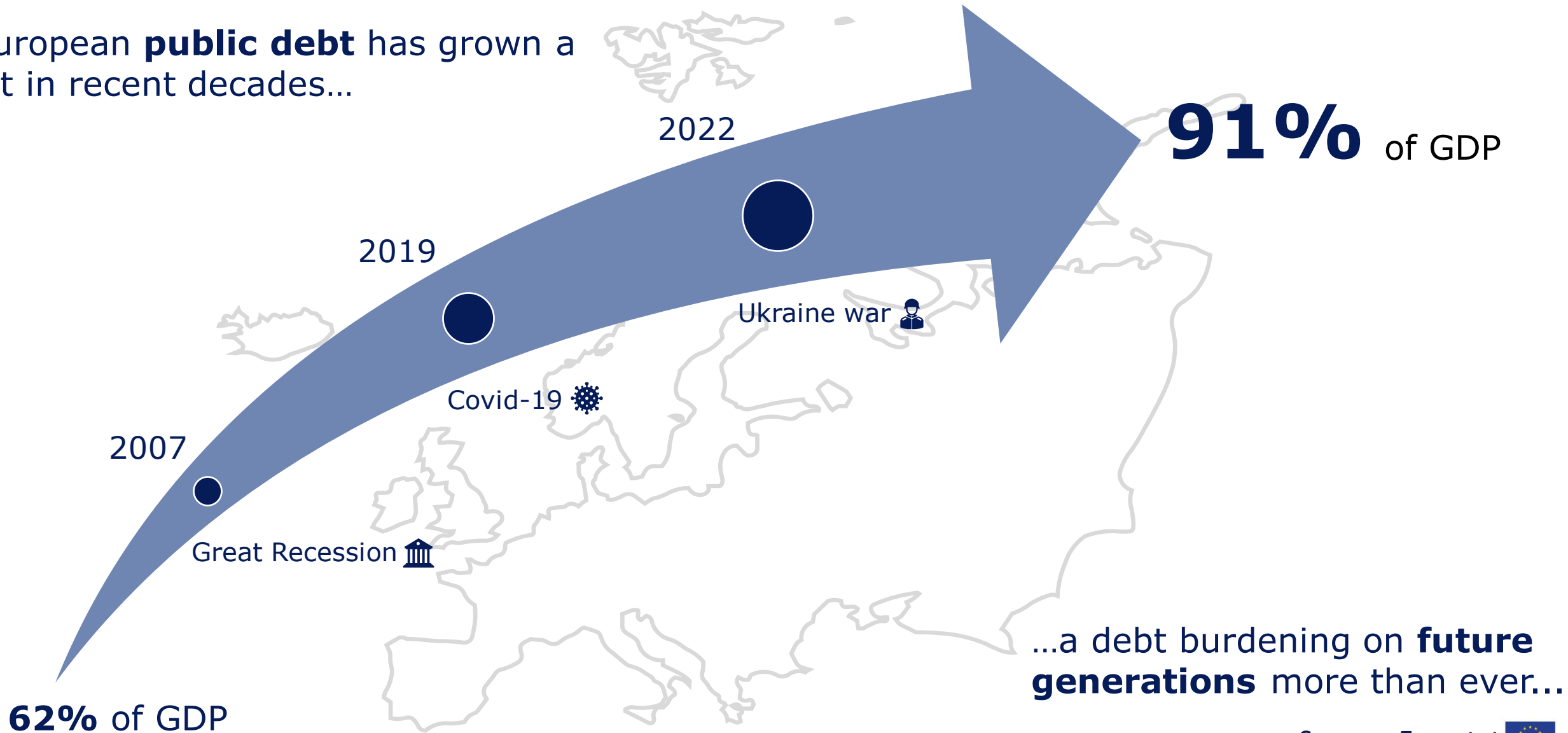**Assistant Supervisor** *cosupervisor*
Prof.ssa Silvia Salini
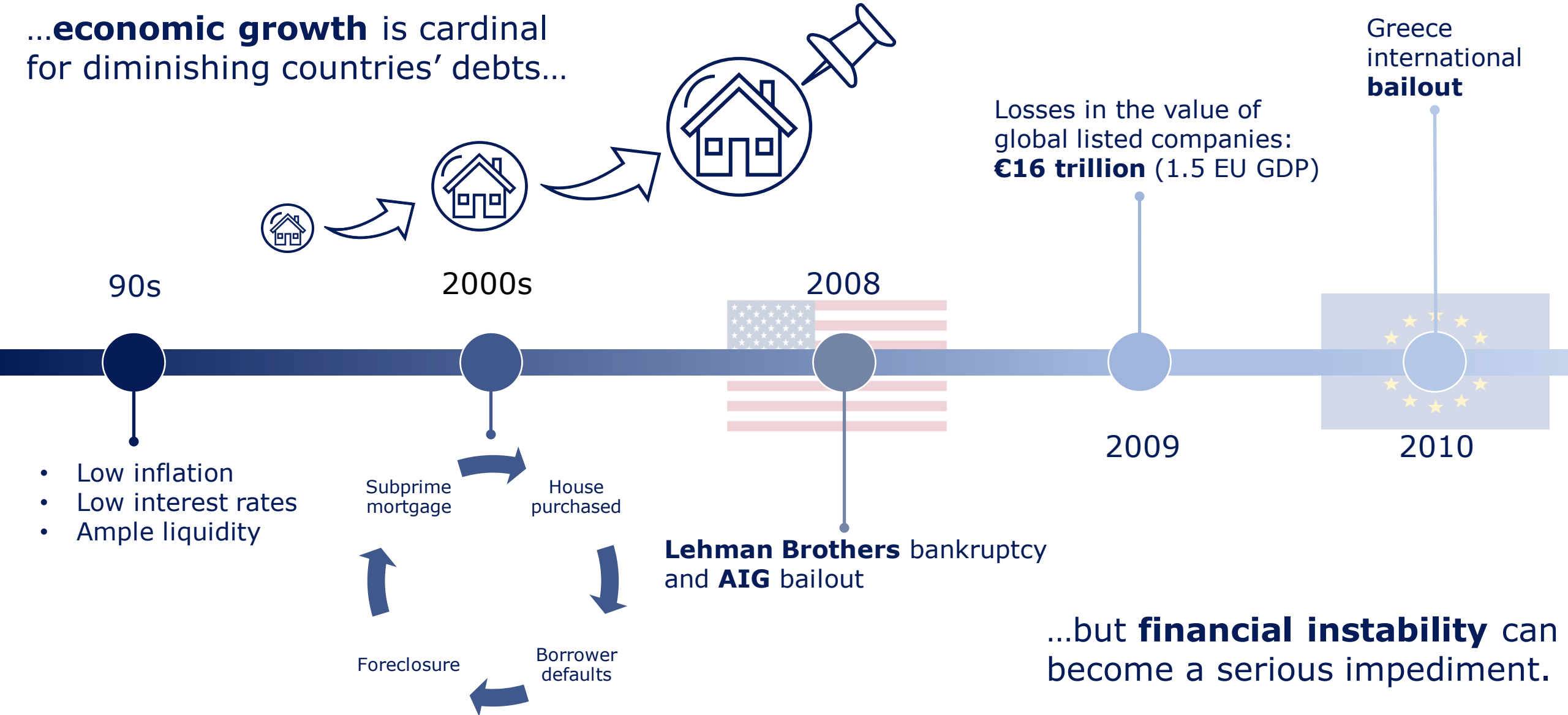**Graduand**
Ivo Bonfanti

Motivations - Great Recession

…**economic growth** is cardinal for diminishing countries' debts…

Greece international **bailout**

Losses in the value of global listed companies: **€16 trillion** (1.5 EU GDP)

90s

2000s

2008

2009

2010

- Low inflation
- Low interest rates
- Ample liquidity

Subprime mortgage

House purchased

Borrower defaults

Foreclosure

Lehman Brothers bankruptcy and **AIG** bailout

…but **financial instability** can become a serious impediment.

Motivations - Systemic Risk

"Regulators and supervisors focused on the micro-prudential supervision of individual financial institutions and not sufficiently on the **macro-systemic risks** of a contagion of correlated horizontal shocks." *-The Larosière Report, EU, 2009-*

"With respect to macroeconomic policy, policymakers failed to take sufficiently into account growing macroeconomic imbalances that contributed to the buildup of **systemic risks** in the financial system and in housing markets." *–Annual Report, IMF, 2009-*

"The financial and economic crisis that has shaken the world economy for more than two years illustrates the relevance of **systemic risk**."…"The objective is to characterise the phenomenon of **systemic risk** from an academic research perspective."…"three main "forms" of **systemic risk**: the contagion risk, the risk of macro shocks causing simultaneous problems and the risk of the unravelling of imbalances that have built up over time." *-Financial Stability Review, ECB, 2009-*

Systemic Risk - Financial Contagion

« *Systemic risk means a risk of **disruption** in the financial system with the potential to have serious negative consequences for the internal market and **the real economy**. All types of financial intermediaries, markets and infrastructure may be potentially systemically important to some degree.*

-EU Regulation 1092/2010-

»
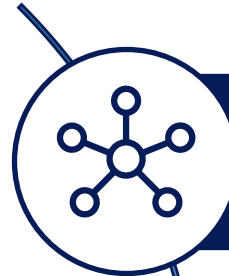
Among the banking instability sources the most prominent is the **dense network** of connection between intermediaries.
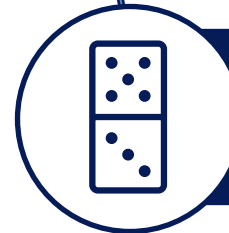
The **concurrent failure** of a handful of large financial institutions could trigger a severe and pervasive economic crisis.

Potentially having serious negative consequences for the **real economy**.

Credit Default Swap - Functioning

It is a financial derivative working similarly to an **insurance** contract.

1

P.B. 🤝 P.S.

