*Welcome to:*

# Introduction to Topology in Condensed Matter

Qing Lin He

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# Symmetry, Topology, and electronic phase of matter

## Symmetry

- ➢ Symmetry protected topological (SPT) phases
- ➢ Topological non-trivial phase:

1. Some exist in absence of any symmetry
2. Most of the phases without intrinsic topological order belong to **Symmetry Protected Topological (SPT) phases**

→ Topology is protected by the symmetry;

→ Almost always possess topologically protected boundary modes;

3. Phases with intrinsic topological order are not necessarily equipped with boundary modes

→ **Symmetry enriched topological (SET) phases**

Questions also to be addressed:

- How to categorize phases of matter by symmetry?
- What operations leave a system invariant?

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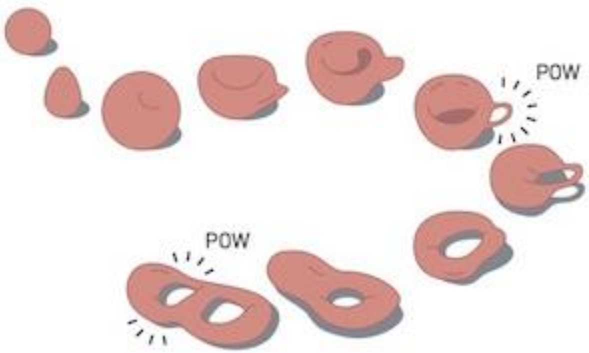
# Symmetry, Topology, and electronic phase of matter

## Topology




- ➢ Topology of the boundary modes;
- ➢ Topological invariants of the phase:
    1. Quantized;
    2. Either 0, positive, or negative integer.

**→ Topological properties are universal;**

**→ Topologically protected.**

- ➢ Berry physics, such as Berry connection, Berry curvature, and Berry phase.
- ➢ Topological properties are defined in cases of:

1. Spectrally gapped ground states of
2. Local Hamiltonians
3. Zero Temperature

Questions also to be addressed:

How to distinguish topological phases by topology?

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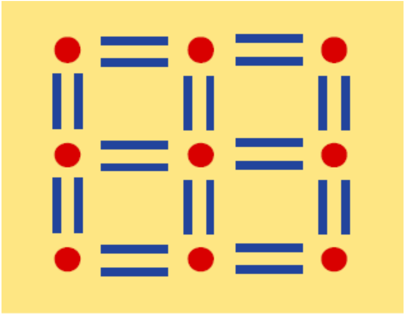
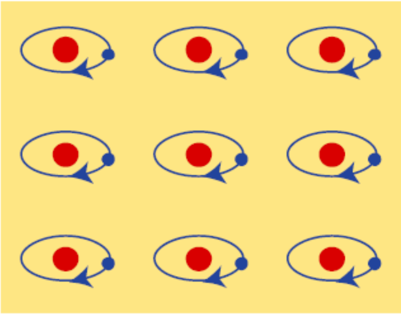
# Topology and Physics

