RotorHead

March 8, 2024

0.0.1 Introduction

I wanted to learn more about how a helicopter works, specifically how the rotors are controlled. So I built a rotor-head out of LEGO and did some math. I also relearned a lot about typesetting equations I knew about a decade ago. I also learned about a method of helicopter control known as cyclic-collective pitch mixing, which is the algorithm I'll be discussing here.

0.0.2 Definitions

Helicopter

An aircraft that uses airflow over a rotating wing to generate lift, rather than using thrust to push a fixed wing through the air as in a traditional airplane. A generic helicopter has a main rotor that generates life, and a tail rotor that counteracts the yaw induced by the main rotor torque on the fuselage and provides directional control. The main rotor of is attached to the fuselage by an assembly known as the rotor-head. The drive shaft for the main rotor emerges from the transmission through the top of the fuselage.

The primary controls of a helicopter are a stick, called the cyclic, a lever, called the collective, and a pair of pedals, called anti-torque pedals. The throttle is typically managed mechanically to keep the rotor RPM constant. The cyclic stick controls roll and pitch of the helicopter. The anti-torque pedals control the thrust of the tail rotor. The collective changes the angle of attack of the main rotor, to regulate the thrust of the rotor, but can really be thought of as the "gas pedal" of the helicopter.

Swashplate

A swashplate is a device that allows a rotating element to be inclined with two degrees of freedom, pitch and roll, and also move vertically. The bottom half is attached to three linear actuators, which provide all degrees of freedom. The rotating top half is linked to the drive shaft through the "scissor-link". Push rods connected to the top of the swashplate vary the pitch of the rotors based on the cyclic and collective inputs.

0.0.3 Coordinate systems

- P0 is origin of coordinate system, centered under the mast of the helicopter.
- Positive X is pointing to the front of the helicopter.
- Positive Y is pointing port.
- Positive Z is pointing up.

