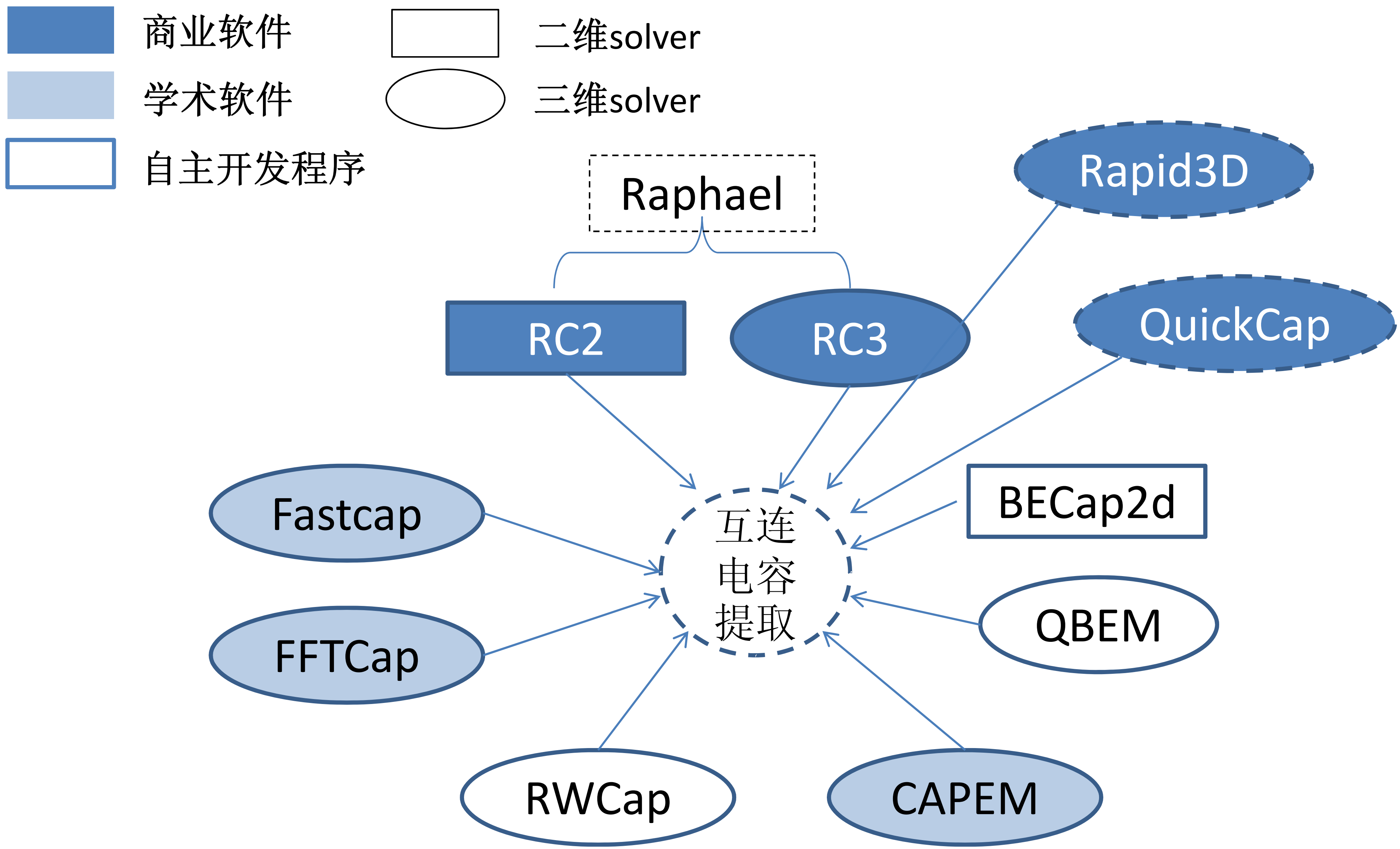
Capacitance Field Solvers

喻文健

几种电容求解器



商业软件
学术软件
自主开发程序
二维solver
三维solver
Raphael
RC2
RC3
Rapid3D
QuickCap
BECap2d
QBEM
CAPEM
RWCap
FFTCap
Fastcap
互连
电容
提取

Capacitance extraction

- 3-D numerical methods – general approach
  - Set voltages on conductor; solve for Q_i

$$Q_i = \sum_{j=1}^{N} C_{ij} U_j \,, \qquad i=1, 2, \ldots, N,$$

Solve the electrostatic field for *u*, then $Q_i = \int_{\Gamma_i} \varepsilon \cdot \frac{\partial u}{\partial n}$