## *An example: Insulating states*

- **Characterized by energy gap without low energy electronic excitations**



**Covalent/ionic insulator (intrinsic semiconductor, solid ionic crystal)**



**Atomic insulator (Solid argon)**



**Vacuum**

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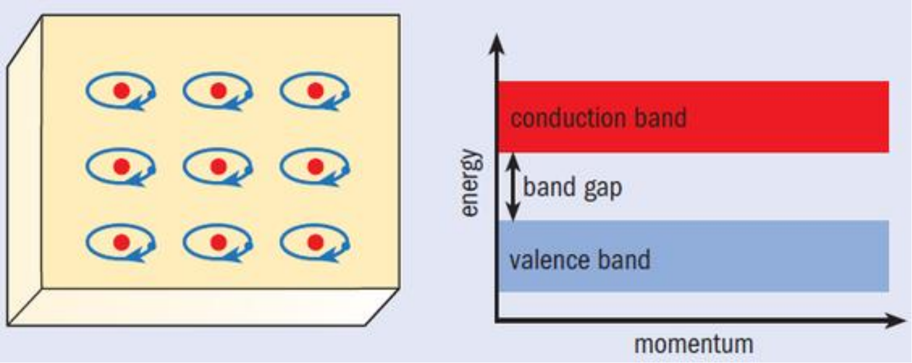
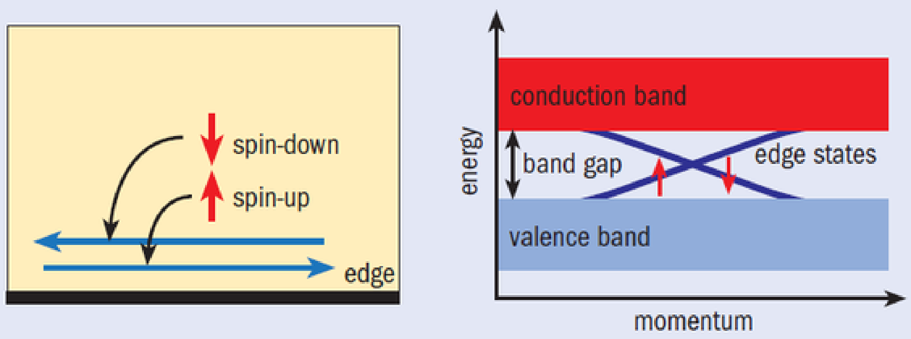
# Topology and Physics

## *An example: Insulating states*

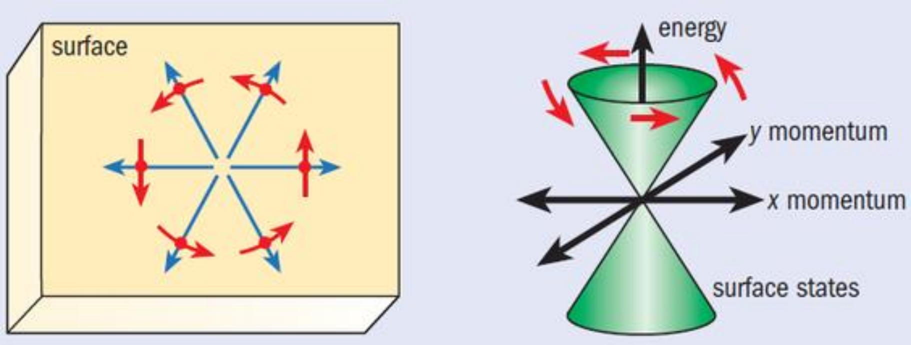
- **Characterized by energy gap without low energy electronic excitations**