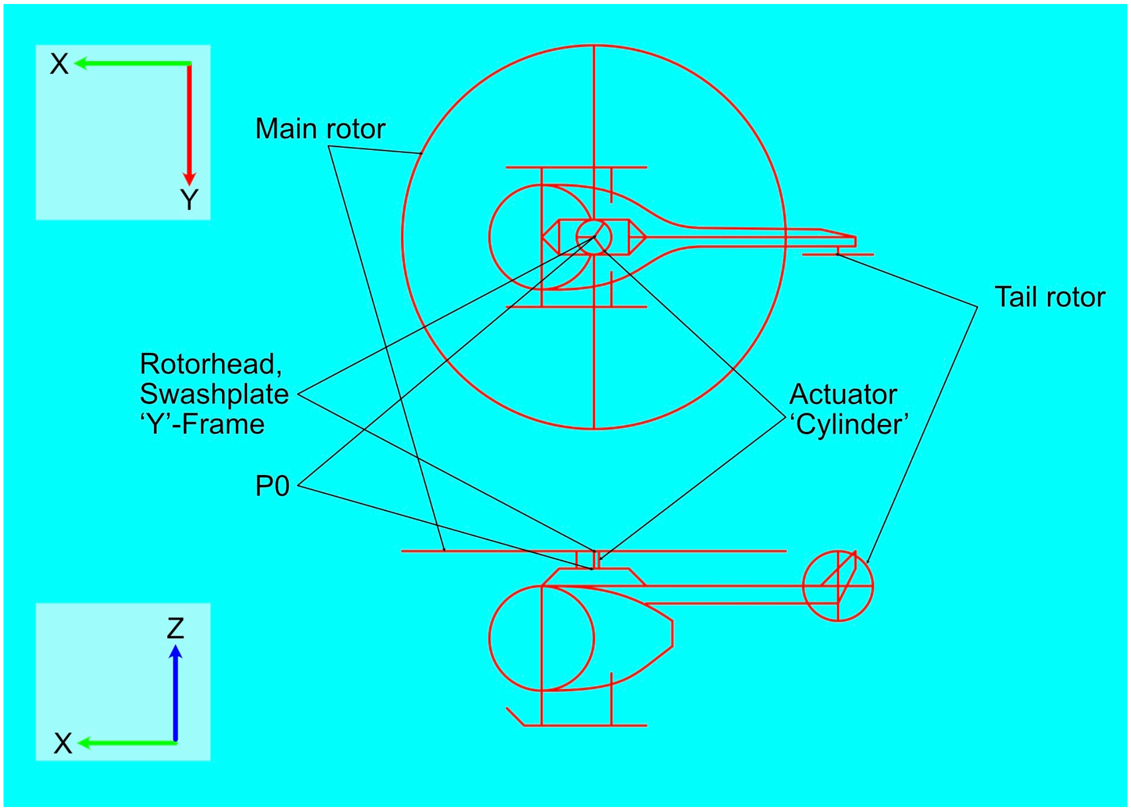


Fig 1: Basic diagram of a helicopter and coordinate system

$$\left(1,\ \vec{V_p} = \left[\cos\left(\angle A_p\right) \quad 0 \quad \sin\left(\angle A_p\right)\right]\right)$$

$$\left(2,\ \hat{V_p} = \frac{\vec{V_p}}{\left|\vec{V_p}\right|}\right)$$

$$\left(3,\ \vec{V_r} = \left[0 \quad \cos\left(\angle A_r\right) \quad \sin\left(\angle A_r\right)\right]\right)$$

$$\left(4,\ \hat{V_r} = \frac{\vec{V_r}}{\left|\vec{V_r}\right|}\right)$$

$$\left(5,\ \vec{F_f} = \left[R_{sw} \quad 0 \quad 0\right]\right)$$

$$\left(6,\ \vec{F_p} = \left[R_{sw}\sin\left(\angle A_{cyl}\right) \quad R_{sw}\cos\left(\angle A_{cyl}\right) \quad 0\right]\right)$$

$$\left(7,\ \vec{F_s} = \left[R_{sw}\sin\left(2\angle A_{cyl}\right) \quad R_{sw}\cos\left(2\angle A_{cyl}\right) \quad 0\right]\right)$$

$$\left(8,\ \vec{V_{mast}} = \left[0 \quad 0 \quad R_{coll}\right]\right)$$

$$\left(9,\ \vec{V_{disk}} = \hat{V_p} \times \hat{V_r}\right)$$

`Then, for every Cylinder` $\vec{C_N}$

$$\left(10,\ \vec{F_N} = P_{fn} - P_O\right)$$

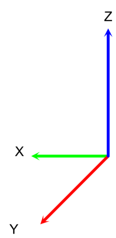
$$\left(11,\ \vec{V_{cN}} = \vec{F_N} \times \vec{V_{mast}}\right)$$

$$\left(12,\ \vec{V_{isectN}} = -\vec{V_{cN}} \times \vec{V_{disk}}\right)$$

$$\left(13,\ \hat{V_{isectN}} = \frac{\vec{V_{isectN}}}{\left|\vec{V_{isectN}}\right|}\right)$$