P.B. ➤ € P.S.

Notional amount
Reference entity
Contingent event
Fixed premium
Tenor

2

P.S. ➤ 💵 ➤ P.B.
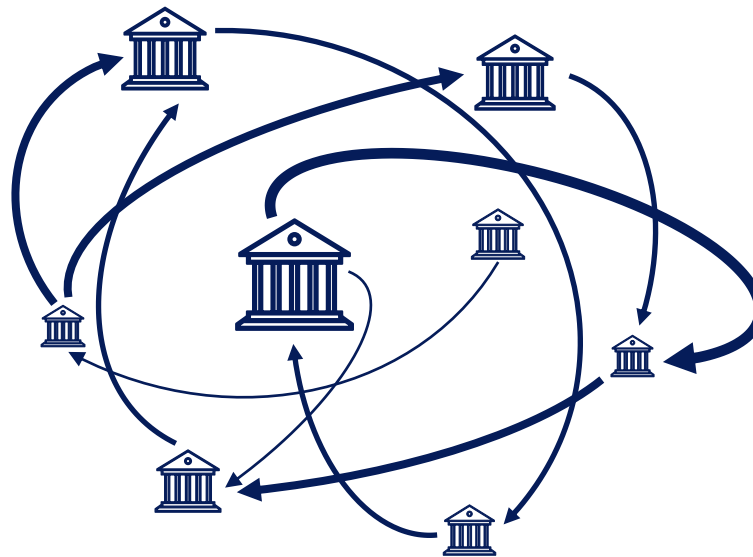
The contract is **settled.**

P.B. ➤ P.S.
€

3
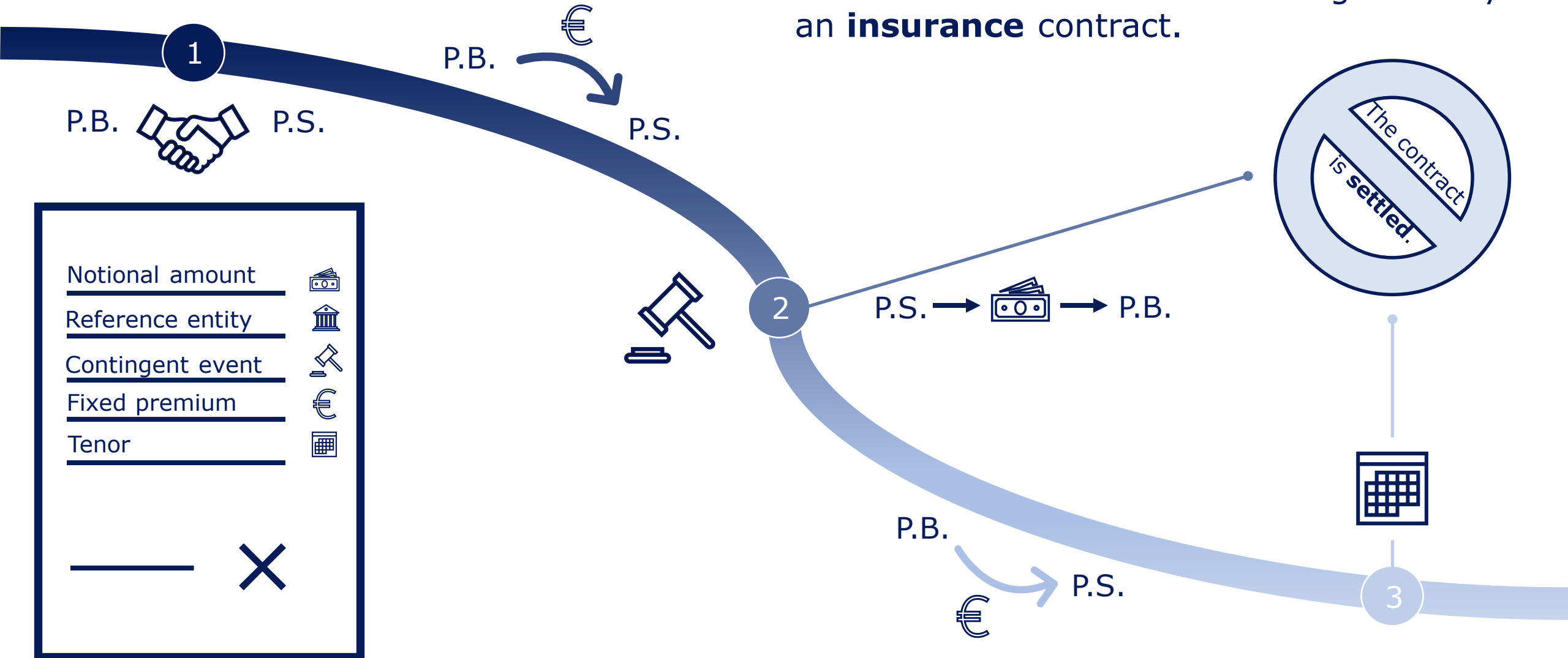
Credit Default Swap - A Proxy for Financial Contagion

The spread incorporates information on how the market perceives the **creditworthiness** of a reference entity.
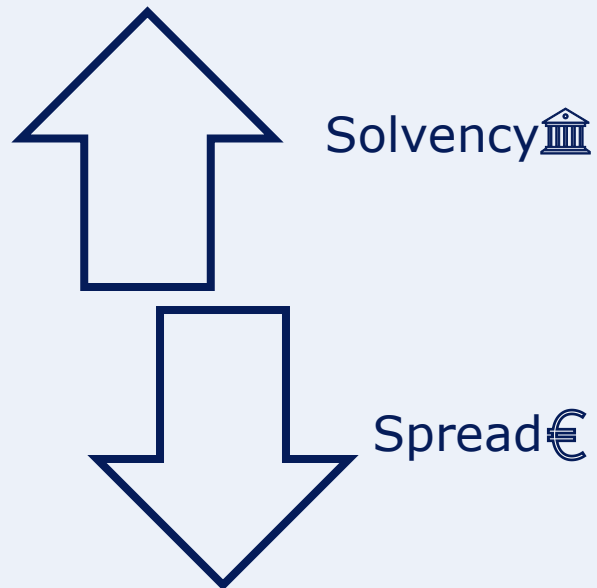
Spreads co-movements carry information on the reference entities level of **interdependence.**

A tightly interconnected network of institutions fosters the rapid spread of **financial contagion**, posing systemic risk. High interdependence among market participants amplifies shocks across the system.

Solvency 🏛

Spread €
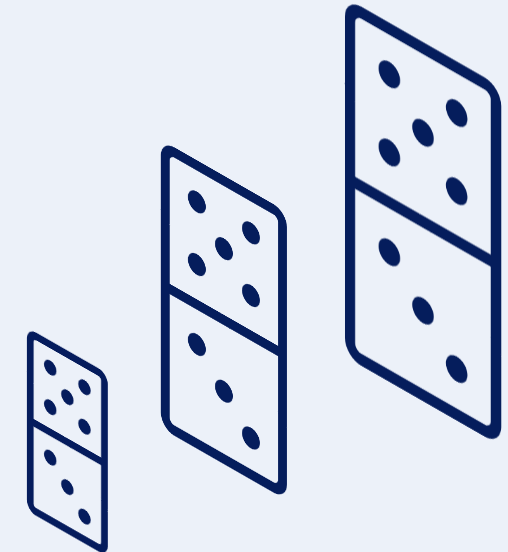
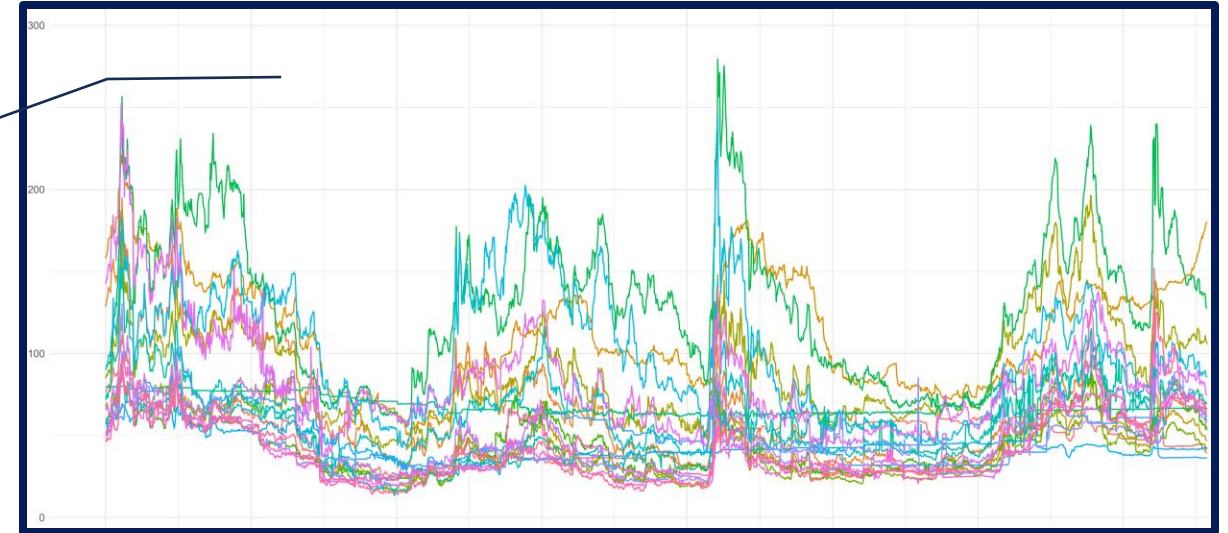# Credit Default Swap - Datasets
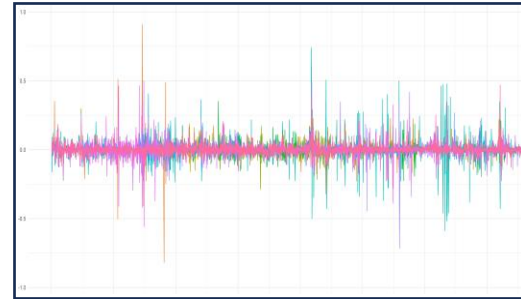