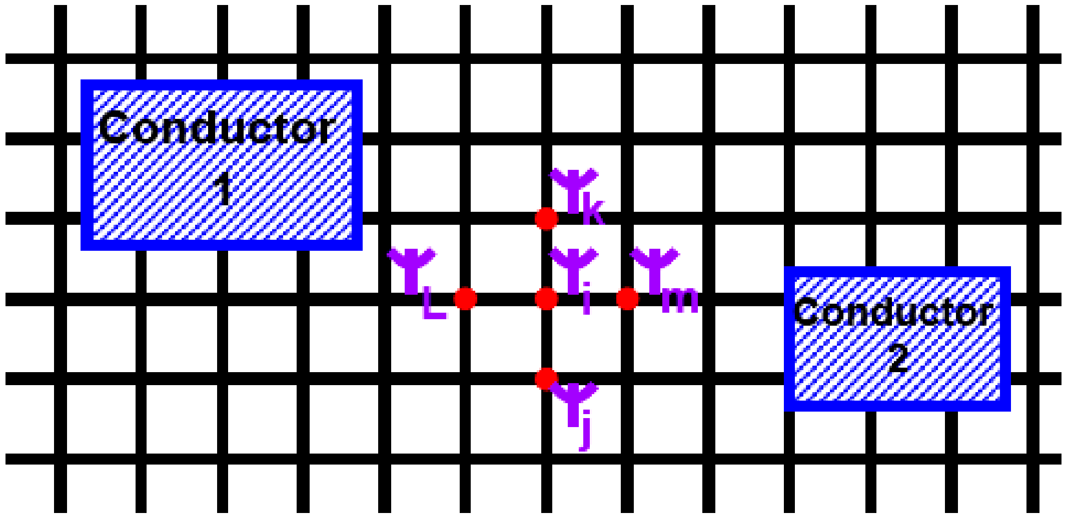
  - Global method to get the whole matrix
- Classification
  - Volume discretization: FDM, FEM
  - Boundary integral (element) method
  - Stochastic method
  - Others – semi-analytical approaches

Raphael's RC3 – Synopsys
Q3D — Ansoft

FastCap, QBEM, HiCap

QuickCap - Magma

Volume Methods - Finite-Differences/Finite-Elements

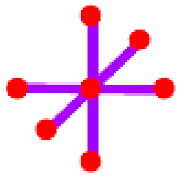


Solve Laplace's equation, $\nabla \cdot \epsilon \nabla \Psi = 0$ in the conductor exterior.

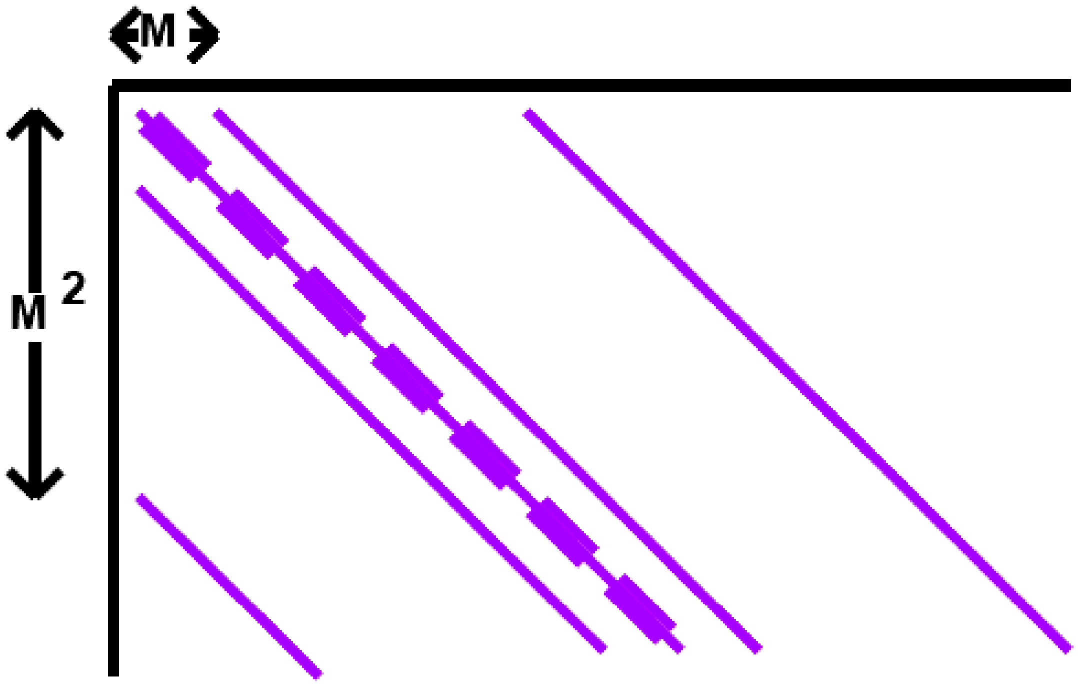
- Approximate derivatives by finite-differences.
- Conductors provide potential boundary conditions (e.g., 1 on conductor 1, zero on conductor 2).
- 2-D example

