Homework -1 (Due on Mar 4, 2020)

1. Calculate the magnetic flux through the classical electron orbit and express it via the filling factor and flux quantum.

2. Consider spinless free 2DEG exposed to a perpendicular magnetic field. Magnetic length is defined as the width of the ground state wave function that lies at the lowest Landau level. Calculate the ratio of the magnetic length to the Larmor radius for an electron orbit via the filling factor in classical picture. Explain the physical meaning of Larmo radius. Hint: Larmor radius can be related to the electron energy for classical orbits.

3. In classical electrodynamics, the potentials $\boldsymbol{A}$ and $\phi$ are not uniquely determined; the physical quantities are the fields, $\boldsymbol{E}$ and $\boldsymbol{B}$.

   (a) Show that the potentials

   $$\phi' = \phi - \frac{\partial \Lambda}{\partial t}, \quad \boldsymbol{A}' = \boldsymbol{A} + \nabla \Lambda$$

   (where $\Lambda$ is an arbitrary real function of spacetime) yield the same $\boldsymbol{E}$ and $\boldsymbol{B}$ fields as $\phi$ and $\boldsymbol{A}$. The above definitions are called a gauge transformation, and the theory is said to be gauge invariant.

   (b) In quantum mechanics the potentials play a more direct role, and it is of interest to know whether the theory remains gauge invariant. Show that

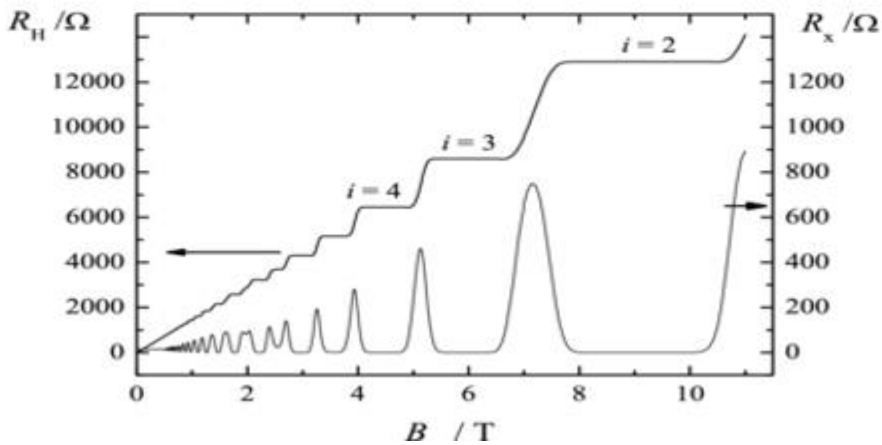
   $$\Psi' = e^{iq\Lambda/\hbar}\Psi$$

   satisfies the Schrödinger's equation of the following form

   $$i\hbar \frac{\partial \Psi'}{\partial t} = \left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\boldsymbol{A}'\right)^2 + q\phi'\right]\Psi'$$

   with the gauge-transformed potentials $\phi'$ and $\boldsymbol{A}'$.

4. In the class, we have seen the data showing the integer quantum Hall effect. In experiments, we changed $B$ with a fixed chemical potential and got the following data:




From $R_x$ , we can see that there are many peaks and zero-plateaus, corresponding to insulating and metallic states, respectively. Why the system look like an perfect insulator and perfect conductor at different $B$-fields?

Hints: we know the Landau levels is given by $E_n = \left(n + \frac{1}{2}\right)\frac{e\hbar B}{\boldsymbol{m}}$. Remember how this Landau levels look like in the drawing.

5. Reading: 25 Years of Quantum Hall Effect (QHE) - A Personal View on the Discovery, Physics and Applications of this Quantum Effect, by Klaus von Klitzing