- IHS Markit
- 5 years
- 21 European **banks**
- 29 European **countries**
- Daily
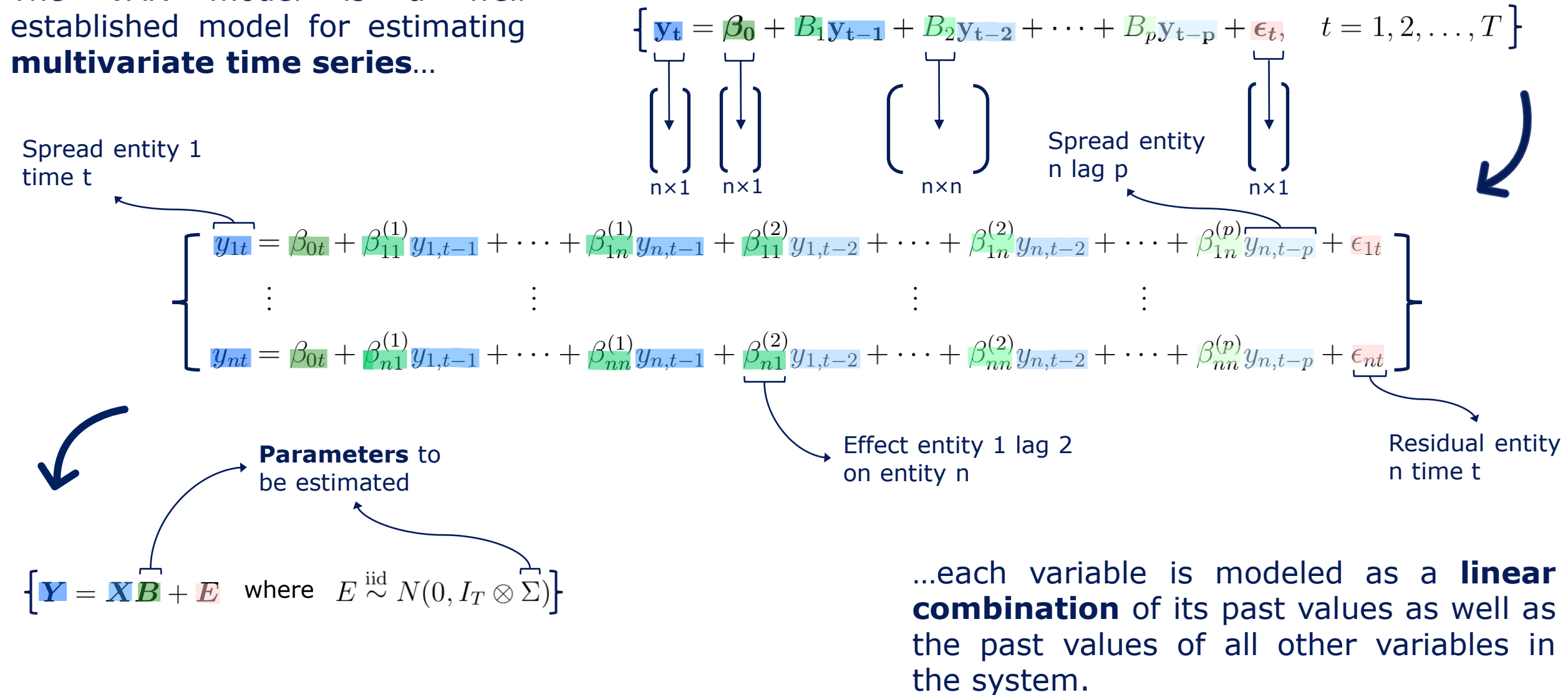- [01/01/2016-31/07/2023]
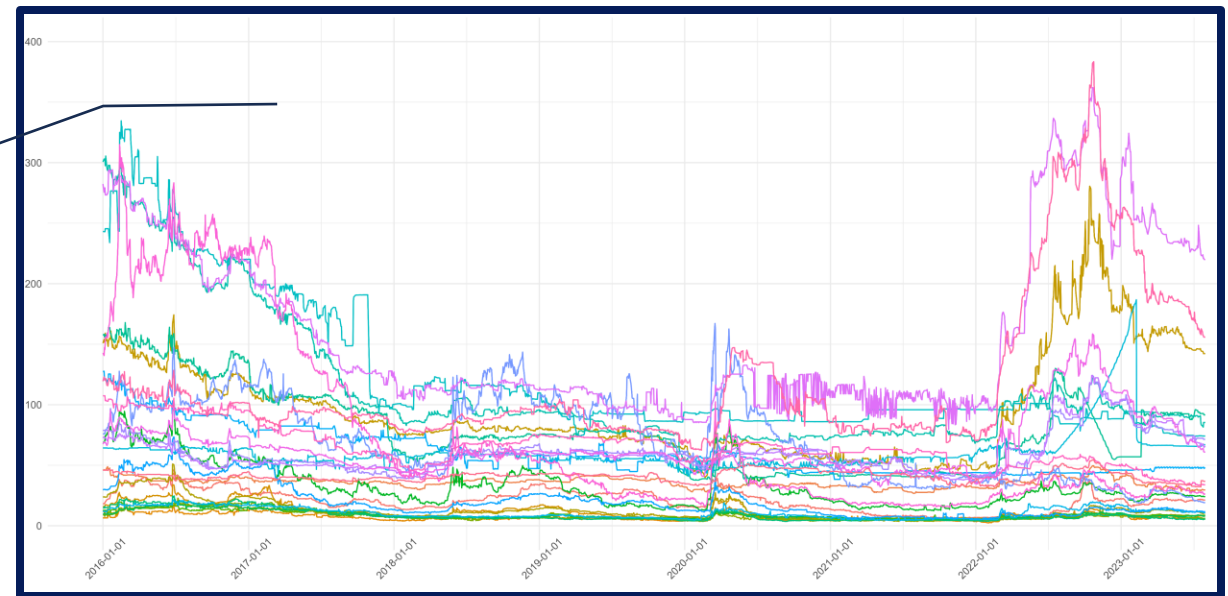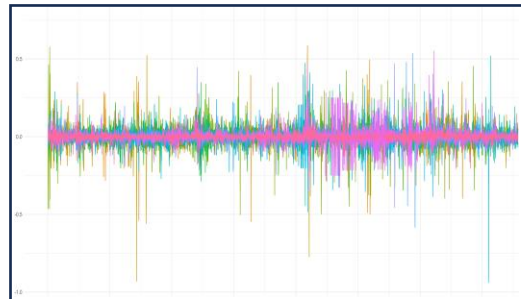
The VAR model is a well established model for estimating **multivariate time series**...

$$\left\{ \mathbf{y_t} = \boldsymbol{\beta_0} + B_1 \mathbf{y_{t-1}} + B_2 \mathbf{y_{t-2}} + \cdots + B_p \mathbf{y_{t-p}} + \boldsymbol{\epsilon_t}, \quad t = 1, 2, \ldots, T \right\}$$

n×1   n×1   n×n   n×1

Spread entity 1 time t

Spread entity n lag p

$$\left[\begin{array}{l} \overline{y_{1t}} = \beta_{0t} + \beta_{11}^{(1)} y_{1,t-1} + \cdots + \beta_{1n}^{(1)} y_{n,t-1} + \beta_{11}^{(2)} y_{1,t-2} + \cdots + \beta_{1n}^{(2)} y_{n,t-2} + \cdots + \beta_{1n}^{(p)} \overline{y_{n,t-p}} + \epsilon_{1t} \\ \quad\vdots \qquad\qquad\qquad\vdots \qquad\qquad\qquad\qquad\vdots \qquad\qquad\qquad\qquad\vdots \\ y_{nt} = \beta_{0t} + \beta_{n1}^{(1)} y_{1,t-1} + \cdots + \beta_{nn}^{(1)} y_{n,t-1} + \beta_{n1}^{(2)} y_{1,t-2} + \cdots + \beta_{nn}^{(2)} y_{n,t-2} + \cdots + \beta_{nn}^{(p)} y_{n,t-p} + \epsilon_{nt} \end{array} \right]$$

Effect entity 1 lag 2 on entity n

Residual entity n time t

Parameters to be estimated

$$\left\{ \boldsymbol{Y} = \boldsymbol{X} \boldsymbol{B} + \boldsymbol{E} \quad \text{where} \quad E \overset{\text{iid}}{\sim} N(0, I_T \otimes \Sigma) \right\}$$

...each variable is modeled as a **linear combination** of its past values as well as the past values of all other variables in the system.