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} \approx \frac{\frac{\Psi_m - \Psi_i}{x_m - x_i} - \frac{\Psi_i - \Psi_L}{x_i - x_L}}{0.5\left((x_m - x_i) + (x_i - x_L)\right)} + \frac{\frac{\Psi_k - \Psi_i}{y_l - y_i} - \frac{\Psi_i - \Psi_j}{y_i - y_j}}{0.5\left((y_l - y_i) + (y_i - y_L)\right)}$$

Volume Methods generate sparse matrices

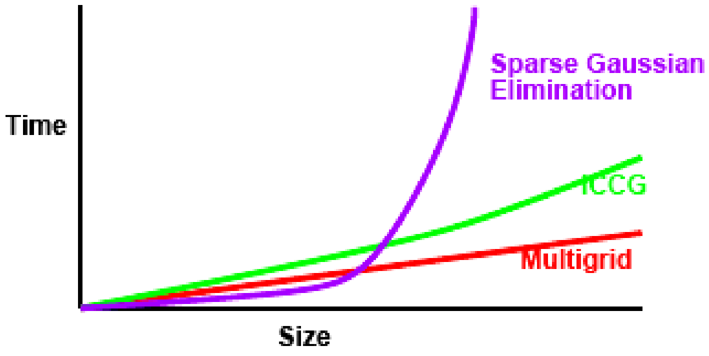


- One equation for each grid node
- In 3-D, each equation involves at least 7 variables
  - Up-Down for $\frac{\partial}{\partial z}$, Left-Right for $\frac{\partial}{\partial x}$, Backward-Forward for $\frac{\partial}{\partial y}$
- Sparse matrix for an $M \times M \times M$ grid is Large



Slides courtesy J. White, MIT

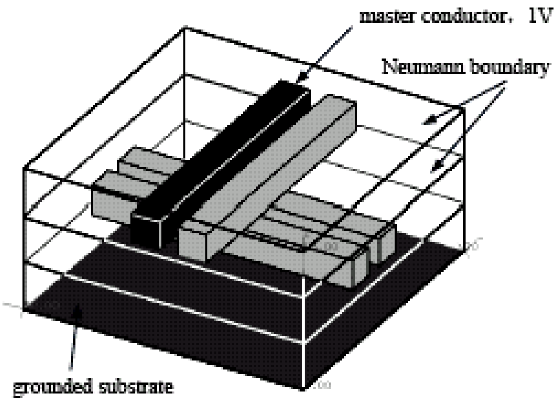
Matrix Solution Methods.

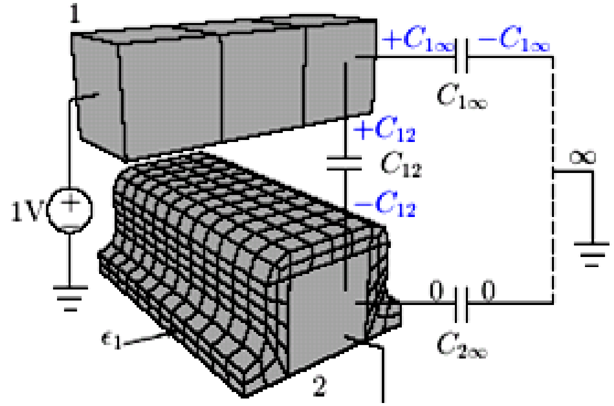


- Sparse Gaussian Elimination
  - Direct, complicated data structures, existing packages.
  - Order N^2 time and storage.
- Incomplete Cholesky Conjugate-Gradient Method (ICCG)
  - Iterative, easy to program, fast when ground planes nearby.
  - Order $NlogN$ time and Order N storage.
- Multigrid methods
  - Iterative, complicated for general grids, fast convergence.
  - Order N time and storage.

Capacitance extraction

- Volume methods
  - What's the size of simulation domain ?
  - Two kinds of problem: finite domain and infinite domain



