

## Environment

The sample game's objective is to conquer a two-dimensional grid using units spawned from nests built by the player. Building nests early offers a material advantage that is hard to overcome later.

However: once a nest is destroyed, all adjacent tiles are permanently captured by the other player, and all overlapping nests are destroyed. This can cause a game ending chain reaction if all a player's nests are connected by overlap. Thus there is a tradeoff between early material advantage and late-game security.

## Method

We employ a packing strategy to optimize growth while minimizing risk. The nests are shaped like "plus" signs with unit area 5 and we want to pack them into the grid without overlap (risk) or wasted space (inefficiency).

The optimal packing seems obvious on paper, but to represent it programmatically without headaches, we move the problem from  $\mathbb{N}^2$  to  $(\mathbb{Z}/5\mathbb{Z})^2$ . Here the optimal packing can be represented by the simple equation  $i = 2j$ , i.e. the points  $(i, j) \in (\mathbb{Z}/5\mathbb{Z})^2$  satisfying  $i = 2j$  are the points we identify with the nest centers.



*Left: nest centers in  $(\mathbb{Z}/5\mathbb{Z})^2$ . Center: the nests themselves, color coded. Right: The nests with centers in blue. The non-central tiles of nests may overlap, but this creates the risk discussed earlier.*

Moving back to  $\mathbb{N}^2$ , we tessellate the above pattern marking each tile  $(i, j)$  as a nest if and only if

$$j \bmod 5 \equiv 2 \cdot i \bmod 5$$

With this marginal adjustment to an otherwise unsophisticated greedy strategy, we get effective behavior with little effort.