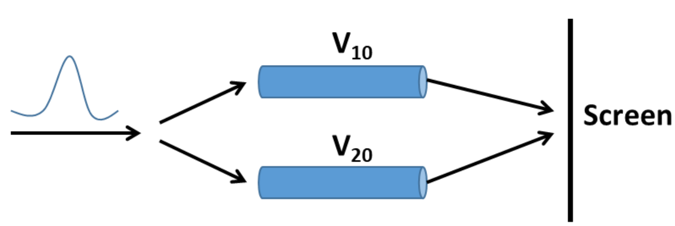
Homework -2 (Due on Mar 18, 2020)

1. While considering the two-level system we showed in the class but without the energy offset: *A Spin in a Magnetic Field*. The Hamiltonian can be described by
$$H = \mathbf{d} \cdot \sigma$$
where $\mathbf{d}$ is a real vector and σ is the vector of Pauli matrices.

   a. Use the spherical coordinates $\mathbf{d} = d(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ to write the Hamiltonian and show that the normalized, orthogonal eigenstates corresponding to $\pm d$ can be written as
$$|+\rangle = \begin{pmatrix} \cos(\theta/2)e^{-i\phi} \\ \sin(\theta/2) \end{pmatrix}, \quad |-\rangle = \begin{pmatrix} \sin(\theta/2)e^{-i\phi} \\ -\cos(\theta/2) \end{pmatrix}.$$
   b. Calculate the Berry connection, $\mathcal{A}_{\theta\phi}$, the Berry curvature, $\mathcal{F}_{\theta\phi}$.
   c. Repeat the calculation in a,b in a different gauge, obtained by the transformation $|\pm\rangle \to e^{i\phi}|\pm\rangle$. Show that although $\mathcal{A}_{\theta\phi}$ change, $\mathcal{F}_{\theta\phi}$ remain the same.
   d. Now calculate Berry curvature directly from the Hamiltonian making use of the formula $\mathcal{F}_{\theta\phi} = \frac{1}{2}\hat{\mathrm{d}} \cdot \left(\partial_\theta \hat{\mathrm{d}} \times \partial_\theta \hat{\mathrm{d}}\right)$.
   e. On the basis of the above results show that the Berry phase acquired by the system when vector $\hat{\mathrm{d}}$ sweeps a closed contour C on the unit sphere is equal to $\frac{1}{2}\Omega$, where Ω is the corresponding solid angle.

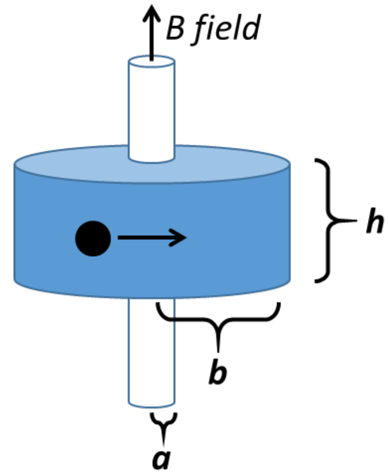
2. Consider a wavepacket starts on the left and is split 50/50 into components that proceed along the upper and lower paths passing through perfectly conducting cylinders. The upper and lower wave packets remain coherent with respect to each other. The voltages V_1 and V_2 are held at zero until the wavepacket enters inside the cylinders, at which time they are set to fixed values V_{10} and V_{20} respectively. Before the wavepacket leave the cylinders, the voltages are turned back to zero. Do you expect to see a shift in the interference pattern? Give quantitative answers.



3. An electron is constrained to move on the surface of a 2D strip keeping at a radius $r = b$ from a cylindrical solenoid of radius $r = a$. The height of the strip is h. The magnetic field inside the solenoid is B. There is zero field outside the solenoid, so that the moving electron on the strip sees zero magnetic field.

(a) What is the energy spectra for the electron due to the presence of the solenoid?

(b) Consider that we have N number of electrons constrained to move on this strip. Take N large and take the system at zero temperature. What happens to the total energy spectrum of the system?



4. Reading: The Berry phase original paper doi:10.1098/rspa.1984.0023