$$\left(14,\ \vec{V_{aN}} = \vec{V_{mast}} + R_{sw}\hat{V_{isectN}}\right)$$

$$\left(15,\ \vec{C_N} = \vec{V_{aN}} - \vec{F_N}\right)$$



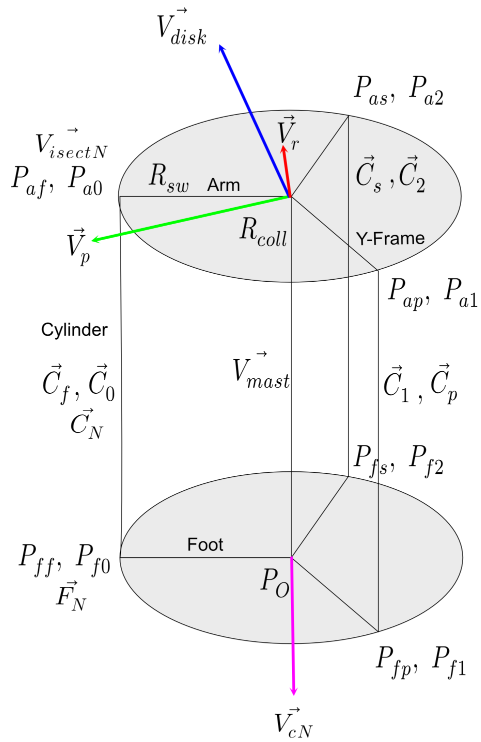


Fig 2: System diagram

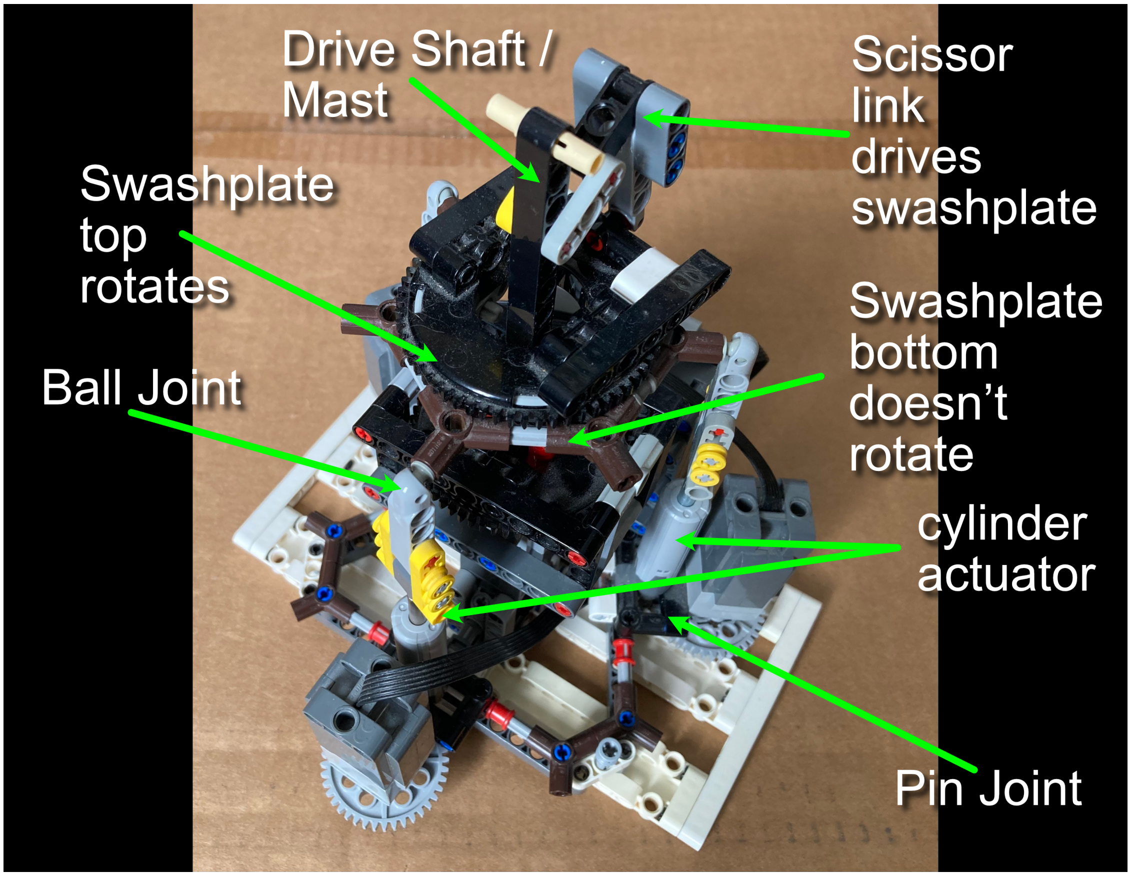


Fig 3: Model rotor head without rotors attached

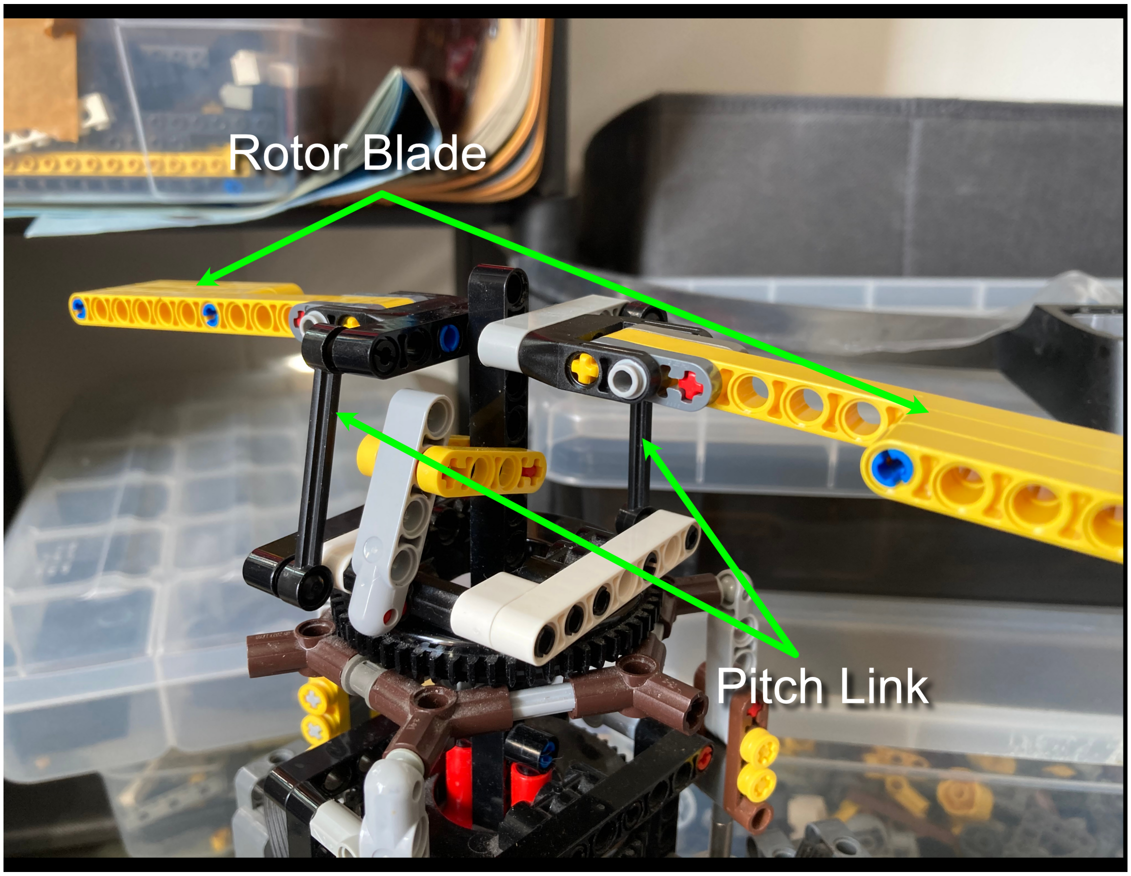


Fig 4: Model rotor head with rotors attached

0.0.4 System Description

A two-part rotating swashplate, the bottom half of which is represented as a Y-shaped frame rotating around a spherical bearing, which in turn surrounds a bushing through which the drive shaft runs. This allows the swashplate to pitch and roll around the spherical bearing, which in turn moves up and down on the drive shaft. The arms of the Y-shaped frame will be referred to as swashpate arms, each with a length of Rsw. The center of the Y-shaped frame is referred to as Pcoll, and represents the hight of the rotorhead over the Origin. Pcoll can vary in Z-height regulated by the collective position. The ends of the arms are referred to as PaN, with N proceeding clockwise from the front. The arms are connected to the helicopter with linear actuators, which will be referred to as cylinders. The cylinders are connected to the ends of the arms with ball joints, and to the base XY plane by pins on "feet", which have an identical Y-shape to the swashplate frame.
Thus, the cylinders can rotate towards the origin along their feet. The length of these feet is the same as the length of the arms Rsw, but needn't be. The cylinders are located 120 around a circle around the Origin, but don't need to be regularly distributed. The ends of the feet are Pfn.