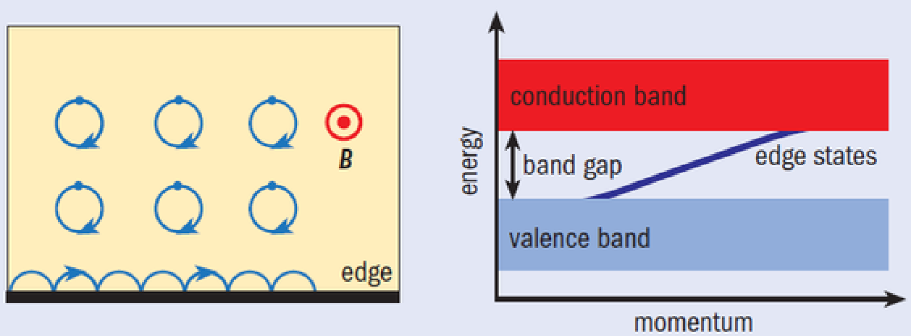

**Normal (trivial) insulator**




**Quantum spin Hall insulator**
**2D topological insulator**




**3D topological insulator**




**Integer quantum Hall insulator; Magnetic topological insulator; Chern insulator**

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# Topology and Physics

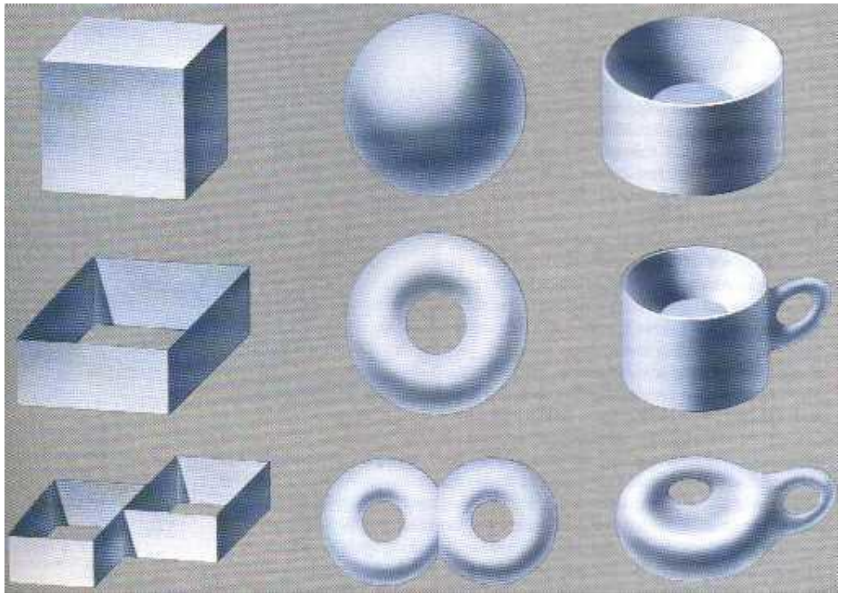
*Topological invariant*

*Continuity deformation*

Genus = 0

Genus = 1

Genus = 2



- **Genus = 0: Insulators are topologically equivalent if they can be continuously deformed into another without closing the energy gap;**
- **Genus ≠ 0: Topological phases that CANNOT be connected to the trivial insulator**

**Objective: Similar to *genus*, find a topological invariant that can describe and categorize the nature of the matter.**

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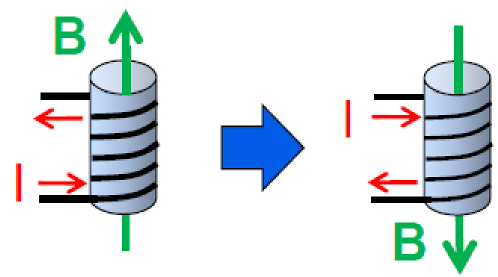
# Symmetry and Physics

## *An example: Time reversal symmetry*

**When the direction of time is reversed:**

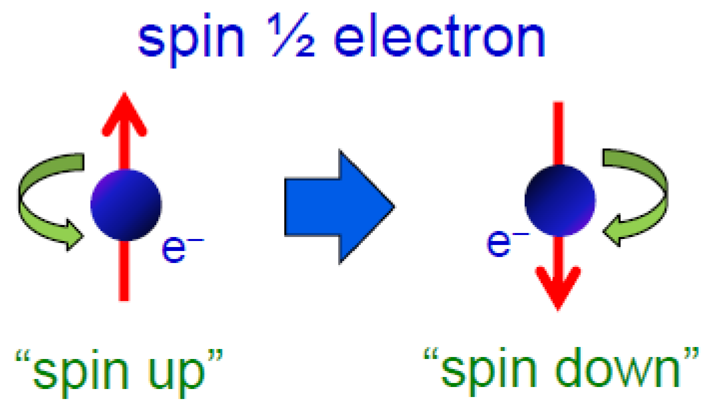
- Magnetic field:

  B → -B




- Spin angular momentum:

  S → -S

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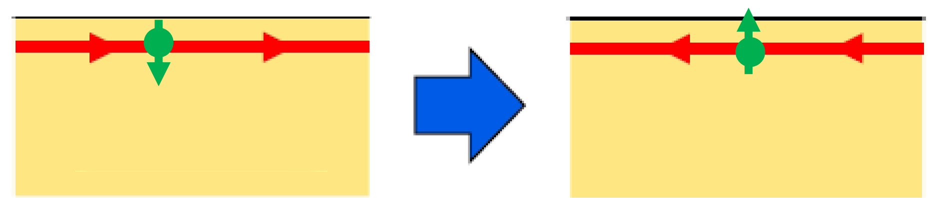
# Symmetry and Physics