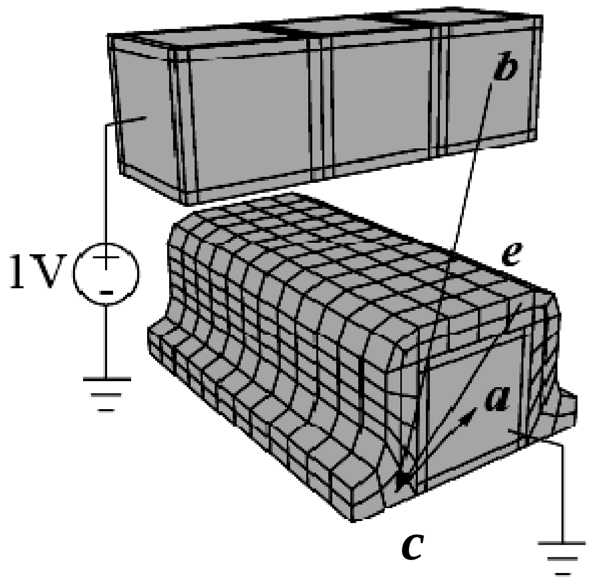


  - Which one is correct ? both in most time
    - 3-D extraction is not performed directly on a "real" case
    - In the chopping & combination procedure, both models used
    - Because of attenuation of electric field, the results from two models can approach to each other

Because of its nature, volume methods use finite-domain model

Integral Formulation Example

inside alg. of FastCap



- Influence of charge on panel c at the center of panel a is

$$\frac{q_c}{A_c} \int_{\text{panel } c} \frac{1}{r_{ac}} dA.$$

- Potential at panel a is sum of all contributions:

$$v_a = \cdots + q_c \left(\frac{1}{A_c} \int_{\text{panel } c} \frac{1}{r_{ac}} dA \right) + \cdots + q_b \left(\frac{1}{A_b} \int_{\text{panel } b} \frac{1}{r_{ab}} dA \right) + \cdots.$$

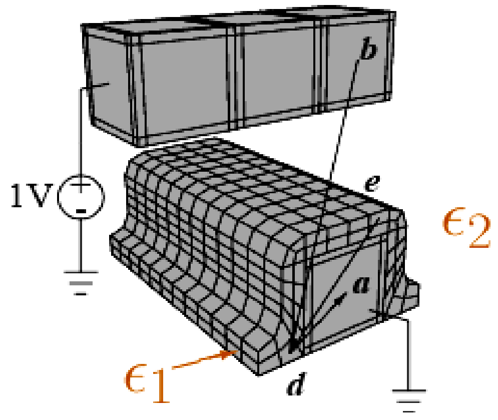
MoM (method of moment)

Method of virtual charge

Indirect boundary element method

Slides courtesy J. White, MIT

Include the Effects of the Dielectric Interfaces



- Dielectric panel d's charge contributes to v_a, as did b and c.
- To force $0 = \epsilon_1 E_{n1} - \epsilon_2 E_{n2}$ at panel d's center:

Polarized charge

$$0 = \cdots + q_e \left[(\epsilon_1 - \epsilon_2) \frac{\partial}{\partial \hat{n}} \frac{1}{A_e} \int_{\text{panel } e} \frac{1}{r_{de}} dA \right]$$
$$+ \cdots + q_b \left[(\epsilon_1 - \epsilon_2) \frac{\partial}{\partial \hat{n}} \frac{1}{A_b} \int_{\text{panel } b} \frac{1}{r_{db}} dA \right] + \cdots .$$

Slides courtesy J. White, MIT

Pack into Matrices

$$\begin{bmatrix} v_1 \\ \vdots \\ v_{n_p} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ \vdots & \vdots & & \vdots \\ P_{n_p 1} & P_{n_p 2} & \cdots & P_{n_p n} \\ E_{n_p+1,\,1} & E_{n_p+1,\,2} & \cdots & E_{n_p+1,\,n} \\ \vdots & \vdots & & \vdots \\ E_{n1} & E_{n2} & \cdots & E_{nn} \end{bmatrix} \begin{bmatrix} q_1 \\ \vdots \\ q_{n_p} \\ q_{n_p+1} \\ \vdots \\ q_n \end{bmatrix}$$

$$P_{ij} \triangleq \frac{1}{A_j} \int_{\text{panel } j} \frac{1}{r_{ij}} \, dA;$$

$$E_{ij} \triangleq \frac{\partial}{\partial \hat{n}} \frac{1}{A_j} \int_{\text{panel } j} \frac{1}{r_{ij}} \, dA, \quad i \neq j.$$

Slides courtesy J. White, MIT

Solve $A\vec{x} = \vec{b}$ System

where $A = \begin{bmatrix} P \\ E \end{bmatrix}$ is a (# of panels)$\times$(# of panels) dense matrix.

- Direct methods like Gaussian Elimination require n^3 operations.
- Iterative methods such as GMRES requires n^2 operations.
- Both direct and iterative methods require n^2 storage.