$\Theta = [\mathbf{B}; \Sigma]$
$Y = $ Data

$$\mathcal{P}(\boldsymbol{B}) = \frac{1}{\sqrt{2\pi\Omega}} exp\left\{-\frac{1}{2}(\boldsymbol{B} - \mu)'\Omega^{-1}(\boldsymbol{B} - \mu)\right\}$$

$$\mathcal{P}(\Sigma) = \frac{|\zeta|^{\frac{\nu}{2}}}{2^{\frac{\nu n}{2}}\Gamma_n(\frac{\nu}{2})}|\Sigma|^{-\frac{\nu+n+1}{2}} exp\left\{-\frac{1}{2}tr(\zeta\Sigma^{-1})\right\}$$

$$\mathcal{P}(A|B) = \frac{\mathcal{P}(B|A)\mathcal{P}(A)}{P(B)} \Rightarrow \Pi(\boldsymbol{\theta}|\boldsymbol{Y}) = \frac{\mathcal{P}(\boldsymbol{\theta})\mathcal{L}(\boldsymbol{Y}|\boldsymbol{\theta})}{P(\boldsymbol{Y})} \propto \mathcal{P}(\boldsymbol{\theta})\mathcal{L}(\boldsymbol{Y}|\boldsymbol{\theta})$$

Bayesian theorem

Posterior distribution

$$\mathcal{L}(\boldsymbol{Y}|\boldsymbol{B},\Sigma) \propto |\Sigma|^{-\frac{T}{2}} exp\left\{-\frac{1}{2}(\boldsymbol{Y} - \boldsymbol{XB})'\Sigma^{-1}(\boldsymbol{Y} - \boldsymbol{XB})\right\}$$

To handle the typical VAR **overparameterization** issue…

$$\mu_{ij}^{(s)} = \begin{cases} 1, & \text{if } i = j \text{ and } s = 1, \\ 0, & \text{otherwise}, \end{cases}$$

$$\beta_{ij}^{(s)} \sim N(\mu_{ij}^{(s)}, \Omega_{ij}^{(s)})$$

$$\Omega_{i,j}^{(s)} = \begin{cases} \omega^2, & \text{if } s = 1 \text{ and } i = j, \\ (\omega \cdot f(s^{-1}))^2, & \text{if } s \geq 2 \text{ and } i = j, \\ (\kappa \cdot \omega \cdot \tau_{i,j} \cdot f(s^{-1}))^2, & \text{if } i \neq j \text{ and } \forall s \geq 1, \end{cases}$$

$j^{(t-s)}$ $\qquad$ $i^{(t)}$

$$\beta_{ii}^{(1)} \sim N(1, \omega^2), \qquad \beta_{ij}^{(1)} \sim N(0, (\kappa \cdot \omega \cdot \tau_{ij})^2),$$

$$\beta_{ii}^{(2)} \sim N(0, (\tfrac{\omega}{2})^2), \qquad \beta_{ij}^{(2)} \sim N(0, (\tfrac{\kappa \cdot \omega \cdot \tau_{ij}}{2})^2),$$

$$\beta_{ii}^{(3)} \sim N(0, (\tfrac{\omega}{3})^2), \qquad \beta_{ij}^{(3)} \sim N(0, (\tfrac{\kappa \cdot \omega \cdot \tau_{i,j}}{3})^2),$$
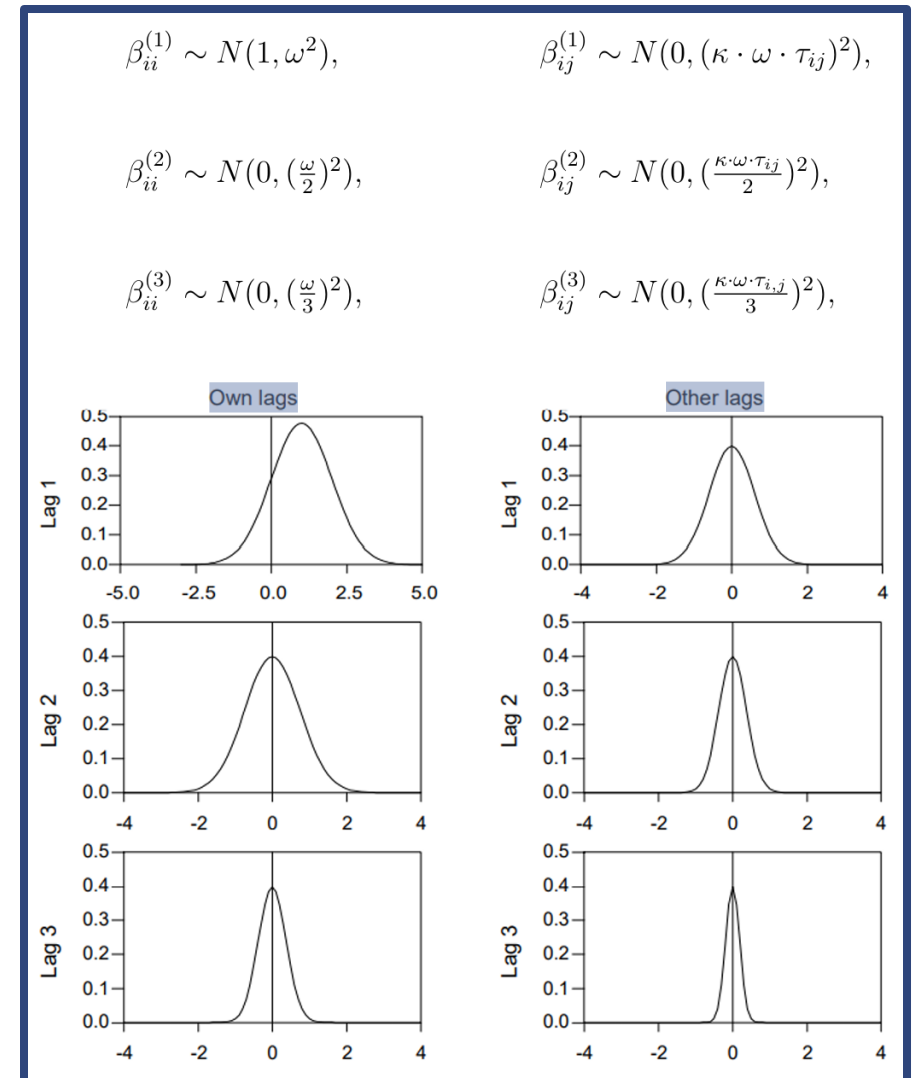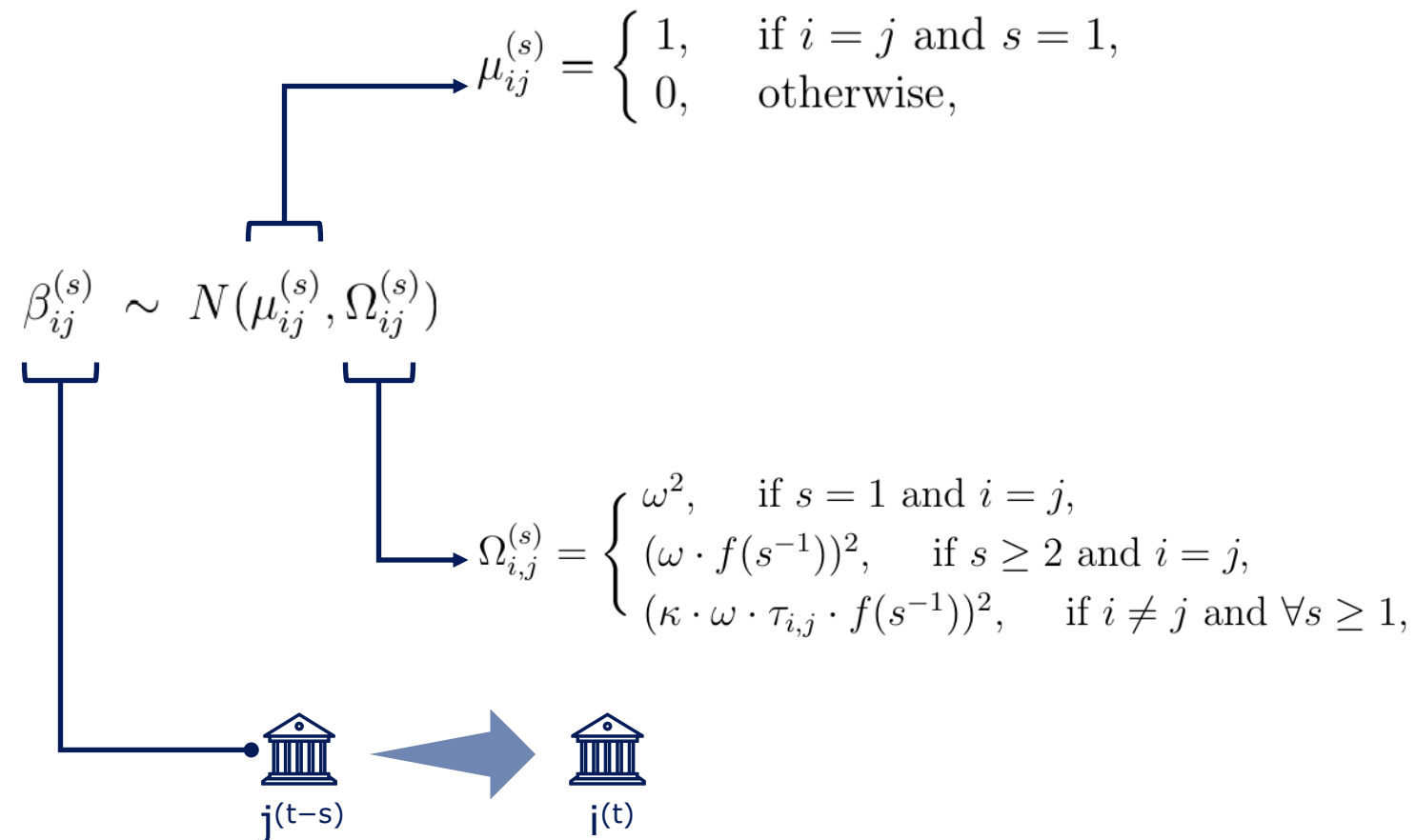
Algorithm 1: Gibbs Sampler

1 Initialize $Draws\boldsymbol{B}$ and $Draws\Sigma$ as empty matrices of dimensionality $[n \cdot (n \cdot p + 1) \times 9000]$ and $[(n \cdot n) \times 9000]$ respectively.

2 Set the counter $i = 1$.

3 Initialize $\Sigma^{(0)} = I_{[n \times n]}$

4 For i in 1:10000 do:

    (I) Draw $\boldsymbol{B}^{(i)}$ from the posterior $\Pi(\boldsymbol{B}^{(i)}|\Sigma^{(0)}, \boldsymbol{Y})$

    (II) Draw $\Sigma^{(i+1)}$ from the posterior $\Pi(\Sigma^{(i+1)}|\boldsymbol{B}^{(i)}, \boldsymbol{Y})$

5 If i>1000 do:

    (I) Store $\boldsymbol{B}^{(i)}$ in $Draws\boldsymbol{B}$.