Pff PO Pfp = 120

Pff PO Pfs = 120

For convenience, cylinders also have position names. C0 is Cf, C1 is Cp, and C2 is Cs. Similarly, Paf is equivalent to Pa0, Pff is Pf0, and so on, with "f", "p", and "s" abbreviating front, port, and starboard.

Each cylinder has a minimum and maximum length, Cmin and Cmax

The cyclic input is used to make Pitch and Roll vectors Vp and Vr, and collective the scalar Pcoll.

The plane of the rotor is defined by Pcoll, the nominal height of the collective, as well as the three points Paf, Pap, and Pas. This plane is manipulated around the Y axis by the pitch input of the cyclic, and around the x axis by the roll input of the cyclic.

$\left(P_O,\ R_{coll},\ R_{sw},\ \angle A_p,\ \angle A_r,\ \vec{C_f},\ \vec{C_0},\ \vec{C_p},\ \vec{C_1},\ \vec{C_s},\ \vec{C_2}\right)$

$\left(P_{af},\ P_{a0},\ P_{ap},\ P_{a1},\ P_{as},\ P_{a2},\ P_{ff},\ P_{f0},\ P_{fn}\right)$

$\left(P_{fp},\ P_{f1},\ P_{fs},\ P_{f2},\ C_{min},\ C_{max},\ \vec{V_{cN}},\ \vec{V_{aN}},\ \vec{C_N},\ \vec{F_N}\right)$

$\left(\vec{V_{isectN}},\ \hat{V_{isectN}},\ \vec{V_{mast}},\ \vec{V_{disk}},\ P_{coll},\ \vec{V_p},\ \vec{V_r},\ \hat{V_r},\ \hat{V_p}\right)$

$$\begin{aligned}
&\text{«eq.1»} & \vec{V_p} &= \begin{bmatrix}\cos\left(\angle A_p\right) & 0 & \sin\left(\angle A_p\right)\end{bmatrix} \\
&\text{«eq.2»} & \hat{V_p} &= \frac{\vec{V_p}}{\left|\vec{V_p}\right|} \\
&\text{«eq.3»} & \vec{V_r} &= \begin{bmatrix}0 & \cos\left(\angle A_r\right) & \sin\left(\angle A_r\right)\end{bmatrix} \\
&\text{«eq.4»} & \hat{V_r} &= \frac{\vec{V_r}}{\left|\vec{V_r}\right|} \\
&\text{«eq.5»} & \vec{F_f} &= \begin{bmatrix}R_{sw} & 0 & 0\end{bmatrix} \\
&\text{«eq.6»} & \vec{F_p} &= \begin{bmatrix}R_{sw}\sin\left(\angle A_{cyl}\right) & R_{sw}\cos\left(\angle A_{cyl}\right) & 0\end{bmatrix} \\
&\text{«eq.7»} & \vec{F_s} &= \begin{bmatrix}R_{sw}\sin\left(2\angle A_{cyl}\right) & R_{sw}\cos\left(2\angle A_{cyl}\right) & 0\end{bmatrix} \\
&\text{«eq.8»} & \vec{V_{mast}} &= \begin{bmatrix}0 & 0 & R_{coll}\end{bmatrix} \\
&\text{«eq.9»} & \vec{V_{disk}} &= \hat{V_p} \times \hat{V_r}
\end{aligned} \tag{1}$$