## *An example: Time reversal symmetry*

**When the direction of time is reversed:**

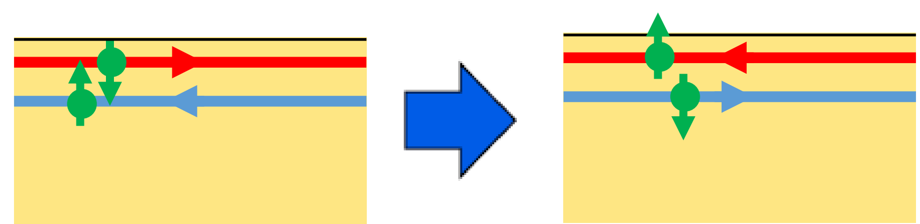
- Chiral edge state:

R → L, S → -S



- Helical edge state:

R → L, S → -S



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# Symmetry and Physics

*Emergent effects*

- **Time reversal symmetry → Magnetism**

*Orbital* **Multiferroics**

- **Inversion symmetry → Ferroelectricity**

*Charge* **Non-centrosymmetric superconductor**

- **Gauge symmetry → Superconductivity**

*Spin* **Magnetic superconductivity**

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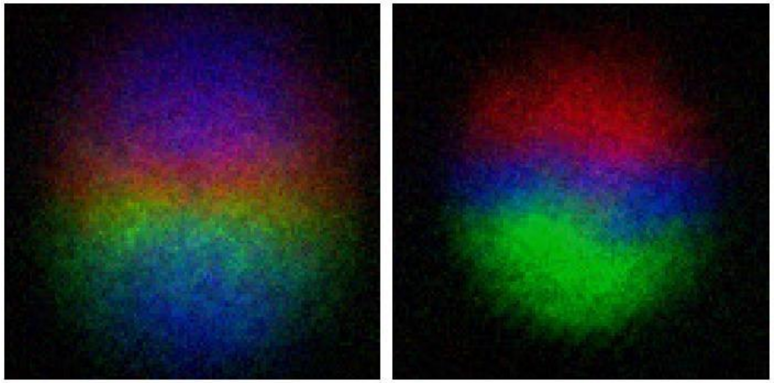
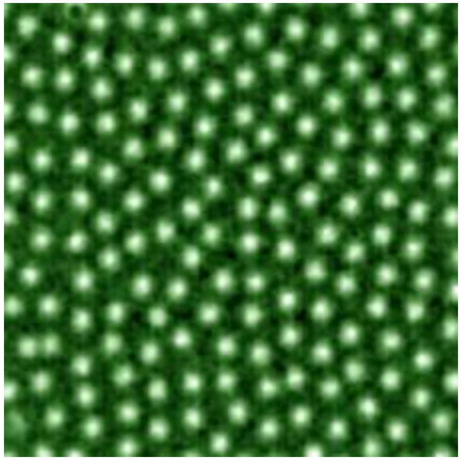
# Historical development

*Before 1970*

- Quantum magnetic monopole (Dirac, 1931)
- Quantum vortex in superfluid (Onsager, 1947)
- Quantum vortex in superconductor (Abrikosov, 1957)
- …





- Experimental side image of the quantum monopole (left). After 0.2s, the quantum monopole has decayed into the Dirac monopole (right).

