Galois representations of abelian surfaces

ICERM Project Summary

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Notation

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$\rho_{A,l}$ - the Galois representation $G_{\mathbb{Q}} \to \mathrm{Aut}(A[l]) = \mathrm{GSp}(4, \mathbb{F}_l)$.

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Compute the finite list of primes $\{l_1, \ldots, l_n\}$ such that ρ_{A,l_i} is not surjective.

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Step 2: Given a prime l, determine if $\rho_{A,l}$ is non-surjective.

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- Mitchell [2] classifies the maximal proper subgroups of $\mathrm{PGSp}(4, \mathbb{F}_l)$
- Use $\mathrm{Frob}\, p \in G_{\mathbb{Q}}$ (for various primes p).
- If the image of $\rho_{A,l}$ is contained in a maximal proper subgroup, then the characteristic polynomial of $\rho_{A,l}(\mathrm{Frob}\, p)$ must satisfy certain conditions.

Progress

Bibliography

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Special Thanks

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