Commentary on Farrington, "The Measurement and Interpretation of Age-Specific Vaccine Efficacy" (1992)

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Background

This important paper investigates the biases in two measures of vaccine effectiveness if the vaccine effect is either "all-or-none" or "leaky". The two measures of vaccine effectiveness, $\mathrm{VE}_1(a)$ and $\mathrm{VE}_2(a)$ are defined by one minus the ratios of the instantaneous attack rates $p_v(a)$, $p_0(a)$ and the probability densities of the ages at infection infection $u_v(a)$, $u_0(a)$, respectively (equations 3 & 4). The instantaneous attack rates are defined as

$$p_v(a) = -\frac{1}{S_v(a)} \frac{dS_v(a)}{da}, \tag{1}$$

corresponding to equation 1 with the index v if vaccinated and 0 if unvaccinated. The probability densities are given as

$$p_v(a) = -\frac{dS_v(a)}{da} \tag{2}$$

in equation 2, where $S_v(a)$ is the probability to "survive", i.e. to remain uninfected at age a. For failure rate λ,

$$S_v(a) = e^{-\lambda a} \tag{3}$$

If $\lambda(a)$ is age-dependent the expression becomes

$$S_v(a) = e^{-\Lambda(a)a}, \tag{4}$$

where $\Lambda(a) = \int_{u=0}^{a} \lambda(u)du$, i.e. the cumulative failure rate.

In Appendix A, Farrington derives the expressions for $\mathrm{VE}_1(a)$ and $\mathrm{VE}_2(a)$ in Table 1, which give rise to the graphs of Figures 1 & 2. Specifically, she derives the following expressions:

$$S_v(a) = l - R(a) + R(0)e^{-\lambda a} - \int_0^a R'(u)e^{-\lambda(a-u)}du \tag{5}$$

and

$$S_v'(a) = -\lambda e^{-\lambda a}\left[\int_0^a R'(u)e^{\lambda u}du + R(0)\right], \tag{6}$$

where $R(a)$ is the

> "[...] proportion of vaccinees without vaccine-induced immunity at age a." (p. 1015)

Specifically, under vaccine failure mode A, according to which the vaccine initially fully immunizes the proportion $1 - R_0$, but that proportion decays at a rate ρ. Thus

$$R(a) = 1 - (1 - R_0)e^{-\rho a} \tag{7}$$

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