    (II) Store $\Sigma^{(i+1)}$ in $Draws\Sigma$.

6 Change the counter i to $i + 1$ and return to step 4.

7 Exit from the loop.

$\Pi(\boldsymbol{B}|\Sigma, Y)$

$\Pi(\Sigma|\boldsymbol{B}, Y)$
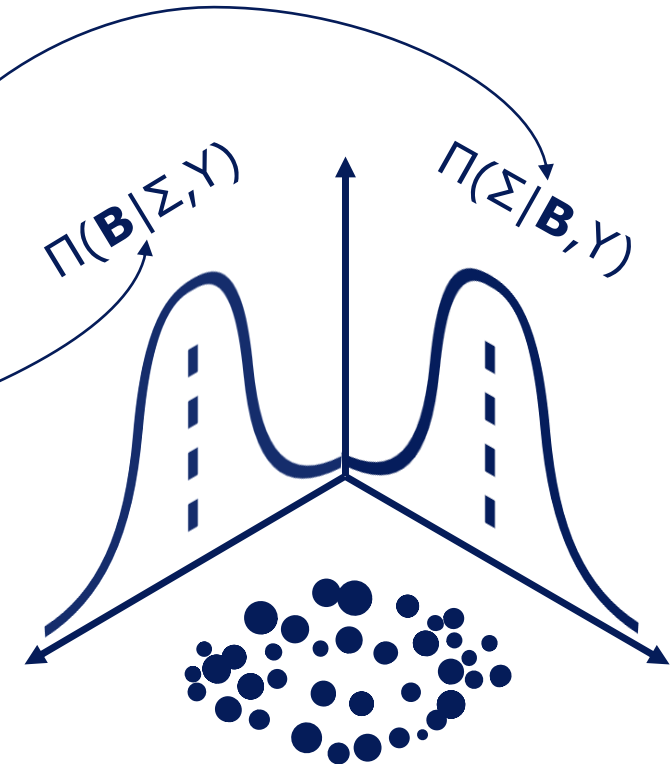
A **M**arkov **C**hain **M**onte **C**arlo method to simulate realizations from the full **posterior distribution**…

Signal Adaptive Variables Selector - Sparsity

…the Signal Adaptive Variable Selector is an automated post-processing technique useful for distinguishing between **signal** and **null** parameters…

Sparse

Shrank

Design matrix

Penalization term

$$\hat{\boldsymbol{B}}^* := \arg\min_{\boldsymbol{B}} \left\{ \frac{1}{2} \|\boldsymbol{X}\hat{\boldsymbol{B}} - \boldsymbol{X}\boldsymbol{B}\|_2^2 + \sum_{i=1}^{r} \sum_{j=1}^{c} \mu_j |\beta_{i,j}| \right\}$$

Algorithm 2: Signal Adaptive Variable Selector

1. Input: posterior estimate $\hat{\boldsymbol{B}}^p$ and design matrix $\boldsymbol{X}$.
2. Set the counter $i = 1$.
3. For i in 1:r do:
   Set the counter $j = 1$.
4. For j in 1:c do:
   $\mu_j = \frac{1}{|\hat{\beta}_{i,j}|^2}$
5. If $|\hat{\beta}_{i,j}|^2 \cdot \|\boldsymbol{X_j}\|_2^2 \leq \mu_j$ do:
   $\hat{\beta}_{i,j}^* = 0$
6. Else do:
   $\hat{\beta}_{i,j}^* = sign(\hat{\beta}_{i,j}) \|\boldsymbol{X_j}\|_2^{-2} (|\hat{\beta}_{i,j}|^2 \cdot \|\boldsymbol{X_j}\|_2^2 - \mu_j)$
7. Change the counter j to $j+1$ and return to step 4.
8. Change the counter i to $i+1$ and return to step 3.
9. Output: sparse estimate $\hat{\boldsymbol{B}}^*$

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

…the matrix of estimated coefficients is **sparsified** to enhance interpretability.
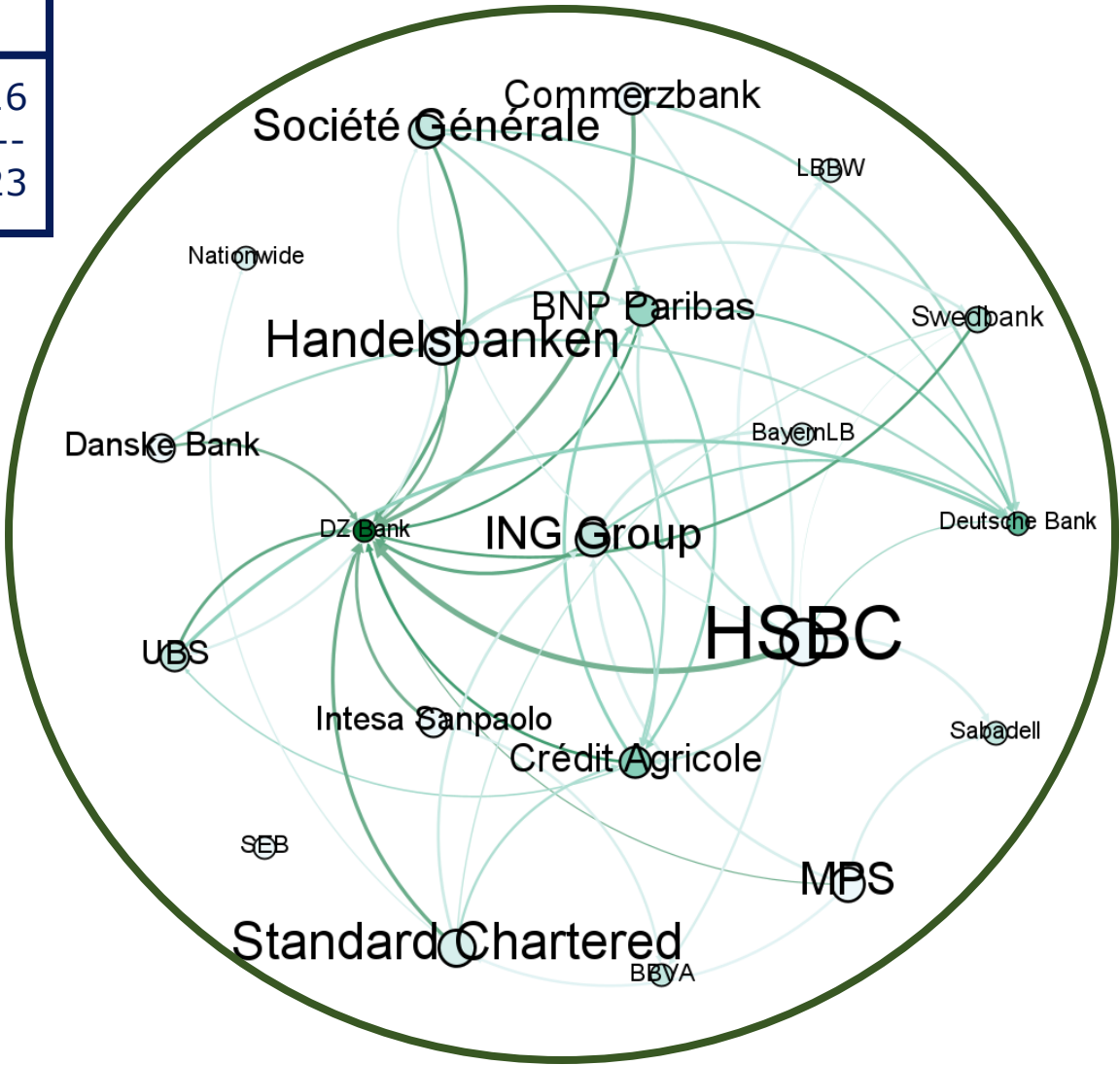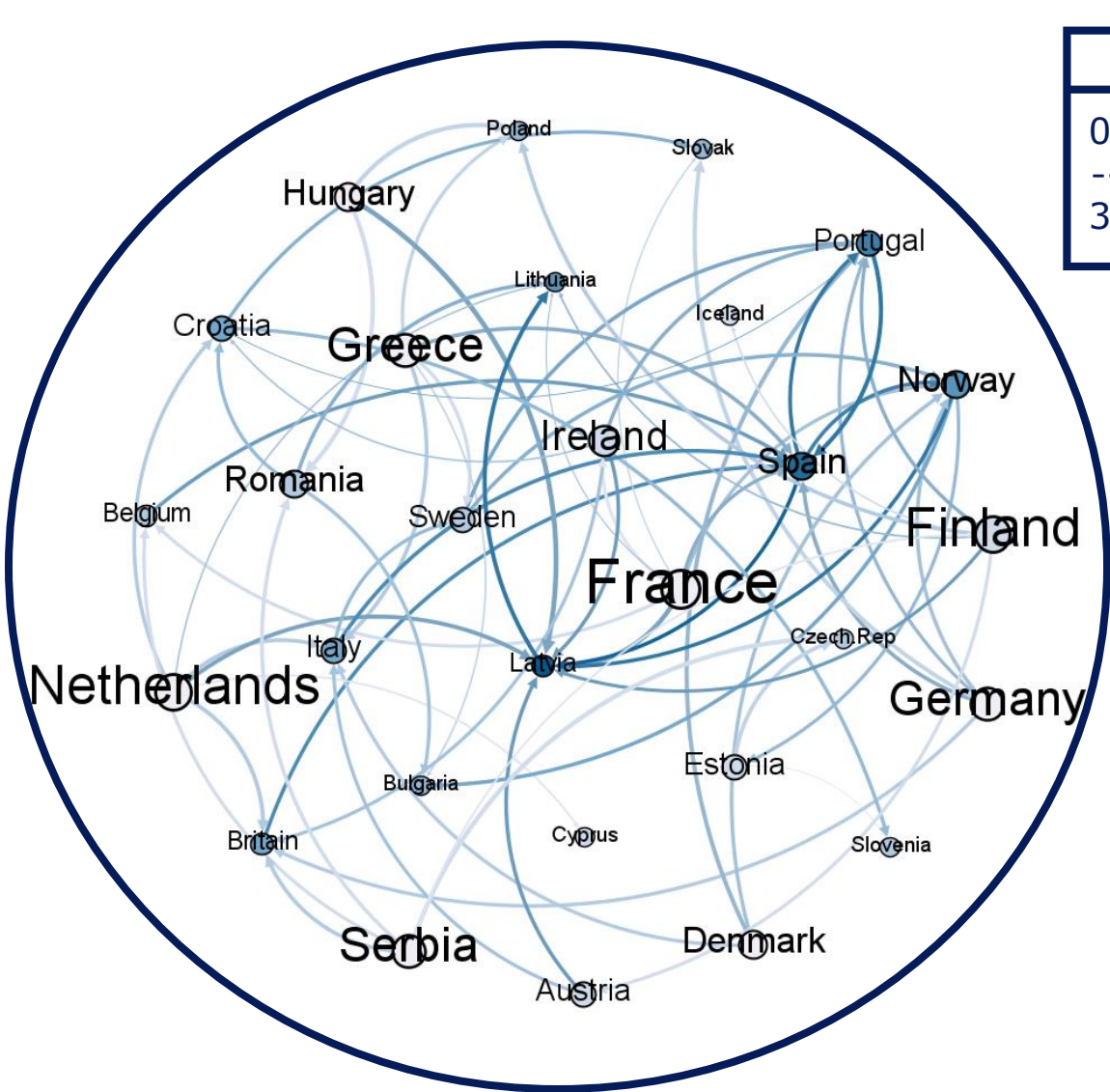
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31/07/2023