- Vortex (in white) are observed here in $NbSe_2$

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# Historical development

*1970's*

- Kosterlitz-Thouless transition (Kosterlitz and Thouless, 1973)
- Non-Abelian magnetic monopole ('t Hooft, Polyakov 1974)
- Instanton (Belavin et al, 1975)
- Gauge theory and fiber bundle (Wu and Yang, 1975)
- Defect in (liquid-)crystal (point, line, disclination…)
- Soliton with charge ½ (Jackiw and Rebbi, 1976)
- Soliton in SSH model (Su, Schrieffer and Heeger 1979)
- …

# Historical development

*1980's*

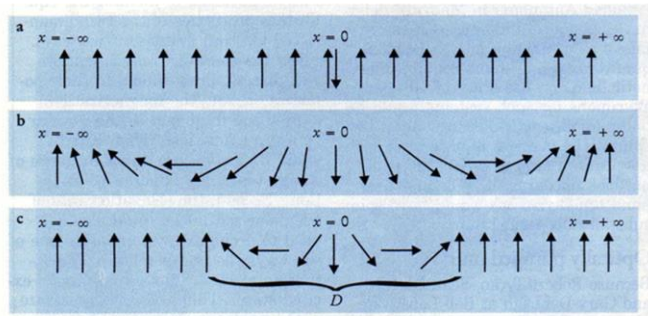
- Discovery of IQHE (1980, von Klitzing) and FQHE (Tsui et al, 1982)
- Fractional statistics in 2D: anyon (Leinaas, 1977; Wilczek, 1982)
- Energy gap in spin-1 chain (Haldane, 1983)
- TKNN theory of IQHE (Thouless et al, 1984)
- Theory of geometric phase (Berry, 1984)
- Anyon with fractional charge in FQHS (Arovas, Schrieffer, and Wilczek, 1985)
- Haldane graphene model (Haldane, 1989)
- Topological field theory (Witten, 1988)
- …

# Historical development

*1990's*

- Ground state degeneracy in FQHS (Wen and Niu, 1990)
- Moore-Read state (spin-polarized p-wave pairing) in FQHS (Moore and Read, 1991)
- Skyrmion (Skyrme, 1962; Sondhi et al, 1993)
- Modern theory of polarization (Resta, King-Smith and Vanderbilt, 1993)
- Berry curvature in Bloch electron dynamics (Chang and Niu, 1995)
- Fault-tolerant quantum computation by anyons (Kitaev, 1997)
- …

    - **Non-interacting**
    - **Plus e-e interaction**
    - **Plus Zeeman energy**

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# Historical development

*2000's*

- Quantum spin Hall effect (Kane and Mele, 2005)
- Theory of topological insulator (Fu and Kane, 2006)
- Majorana fermion in hybrid TI-SC (Majorana, 1937; Fu and Kane 2008)
- Topological photonics (Haldane and Raghu; Wang et al, 2008)
- Topological phononics (Prodan, 2009)
- Classification of TI/TSC (Schnyder et al, Kitaev, 2009)
- Magnetic skyrmion in MnSi (Mühlbauer et al, 2009)
- …

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# Historical development

*2010's*

- Weyl semi-metal (Wan et al, Burkov and Balent, 2011)
- Topological mechanics (Kane and Lubensky, 2014)
- Topological quantum chemistry (…)
- Classification of the topology of interacting systems (…)
- …

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# Historical development

*2020 and beyond*

- Quantum device using majorana fermion
- Topological quantum computation
- Classification of the topology of interacting systems
- …

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# Importance of Topology


Kosterlitz Thouless

**Kosterlitz, Thouless**

- **Kosterlitz–Thouless transition (1973)**
- **“for theoretical discoveries of topological phase transitions and topological phases of matter” (2016)**

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# Importance of Topology


Kosterlitz Thouless

Klitzing

**Klitzing**

- **Integer plateaus are seen experimentally (1980)**
- **“for the discovery of the quantized Hall effect” (1985)**

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# Importance of Topology


Kosterlitz Thouless Klitzing

**Thouless, Kohmoto, Nightingale, and Nijs**

- **Theorists find profound explanation why integers will always be seen.**
- **The picture involves nearly free electrons with ordinary fermionic statistics.**
- **“for theoretical discoveries of topological phase transitions and topological phases of matter” (2016)**