Then, for every Cylinder $\vec{C_N}$

$$\begin{aligned}
&\text{«eq.10»} & \vec{F_N} &= P_{fn} - P_O \\
&\text{«eq.11»} & \vec{V_{cN}} &= \vec{F_N} \times \vec{V_{mast}} \\
&\text{«eq.12»} & \vec{V_{isectN}} &= -\vec{V_{cN}} \times \vec{V_{disk}} \\
&\text{«eq.13»} & \hat{V_{isectN}} &= \frac{\vec{V_{isectN}}}{\left|\vec{V_{isectN}}\right|} \\
&\text{«eq.14»} & \vec{V_{aN}} &= \vec{V_{mast}} + R_{sw}\hat{V_{isectN}} \\
&\text{«eq.15»} & \vec{C_N} &= \vec{V_{aN}} - \vec{F_N}
\end{aligned} \tag{2}$$

0.0.5 Source Code of Solve Function

```python
def solve(self, pitch, roll, collpct):
    Vp = lin.vector(m.cos(m.radians(pitch)), 0, m.sin(m.radians(pitch)))
    Vr = lin.vector(0, m.cos(m.radians(roll)), m.sin(m.radians(roll)))

    # Normal of rotor disk
    Vdisk = lin.cross(Vp, Vr)
    Vdisk_n = lin.normalize(Vdisk)

    # top of mast at collective setting
    Vmast = lin.vector(0, 0, self.Cmin + collpct*self.Crange)
    arms = []
    for i, Fn in enumerate(self.feet):
        # Vcn is the plane the cylinder rotates on its foot in, Foot X Mast
        Vcn = lin.cross(Fn, Vmast)
        Vcn_n = lin.normalize(Vcn)

        # Visect is the intersection of the Rotor Disk plane and the Cylinder rotation plane
        Visect = lin.cross(Vdisk_n, Vcn_n) # should be plane intersection
        Visect_n = lin.normalize(Visect)

        # Va is the arm vector as rotated in the cylinder rotation plane
        Va = (self.Rsw * Visect_n) + Vmast

        arms.append(Va)
        cyl_len = lin.vmag(Va - Fn)
        if cyl_len < self.Cmin:
            raise ValueError(f"too short! Cyl: {cyl_len:{4}.{4}} min: {self.Cmin:{4}}")
        elif cyl_len > self.Cmax:
            raise ValueError(f"too long! Cyl: {cyl_len:{4}.{4}} max: {self.Cmax:{4}}")

    (Cf, Cp, Cs) = arms

    #old validation code, cyl_len test above is sufficient
    #self.validate(Cf, Cp, Cs, Vmast, pitch, roll, collpct)

    #successfully validated, save old values and calculate cylinder lengths
    self.s_Cf = Cf
    self.s_Cp = Cp
    self.s_Cs = Cs
    self.s_Vmast = Vmast

    #cylinder lengths in mm
    self.cfc = lin.vmag(Cf - self.Ff)
    self.cpc = lin.vmag(Cp - self.Fp)
    self.csc = lin.vmag(Cs - self.Fs)
    return (Cf, Cp, Cs, Vmast)
```

0.0.6 Algorithm

The point P_{coll} is first chosen from the collective input, yielding $\vec{V_{mast}}$.
The pitch vector $\vec{V_p}$ is the x axis rotated around the y axis by the pitch input.
The roll vector $\vec{V_r}$ is the y axis rotated along the x axis by the roll input.

The normal vector $\vec{V_{disk}}$ representing the plane of the rotor disk is then $\vec{V_p}$ X $\vec{V_r}$ [eq. 9].

Then for each of the cylinders $\vec{C_N}$, the foot vector $\vec{F_N}$ is the the line between the origin P_O and the base of the cylinder P_{fn}.