01/01/2016   31/07/2023
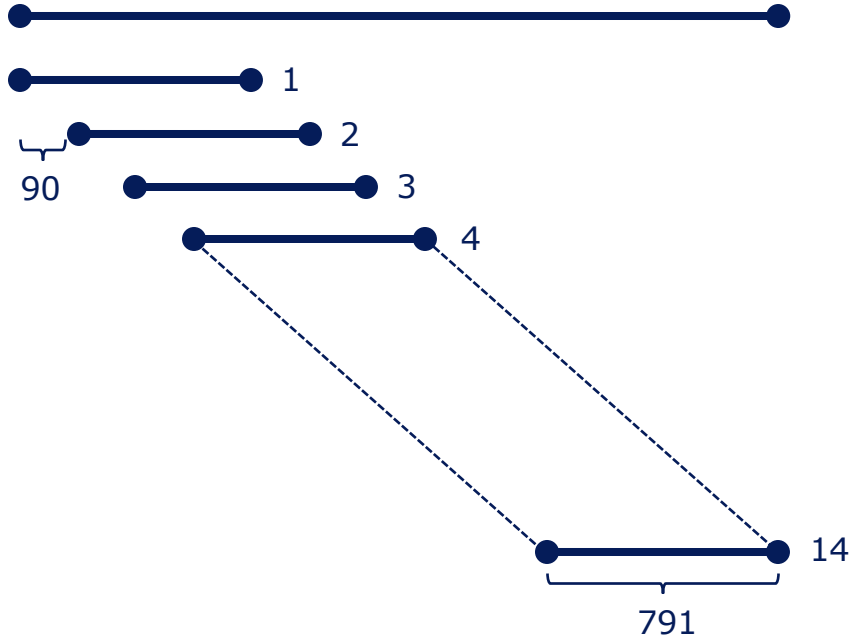
To determine how the networks describing spreads co-movements **evolve** through time.
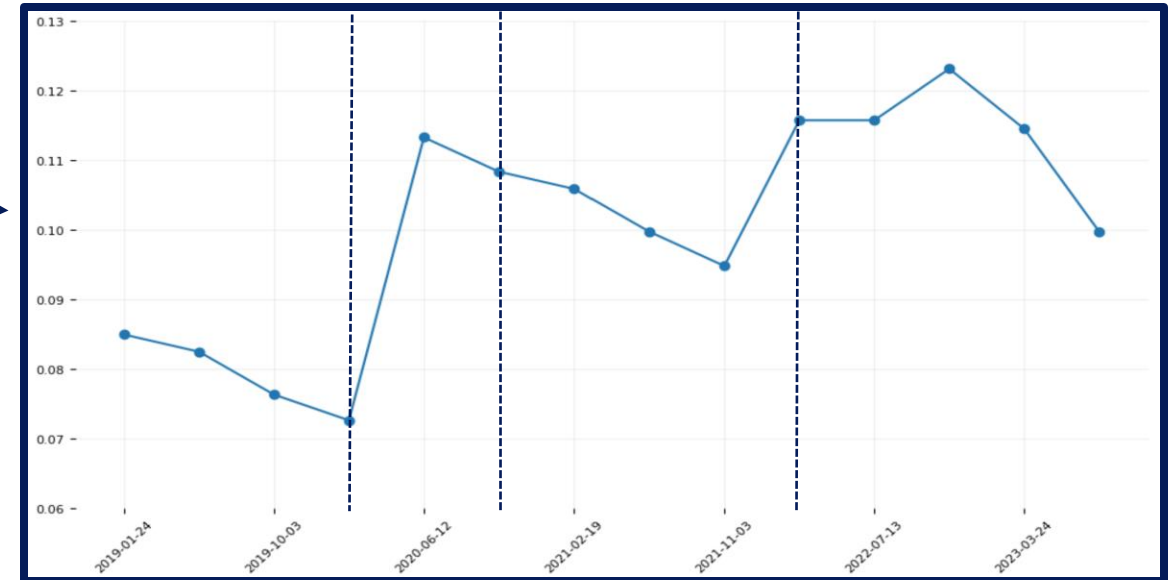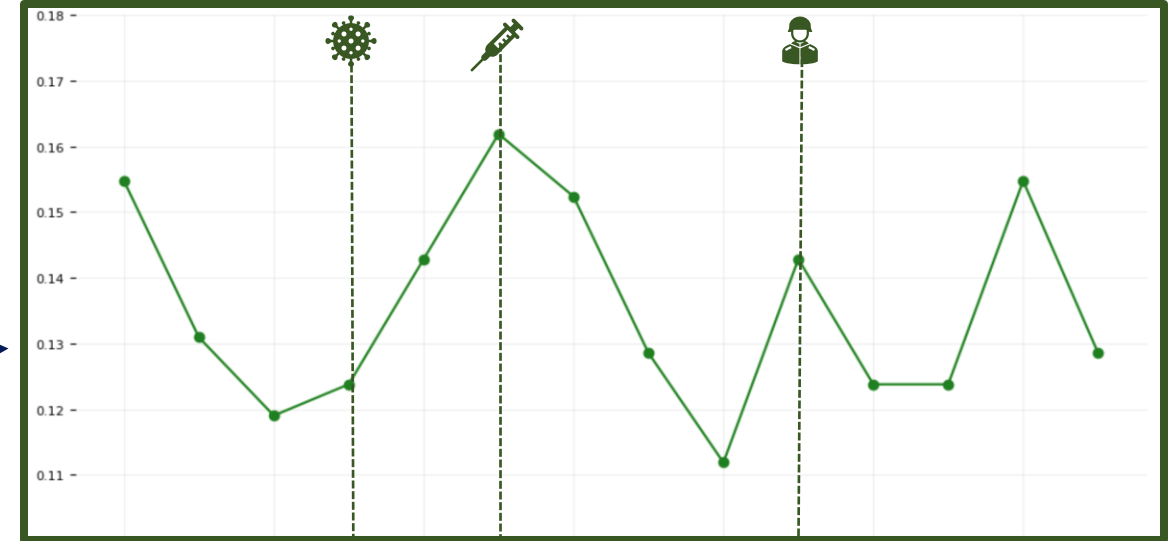
Measures the extent to which a node lies on **paths** between other vertices…
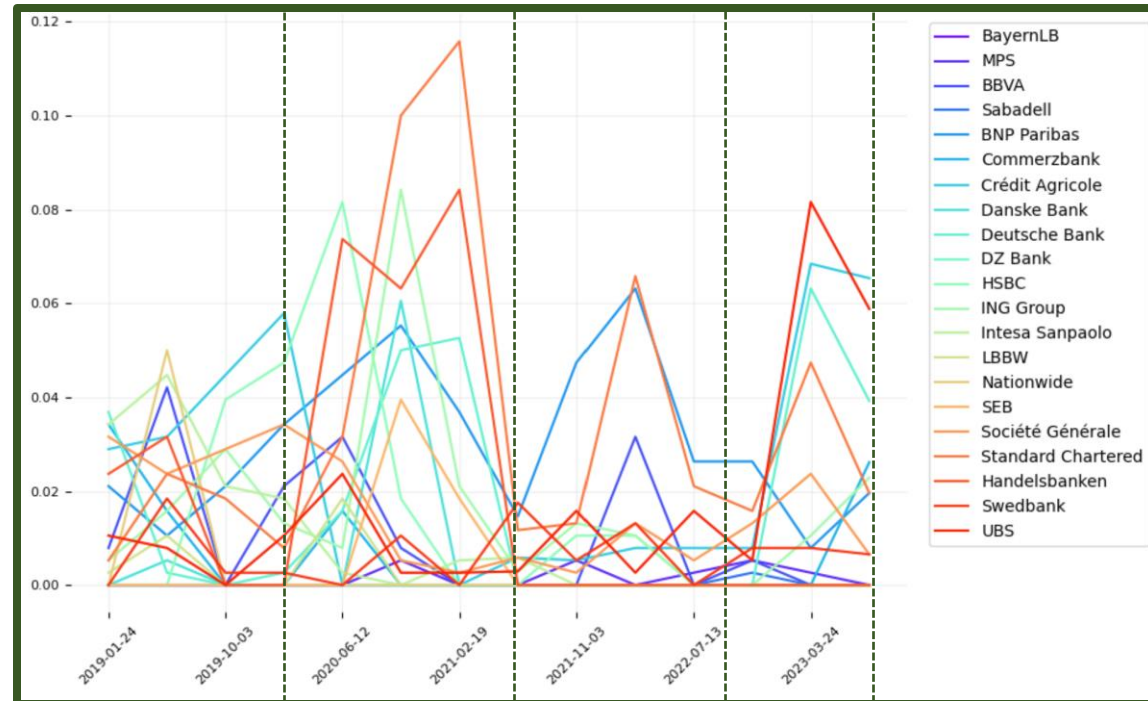
1-Greece
2-Spain