Slides courtesy J. White, MIT

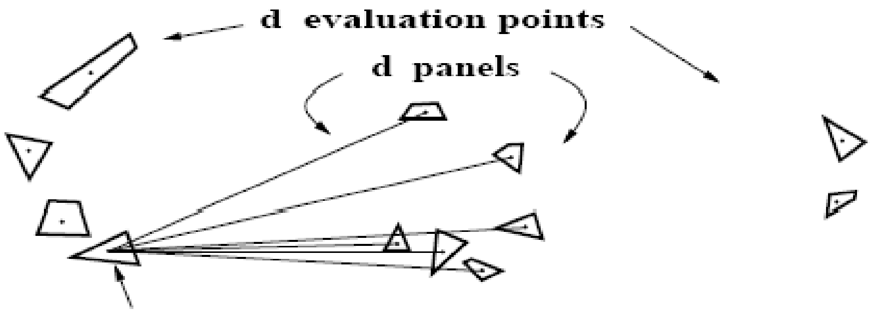
Solve $A\vec{x} = \vec{b}$ Iteratively

1. Guess charges, $\tilde{\vec{x}}$.
2. Calculate $A\tilde{\vec{x}}$—costs $O(n^2)$.
3. If $A\tilde{\vec{x}}$ does not equal b, fix $\tilde{\vec{x}}$.

Speed Up $A\vec{x}$ Product

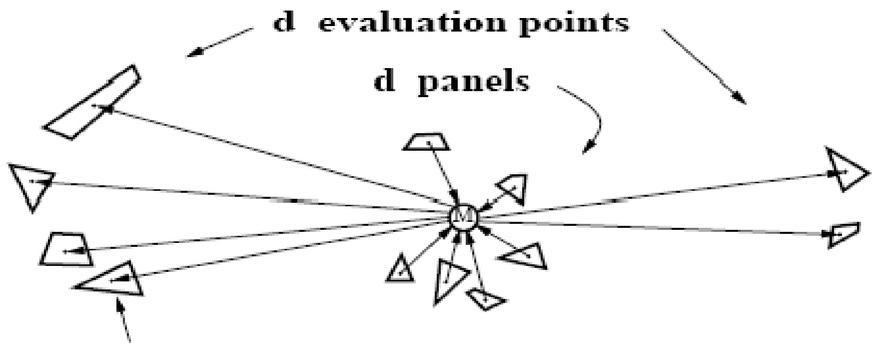
- Computing $A\vec{x}$ is equivalent to computing n potentials and electric fields from n charges.
- Accelerate matrix-vector products using potential approximations.
- Save Memory by not forming A

Direct Potential Evaluation



- Computing d potentials due to d panels costs d^2 operations.

Multipole Potential Evaluation



- Multipole Approximations compute d potentials due to d panels is order d operations.

Multipole expansion with order l

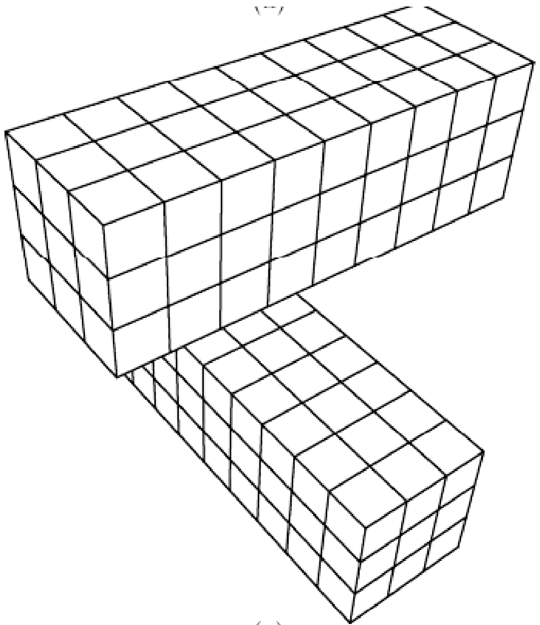
l=0: $$\frac{\sum_{i=1}^{n_2} q_i}{r_j},$$