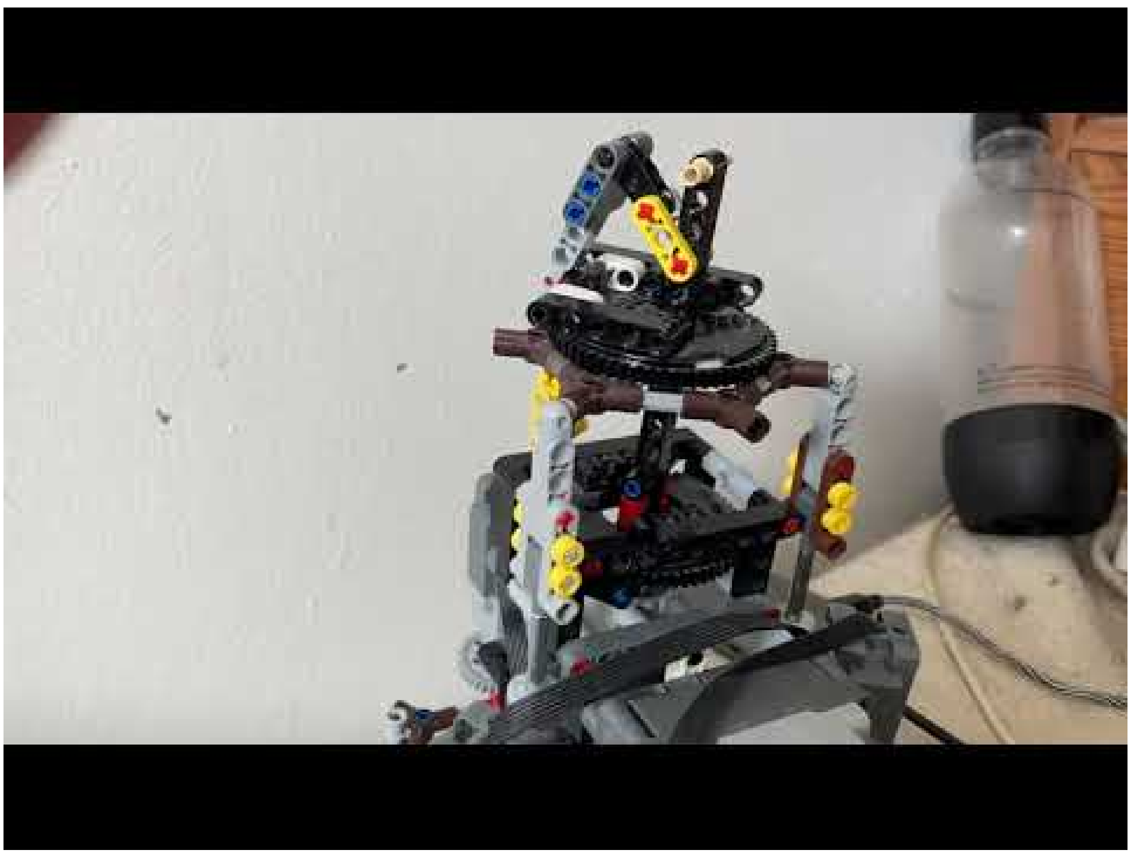
The cylinder plane vector $\vec{V_{cN}}$ is then $\vec{F_N}$ X $\vec{V_{mast}}$ [eq. 11].

The cross product of $\vec{V_{disk}}$ and $\vec{V_{cN}}$ is the intersection of the rotor disk and the cylinder plane, which is our answer, normalized to $\hat{V_{isectN}}$ [eq. 12].

$\hat{V_{isectN}}$ is multiplied by R_{sw} and added to $\vec{V_{mast}}$, yielding the arm vector $\vec{V_{aN}}$ [eq. 14].