1-S.Chartered
2-Handlesbanken
3-ING Group
4-HSBC

1-S.Chartered
2-BNP Paribas

1-UBS
2-Cr.Agricole
3-Deutsche Bk
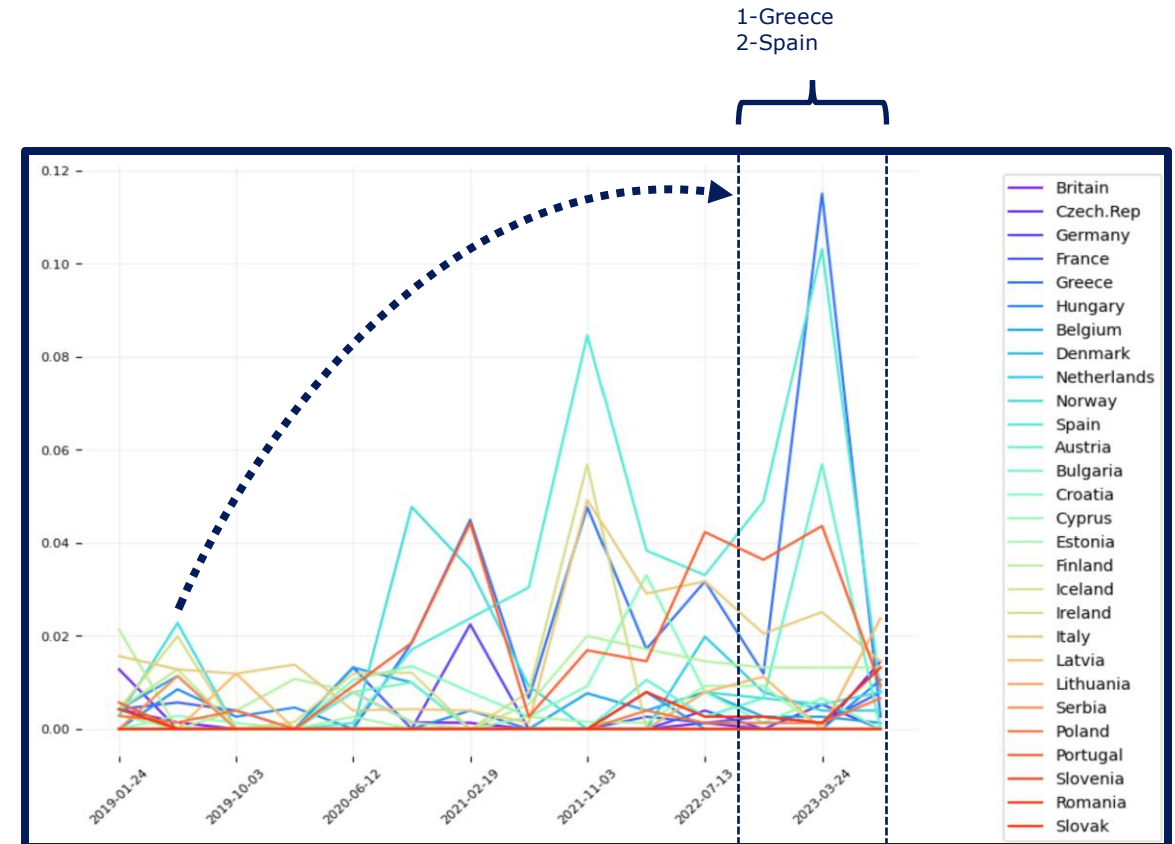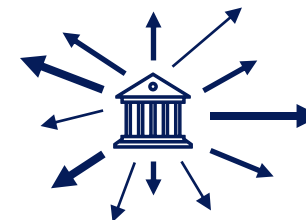
…nodes with high betweenness have considerable **influence** within a network.
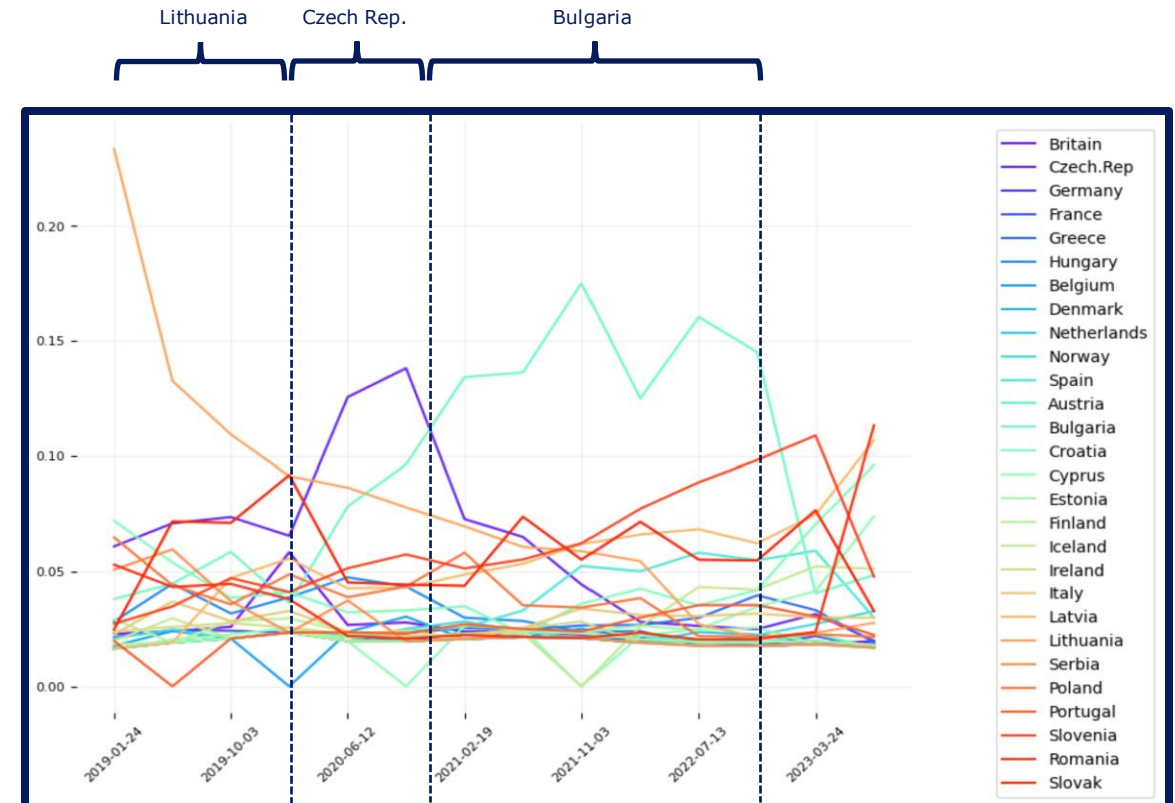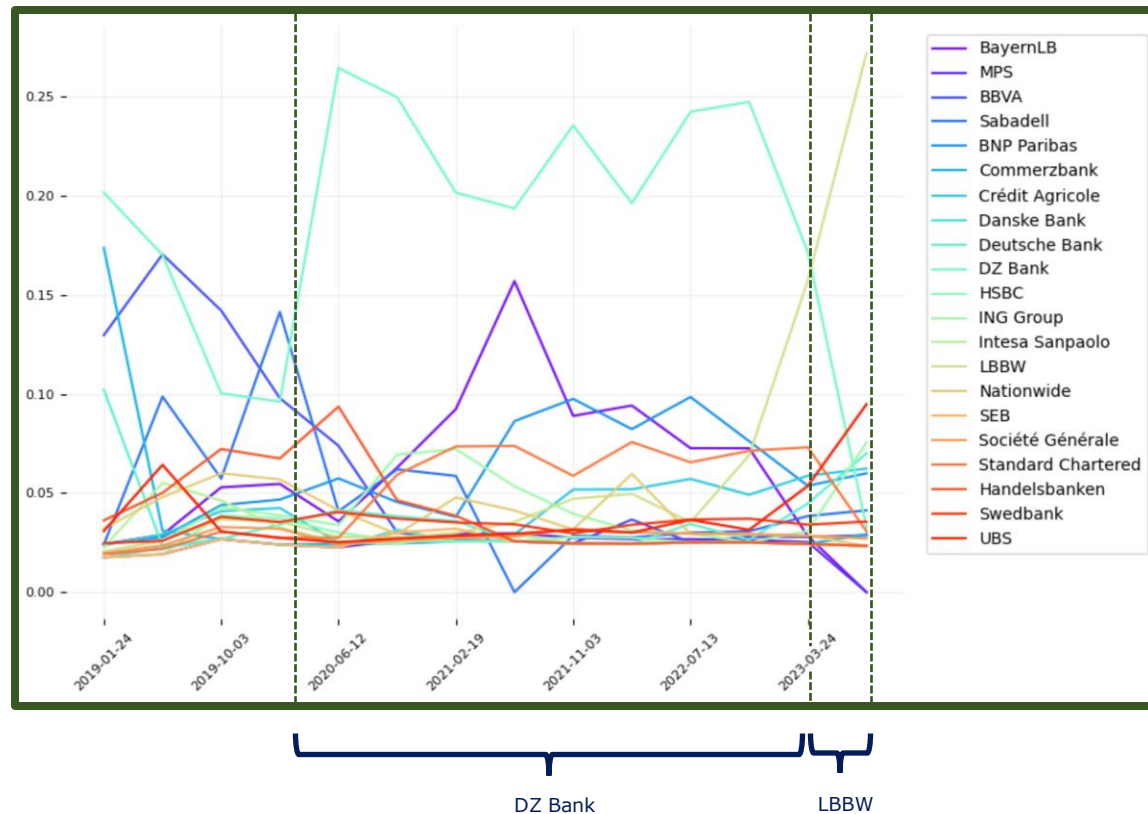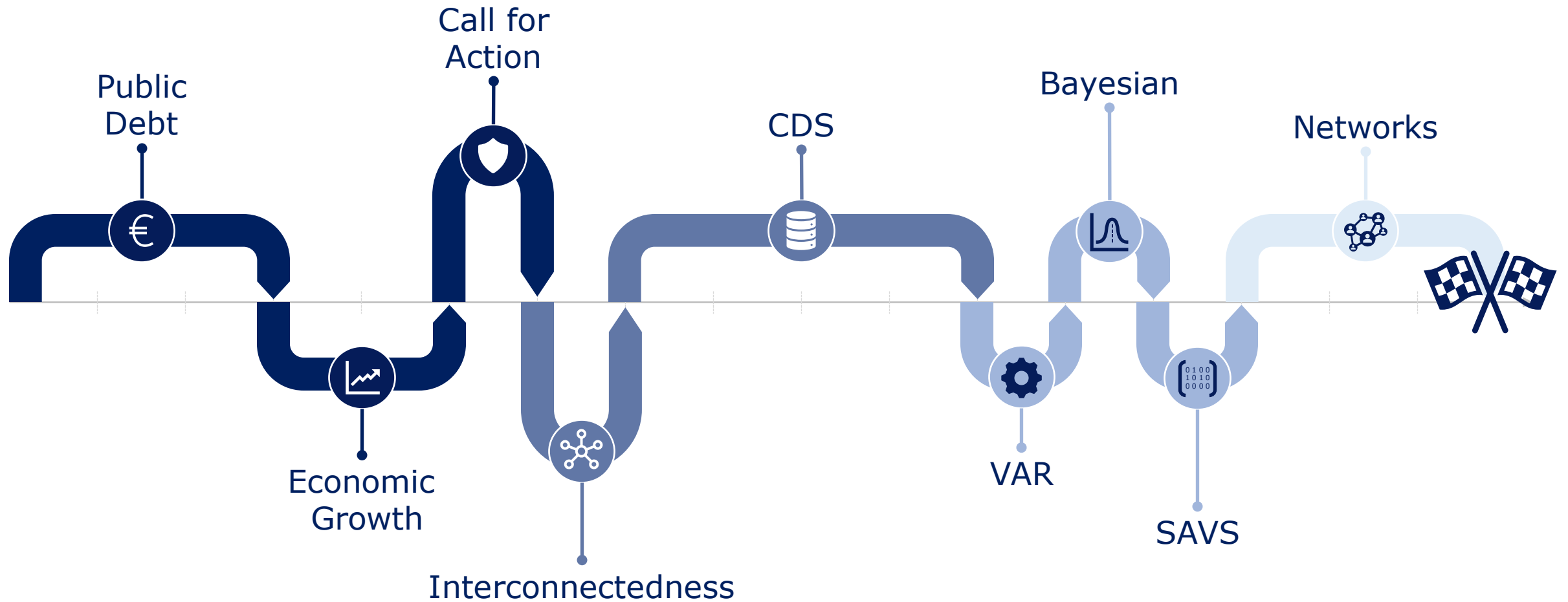
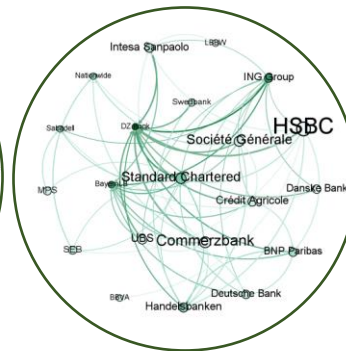# **Appendix** - Sovereign Networks
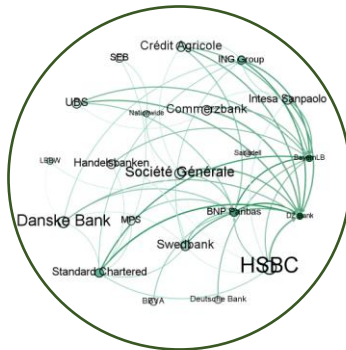