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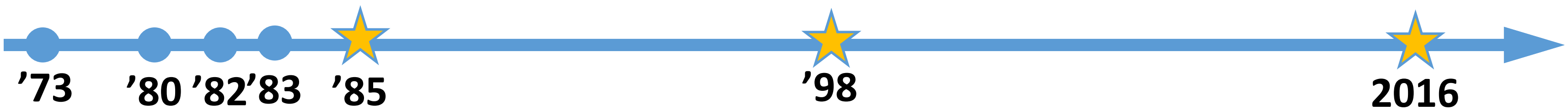
# Importance of Topology




Kosterlitz Thouless Klitzing Laughlin Störmer Tsui

**Laughlin, Störmer, Tsui**

- **Fractional plateaus are seen experimentally (1983)**
- **Eventually many fractions are seen, all with odd denominators**
- **“for their discovery of a new form of quantum fluid with fractionally charged excitations” (1998)**

✓ **Theorists find profound explanation why odd denominators will always be seen.**
✓ **The picture (Laughlin) involves an interacting electron liquid that hosts**
✓ **“Quasiparticles” with fractional charge and fractional “anyonic” statistics.**

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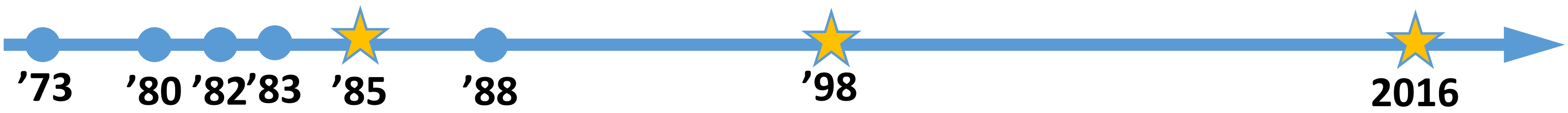
# Importance of Topology

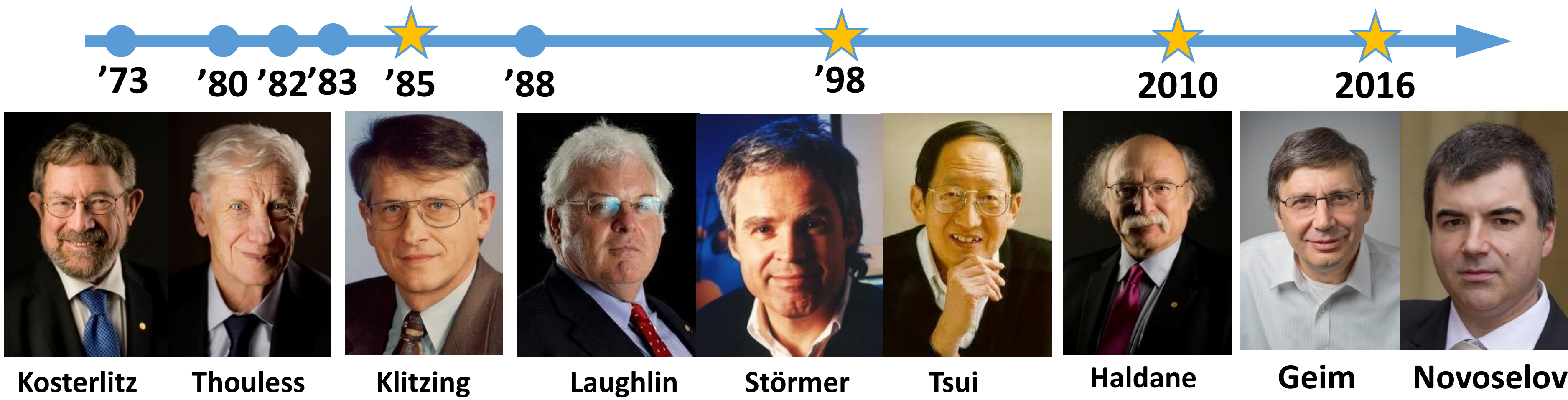



Kosterlitz Thouless Klitzing Laughlin Störmer Tsui Haldane

**Haldane**

- **Haldane model**
- **“for theoretical discoveries of topological phase transitions and topological phases of matter” (2016)**