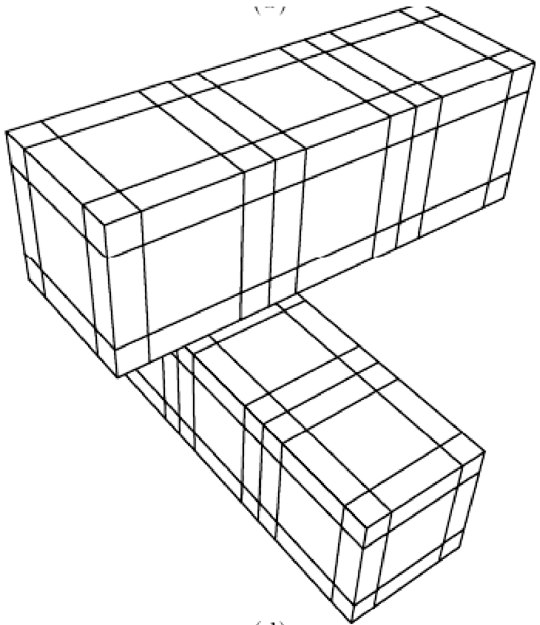
Slides courtesy J. White, MIT

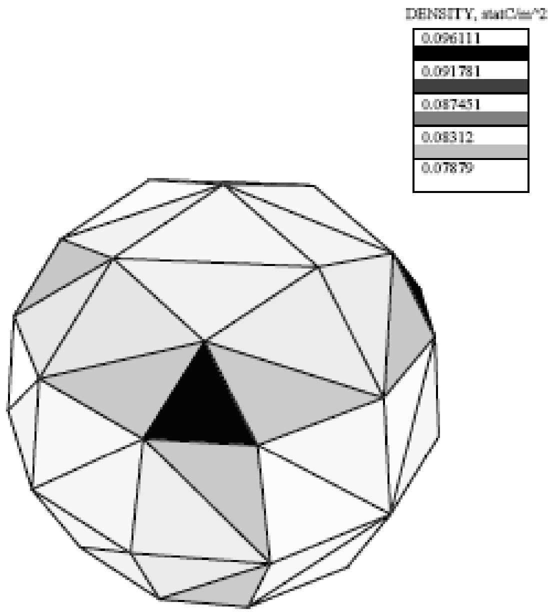
FastCap简介

- 功能
  - 输入格式：形体描述、边界划分描述
  - 计算功能：指定主导体、整个矩阵
  - 附加功能：形体可视化、计算结果可视化

开放源码!


(c)


(d)



FastCap简介

- 输入格式
  - 两种表面：导体-介质交界面，介质-介质交界面
  - 需要指定所有表面的离散化情况(**panel**)
  - **Fastcap generic file format, PATRAN neutral file**
- **Generic file format**
  - **.qui**文件**,** 文本格式，描述每个**panel**的顶点坐标
  - 第一行为标题行，以**"0"**开始**; "*"**开始行为注释
  - 每行描述一个**panel,** 两种开始字符**: T (**三角形**), Q (**四边形**)**

```
Q  <cond. name>  <x1> <y1> <z1>  <x2> <y2> <z2>  <x3> <y3> <z3>  <x4> <y4> <z4>
T  <cond. name>  <x1> <y1> <z1>  <x2> <y2> <z2>  <x3> <y3> <z3>
```