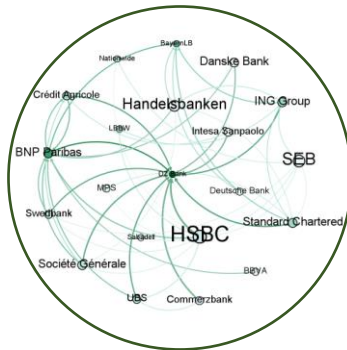2019-01-24   2019-05-30   2019-10-03   2020-02-06   2020-06-12   2020-10-16   2021-02-19
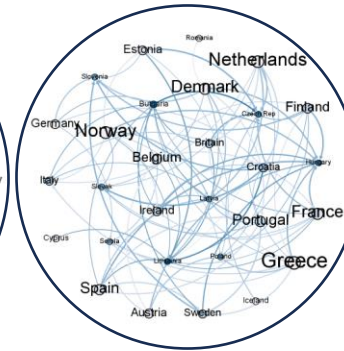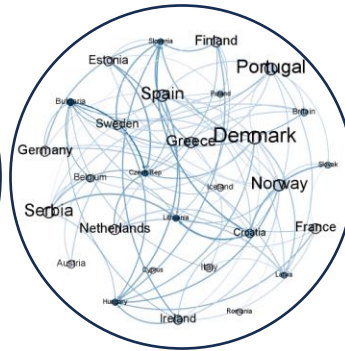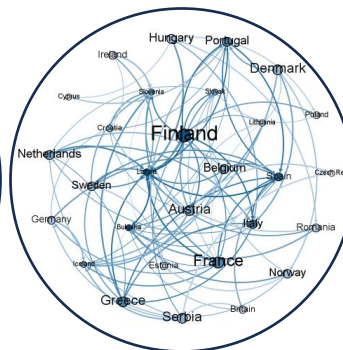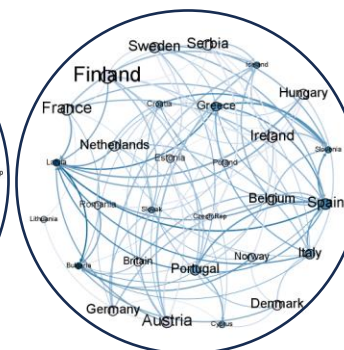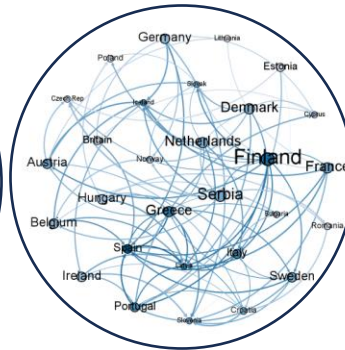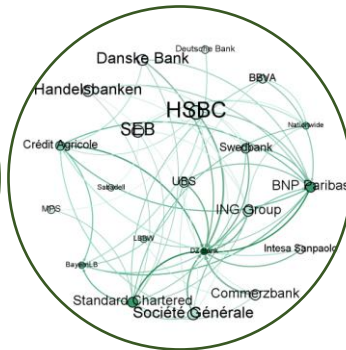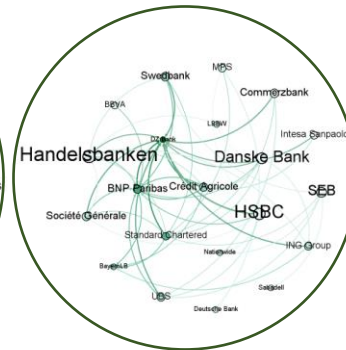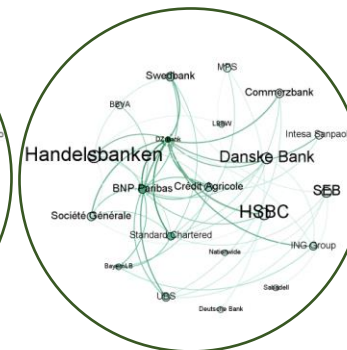
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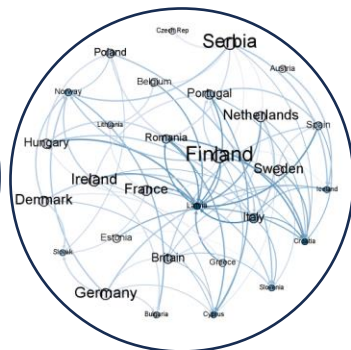
Appendix - Banking Networks

2019-01-24 2019-05-30 2019-10-03 2020-02-06 2020-06-12 2020-10-16 2021-02-19
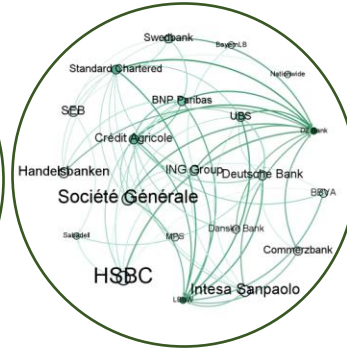
2021-06-29 2021-11-03 2022-03-09 2022-07-13 2022-11-18 2023-03-24 2023-07-28