# Importance of Topology




**Geim, Novoselov**

- **"for groundbreaking experiments regarding the two-dimensional material graphene“ (2010)**

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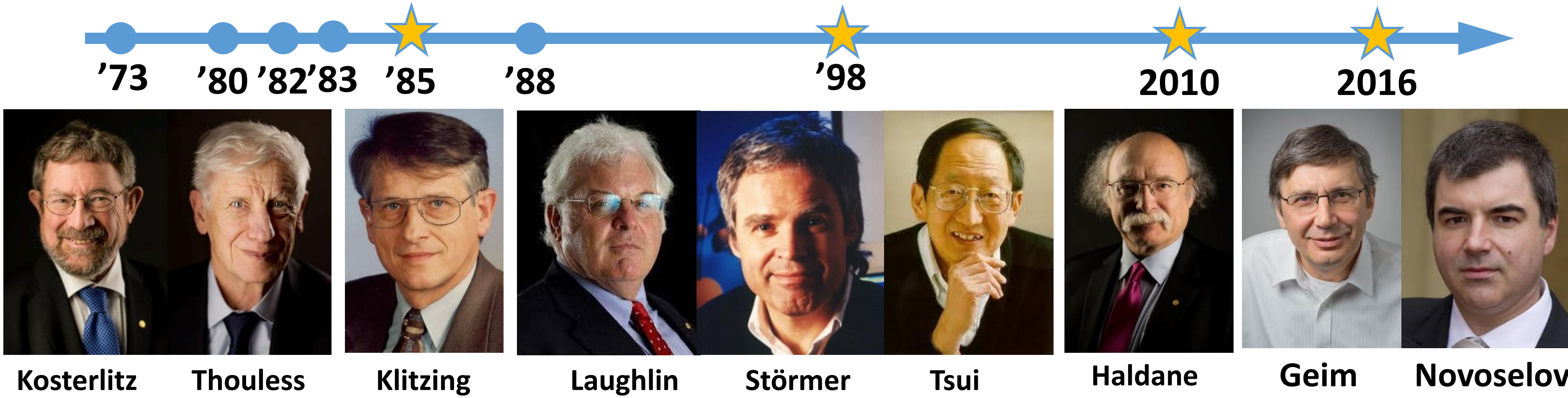
# Importance of Topology




- **A new view to nature**
- **What are they good for?**
  - **Future electronics/spintronics: Dissipationless**
  - **New physics in hybrid: Majorana, magnetic monopoles, etc.**
  - **Topological quantum computation**

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# Questions we are going to address

- What’s the linkage between topology and physics?
- What’s Berry physics, topological invariant, and what are their roles?
- Application of topology in condensed matter: what material systems?
- How to understand the topological physics in emergent materials?

# Material systems we are going to study

- Integer quantum Hall insulator
- Square lattice Chern insulator
- Haldane Chern insulator
- Kane-Mele model
- BHZ model
- 2/3D TI (strong and weak)
- Majorana in Kitaev model
- P-wave superconductor
- Detection of Majorana
- Helical/chiral topological superconductors

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# Topological Material systems but not being involved in this course

- Topological crystalline insulator
- Topological (Weyl and Dirac) semimetals
- Fractional quantum Hall insulator
- Bosonic system
- Cold atom
- …