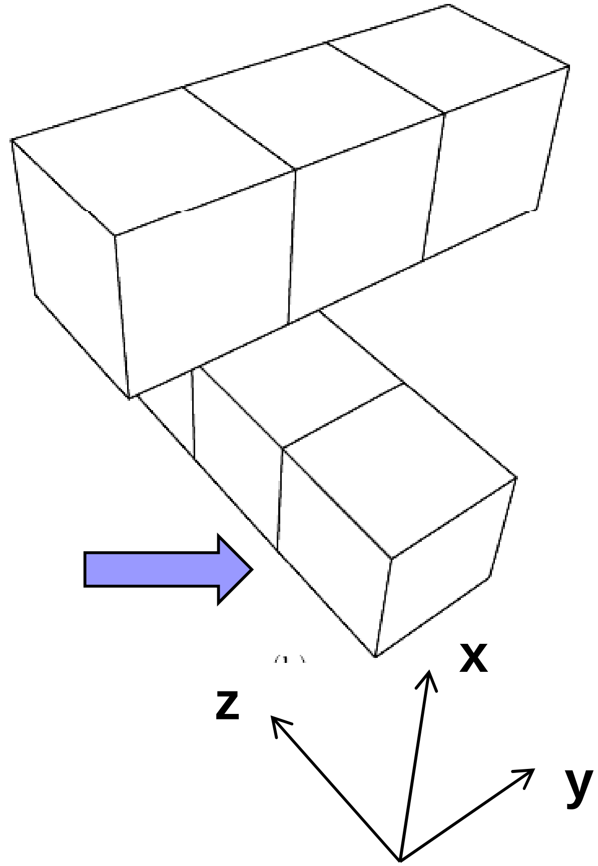
  - 导体名字常用数字表示**,** 点必须按顺时针或逆时针顺序

FastCap简介

- **Generic file format**

```
Q  1   0.0 0.0 0.0  1.0 0.0 0.0  1.0 1.0 0.0  0.0 1.0 0.0
Q  1   0.0 0.0 0.0  1.0 0.0 0.0  1.0 0.0 1.0  0.0 0.0 1.0
Q  1   0.0 0.0 1.0  1.0 0.0 1.0  1.0 0.0 2.0  0.0 0.0 2.0
Q  1   0.0 0.0 2.0  1.0 0.0 2.0  1.0 0.0 3.0  0.0 0.0 3.0
Q  1   0.0 1.0 0.0  1.0 1.0 0.0  1.0 1.0 1.0  0.0 1.0 1.0
Q  1   0.0 1.0 1.0  1.0 1.0 1.0  1.0 1.0 2.0  0.0 1.0 2.0
Q  1   0.0 1.0 2.0  1.0 1.0 2.0  1.0 1.0 3.0  0.0 1.0 3.0
Q  1   1.0 1.0 0.0  1.0 0.0 0.0  1.0 0.0 1.0  1.0 1.0 1.0
Q  1   1.0 1.0 1.0  1.0 0.0 1.0  1.0 0.0 2.0  1.0 1.0 2.0
Q  1   1.0 1.0 2.0  1.0 0.0 2.0  1.0 0.0 3.0  1.0 1.0 3.0
Q  1   0.0 1.0 0.0  0.0 0.0 0.0  0.0 0.0 1.0  0.0 1.0 1.0
Q  1   0.0 1.0 1.0  0.0 0.0 1.0  0.0 0.0 2.0  0.0 1.0 2.0
Q  1   0.0 1.0 2.0  0.0 0.0 2.0  0.0 0.0 3.0  0.0 1.0 3.0
Q  1   0.0 0.0 3.0  1.0 0.0 3.0  1.0 1.0 3.0  0.0 1.0 3.0
N  1   Big
```

(把导体"1"重命名为Big)



- 描述复杂形体的**List**文件
  - 用**List (.lst)**文件将多个**.qui**文件组合起来
  - 可描述多介质结构和复杂形体

FastCap简介

- 描述复杂形体的List文件

G <group name> **(给后续"+"连接的group重命名)**

C <file> <outperm> <xtran> <ytran> <ztran> [+]

D <file> <outperm> <inperm> <xtran> <ytran> <ztran> <xref> <yref> <zref> [-]

B <file> <outperm> <inperm> <xtran> <ytran> <ztran> <xref> <yref> <zref> [-][+]

  - **C:** 导体-介质交界面，一般的导体表面
  - **D:** 介质-介质交界面
  - **B:** 无限薄的导体板，这个面兼具**C, D**的特点
  - **x/y/ztran:** 将**.qui**文件中所有坐标偏移这个量
  - **Out/inperm:** 介质的介电常数
  - **x/y/zref:** 指定一个参考点坐标，该点在**outperm**的介质那边**,** 若写了**"-"**则该点在**inperm**的介质那边
  - **+**使不同行文件中同名导体成为一个导体**,** 同属一**group**

FastCap简介

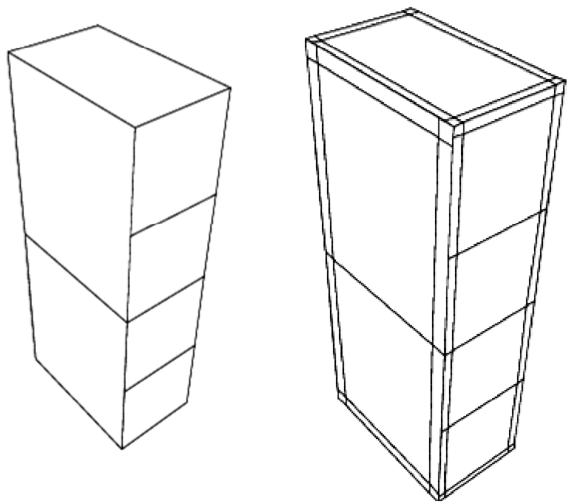
- 描述复杂形体的**List**文件

```
* 1x1 bus crossing problem with dielectric on lower conductors
*
* conductor to air interfaces
C cond_air_1x1.qui 1.0 0.0 0.0 0.0 +
*
* conductor to dielectric interfaces
C cond_dielec_1x1.qui 7.5 0.0 0.0 0.0
```

  - **cond_air_1x1.qui**和**cond_dielec_1x1.qui**中都包含名字为**1**的导体
  - **“+”**使这两个导体合成**1**个，全局名字为**1%GROUP1**
  - 若没有**“+”**，它们属于不同组，名字为**1%GROUP1, 1%GROUP2**

FastCap简介

- **Generic file generator**程序
  - **Busgen:** 平行总线结构(多根平行线); **Cubegen:** 立方体导体; **Capgen:** 平板电容器(两块导体板)
  - 生成**.qui**文件到标准输出
  - **$ cubegen -n1 -xh1 -yh1 -zh3 > cube113.qui (pp. 16)**
  - **-n:** 每个矩形面最短边分的份数, **-x/y/zh**三个方向长度
  - **$ cubegen -n1 -xh1.5 -yh2.5 -zh6**
  - **$ cubegen -n3 -xh1.5 -yh2.5 -zh6**
  - **-e: edge to inner panel width ratio** 缺省值为**0.1**
  - 详细命令参数设定，见**[1][2]**