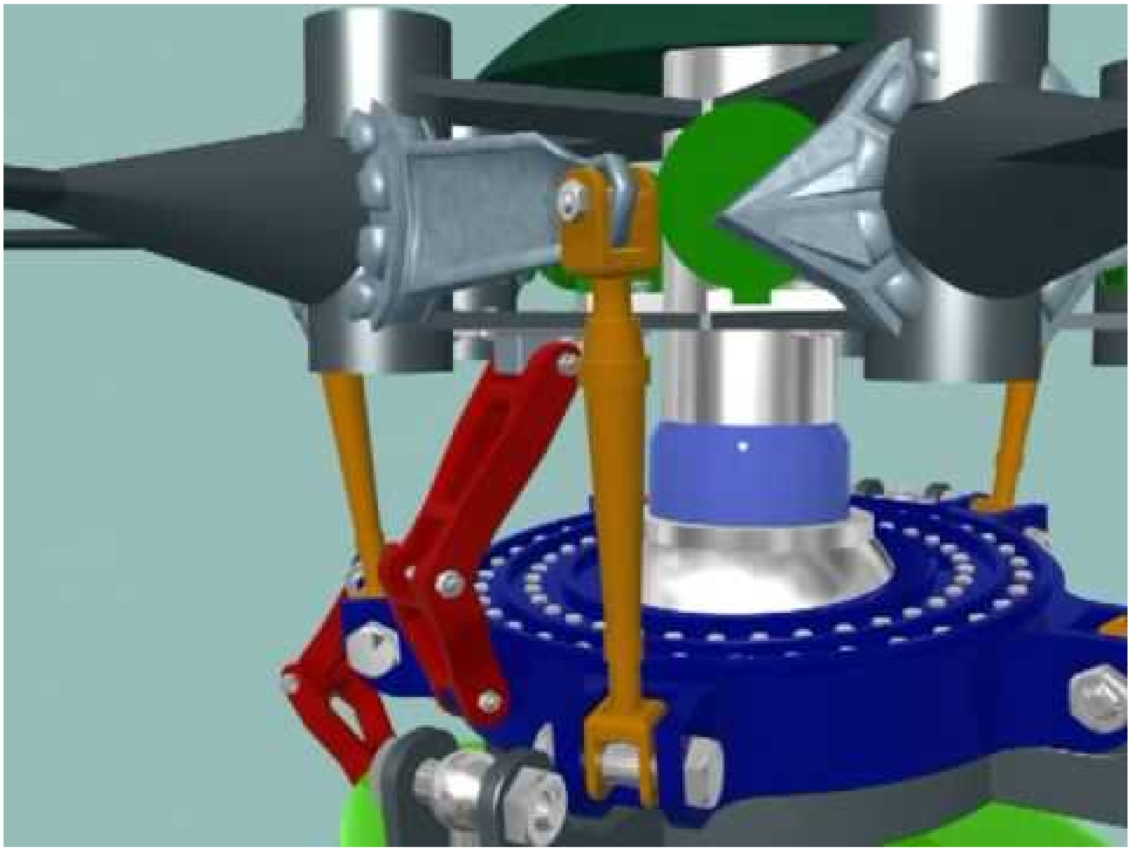
The cylinder vector $\vec{C_N}$ is then $\vec{V_{aN}}$ - $\vec{F_N}$ [eq. 15].

Finally, the length of the cylinder vector is checked to make sure it fits within the limits.



0.0.7 References

This animation of an S-61 Sea King rotor head was invaluable in my understanding of how a helicopter works.



- What Is CCPM, on an RC Helicopter & Why it's Important?
- Cyclic/collective pitch mixing (wikipedia)

Table of Variables #####invariants PO: Origin of XYZ system Rsw: radius of swashplate arms from Pcoll Cmin: cylinder minimum length Cmax: cylinder maximum length Acyl: separation of cylinders, 120 Ff, Fs, Fp, Fn: Foot vectors Vmast: Vector of mast Pff, Pf0: End of front swashplate foot Pfp, Pf1: End of port swashplate foot Pfs, Pf2: End of starboard swashplate foot VcN: normal vector plane of cylinder foot and mast #####inputs Pcoll: height of collective above origin Ap, Ar: Pitch and Roll angle derived from collective

#####outputs Cf, C0: Front linear actuator, vector representing length Cp, C1: Port linear actuator Cs, C2: Starboard linear actuator Paf, Pa0: End of front swashplate arm Pap, Pa1: End of port swashplate arm Pas, Pa2: End of starboard swashplate arm

#####calculations Vp, V^p: Pitch vector from cyclic, normalized Vr, V^p: Roll vector from cyclic, normalized Vdisk: Normal vector of rotor plane, Vp X Vr Visectn, hat: intersection of rotor plane and cylinder plane, Vdisk X VcN Van: vector of arm N, Rsw*Visctn+ Vvmast Cn: Cylinder Vector: Van-Vf