<u>Given</u>: (x_1, y_1) (x_2, y_2) (x_3, y_3)

<u>Find</u>: (x_c, y_c) ... The center of a circle passing through all 3 points

<u>Solution</u>: For a circle

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

So

$$x^2 - 2x x_c + x_c^2 + y^2 - 2y y_c + y_c^2 = r^2$$

1 equation with 3 unknowns

Rewrite as

$$x_c^2 + y_c^2 - r^2 = 2x x_c + 2y y_c - (x^2 + y^2)$$

For point 1

$$x_c^2 + y_c^2 - r^2 = 2x_1 x_c + 2y_1 y_c - \underbrace{(x_1^2 + y_1^2)}_{r_{01}^2}$$

From origin to 1

So, for points 2 & 3

$$2x_2 x_c + 2y_2 y_c - r_{02}^2 = 2x_1 x_c + 2y_1 y_c - r_{01}^2$$

or

$$(2)(x_2 - x_1)\, x_c + (2)(y_2 - y_1)\, y_c = r_{02}^2 - r_{01}^2$$

and similarly

$$(2)(x_3 - x_1)\, x_c + (2)(y_3 - y_1)\, y_c = r_{03}^2 - r_{01}^2$$

so

$$\underbrace{\begin{bmatrix} (x_2 - x_1) & (y_2 - y_1) \\ (x_3 - x_1) & (y_3 - y_1) \end{bmatrix}}_{A} \begin{bmatrix} x_c \\ y_c \end{bmatrix} = \frac{1}{2} \underbrace{\begin{bmatrix} r_{02}^2 - r_{01}^2 \\ r_{03}^2 - r_{01}^2 \end{bmatrix}}_{b}$$

$$\boxed{\begin{bmatrix} x_c \\ y_c \end{bmatrix} = A^{-1} b}$$

Format for vectors

zeros (2x2)

$$
\begin{bmatrix}
x_{2-1} & y_{2-1} & & (1) & & (2) \\
x_{3-2} & y_{3-2} & & & & \\
& (1) & x_{3-2} & y_{3-2} & & \\
& & x_{4-3} & y_{4-3} & & \\
& (2) & & & x_{4-3} & y_{4-3} \\
& & & & x_{5-4} & y_{5-4}
\end{bmatrix}
$$