FastCap简介

- 构造**FastCap**输入的一般方法
  - 小规模例子：用**Generator**程序生成**.qui**文件，手写**.lst**文件组合多个**.qui**文件
  - 较大规模例子：编程写**.qui**文件 **(**单介质问题**)**；编程写**.lst**文件**(**多介质、或复杂问题**)**
- **FastCap**使用命令

```
fastcap [-o<expansion order>] [-d<partitioning depth>] [<input file>]
        [-p<permittivity factor>] [-rs<cond list>] [-ri<cond list>]
        [-] [-l<list file>] [-t<iter tol>]
```

  - **-o, -d, -t:** 影响多极展开、方程求解计算效率，不用设置
  - **-p:** 介电常数加倍比例
  - **-rs:** 求解过程中去掉主导体，否则每个都做一次主导体
  - **-ri:** 从输入文件中去掉导体，可方便求解一个子结构

FastCap简介

[yuwj@linux90 bin]$ fastcap cube1.qui *(cubegen -n1 -xh1.5 -yh2.5 -zh6)*
Running fastcap 2.0 (18Sep92)
  Input: cube1.qui
  Input surfaces:
   GROUP1
    cube1.qui, conductor
      title: `1.5mX2.5mX6m cube (n=1 e=0.1)'
      outer permittivity: 1
      number of panels: 14
      number of extra evaluation points: 0
      translation: (0 0 0)
  Date: Tue Jan 18 12:09:15 2011
  Host: linux90

INPUT SUMMARY
  Expansion order: 2
  Number of partitioning levels: 3
  Overall permittivity factor: 1
  Total number of panels: 14
  Number of conductor panels: 14
    Number of dielectric interface panels: 0
    Number of thin conductor on dielectric interface panels: 0
  Number of conductors: 1
No expansions at level 3 (lowest)
No expansions at level 2
Percentage of multiplies done by multipole: 0%
Warning: no multipole acceleration

ITERATION DATA
Starting on column 1 (1%GROUP1)
1 2 3 4
CAPACITANCE MATRIX, nanofarads
                    1
1%GROUP1 1   0.2185

FastCap简介

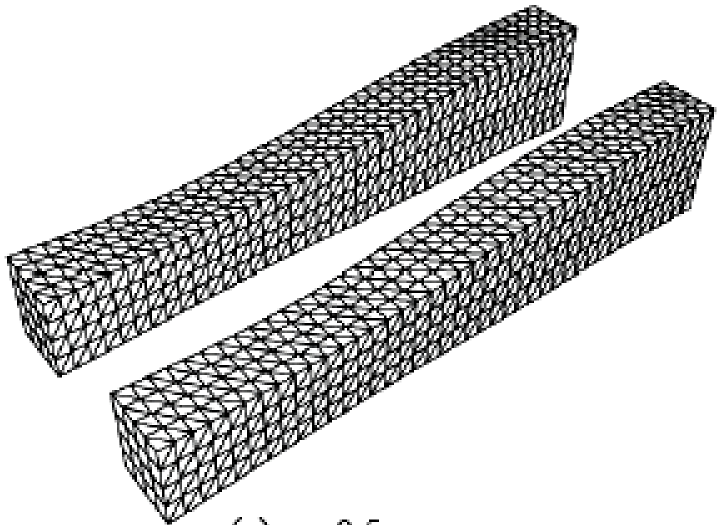
- **FastCap使用命令**

```
fastcap [-o<expansion order>] [-d<partitioning depth>] [<input file>]
        [-p<permittivity factor>] [-rs<cond list>] [-ri<cond list>]
        [-] [-l<list file>] [-t<iter tol>] [-a<azimuth>] [-e<elevation>]
        [-r<rotation>] [-h<distance>] [-s<scale>] [-w<linewidth>]
        [-u<upaxis>] [-q<cond list>] [-rc<cond list>] [-x<axeslength>]
        [-b<.figfile>] [-m] [-rk] [-rd] [-dc] [-c] [-v] [-n] [-f] [-g]
```

  - 除了**8**个基本参数，后面的都用于图形可视化
  - 最重要的是**-m**参数**,** 生成形体和划分的**.ps**文件
  - **-q:** 显示电荷密度分布图**,** 针对每种主导体设置，生成一个**.ps**文件

FastCap简介

- 参考资料
  - 获取源代码: **Google “Fastcap MIT”**
  - 在**Linux**环境下编译、安装
  - 随软件包发布的文档资料：
  - **“FastCap USER’S GUIDE”**
  - “A nice supplementary user's g



(a) η =0.5μm