Galois representations of abelian surfaces

ICERM Project Summary

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OUTPUT

$[\ell_1, \ldots, \ell_n]$ where ρ_{A,ℓ_i} is not surjective.

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If End(A) $= \mathbb{Z}$ *and A is principally polarized, then* $\rho_{A,\ell}$ *is surjective for almost every prime* ℓ

OUTPUT

The finite list of primes $\{\ell_1, \ldots, \ell_n\}$ *such that* ρ_{A,ℓ_i} *is not surjective.*

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e.g.

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Step 2(a): Rule out each maximal subgroup by sampling characteristic polynomials of $\rho_\ell(\mathrm{Frob}\, p)$

Progress

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Determined whether or not $\rho_{A,\ell}$ is surjective for $\ell = 2, 3, 5$ and C in the lmfdb.

Bibliography

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