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# Outline of this course



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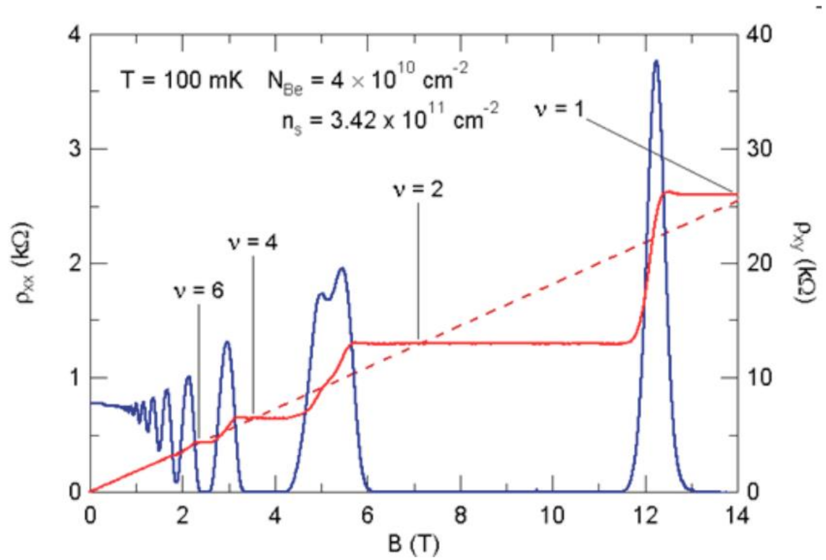
# Question from IQHE

- In 1980, the ”Landau paradigm” was challenged by the discovery of the (integer) quantum Hall effect
- Electrons confined to a plane and in a strong magnetic field show, at low enough temperature, plateaus in the “Hall conductance”:
- $\sigma_{xy} = n\dfrac{e^2}{h}$




**What type of order causes this quantization?**

**Topological Order !**

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# Topological Order

***What mechanism causes the quantization?***

- Definition I:

In a topologically ordered phase, some physical response function is given by a "topological invariant". → Hall measurement probes topological invariants

***What is a topological invariant? How does it explain the observations?***

- Definition II:

A topological phase is insulating (defined by the gap) but always has metallic edges/surfaces when proximity to vacuum or an ordinary phase.

→ Spectrally gap by the strong B field and QH edge states arise

***What does this have to do with Definition I?***

"Topological invariant" = quantity that does not change under continuous deformation→ Topological protection (spin independent impurity and defect)

(A third definition: phase is described by a "topological field theory")

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# Outline of this course



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# Outline of this course

**Ch5**

**5. Majorana in topological superconductor**

- **Classification with respect to time-reversal and particle-hole symmetry**

**5.1 Topological Phases in the SSH and Kitaev Models**

**5.1.1 Kitaev chain and bulk-edge correspondence**

**5.1.2 Unpaired Majorana modes in a model of dominoes**

**5.1.3 The Kitaev chain model**

**5.1.4 Continuum model and phase diagram**

**5.1.5 Topological protection of Majorana modes**

**5.2 Realization of Kitaev model**

**5.2.1 The need for spin**

**5.2.2 Realistic superconducting pairing**

**5.2.3 How to open the gap?**

**5.3 Topological insulator edges**

**5.4 The two-dimensional p-wave superconductor**

**5.5 Majorana bound states on vortices**

**5.6 How to detect Majoranas**

**5.6.1 Andreev reflection off a Majorana zero mode**

**5.6.2 Majorana resonance**

**5.6.3 Conductance signature**

**5.6.4 *Flux-induced fermion parity switch in topological superconductors**

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# Outline of this course

**Ch6**
**6. Non-Abelian statistics**
**6.1 Majorana zero modes in nanowire networks**
**6.2 Non-Abelian statistics of Majoranas**
**6.3 Manipulation of Majorana bound states**
**6.3.1 Non-Abelian Berry phase**
**6.3.2 Braiding Majorana zero modes**
**6.3.3 Braiding Majorana chiral modes**

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# Tasks

- Homework
  - ❖ 6 problem sets to be assigned every two weeks; **50%**)
  - ❖ Late homework will be accepted up to one week late at 50% credit.
- Academic report (assigned on ~7th week; **20%**)
- Final presentation (**30%**)

Remarks

- Office hour: Email appointment (qlhe@pku.edu.cn)
- Grading: A+/A/A–/B+/B/B–/C+/C/C–/D+/D/F
- Final Presentation in English, per person or group
- No roll call